GATE ASSIGNMENT-3

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https://github.com/V−gopireddy/EE3900/blob/main/GATE_Assignment3/GateAssignment−3.tex

1 QUESTION EC-2001/Q.1.3

The transfer function of a system is given by

$$H(s) = \frac{1}{s^2(s-2)} \tag{1.0.1}$$

The impulse response of the system is

1) $(t2 * e^{-2t})U(t)$
2) $(t * e^{2t})U(t)$
3) $(te^{-2t})U(t)$
4) $(te^{-2t})U(t)$

2 SOLUTION

Lemma 2.1 (Table of Laplace Transforms)**.**

Time Function $f(t) = \mathcal{L}^{-1}\{F(s)\}$	***Laplace transform*** *of f(t)* $F(s) = \mathcal{L}\{f(t)\}$
$t^n U(t),\ n \geq 1$	$\frac{n!}{s^{n+1}},\ s > 0$
$\frac{t^{n-1}}{(n-1)!}U(t),\ n \geq 2$	$\frac{1}{s^n},\ s > 0$
$e^{-at}U(t)$	$\frac{1}{s+a},\ s + a > 0$

Theorem 2.1 (Convolution theorem)**.** *Suppose* $F(s) = \mathcal{L}\{f(t)\}, G(s) = \mathcal{L}\{g(t)\}$ *exist, then,*

$$\mathcal{L}^{-1}\{F(s)G(s)\} = f(t) * g(t) \tag{2.0.1}$$

Let,

$$F(s) = \frac{1}{s^2}, G(s) = \frac{1}{s-2} \tag{2.0.2}$$

$$\implies f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = \frac{t}{1!} = tU(t) \tag{2.0.3}$$

$$\implies g(t) = \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} = e^{2t}U(t) \tag{2.0.4}$$

Given transfer function,

$$H(s) = \frac{1}{s^2(s-2)} \tag{2.0.5}$$

$$\implies h(t) = \mathcal{L}^{-1}\{H(s)\} \tag{2.0.6}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2(s-2)}\right\} \tag{2.0.7}$$

$$= \mathcal{L}^{-1}\{F(s)G(s)\} \tag{2.0.8}$$

$$= f(t) * g(t) = (t * e^{2t})U(t) \tag{2.0.9}$$

Correct Option is (2)