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# Introduction

This project involves evaluating a proposed temporary lifting apparatus for transporting goods to upper levels of a multi-story building. The design utilizes a pulley system constructed from repurposed steel automobile wheels and climbing rope, with a manual crank mechanism for lifting payloads. The objective is to develop a mathematical model of the system's dynamic behavior and assess its vibration characteristics using engineering principles and computational tools. Through this project, students will demonstrate their ability to apply scientific and engineering knowledge to solve complex engineering problems, in alignment with ECSA graduate attribute 2.

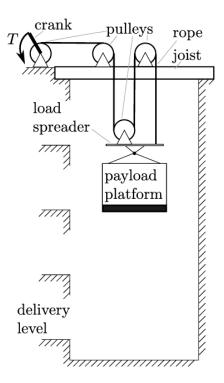
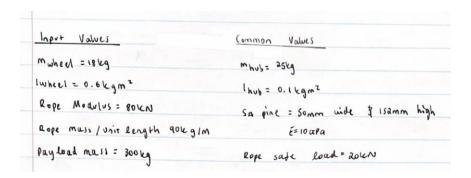
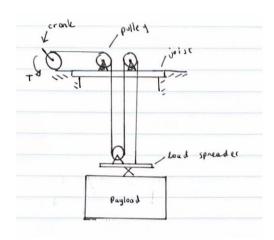


Figure 1: Make-shift lifting system.

# Overview and Values



The above are the respective values and information given for the lifting mechanism.



There is a total of 4 pullies, 1 payload, 1 load spreader and 1 rope. The torque is applied through a mechanism consisting of an arm and handle that will be investigated later. This mechanism is the crank mechanism.

The next step is to establish the number of degrees of freedom.

# **Assumptions & Simplifications**

To determine the degrees of freedom to determine accurate equations of motion each part of the mechanism will be investigated while making logical assumptions.

# Load-spreader & Payload

It will be assumed that the load spreader and payload will not deform throughout operation and are connected by a light inextensible rope.

Therefore, the load-spreader, payload and rope connecting them will be neglected as degrees of freedom.

# **Pulleys**

The pulleys are repurposed automotive wheels with hub inside. The hub axle assembly will be modelled as a solid cylinder and the wheel as a hollow cylinder. From the given values and model assumption the radius of the hub and wheel can be calculated.

Thub = 
$$\frac{1}{2} mr^2$$

O.1 =  $\frac{1}{2} (2s) (rhub)$  ...  $rhub = 0.089m$ 

Thubel =  $\frac{1}{2} m (rh^2 + rw^2)$ 

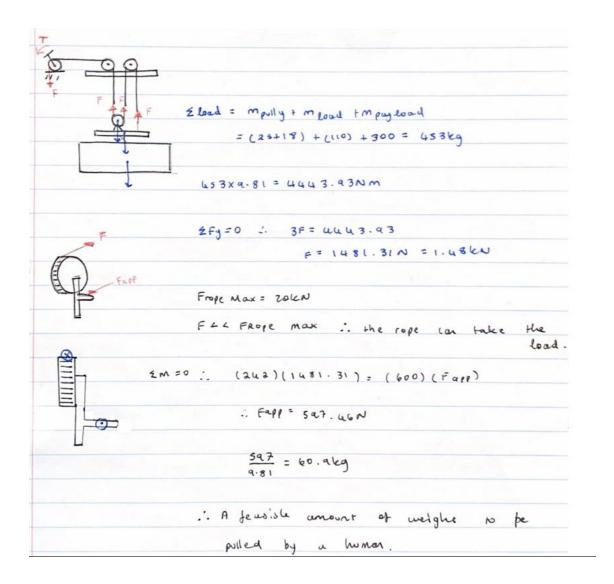
O.6 =  $\frac{1}{2} (18)(0.089^2 + rw^2)$ 

...  $rw = 0.242m$ 

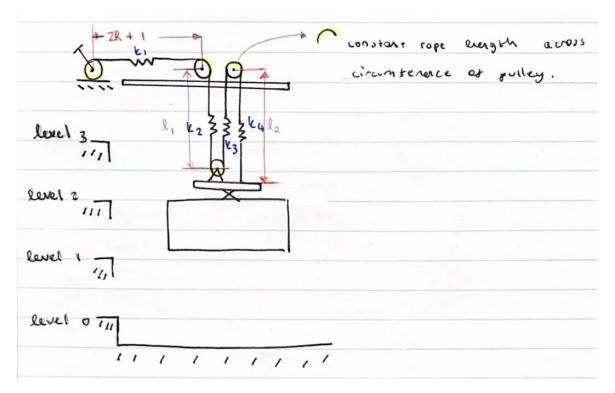
The pulleys are assumed to remained in a fixed position and are frictionless throughout operation.

