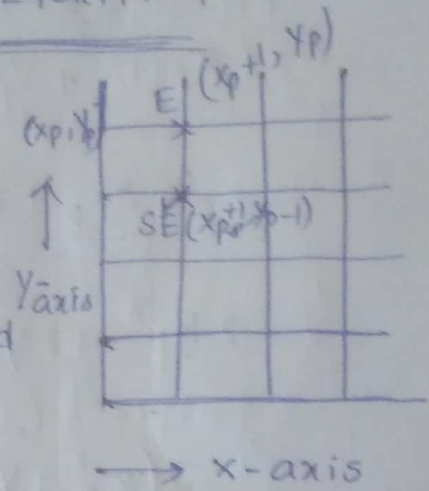


BRESSENHAM'S CIRCLE DRAWING ALGORITHM

→ Suppose we are standing at the point (x_p, y_p)

→ We know that equation of circle using the coordinates (x, y) and radius r is



i.e. we can obtain circle using coordinate axis:-

$$F(x, y) = x^2 + y^2 - r^2$$

$$= x^2 + y^2 - r^2 = 0$$

$$\Rightarrow F(x_p, y_p) = x_p^2 + y_p^2 - r^2 = 0$$

Step 2:- Now For selecting one coordinate among (x_p+1, y_p) , (x_p+1, y_p-1)

we need to consider the concept

$$F(x, y) = \begin{cases} < 0 \Rightarrow (x, y) \text{ is inside the circle} \\ = 0 \Rightarrow (x, y) \text{ is on the circle} \\ > 0 \Rightarrow (x, y) \text{ lies outside circle} \end{cases}$$

$$d_E = F(x_p+1, y_p) = (x_p+1)^2 + y_p^2 - r^2 \rightarrow ①$$

$$d_{SE} = F(x_p+1, y_p-1) = (x_p+1)^2 + (y_p-1)^2 - r^2 \rightarrow ②$$

decision parameter

$$d = d_E + d_{SE}$$

i.e. we want less distance

Step 3:- If $(d > 0)$ select SE pt // Resultant is +ve ($d_E > d_{SE}$)

If $(d < 0)$ select E pt // $d_{SE} > d_E$

Step 4:- Solve 'd'

$$\begin{aligned}
 d &= (x_p+1)^2 + y_p^2 - r^2 + (x_p+1)^2 + (y_p-1)^2 - r^2 \\
 &= x_p^2 + 2x_p + 1 + y_p^2 - r^2 + x_p^2 + 2x_p + 1 + y_p^2 - 2y_p + 1 - r^2 \\
 &= 2x_p^2 + 2y_p^2 + 4x_p - 2y_p - 2r^2 + 3
 \end{aligned}$$

$$\Rightarrow 2(x_p^2 + y_p^2 - r^2) + 4x_p - 2y_p + 3$$

$$= 2(0) + 4x_p - 2y_p + 3$$

$$\boxed{d = 4x_p - 2y_p + 3}$$

Step 5:- considering initial point as

$$(x_p, y_p) = (0, r)$$

$$\Rightarrow d = 0 - 2r + 3$$

$$d = 3 - 2r$$

Step 6:-

Generalisation:-

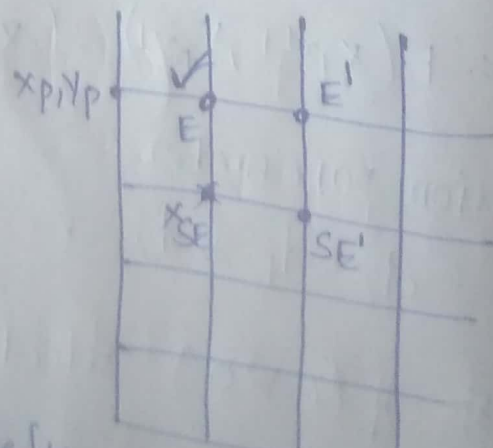
Suppose E got selected as valid

pixel

Next pixel that should

be selected is

one among



E' coordinates $(x_p + 2, y_p)$

SE' coordinates $(x_p + 2, y_p - 1)$

$$d_{\text{new}}^2 = (x_p + 2)^2 + (y_p)^2 - r^2 + (x_p + 2)^2 + (y_p - 1)^2$$
$$= 9 + 8x_p - 2y_p$$

$$\Delta = d_{\text{new}} - d$$

$$\Delta = 6 + 4x_p$$

$$d_{\text{new}} = \Delta + d$$

$$d_{\text{new}} = 6 + 4 * x_p + d$$

When east gets selected

ply For SE got selected previously
next pixel to be selected is

$$d_{\text{new}} = d + 4 * (x_p - y_p) + 10$$

Bresenham's ^{Circle} ~~Line~~ Drawing Algorithm

Pseudo code :-

$x = 0$, $y = r$, decision parameter $= 3 - 2r$

putpixel(x, y)

while($y > x$) do

$x++$

 if ($d < 0$)

$d = d + 4x + 6$

 else

$d = d + 4(x - y) + 10$; $y--$;

end

putpixel(x, y)

end while

This is when centre ^{considering} $(0, 0)$

when centre is (x_c, y_c) ⇒

putpixel($x + x_c, y + y_c$),

Bresenham's Line Drawing

→ Drawback of DDA algorithm:-
using the round function

→ Here we will avoid rounding.

