CS2208 Assignment 1 vlam54

Question 1

Q1 a)

$1x2^7 + 1x2^6 + 0x2^5 + 0x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 0x2^0$
= 128 + 64 + 8 + 4
= $\mathbf{204_{10}}$

%%

Q1 b)

$1x3^7 + 1x3^6 + 1x3^3 + 1x3^2$
= 2187 + 729 + 27 + 9
= $\mathbf{2952_{10}}$

%%

Q1 c)

$1x4^7 + 1x4^6 + 1x4^3 + 1x4^2$
= 16384 + 4096 + 64 + 16
= $\mathbf{20560_{10}}$

$$

Question 2

Q2 a)

```
10000/2 = 5000 remainder 0
5000/2  = 2500 remainder 0
2500/2  = 1250 remainder 0
1250/2  =  625 remainder 0
625/2   =  312 remainder 1
312/2   =  156 remainder 0
156/2   =   78 remainder 0
78/2    =   39 remainder 0
39/2    =   19 remainder 1
19/2    =    9 remainder 1
9/2     =    4 remainder 1
4/2     =    2 remainder 0
2/2     =    1 remainder 0
1/2     =    0 remainder 1
```

Thus the answer is $\mathbf{10011100010000_2}$

%%

Q2 b)

First convert hex to binary:

$= 1111\ 1110\ 1101\ 1100\ .\ 1011\ 1010\ _2$
$= 1111111011011100.10111010_2$

Now convert binary to octal by grouping and adding zeroes:
$= \mathbf{00}1\ 111\ 111\ 011\ 011\ 100\ .\ 101\ 110\ 10\mathbf{0}_2$

Thus the answer is: $\mathbf{177334.564_8}$

%%

Q2 c)

First convert octal to binary:

$= 001\ 010\ 011\ 100\ 101\ .\ 110\ 111\ _2$
$= 001010011100101.110111_2$

Now convert binary to hex by grouping and adding zeroes:

$= \mathbf{0}001\ 0100\ 1110\ 0101\ .\ 1101\ 11\mathbf{00_2}$

Thus the answer is $\mathbf{14E5.DC_{16}}$

$$

Question 3

Q3 a)

We can expand the left side:

$$1xS^3 + 0xS^2 + 0xS^1 + 1xS^0 = 19684_{10}$$

We know that $1xS^0$ is = 1 so:

$$1xS^3 + 1 = 19683 + 1$$

Now take the cubic root of each term to find S:

S = 27

%%%

Q3 b)

We can expand the left side:

$$1xT^3 + 0xT^2 + 1xT^1 + 1xT^0 = 4931_{10}$$

We know that $1xT^0$ is = 1 so:

$$1xT^3 + 1xT^1 + 1 = 4930 + 1$$

Through factoring, trial and error and rounding:

T = 17

$$

Question 4

Q4 a)

Convert to binary:

```
9876/2 = 4938 remainder 0
4938/2 = 2469 remainder 0
2469/2 = 1234 remainder 1
1234/2 =  617 remainder 0
617/2  =  308 remainder 1
308/2  =  154 remainder 0
154/2  =   77 remainder 0
77/2   =   38 remainder 1
38/2   =   19 remainder 0
19/2   =    9 remainder 1
9/2    =    4 remainder 1
4/2    =    2 remainder 0
2/2    =    1 remainder 0
1/2    =    0 remainder 1
```

We get $9876_{10} = 10011010010100_2$ and since it's unsigned, add a sign:

$+9876_{10} = \mathbf{0}10011010010100_2$

Now two's complement

$-9876_{10} = \mathbf{1}01100101101100_2$

So the answer is $\mathbf{101100101101100_2}$

%%

Q4 b)

First convert the number to binary (ignore the negative, we will consider the sign when we do the two's complement):

```
98/2 = 49 remainder 0
49/2 = 24 remainder 1
24/2 = 12 remainder 0
12/6 =  6 remainder 0
6/2  =  3 remainder 0
3/2  =  1 remainder 1
1/2  =  0 remainder 1
```

