GATE ASSIGNMENT 1

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Download all python codes from

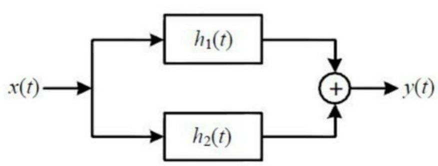
```
https://github.com/V−Gopireddy/EE3900/blob/
    main/GATE_Assignment1/codes/
    GATE_Assignment−1.py
```

and latex-tikz codes from

```
https://github.com/V−Gopireddy/EE3900/blob/
    main/GATE_Assignment1/GATE_Assignment
    −1.tex
```

1 GATE EC 2017 Q.35

Consider the parallel combination of two LTI systems shown in the figure.



The impulse responses of the systems are

$$h_1(t) = 2\delta(t+2) - 3\delta(t+1) \tag{1.0.1}$$
$$h_2(t) = \delta(t-2) \tag{1.0.2}$$

If the input $x(t)$ is a unit step signal, then find the energy of $y(t)$.

2 Solution

Theorem 2.1 (Convolution Theorem)**.** *Let $F(z)$ and $G(z)$ be the Z-transform of two functions f and g respectively. Then*

$$\mathcal{Z}(f * g) = F(z)G(z) \tag{2.0.1}$$

Lemma 2.1. *Let $u(t)$ be unit step funtion and $\delta(t)$ be dirac-delta function. Then for any real number c,*

$$u(t) * \delta(t-c) = u(t-c) \tag{2.0.2}$$

Proof. We know,

$$\delta(t) \overset{\mathcal{Z}}{\rightleftharpoons} 1 \tag{2.0.3}$$
$$\implies \mathcal{Z}\{\delta(t-c)\} = z^{-c}\mathcal{Z}\{\delta(t)\} = z^{-c} \tag{2.0.4}$$

And

$$U(z) = \frac{1}{1-z^{-1}}, \quad |z| > 1 \tag{2.0.5}$$

We have,

$$\mathcal{Z}\{u(t) * \delta(t-c)\} = \mathcal{Z}\{\delta(t-c)\}\, U(z) \tag{2.0.6}$$
$$= \frac{z^{-c}}{1-z^{-1}} \tag{2.0.7}$$
$$= z^{-c}\sum_{n=0}^{\infty} z^{n} \tag{2.0.8}$$
$$= \mathcal{Z}\{u(t-c)\} \tag{2.0.9}$$
$$\implies u(t) * \delta(t-c) = u(t-c) \tag{2.0.10}$$

□

Given that,

$$h_1(t) = 2\delta(t+2) - 3\delta(t+1) \tag{2.0.11}$$
$$h_2(t) = \delta(t-2) \tag{2.0.12}$$

Overall impulse response of the system is,

$$h(t) = h_1(t) + h_2(t) \tag{2.0.13}$$
$$= 2\delta(t+2) - 3\delta(t+1) + \delta(t-2) \tag{2.0.14}$$

Given input, $x(t) = u(t)$, then output signal $y(t)$ is given by,

$$y(t) = x(t) * h(t) \tag{2.0.15}$$
$$= u(t) * [2\delta(t+2) - 3\delta(t+1) + \delta(t-2)] \tag{2.0.16}$$

From (2.0.2) we have,

$$y(t) = 2u(t+2) - 3u(t+1) + u(t-2) \tag{2.0.17}$$

Solving we get,

$$y(t) = \begin{cases} 2, & -2 \leq t < -1 \\ -1, & -1 \leq t < 2 \\ 0, & \text{otherwise} \end{cases} \tag{2.0.18}$$

Energy of the output signal E_y is given by,

$$E_y = \int_{-\infty}^{\infty} |y(t)|^2 dt \tag{2.0.19}$$

$$= \int_{-2}^{-1} 4dt + \int_{-1}^{2} dt \tag{2.0.20}$$

$$= 7 \tag{2.0.21}$$

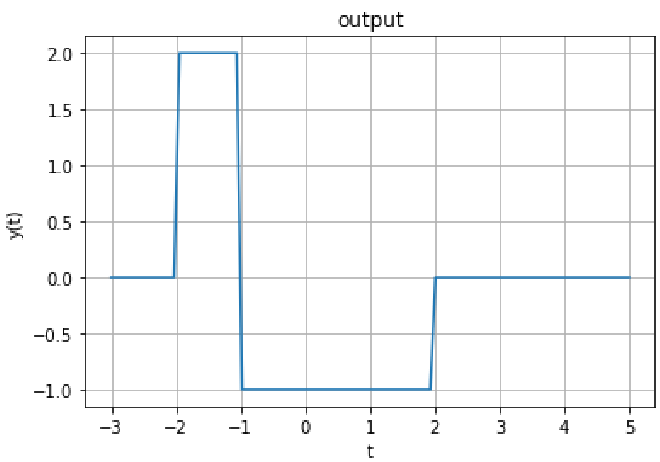


Fig. 0: Output signal $y(t)$