# **Rope Configuration**

The rope being used to pull up the load has a can withstand a maximum load of 20kN, the safety of the rope will be investigated.



Therefore, the rope can safely carry the load and will not break through operation.



The distance between the bottom of the joist to level 0 is 16m, each level is 4m apart.

The rope in contact with the circumference of the pulleys is content and does not change.

The distance between the 2 pulleys is assumed to be 2R + 1m = 1.484m, this length is constant at all levels therefore K1 is constant at all 4 levels.

$$l_1 = r_{pulley} + l_{rope} + h_{joist}$$

$$l_2 = 2r_{pulley} + l_{rope} + h_{joist}$$

<u>Level</u>	<u>L1</u>	<u>L2</u>
0	15.394	15.636
1	11.394	11. 636
2	7.394	7. 636
3	3.394	3. 636

$$k_x = AE/l_y$$

 $Rope\ Modulus = AE = 80kN$ 

Level	<u>K1</u>	<u>K2</u>	<u>K3</u>	<u>K4</u>
<u>0</u>	53.91	5.20	5.20	5.11
<u>1</u>	53.91	7.02	7.02	6.87
2	53.91	10.82	10.82	10.48
<u>3</u>	53.91	23.57	23.57	22.00

At level 3 the largest difference between k2/3 and k4. There is a 6.66% difference between them when looking at the most extreme case, this is smaller than the difference with k1 by more than a factor of 10.

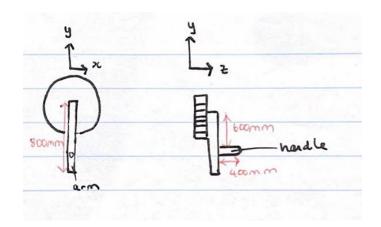
#### Rope masses at lowest level

$$m_{horizontal} = (rope\ mass\ length) \left(2\ r_{pulley} + 1\right) = (90)(1.484) = 133.56\ g$$
 $m_{vertical\ 1} = 2(rope\ mass\ length) \left(r_{pulley} + 1\right) = (90)(15.394) = 2.77\ kg$ 
 $m_{vertical\ 2} = (rope\ mass\ length) \left(2r_{pulley} + 1\right) = (90)(15.636) = 1.41\ kg$ 
 $m_{vertical\ 1} + m_{vertical\ 2} = 4.18\ kg$ 

The sum of the ropes mass at the maximum case is less than the payload and load spreader by more than a factor of 10 therefore the mas is not significant enough to change the load rating of the system. Therefore, it will be ignored in the analysis.

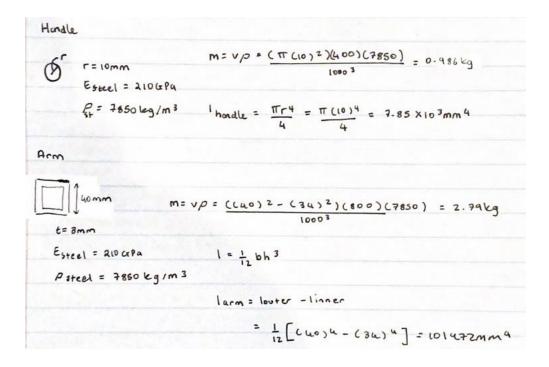
### Crank Mechanism

The crank mechanism consists of an arm and handle. The handle is a solid circular steel shaft while the arm is a hollow steel shaft with significant thickness. The orientation of the handle is shown in the illustration below.



The arm is welded to the pulley, making it rigid.

The orientation of the handle has been established. Now the stiffness and maximum deflection of the arm and handle will be investigated if the arm and handle need to be considered as degrees of freedom.