Convert the decimal part to binary (using multiplication method!)

```
0.7654310x2 = 1.530862
0.530862x2  = 1.061724
0.061724x2  = 0.123448
0.123448x2  = 0.246896
```

0.246896x2 = 0.493792
0.493792x2 = 0.987584
0.987584x2 = 1.975168
0.975168x2 = 1.950336
0.950336x2 = 1.900672
0.900672x2 = 1.801344
0.801344x2 = 1.602688
0.602688x2 = 1.205376
0.205376x2 = 0.410752
0.410752x2 = 0.821504

Round to 10 decimal digits (10 digits after the decimal)

01100010.1100001111_2

Take two's complement of the number since it's negative

Thus the final answer is $\mathbf{10011101.0011110001_2}$

$$

Question 5

Q5 a)

$1010.1010_2 = 1x2^3 + 0x2^2 + 1x2^1 + 0x2^0 + 1x2^{-1} + 1x2^{-3}$
= 8 + 2 + 0.5 + 0.125
= **10.625_{10}**

%%

Q5 b)

If it is a sign and magnitude number, then the first bit will represent the sign, which is negative. Taking the work shown from above, we can see that the 8 is omitted (as the sign takes its place).

So the answer is **-2.625_{10}**

%%

Q5 c)

The two's complement is -101.0110_2

$= 1x2^2 + 1x2^{-2} + 1x2^{-3}$
= **-5.375_{10}**

$$

Question 6

Q6 a)

```
  1111111
   10101010
+  11111111
___________
   10101001₂
```

The answer is: $\mathbf{10101001_2}$
No overflow because last two carries are the same.

%%

Q6 b)

```
   1111111
   01011111
+  01110101
___________
   11010100₂
```

The answer is: $\mathbf{11010100_2}$
Overflow because adding two positives and getting a negative!

%%

Q6 c)

```
  1111 1 1
   11110101
+  01010101
___________
   01001010₂
```

The answer is: $\mathbf{01001010_2}$
No overflow because we are adding numbers of different signs. And last two carries are the same.

$$

Question 7

Q7 a)

First convert to binary:

```
1234/2 = 617 remainder 0
617/2  = 308 remainder 1
308/2  = 154 remainder 0
154/2  =  77 remainder 0
77/2   =  38 remainder 1
38/2   =  19 remainder 0
19/2   =   9 remainder 1
9/2    =   4 remainder 1
4/2    =   2 remainder 0
2/2    =   1 remainder 0
1/2    =   0 remainder 1
```

Converting the decimal (via multiplication):

```
0.875x2 = 1.75
0.75x2  = 1.5
0.5x2   = 1.0
```

The number in binary is 10011010010.111_2 so now normalize it:

$1.0011010010111_2 \times 2^{10}$ This is the real exponent
$1.0011010010111_2 \times 2^{136}$ This is adding the bias 127

Now convert the exponent 137 to binary:

```
137/2 = 68 remainder 1
68/2  = 34 remainder 0
34/2  = 17 remainder 0
17/2  =  8 remainder 1
8/2   =  4 remainder 0
4/2   =  2 remainder 0
2/2   =  1 remainder 0
1/2   =  0 remainder 1
```

Now convert to 32-bit IEEE format (and add zeros at the end so that it's 32 bits). Also recall that the original number is negative, so the first bit is a 1:

1 10001001 0011010010111 0000000000 $_2$

Now convert to hex via grouping:

1100 0100 1001 1010 0101 1100 0000 0000 $_2$

Therefore the answer is $\mathbf{C49A5C00_{16}}$

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Q7 b)

First convert to binary:

```
7654/2 = 3827 remainder 0
3827/2 = 1913 remainder 1
1913/2 =  956 remainder 1
956/2  =  478 remainder 0
478/2  =  239 remainder 0
239/2  =  119 remainder 1
119/2  =   59 remainder 1
59/2   =   29 remainder 1
29/2   =   14 remainder 1
14/2   =    7 remainder 0
7/2    =    3 remainder 1
3/2    =    1 remainder 1
1/2    =    0 remainder 1
```

Converting the decimal (via multiplication):

```
0.3x2 = 0.6
0.6x2 = 1.2
0.2x2 = 0.4
0.4x2 = 0.8
0.8x2 = 1.6
0.6x2 = 1.2
.
.
.
```

As you can see, the numbers follow a pattern (we get 01**0011**, where the bolded numbers repeat). Since it is repeating, we must truncate to round.