Step 1:-

Calculate the slope from
the start point and end point

$m < 1$	$m > 1$	$m = 1$
$x_{k+1} = x_k + 1$	$y_{k+1} = y_k + 1$	$x_{k+1} = x_k + 1$
$y_k = ?$	$x_{k+1} = ?$	$y_{k+1} = y_k + 1$

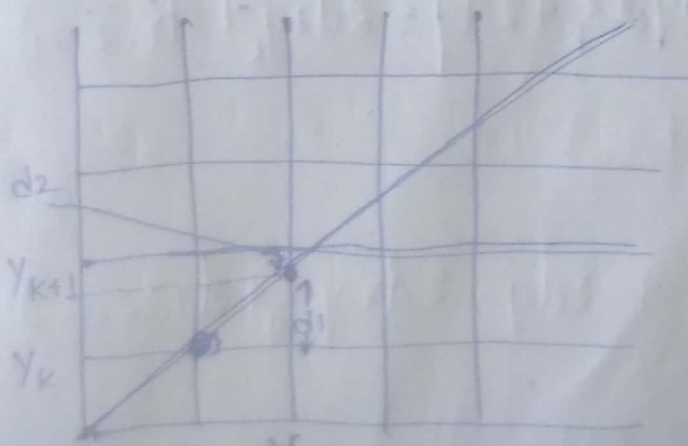
Step 2:-

Bresenham's ($m < 1$) :- m : Slope

$$y = mx + c$$

$$y = m(x_{k+1}) + c \rightarrow (1)$$

i.e we have to find y at x_{k+1}



at: x_{k+1}
Now we
need to
select y_k or
 $y_{k+1} = ?$

Step 3: To select y_k, y_{k+1} we need to calculate Decision parameter (d_p)

$$d_p = (x_{k+1} - x_k)(d_1 - d_2)$$

$$\boxed{d_p = dx (d_1 - d_2)}$$

From graph

$$d_1 = y - y_k$$

$$d_1 = m(x_{k+1}) - y_k \quad [\because \text{From } \textcircled{D}]$$

$$d_2 = y_{k+1} - y$$

$$d_2 = y_{k+1} - m(x_{k+1}) - c$$

$$\boxed{d_1 - d_2 = 2m(x_{k+1}) - 2y_k + 2c - 1}$$

$$\frac{d_p}{dx} = dx (d_1 - d_2) = dx \left(2 \cdot \frac{dy}{dx} (x_{k+1}) - 2y_k + 2c - 1 \right)$$

$$p_k = \frac{d_p}{dx} = 2dy dx_k + 2dy - 2dx y_k + dx(2c-1)$$

d_p at next point.

$$p_{k+1} = 2dy x_{k+1} + 2dy - 2dx y_{k+1} + dx(2c-1)$$

$$(P_{k+1} - P_k) = \underbrace{2dy}_{x_{k+1}} (x_{k+1} - x_k) - \underbrace{2dx}_{\text{we need to divide}} (y_{k+1} - y_k)$$

$$= 2dy(x_{k+1} - x_k) - 2dx(y_{k+1} - y_k)$$

$$\boxed{P_{k+1} = P_k + 2dy - 2dx(y_{k+1} - y_k)} \quad \text{--- (2)}$$

We

$$P_k = d_f = 2dyx_k + 2dy - 2dx y_k + dx(2c - 1)$$

we have $y = mx + c$

$$= 2dyx_k + 2dy - 2dx y_k + dx(2(3-mx) - 1)$$

$$= 2dyx_k + 2dy - 2dx y_k + 2dx y - \cancel{2dx \frac{dy}{dx} x} - dx$$

$$= 2dyx_k + 2dy - 2dx y_k + 2dx y - \frac{dx}{dx} - dx$$

at pt (x_k, y_k)

$$\Rightarrow 2dyx_k + 2dy - 2dx y_k + 2dx y_k - \frac{dx}{dx} - dx$$

if $P_k \geq 0$	$P_k < 0$
$X_{k+1} = X_k + 1$	$X_{k+1} = X_k$
$Y_{k+1} = Y_k + 1$	$Y_{k+1} = Y_k$

\therefore We do not require rounding function bcoz we are selecting either (Y_k, Y_{k+1})

Pseudocode:- For $(x_1, y_1), (x_2, y_2)$

$$dx = (x_2 - x_1)$$

$$dy = (y_2 - y_1)$$

to draw line

$$[d = 2 * dy - dx] \text{ // Decision parameter }$$

$$x = x_1$$

$$y = y_1$$

putpixel (x, y)

while $(x \leq x_2)$

{
if $(d < 0)$

{
 $x = x + 1$;

$y = y$

$d = d + 2 * (dy)$

}

else

{
 $x = x + 1$;

$y = y + 1$

$d = d + 2 * (dy - dx)$;

}

DDA - Pseudocode

→ coordinates (x_1, y_1) , (x_2, y_2)

for this coordinates if we want to draw a line.

$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$x = x_1, y = y_1$$

while ($y \leq y_2$ || $x \leq x_2$)

{

put pixel (round (x), round (y))

$$x = x + 1$$

$$y = y + m$$

}