#### **Handle**

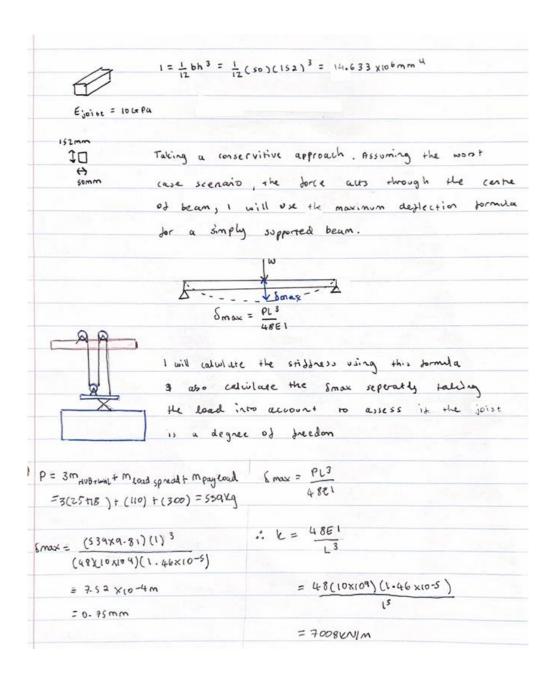
#### <u>Arm</u>

Arm
$$\frac{1}{1 + \frac{1}{1 + \frac{1}{$$

The arm and handle are stiff when compared to the stiffness of the rope by a factor of 10 therefore both the arm and the handle will not be considered as degrees of freedom. Despite the handle having a lower stiffness, if the handle bends the arm will not necessarily. This also solidifies the assumption to not include these as DOF. The minimum distance the ropes will extend to is 4m therefore the calculated deflections are less than a factor of 10 for both the arm and handle this also solidifies the assumption and decision to not include the arms and handles as degrees of freedom.

## **Joist**

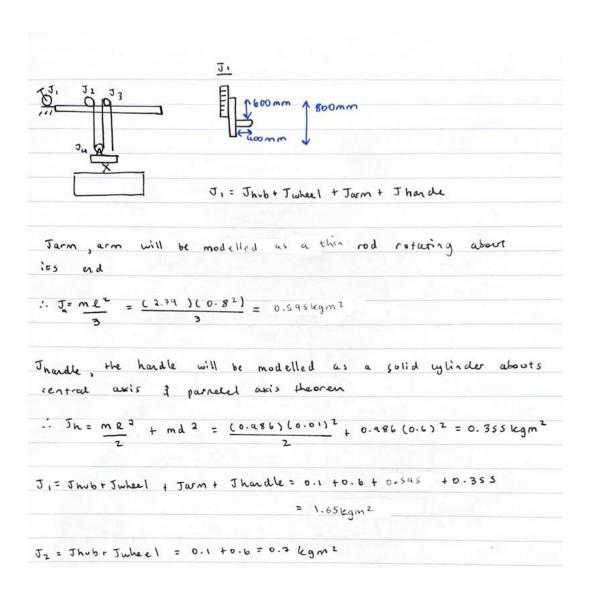
2 pulleys are mounted in a one wood joist, the integrity of the joist will be investigated to establish if it needs to be included as a degree of freedom in the lifting system.



The joist when compared to the stiffness of the rope **(what)** is greater by a factor of 10 therefore the joist will not be considered as degrees of freedom. The calculated deflection is less by a factor than the minimum length the ropes will extend (4m) solidifying this assumption and decision.

### Moments of Inertia

Each pulley will have a moment of inertia. 3 of the 4 pulleys will have the same moment of inertia value however the pulley with crank mechanism will have a different moment of inertia this will be explored later after the crank mechanism has been investigated.



# Outcome

The investigation has led to the following outcomes and assumptions.

It will be assumed that the load spreader and payload will not deform throughout operation and are connected by a light inextensible rope. Therefore, will not be considered a degree of freedom.

The joist is stiff when compared to the rope by more than a factor of 10 therefore it will not deform and does not need to be considered as a degree of freedom.

The pulleys are assumed to remained in a fixed position and are frictionless throughout operation and the length of the rope in contact with pulleys remain constant.

The crank arm and handle are stiff when compared to rope by more than a factor of 10 therefore will not be considered a degree of freedom and will not deform. The arm is welded the pulley and is considered rigid.

The pulley with the crank mechanism has a moment of inertia different to the other 3 which all share the same value.