The number in binary is $1110111100110.010011\ldots_2$ so now normalize it:

$1.110111100110010011_2 \times 2^{12}$ This is the real exponent
$1.110111100110010011_2 \times 2^{139}$ This is adding the bias 127

Now convert the exponent 139 to binary:

```
139/2 = 69 remainder 1
69/2  = 34 remainder 1
34/2  = 17 remainder 0
17/2  =  8 remainder 1
8/2   =  4 remainder 0
4/2   =  2 remainder 0
2/2   =  1 remainder 0
1/2   =  0 remainder 1
```

Now convert to 32-bit IEEE format (and add the repeating digits at the end so that it's 32 bits). Also recall that the original number is positive, so the first bit is a 0:

$0\ 10001011\ 110111100110010011\ 00110_{2}$

Now convert to hex via grouping:

$0100\ 0101\ 1110\ 1111\ 0011\ 0010\ 0110\ 0110_{2}$

Therefore the answer is $\mathbf{45EF3266_{16}}$

$$$

Question 8

Q8 a)

Convert $FEDCBA98_{16}$ to binary: $1111\ 1110\ 1101\ 1100\ 1011\ 1010\ 1001\ 1000_2$

Rewrite into significand form and normalize:

$1\ 11111101\ 10111001011101010011000_2$

$-\ 1.10111001011101010011000_2 \times 2^{253}$ We must subtract the bias 127
$-\ 1.10111001011101010011000_2 \times 2^{126}$ This is the true exponent

Convert $-1.10111001011101010011000_2$ to decimal

$= -\ 1x2^{0} + 1x2^{-1} + 1x2^{-3} + 1x2^{-4} + 1x2^{-5} + 1x2^{-8} + 1x2^{-10} + 1x2^{-11} + 1x2^{-12} + 1x2^{-14} + 1x2^{-16} + 1x2^{-19} + 1x2^{-20}$

$= -\ 1.72444438934$

Now to make the exponent more readable

$2^{126} = 10^{z}$
$Log_{10}(2^{126}) = z$
$z = 37.9297794537$

$2^{126} = 10^{37.9297794537}$
$2^{126} = 10^{37} \times 10^{0.9297794537}$
$= 10^{37} \times 8.50705917378$

Now to make simplify the whole number

$-1.72444438934 \times 2^{126}$
$= -1.72444438934 \times 8.50705917378 \times 10^{37}$
$= -\ 14.669950462 \times 10^{37}$

Therefore the answer is: $\mathbf{-\ 1.4669950462_{10} \times 10^{38}}$

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Q8 b)

Convert $89ABCDEF_{16}$ to binary: $1000\ 1001\ 1010\ 1011\ 1100\ 1101\ 1110\ 1111_2$

Rewrite into significand form and normalize:

$1\ 00010011\ 01010111100110111101111_2$

$-\ 1.01010111100110111101111_2 \times 2^{19}$ We must subtract the bias 127
$-\ 1.01010111100110111101111_2 \times 2^{-108}$ This is the true exponent

Convert $-1.01010111100110111101111_2$ to decimal

$= -\ 1x2^{0} + 1x2^{-2} + 1x2^{-4} + 1x2^{-6} + 1x2^{-7} + 1x2^{-8} + 1x2^{-9} + 1x2^{-12} + 1x2^{-13} + 1x2^{-15} + 1x2^{-16} + 1x2^{-17} + 1x2^{-18} + 1x2^{-20} + 1x2^{-21} + 1x2^{-22} + 1x2^{-23}$

