<u>COL100</u>

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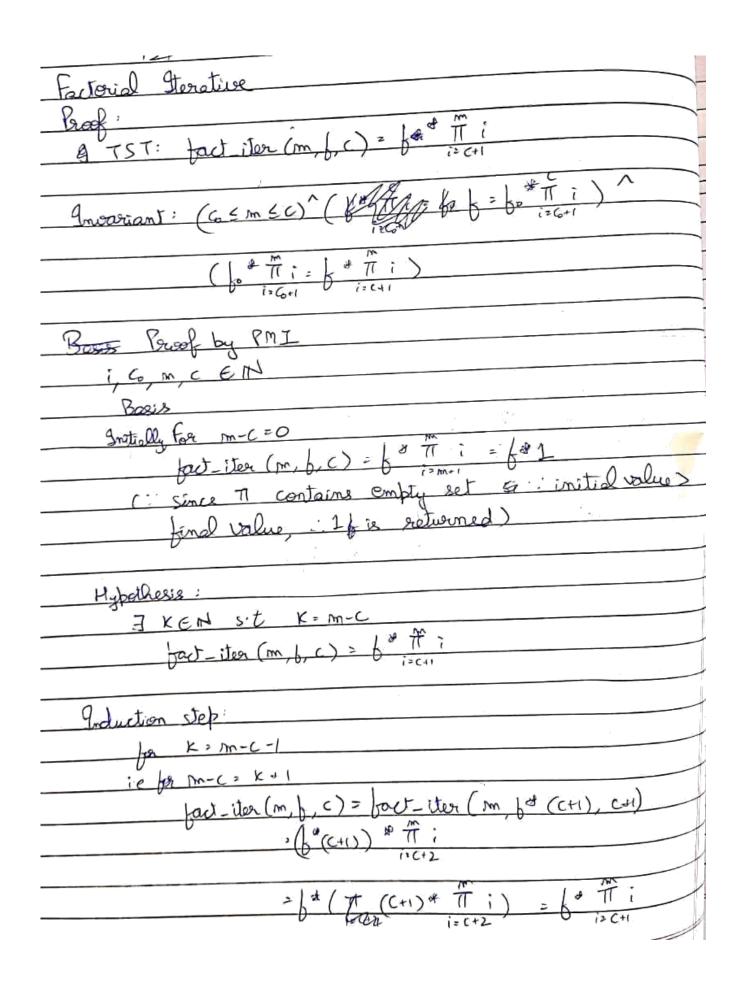
Assignment 1

Q1. Computing factorial of a given integer using both recursive and iterative procedures $\mbox{\sc A1.}$

Recursive:

Assignment 2	
(1) Factorial - Recursive	
k: N→N (x)= x!	
Eroof:	Algo:
Induction Basis:	fun f (m) =
1(0) = 1	if m=0 then 1
Induction Hypothesis:	else
k(n-1)= (m-1)! FmEN	m & (m-1);
Induction Step:	U
k(m)= m = k(m-1) ~ by Algo	
= $m(m-1)!$ ~ by ind	uch hybothesis
	computes m! for m EIN

Iterative:



By Substitution	
fact-iler (M,1,0) = 1× TT i =1×1×2×3× × M	
is i	
= m!	
Hence browd	

Q2. Computing x^n . Write both recursive and iterative versions.

A2.

Recursive:

(2) Your Recursive $f: N^{2} N \rightarrow N \qquad f(2, m) = x^{m}$ Broof:

Induction Besis: $f(0) = 1 \qquad \qquad f(x, m) = 1$ $f(2, m) = x^{m-1} \qquad \text{else}$ $f(x, m) = x^{m} f(x, m-1) = by Algo$ $= x - x^{m-1} \qquad by Def.$ $= x^{m}$ Func m Computes x^{m} for $x^{m} \in N$

Iterative:

= (pt) (bxc)x(ci) [By Ago]	
= bx (cixc) = bx citi	
Hence browled b iter (b, c,m) = b+cm	
Now,	
When bitton is called is with arguments 1, 2 and n	
When p-iton is collect is with arguments 1, 2 and n the france computes the value xem which is desired	Ĺ

Q3. Computing the n^{th} fibonacci number. First use the algorithm given by the following functional description: fib(1) = 1; fib(2) = 1; fib(n) = fib(n-1)+fib(n-2) for n > 2. Also develop iterative algorithms for the same problem.