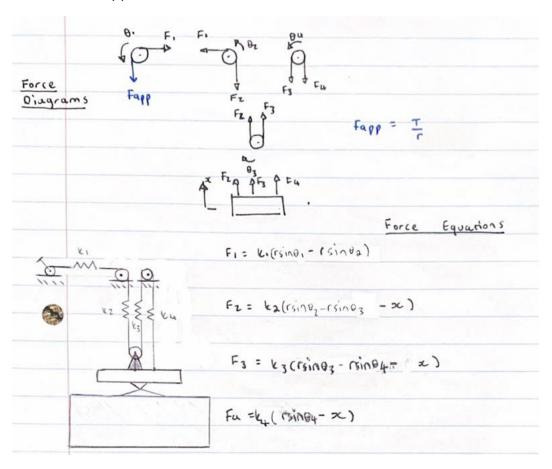
The rope can safely carry the load and the mass of the rope can be neglected as it less than the payload mass by a factor of 10.

There will be 3 K values in the model.

Therefore, the chosen degrees of freedom are 5. One DOF for each pulley and a degree of freedom for the payload and load spreader together.

# **Equations of Motion**

A Newtonian approach was used.



x	2Fx = m bram x = F4 + F3 + F2
	= ku(rsiney -x) + kg (rsineg-rsiney-x) + kg (rsineg-rsineg-x)
	(DM = - F4 r + F3 r = - ku (rsiney - x) r + k3 (rsinea - rsineg - x) r  10 u = - ku (rsinea - x) r + k3 (rsinea - rsiney - x) r
4	104 = - ky (rsine, - oc) + kz (rsinez - rsiney - x) r
	4 M =-F3r + FAr =- k3 (rsinb3 - rsin04 - x) r + k2 (rsin02 - rsin03-x) r
93	€ M = -F3r + FAr = -k3(rsin83 - (sin84 - \pi) r + k2(rsin82 - rsin83 - \pi) r  Jôi3 = k3(-rsin83+rsin84+x)r+k2(rsin82 - rsin83 - \pi) r
	COM = - Far + Fir = -kacrsinez-rsine3 - x) r + ki(rsine,-rsinea)r
O 2	$     \int \frac{\partial u}{\partial x} = -k_2(r\sin\theta_2 - r\sin\theta_3 - x) r + k_1(r\sin\theta_1 - r\sin\theta_2) r $ $     \int \frac{\partial u}{\partial x} = -k_2(r\sin\theta_2 - r\sin\theta_3 - x) r + k_1(r\sin\theta_1 - r\sin\theta_2) r $
	Jö. = - Ki (rsing) r
91	100

mi - kureu + kux -kgrez + kzr	
mž -kureu trzceu -kzrez +kzr	03 + k4 x + k2x - k2 r 02 = 0
mi -kurou tezrou -kzroz +kzr	03 + k4 x + k2x - k2 r 02 = 0
4 + kurour-kuxr - 431031 +k 31041	+k3x1 =0
- 43 1203 + 43 120 4+ 43 21 + 421202 -	-k212 83 -k2xr =0
+ 21202 - 421203 - 42x1 - 4,10, +4	11 FB 2 = 0
1 + k2 + 2 + k1 + 2 0 2 - k2 + 2 0 3 - k;	xr - k, r 20, = 0
	4 + k4 r 2 0 4 - k4 xr + k3 xr - k3 r 2 0 2 - k3 r 2 0 3 + k3 r 2 0 4 + k3 xr + k2 r 2 0 2 - k3 r 2 0 4 - k2 r 2 0 2 - k3 r 2 0 4 - k2 r 2 0 2 - k2 r 2 0 3 - k2 xr - k r 2 0 3 - k3 r 2 0 2 - k2 r 2 0 3 - k3 r 2 0 2 - k2 r 2 0 3 - k3 r 2 0 2 - k2 r 2 0 3 - k3 r 2 0 2 - k2 r 2 0 3 - k3 r 2 0 2 - k2 r 2 0 3 - k3 r 2 0 2 - k2 r 2 0 3 - k3 r 2 0 2 - k2 r 2 0 3 - k3 r 2 0 2 - k2 r 2 0 3 - k3 r 2 0 2 - k2 r 2 0 3 - k3 r 2 0 2 - k2 r 2 0 3 - k3 r 2 0 2 - k3 r 2 0

