QUIZ-1

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Download all latex-tikz codes from

https://github.com/V−gopireddy/EE3900/blob/main/Quiz1/Quiz−1.tex

1 Question 2.19 (d,e,f)

For each of the following impulse responses of LTI systems, indicate whether or not the system is stable:

1) $h[n] = \sin(n\pi/3)u[n]$
2) $h[n] = (3/4)^{|n|}\cos(n\pi/4 + \pi/4)$
3) $h[n] = 2u[n+5] - u[n] - u[n-5]$

2 Solution

Definition 1. *We say that a system is **stable** if it produces a bounded output for every possible bounded input, i.e it satisfies the BIBO(Bounded-input-Bounded-output) condition.*

Lemma 2.1. *A system with impulse response $h[n]$ is said to be BIBO stable if and only if $h[n]$ is absolutely summable, i.e.*

$$S = \sum_{n=-\infty}^{\infty} |h[n]| < \infty \tag{2.0.1}$$

Lemma 2.2. *LTI system with impulse response*

$$h[n] = \sin(n\pi/3)u[n] \tag{2.0.2}$$

is unstable

Proof. We have,

$$h[n] = \sin\left(\frac{n\pi}{3}\right)u[n] \tag{2.0.3}$$

$$= \begin{cases} \sin\left(\frac{n\pi}{3}\right), & n \geq 0 \\ 0, & n < 0 \end{cases} \tag{2.0.4}$$

Therefore

$$S = \sum_{n=-\infty}^{\infty} |h[n]| \tag{2.0.5}$$

$$= \sum_{n=0}^{\infty} |\sin\left(\frac{n\pi}{3}\right)| \tag{2.0.6}$$

$$= \sum_{n=0}^{\infty} \sqrt{3} = \infty \tag{2.0.7}$$

Since,

$$S = \infty \tag{2.0.8}$$

The system is unstable □

Lemma 2.3. *LTI system with impulse response*

$$h[n] = (3/4)^{|n|}\cos(n\pi/4 + \pi/4) \tag{2.0.9}$$

is stable

Proof. Since $-1 \leq \cos(n\pi/4 + \pi/4) \leq 1$

$$S = \sum_{n=-\infty}^{\infty} |h[n]| \tag{2.0.10}$$

$$= \sum_{n=-\infty}^{\infty} |(3/4)^{|n|}\cos(n\pi/4 + \pi/4)| \tag{2.0.11}$$

$$\leq \sum_{n=-\infty}^{\infty} |\left(\frac{3}{4}\right)^{|n|}| \tag{2.0.12}$$

$$= 1 + 2\sum_{n=1}^{\infty} |\left(\frac{3}{4}\right)^{n}| = 7 \tag{2.0.13}$$

Since,

$$S \leq 7 < \infty \tag{2.0.14}$$

The system is stable □

Lemma 2.4. *LTI system with impulse response*

$$h[n] = 2u[n+5] - u[n] - u[n-5] \tag{2.0.15}$$

is stable

Proof. We have

$$h[n] = 2u[n+5] - u[n] - u[n-5] \tag{2.0.16}$$

$$= \begin{cases} 2, & -5 \le n < 0 \\ 1, & 0 \le n < 5 \\ 0, & \text{otherwise} \end{cases} \tag{2.0.17}$$

Therefore

$$S = \sum_{n=-\infty}^{\infty} |h[n]| \tag{2.0.18}$$

$$= \sum_{n=-5}^{0} |2| + \sum_{n=0}^{-5} |1| = 15 \tag{2.0.19}$$

Since,

$$S = 15 < \infty \tag{2.0.20}$$

The system is stable $\square$