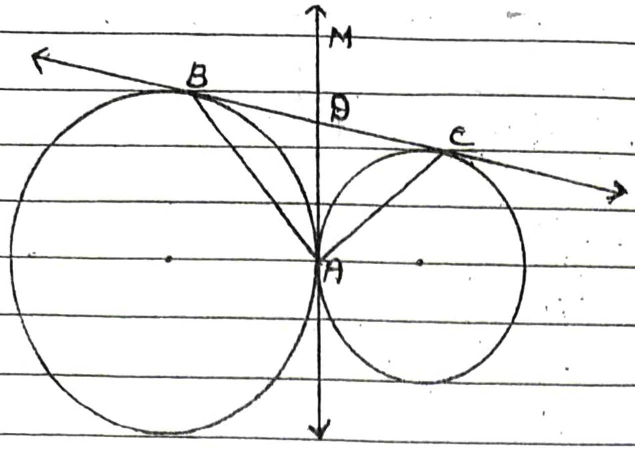
Chp 3. Circle

1 (9)



Proof:-

(i) Seg DB and seg DA are the tangent segments to the circle from external point D.

(ii) $\therefore$ Seg DB = Seg DA ... (tangent segments from external point)

(iii) $\therefore m\angle DAB = m\angle DBA$ ... (isosceles triangle theorem)

(iv) let $m\angle DAB = m\angle DBA = x$

(v) Seg DC and Seg DA are the tangent segments to the circle from external point D.

(vi) $\therefore$ Seg DC = Seg DA ... (tangents drawn from external point)

(vii) $\therefore m\angle DAC = m\angle DCA$ ... (isosceles triangle theorem)

(viii) let $m\angle DAC = m\angle DCA = y$.

In $\triangle ABC$,

(ix) $m\angle ABC + m\angle BCA + m\angle BAC = 180^\circ$ ... (Sum of measure of angle of triangle is 180°)

$\therefore m\angle ABC + m\angle ACB + [m\angle BAD + m\angle CAD] = 180^\circ$

$\therefore x + y + x + y = 180^\circ$ ... [from (iv) & (viii)]

$\therefore 2x + 2y = 180^\circ$

$\therefore 2(x+y) = 180^\circ$

$\therefore x + y = \frac{180}{2}$

$\therefore x + y = 90^\circ$

$\therefore m\angle BAD + \angle CAD = 90^\circ$ ... [from (iv) & (viii)]

(x) $\therefore m\angle BAC = 90^\circ$

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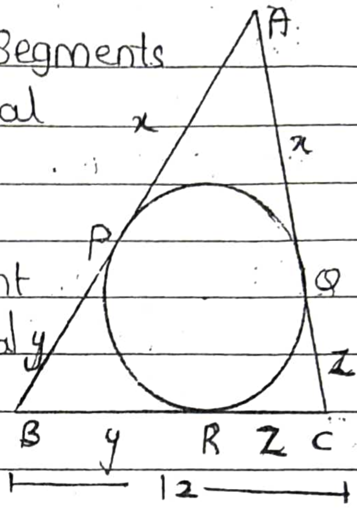
(xi) Seg DB = seg DC ... [from (ii) & (vi)]

(xii) $\therefore$ Point D is the midpoint of seg BC.

2 (i) Seg AP and seg AQ are the tangent segments drawn to the circle from external point A ...... (given)



(ii) $\therefore$ Seg AP = Seg AQ ... (tangent segment drawn from external point)

Similarly,

(iii) Seg BP = Seg BR } ... (tangent segment drawn from external point)
(iv) Seg CQ = Seg CR }

(v) let AP = AQ = x

(vi) BP = BR = y

(vii) CQ = CR = z

(viii) Perimeter of ΔABC = 44 cm ... (given)

$\therefore AB + BC + AC = 44$

$\therefore AP + PB + BR + RC + AQ + AC = 44$ ... (A-P-B, B-R-C, A-Q-C)

$\therefore x + y + y + z + x + z = 44$

$\therefore 2x + 2y + 2z = 44$

$\therefore 2(x + y + z) = 44$

$\therefore x + y + z = 22$

But BC = $y + z = 12$ ... (given)

