Problem : 3

Basis : If $(x = 0)$, the function divide(x, y) returns $(0, 0)$

s.t. $x = 0(y) + 0 = 0$

$\therefore$ Basis holds

Induction Hypothesis :

Let divide $(k, y) = (a, b)$

s.t. $k = ya + b$; $a \geq 0$, $0 \leq b \leq y$

Induction Step :

[From here, BA means By Algo]

~~(q, r)~~ =

I : $x = 2k$

$(q, r) = \text{divide}(k, y)$ | BA

$= (a, b)$ | By IH

$q1 = 2a$

$r1 = 2b$

I.1 if $2b < y$

then $(2a, 2b)$

i.e. $2k = (2a) \times y + 2b$

by IH $k = ay + b$ $\Rightarrow 2k = 2ay + 2b$

Also $2a \geq 0$ $\because a \geq 0$ $0 \leq 2b \leq y$ (By Case)

I.2 if $2b \geq y$

then $(2a+1, 2b-y)$

By IH $k = ay + b$ $2k = 2ay + 2b$

$2k = (2a+1)y + (2b - y)$

s.t. $2a+1 \geq 0$ $0 \leq 2b - y \leq y$ ($\because 2b \geq y$ & $b \leq y$)

II : $x = 2k+1$

$r2 = 2b+1$

II.1 $(2b+1 < y)$ then $(2a, 2b+1)$

Scanned with CamScanner

Teacher's Signature..............

s.t. $2k+1 = 2ay + (2b+1)$

By IH $k = ay + b$

$\Rightarrow 2k = 2ay + 2b \quad \therefore 2k+1 = 2ay + (2b+1)$

s.t. $2a \geq 0$ $\quad 0 \leq 2b+1 < y$

(By case condⁿ)

II.2 $(2b+1 \geq y)$ then $(2a+1, 2b+1-y)$

By IH $k = ay + b$

$2k+1 = 2ay + 2b+1$

$= (2a-1)y + (2b+1-y)$

$2a-1 \geq 0$

$0 \leq 2b+1-y < y$

$\because 2b+1 \geq y$ & $2b+1 < 2y$

Since the funcⁿ is recurring

$\therefore T(m) = T(m \text{ div } 2) + O(1)$

Let $T(0) = 0$

$T(m) = O(\log_2 m)$