QUIZ 2

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Download all python codes from

```
https://github.com/V−Gopireddy/EE3900/blob/
    main/Quiz2/codes/Quiz−2.py
```

and latex-tikz codes from

```
https://github.com/V−gopireddy/EE3900/blob/main
    /Quiz2/Quiz−2.tex
```

1 QUESTION 3.3(c)

Determine the z-transform of the following sequence. Include with your answer the region of convergence in the z-plane and a sketch of the pole-zero plot. Express all sums in closed form.

$$x[n] = \begin{cases} n, & 0 \le n \le N \\ 2N - n, & N+1 \le n \le 2N \\ 0, & \text{otherwise} \end{cases} \tag{1.0.1}$$

2 SOLUTION

Definition 1. *The z tansform of a function is defined as*

$$x[n] \overset{\mathcal{Z}}{\rightleftharpoons} X(z) \tag{2.0.1}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{2.0.2}$$

Theorem 2.1 (Convolution Theorem)**.** *Let $F(z)$ and $G(z)$ be the Z-transform of two functions f and g respectively. Then*

$$\mathcal{Z}(f * g) = F(z)G(z) \tag{2.0.3}$$

Given sequence is

$$x[n] = \begin{cases} n, & 0 \le n \le N \\ 2N - n, & N+1 \le n \le 2N \\ 0, & \text{otherwise} \end{cases} \tag{2.0.4}$$

Let's define a new sequence

$$x_1[n] = \begin{cases} n, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases} \tag{2.0.5}$$

Consider

$$x_1[n] * x_1[n-1] = \sum_{k=-\infty}^{\infty} x_1[k] x_1[n-1-k] \tag{2.0.6}$$

On solving we get,

$$x_1[n] * x_1[n-1] = \begin{cases} n, & 0 \le n \le N \\ 2N - n, & N+1 \le n \le 2N \\ 0, & \text{otherwise} \end{cases} \tag{2.0.7}$$

$$\implies x[n] = x_1[n] * x_1[n-1] \tag{2.0.8}$$

And,

$$X_1(z) = \mathcal{Z}\{x_1(n)\} = \sum_{n=0}^{\infty} z^{-n} \tag{2.0.9}$$

$$= \frac{1 - z^{-N}}{1 - z^{-1}}, z \neq 0 \tag{2.0.10}$$

Therefore

$$X_1(z) = \frac{1 - z^{-N}}{1 - z^{-1}}, ROC : z \neq 0 \tag{2.0.11}$$

From (2.0.8) we have,

$$x[n] = x_1[n] * x_1[n-1] \tag{2.0.12}$$

$$\implies X(z) = \mathcal{Z}\{x_1[n] * x_1[n-1]\} \tag{2.0.13}$$

$$= X_1(z) \left(z^{-1} X_1(z) \right) \tag{2.0.14}$$

$$= z^{-1} \left(X_1(z) \right)^2 \tag{2.0.15}$$

$$= z^{-1} \frac{(1 - z^{-N})^2}{(1 - z^{-1})^2}, z \neq 0 \tag{2.0.16}$$

Therefore,

$$X(z) = \mathcal{Z}\{x(n)\} = z^{-1} \frac{(1 - z^{-N})^2}{(1 - z^{-1})^2} \tag{2.0.17}$$

ROC in z-plane : $z \neq 0$

Poles are,

$$z = 0 \tag{2.0.18}$$

Zeros exsist if N is even and they are:,

$$z = -1 \tag{2.0.19}$$

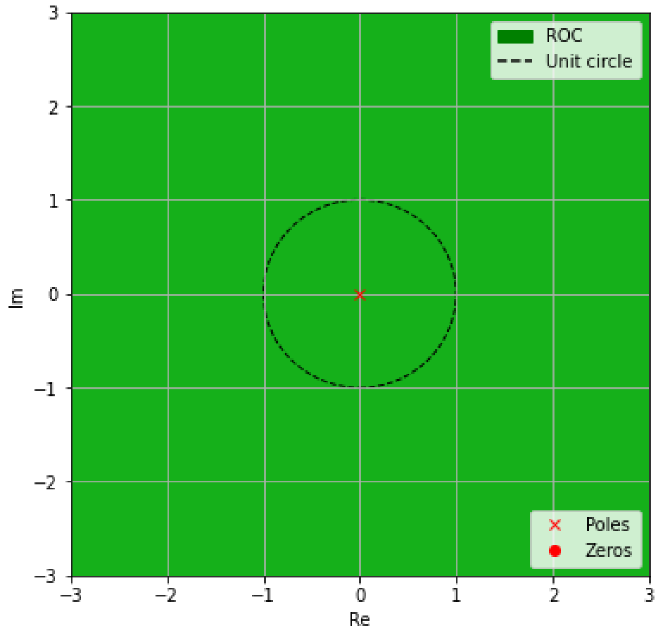

Fig. 0: Pole-zero Plot of the given sequence