$\therefore x + 12 = 22$

$x = 22 - 12$

$x = 10$

AP = 10

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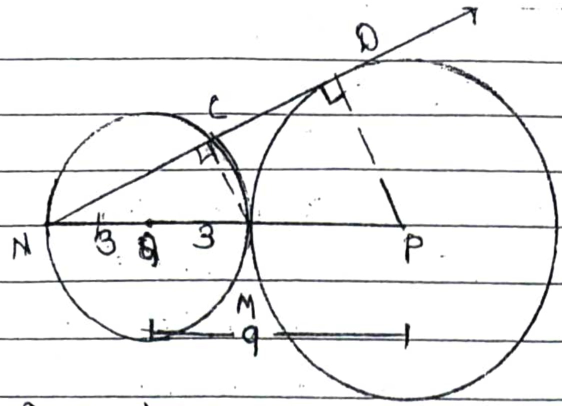

Join PD and MC

(i) MQ = QN = 3 ... (Radii of circle with centre Q)

(ii) NM = NQ + QM ... (N-Q-M)
= 3 + 3
= 6

(iii) Circle with the centre Q and circle with centre P touches externally in point M ... (given)

(iv) ∴ Q, M, P are collinear [If two circles are touching circle than the common point lies on line joining it]

(v) QM + PM = QP ... (Q-M-P)
∴ 3 + PM = 9
∴ PM = 6

(vi) PN = PQ + QN ... (P-Q-N)
∴ PN = 9 + 3
∴ PN = 12

(vii) DP = PM = 6 ... (Radii of circle with centre P)

(viii) line ND is a tangent to the circle with centre P and seg PD is the radius ... (given)

(ix) ∴ ∠PDN = 90° ... (tangent perpendicular to radius)

In right angled $\triangle PDN$,

$ND^2 + PD^2 = NP^2$ ... (By pythagoras theorem)

12

$\therefore ND^2 + (6)^2 = (12)^2$

$\therefore ND^2 + 36 = 144$

$\therefore ND^2 = 144 - 36$

$\therefore ND^2 = 108$

(x) $\therefore ND = 6\sqrt{3}$ ... (taking square root)

(xi) NM is a diameter with centre Q.

(xii) $\therefore \angle MCN = 90^\circ$ ... [Angle subtended by diameter)

(xiii) Seg MC || seg PD ... (from (ix) & (xii) and corresponding angle test

In $\triangle NDP$

(xiv) Seg MC || seg PD ... [from (xiii)]

(xv) $\frac{NC}{CD} = \frac{NM}{MP}$ ... [Basic Proportionality theorem]

$\therefore \frac{NC}{CD} = \frac{6}{6}$

$\therefore \frac{NC}{CD} = \frac{1}{1}$

(xvi) $\therefore NC = CD$

(xvii) $NC + CD = ND$ .... $(N - C - D)$

$\therefore NC + NC = ND$ ... [from (xvi)]

$\therefore 2NC = 6\sqrt{3}$

$\therefore NC = \frac{6\sqrt{3}}{2}$

$\therefore NC = 3\sqrt{3}$ unit

$\therefore CD = 3\sqrt{3}$ unit

Ans: $ND = 6\sqrt{3}$ unit and $NC = CD = 3\sqrt{3}$ unit

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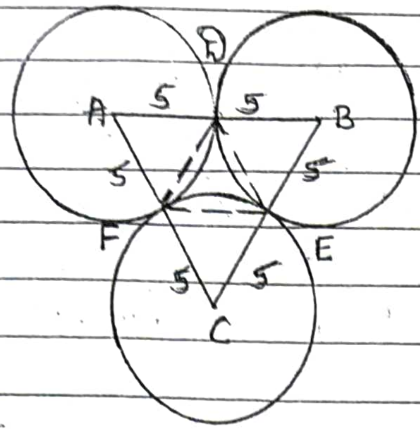

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Circle with centre A and B touch each other externally in point D ... (given)

$\therefore$ Point A, D, B are collinear [If two circle are touching circle than the common point lies on line joining their centre]

$\therefore AB = AD + DB$ ... [A - D - B]

$\therefore AB = 5 + 5$ ... (Radius of each circle is 5]

(i) $\therefore AB = 10$ cm

Similarly,

(ii) $BC = 10$ cm

(iii) $AC = 10$ cm

(iv) Perimeter of $\Delta ABC = AB + BC + AC$

$= 10 + 10 + 10$

$= 30$ cm

In ΔABC

$AD = BD$ ... (Each radius is 5 cm)

(v) $\therefore$ D is midpoint of side AB

$BE = EC$ ... (Each radius is 5 cm)