$= -\ 1.34222209453$

Now to make the exponent more readable

$2^{-108} = 10^{z}$
$Log_{10}(2^{-108}) = z$
$z = -32.5112395317$

$2^{-108} = 10^{-32.5112395317}$
$2^{-108} = 10^{-32} \times 10^{-0.5112395317}$
$= 10^{-32} \times 8.50705917378$

Now to make simplify the whole number

$-1.34222209453 \times 2^{-108}$
$= -1.34222209453 \times 0.3081487911 \times 10^{-32}$
$= -0.41360411582 \times 10^{-32}$

Therefore the answer is: $\mathbf{-4.1360411582_{10} \times 10^{-31}}$

$$$

Question 9

Q9 a)

First convert each to binary:

$FEDCBA98_{16}$: $1\ 11111101\ 10111001011101010011000_2$
$89ABCDEF_{16}$: $1\ 00010011\ 01010111100110111101111_2$

Now rewrite into 32 bit IEEE format (work shown in previous question):

$FEDCBA98_{16}$: $-\ 1.10111001011101010011000_2 \times 2^{126}$
$89ABCDEF_{16}$: $-\ 1.01010111100110111101111_2 \times 2^{-108}$

Now to make the number with the smaller exponent equal to the larger one. However, since the difference between the two exponents is larger than the number of significant bits+1, the result will just equal the larger number ($FEDCBA98_{16}$)

Therefore the answer is $\mathbf{FEDCBA98_{16}}$

%%

Q9 b)

First convert each number to binary:

$00FCD6EB_{16}$: $0000\ 0000\ 1111\ 1100\ 1101\ 0110\ 1110\ 1011_2$
$80FCD6EA_{16}$: $1000\ 0000\ 1111\ 1100\ 1101\ 0110\ 1110\ 1010_2$

Rewrite into 32bit IEEE format (don't forget to subtract the bias 127)

$00FCD6EB_{16}$: $+\ 1.11111001101011011101011_2\ x2^{-126}$
$80FCD6EA_{16}$: $-\ 1.11111001101011011101010_2\ x2^{-126}$

The numbers are the same exponent. If we were to sum them, we can see that they differ by $0.00000000000000000000001_2\ x2^{-126}$, which is $0.000000000000000000000010_2\ x2^{-127}$.

Convert to proper IEEE. (Remember to consider the bias. Note that this is a case of underflow as the smallest exponent we can have is -127) We can now convert this to hex via grouping.

0000 0000 0000 0000 0000 0000 0000 0010

Therefore the answer is: **00000002_{16}**

%%

Q9 c)

First convert each number to binary:

$00FCD6EB_{16}$: $0000\ 0000\ 1111\ 1100\ 1101\ 0110\ 1110\ 1011_2$
$09ABCDEF_{16}$: $0000\ 1001\ 1010\ 1011\ 1100\ 1101\ 1110\ 1111_2$

Rewrite into 32bit IEEE format (don't forget to subtract the bias 127)

$00FCD6EB_{16}$: $+\ 1.11111001101011011101011_2\ x2^{-126}$
$09ABCDEF_{16}$: $+\ 1.01010111100110111101111_2\ x2^{-108}$

Now to make the number with the smaller exponent equal to the larger one via division.

$00FCD6EB_{16}$: $+\ 1.11111001101011011101011_2\ x2^{-126}$
$00FCD6EB_{16}$: $+\ 0.00000000000000000111111_2\ x2^{-108}$

Now to sum:

$$
\begin{array}{rl}
 & \mathbf{111111111} \\
 & 1.01010111100110111101111_2\ x2^{-108} \\
+ & 0.00000000000000000111111_2\ x2^{-108} \\
\hline
 & 1.01010111100111000101110_2\ x2^{-108}
\end{array}
$$

Now convert to binary (remember to add the bias 127 to the exponent and then convert to binary):

0 0001 1001 $01010111100111000101110_2$

Now convert to hex. Therefore the answer is: **$09ABCE2E_{16}$**

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