A3.

Since Fibonacci Sequence is defined inductively, therefore its proof is trivial.

- Q4. The integer square root of n is the integer k such that $k^2 \le n \le (k+1)^2$. The integer square root can be computed using the following inductive process:
 - •Compute the integer square root I of m = n div 4 recursively. We then have that $i^2 <= m < (i+1)^2$.
 - •Since m and I are integers we have that $(m+1) <= (i+1)^2$. We thus have $(2i)^2 <= 4m <= n < 4m + 4 <= (2i + 2)^2$. Hence we have that the integer square root of n is either 2i or 2i+1.
 - •Write a recursive ML program corresponding to the above algorithm. Indicate the type of the function and derive the number of steps required.

A4. The type of function is recursive and the total steps it takes is $(log_4(n) + 2)$

A4. The type of function is recur

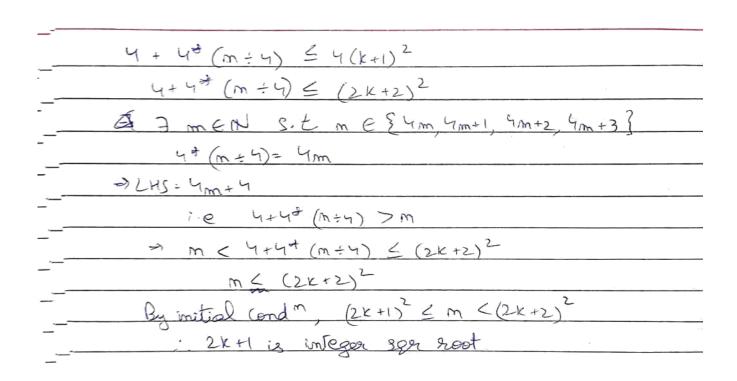
$$(log_4(n) + 2)$$

 $T(4^n) = T(4^n-1) + 1$
 $= T(4^n-2) + 2$
.
.
 $= T(1) + n$
 $= T(0) + n + 1$

Hence number of steps = $log_4(n) + 2$

Proof:

(1) Integer Sq. 9000t 6:1N -> IN (Recursive)
$S(x) \ge M$
Person Algo.
fun & (m):
if med theno
else if (2" 6(mdio 4) 11) + (2" 6 (m dio 4) +1) <= m then
2 6 (m die 4) +1
else 2 (m div 4)
Proof:
Busin (Let K be sparage root of m die 4)
1.e K2 < (m dw 4) < (K+1)2) K, m E /N
Induction Hypothesis:
Basis: (0)=0 :e sgrt(0)=0
People 02 5 0 < (0+1)2 0 5 0 < 1
Induction stop
- Coop I: m < (2K+1)2
From IH, K2 \le (m die 4)
- > 4k² ≤ (mdie 4) * 4
or some men (in die 9 can give values mont, mos mos
: (m die4) * 4 ≤ m
m can be 4 m, 4 m + 1, 4 m + 2, 4 m + 3
mais 4 = m (mais 4) 9 = 4m
m div 4) *4 < m
$\frac{1}{2} \frac{1}{2} \frac{1}$
Since national numbers and considered by the specific are
$(2k)^2 \leq m \leq (2k+1)^2$ $God T : m > (2k+1)^2$
and the state of t
Fince we me are considering matural numbers
1+ (m=4) A = (K+1)2



- Q5. Study the problem of computing *perfect numbers* from the Lecture notes Example 3.13 and implement the ML program. Also study the following discussion on *scope rules*. You will be questioned on this problem during the demonstration.
- A5. <Studied in class and Implemented the program and submitted>