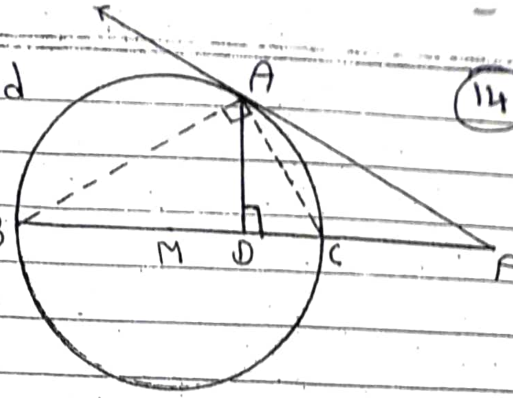
(vi) $\therefore$ E is midpoint of side BC

$DE = \frac{1}{2} AC$ ... [from (v), (vi) and midpoint theorem]

$\therefore DE = \frac{1}{2} \times 10$

$\therefore DE = 5$ cm

5 Construction : Draw seg AB and seg AC.



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Proof : In ΔABC,

① $\angle BAC = 90^\circ$ [Angle inscribed in a semicircle]

② seg $AD \perp$ side BC [Given]

③ $\therefore AD^2 = BD \times DC$ [Geometric Mean Property]

④ Ray PA is a tangent and PB is a secant.

⑤ $PA^2 = PC \times PB$ [Tangent - secant theorem]

In right - angled ΔADP,

⑥ $PA^2 = AD^2 + DP^2$ [Pythagoras theorem]

⑦ $\therefore PA^2 - AD^2 = DP^2$

⑧ $\therefore DP^2 = PA^2 - AD^2$

⑨ $\therefore DP^2 = PC \times PB - (BD \times DC)$ [from ③ and ⑤]

⑩ $\therefore DP^2 = PC \times PB - BD \times DC$

i.e. $DP^2 = BP \times CP - BD \times CD$.

(2) $\angle CAB \cong \angle CDB$ ... (Angles inscribed in the same arc are congruent)

$\therefore\ \angle CAB = \angle CDB = y$

$\therefore\ \angle MDB = y$ ... (C–M–D) ~~($\frac{1}{2}$ mark)~~

6 In ΔMDB,

$\angle MDB + \angle DMB + \angle DBM = 180°$

... (Angle sum property of a triangle)

$\therefore\ y + 90° + x = 180°$

$\therefore\ x + y = 180° - 90°$

$\therefore\ x + y = 90°$ ... (1) *($\frac{1}{2}$ mark)*

Substituting $x = 40$ in (1), we get,

$40° + y = 90°$ $\therefore\ y = 90° - 40°$

$\therefore\ y = 50°$

$\therefore$ If $x = 40°$ then $y = 50°$ *(1 mark)*

Substituting $x = 35°$ in (1), we get,

$35° + y = 90°$

$\therefore\ y = 90° - 35°$

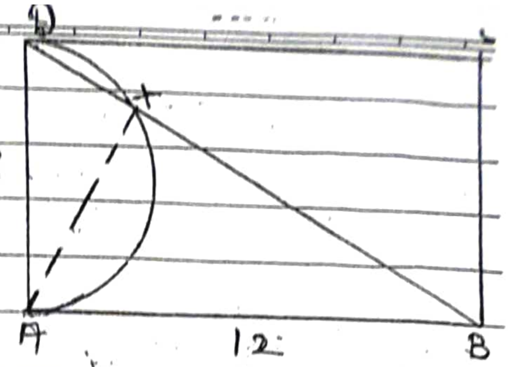
$\therefore\ y = 55°$

$\therefore$ If $x = 35°$ then $y = 55°$ *(1 mark)*

Ans. ($x = \mathbf{40°}$ and $y = \mathbf{50°}$); ($x = \mathbf{35°}$ and $y = \mathbf{55°}$) are two pairs of possible values of x and y.

[**Note :** Student can select any two pairs of values of x and y satisfying the equation $x + y = 90°$]

8] Construction: Join AX



Solution:

In $\triangle BAD$

(i) $\angle BAD = 90°$ ... [Angle of Rectangle]

(ii) $\therefore BD^2 = AB^2 + AD^2$ ... (By pythagoras thm)

$\therefore BD^2 = (12)^2 + (9)^2$

$\therefore BD^2 = 144 + 81$

$\therefore BD^2 = 225$

$\therefore BD = 15cm$ ... (Taking square root)

(iv) BA is a tangent and BXD is a secant

(v) $AB^2 = BX \times BD$ (tangent secant property)

$\therefore (12)^2 = BX \times 15$

$\therefore 144 = BX \times 15$

$\therefore \frac{144}{15} = BX$

$\therefore 9.6 = BX$

$\therefore BX = 9.6$ cm

ANS] BD = 15 cm , BX = 9.6 cm