ASSIGNMENT 5

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Download all python codes from

https://github.com/V−Gopireddy/EE3900/blob/main/Assignment5/codes/Assignment−5.py

and latex-tikz codes from

https://github.com/V−gopireddy/EE3900/blob/main/Assignment5/Assignment−5.tex

1 Quadratic forms 2.27

Find the coordinates of the foci, the vertices, the lengths of major and minor axes and the eccentricity of the ellipse

$$\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 36 \tag{1.0.1}$$

2 SOLUTION

Given ellipse is

$$\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 36 \tag{2.0.1}$$

On comparing it with standard form we have,

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 36 \tag{2.0.3}$$

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.4}$$

$$\lambda_1 = 9, \lambda_2 = 4 \tag{2.0.5}$$

Semi major and minor axes of ellipse are

$$a = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} = 3 \tag{2.0.6}$$

$$b = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} = 2 \tag{2.0.7}$$

For $\lambda_1 = 9$,

$$\mathbf{V} - \lambda_1 \mathbf{I} = \begin{pmatrix} 0 & 0 \\ 0 & -5 \end{pmatrix} \tag{2.0.8}$$

$$\implies \mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.9}$$

For $\lambda_2 = 4$,

$$\mathbf{V} - \lambda_2 \mathbf{I} = \begin{pmatrix} 5 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.10}$$

$$\implies \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.11}$$

The vertices are

$$\pm \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{2.0.12}$$

Since

$$\lambda_1 > \lambda_2 \tag{2.0.13}$$

Coordinates of the foci are given by,

$$\mathbf{F} = \pm \left(\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1}\mathbf{u} - f)(\lambda_1 - \lambda_2)}{\lambda_1 \lambda_2}} \right) \mathbf{p_2} \tag{2.0.14}$$

$$= \pm \begin{pmatrix} 0 \\ \sqrt{5} \end{pmatrix}. \tag{2.0.15}$$

Eccentricity of the ellipse is given by,

$$e = \frac{\sqrt{\frac{(\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u})(\lambda_1 - \lambda_2)}{\lambda_1 \lambda_2}}}{\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}}} = \frac{\sqrt{5}}{3}. \tag{2.0.16}$$

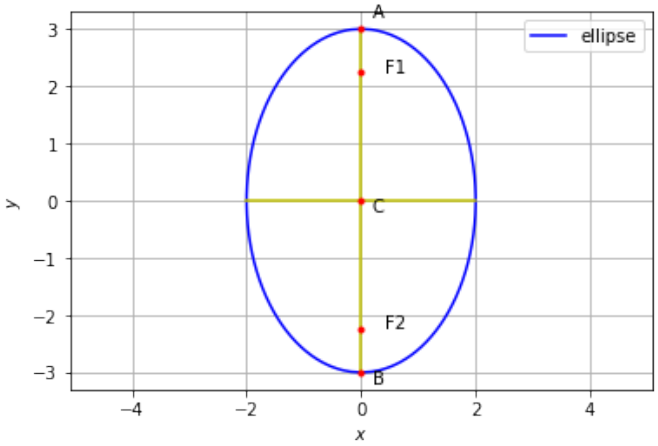

Fig. 0: Plot of the ellipse