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0 0	20 0 0	d 83 p +	0	r2(k,+k2)	r2(k3+k2)	- k3r2	((k2-k3)	9,
0 0	о Јо о	ë 4	0	0	- k 3 r 2	12 (k3+ku)	1(k3 - k4)	9
0 0	0 0 m	[ x ]	0	-½r	r(k2-k3)			11
				rit	77			7.(
				= 0				
				0				

# Modal Analysis

#### Level 0

#### Laden

```
Modal Frequencies (Hz):
['0.0000', '8.6868', '22.5269', '36.7840', '99.3305']
Mode Shapes (normalized eigenvectors):
[['-0.4301' '-0.4176' '-0.2606' '0.1036' '0.4100']
['-0.4301' '-0.4065' '-0.2140' '0.0542' '-1.0147']
['-0.2867' '-0.3164' '0.7220' '-0.8500' '0.0506']
['-0.1434' '-0.1728' '0.8368' '0.8233' '-0.0025']
['-0.0347' '0.0351' '0.0014' '-0.0001' '0.0003']]
```

#### <u>Unladen</u>

```
Modal Frequencies (Hz):
['0.0000', '13.3843', '22.5611', '36.7844', '99.3364']
Mode Shapes (normalized eigenvectors):
[['0.5380' '-0.2592' '-0.2657' '-0.1038' '-0.4099']
['0.5380' '-0.2429' '-0.2180' '-0.0543' '1.0147']
['0.3587' '-0.2496' '0.7160' '0.8500' '-0.0506']
['0.1793' '-0.1562' '0.8332' '-0.8232' '0.0025']
['0.0434' '0.0846' '0.0070' '0.0005' '-0.0012']]
```

#### Level 1

```
Laden
Modal Frequencies (Hz):
 ['0.0000', '10.1298', '26.2088', '42.8860', '99.9964']
Mode Shapes (normalized eigenvectors):
 [['-0.4301' '-0.4193' '-0.2652' '0.1080' '0.4041']
 ['-0.4301' '-0.4042' '-0.2010' '0.0380' '-1.0190']
 ['-0.2867' '-0.3144' '0.7259' '-0.8459' '0.0709']
 ['-0.1434' '-0.1717' '0.8333' '0.8271' '-0.0049']
 ['-0.0347' '0.0351' '0.0014' '-0.0001' '0.0004']]
```

#### <u>Unladen</u>

```
Modal Frequencies (Hz):

['0.0000', '15.6095', '26.2446', '42.8862', '100.0074']

Mode Shapes (normalized eigenvectors):

[['0.5380' '-0.2621' '-0.2702' '-0.1081' '-0.4040']
['0.5380' '-0.2396' '-0.2046' '-0.0381' '1.0190']
['0.3587' '-0.2459' '0.7203' '0.8459' '-0.0709']
['0.1793' '-0.1537' '0.8299' '-0.8271' '0.0049']
['0.0434' '0.0846' '0.0067' '0.0004' '-0.0017']]
```

#### Level 2

#### Laden

```
Modal Frequencies (Hz):

['0.0000', '12.6602', '32.5844', '53.5515', '101.5237']

Mode Shapes (normalized eigenvectors):

[['-0.4301' '-0.4230' '-0.2751' '0.1184' '0.3905']
['-0.4301' '-0.3991' '-0.1722' '-0.0012' '-1.0270']
['-0.2867' '-0.3102' '0.7341' '-0.8350' '0.1185']
['-0.1434' '-0.1693' '0.8252' '0.8356' '-0.0135']
['-0.0347' '0.0351' '0.0012' '0.0000' '0.0007']]
```

#### <u>Unladen</u>

```
Modal Frequencies (Hz):

['0.0000', '19.5120', '32.6184', '53.5515', '101.5504']

Mode Shapes (normalized eigenvectors):

[['0.5380' '-0.2684' '-0.2798' '-0.1184' '-0.3902']
['0.5380' '-0.2324' '-0.1750' '0.0012' '1.0269']
['0.3587' '-0.2378' '0.7295' '0.8350' '-0.1184']
['0.1793' '-0.1484' '0.8222' '-0.8356' '0.0135']
['0.0434' '0.0847' '0.0058' '-0.0000' '-0.0026']]
```

#### Level 3

#### Laden

```
Modal Frequencies (Hz):
 ['0.0000', '19.1207', '48.1024', '79.9978', '108.6443']
Mode Shapes (normalized eigenvectors):
 [['-0.4301' '-0.4365' '-0.3104' '0.1697' '0.3246']
 ['-0.4301' '-0.3803' '-0.0575' '-0.2128' '-1.0249']
 ['-0.2867' '-0.2945' '0.7592' '-0.7520' '0.3431']
 ['-0.1434' '-0.1604' '0.7885' '0.8659' '-0.1043']
 ['-0.0347' '0.0351' '0.0004' '0.0005' '0.0014']]
Unladen
```

```
Modal Frequencies (Hz):
['0.0000', '29.4649', '48.1090', '80.0120', '108.7669']
Mode Shapes (normalized eigenvectors):
[['0.5380' '-0.2929' '-0.3126' '-0.1683' '-0.3236']
['0.5380' '-0.2034' '-0.0578' '0.2112' '1.0246']
['0.3587' '-0.2054' '0.7578' '0.7528' '-0.3415']
['0.1793' '-0.1275' '0.7873' '-0.8658' '0.1034']
['0.0434' '0.0847' '0.0021' '-0.0022' '-0.0054']]
```

# Discussion

The modal frequencies and mode shapes explain the behavior of the lifting operation at different levels and under different loading conditions.

#### Level 3

The modal frequencies at the top level are similar between the laden and unladen systems suggesting that the presence or absence of a payload has minimal effect on behavior of the system at this level.

The mode shapes at the top level also show similarities between the laden and unladen systems. The primary mode shapes involve large vertical displacements (X1 and X2) and little lateral displacements (X3 and X4), with minimal involvement of the payload (X5).

#### Level 2

The modal frequencies show little variation between the laden and unladen systems. This indicates that the dynamic behavior remains consistent regardless of the presence of a payload.

Similar behavior to the top level, the mode shapes show trends between the laden and unladen systems, with vertical displacements (X1 and X2) and small lateral displacements (X3 and X4).

#### Level 1

The modal frequencies at the 1st level show minimal differences between the laden and unladen systems, suggesting consistent dynamic behavior.

The mode shapes at the 2nd level also show similarities between the laden and unladen systems, with significant vertical displacements and minor lateral displacements.

#### Level 0

The modal frequencies at the bottom level show marginal changes between the laden and unladen systems.

The mode shapes at the bottom level also demonstrate similarities between the laden and unladen systems, with dominant vertical displacements and minor lateral displacements.

#### **Results Discussion**

Mode shapes remain relatively consistent across different levels and loading conditions, suggesting that the dynamic behavior of the lifting operation is primarily influenced by structural and geometric characteristics rather than the presence of a payload.

# Part 2

# **Numerical Approximation**

# **Dropped Payload Investigation**

#### Discussion of model variants

#### Unsuitable scenarios

To conduct a MDOF analysis a linear system is assumed, meaning components in the system affect each other proportionally. However, thus us not the case but modelling systems in this way is possible if on a low scale and the characteristics of the system match a linear system.

When a system's characteristics are far from linear or affect each other in a nonlinear fashion the assumption can no longer stand as the results of the model will be very in accurate. An alternative approach to this system is to conduct a finite element analysis.

Another important consideration is the topic of deflections. In our model the deflection of the joist could be ignored because it was far less than the length of the rope therefore will not influence the results by a large degree however if it was similar this deflection cannot be ignored and will result in the system not following a linear pattern.

#### Rope

When the platform is at the lowest level deflections are most severe this is the most relevant case. If a stiffer rope results in a better insight into the deflection between each level, this will make the system and model more precise. A stiffer rope also improves the safety of the system ensuring the rope does not deflect laterally.

A stiffer rope will transmit higher loads to the pulley where they are attached. This means stronger pulleys will need to be sourced and could make for a more complex system. The maximum deflection is decreased but now more rope length is required which can increase the cost of the system.

#### Bicycle Wheels

If bicycle wheels are incorporated into the system the system will need to be modified due to different mass, stiffness, and geometric properties.

Bicycles have a lower mass and are narrower than car wheels, they have spues and have less volume. The empty spaces vs the thick rims results in the bicycle having a smaller moment of inertia, negatively affecting the performance of the pulley system.

The bike wheels also differ in stiffness and damping properties and are designed to operate at lower forces and torques. The system will have to be modelled differently to account for this.

Bicycles also have a bigger radius on average than car wheels, this means a different length rope is required to reach the same levels. Also, bicycles are lighter and therefore they will need to be mounted in a different way to ensure they do not shift during operation.