2 $y_i = \beta + u_i$

МНК: $\hat{y} = \hat{\beta}$

$$RSS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} (y_i - \hat{\beta})^2 \rightarrow \min_{\hat{\beta}}$$

$$\cancel{-2} \sum_{i=1}^{n} (y_i - \hat{\beta}) = 0$$

$$\hat{\beta} = \frac{\sum_{i=1}^{n} y_i}{n} = \bar{y}$$

$$RSS = \sum_{i=1}^{n} (y_i - \bar{y})^2 \qquad TSS = \underset{\substack{\parallel \\ 0}}{ESS} + RSS$$

$$\parallel$$

$$TSS = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$ESS = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 = \sum_{i=1}^{n} (\bar{y} - \bar{y})^2 = 0$$

$$R^2 = \frac{ESS}{TSS} = 0$$

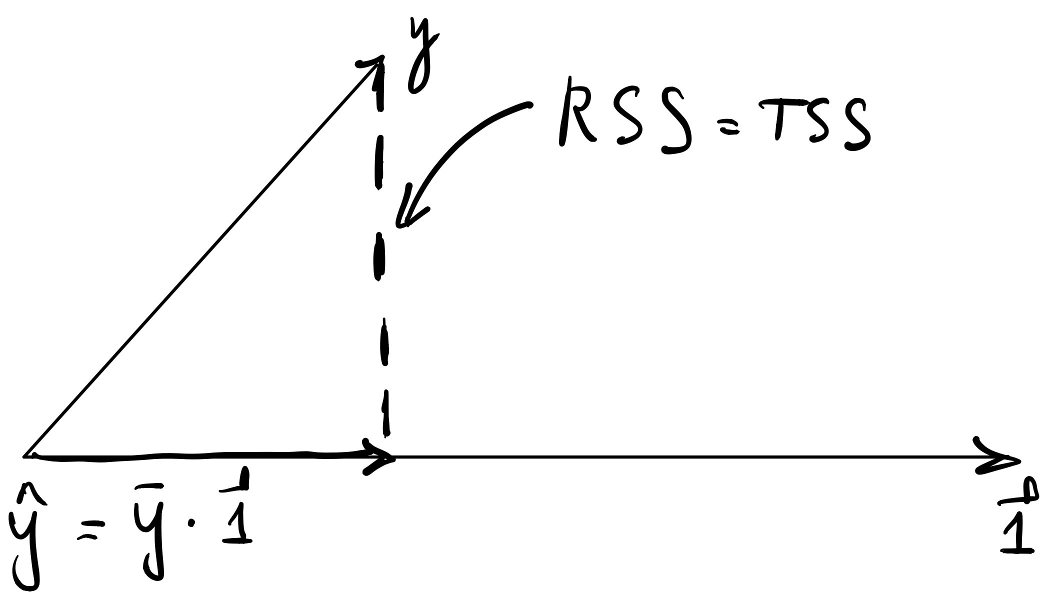
$$R^2 = scorr^2(y, \hat{y}) = 0 = scov^2(y - \hat{y})$$

$$\mathrm{scov}(y, x) = \sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})$$

$$\mathrm{scorr}(y, x) = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2} \sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}}$$

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

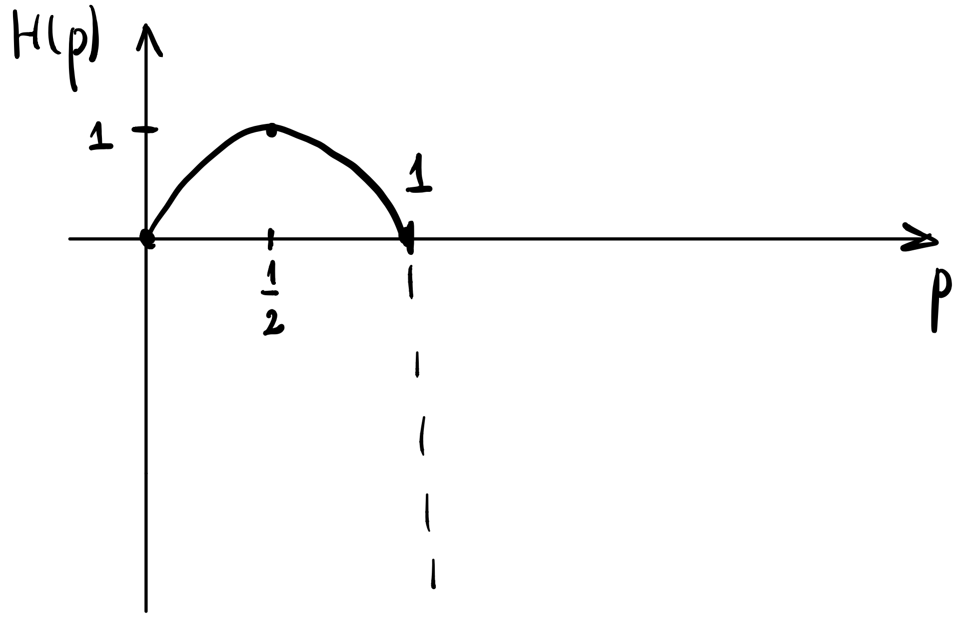
$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$



4

$$X = \begin{cases} 1, & p \\ 0, & 1-p \end{cases}$$

$$H(p) = p \log_{\frac{1}{2}} p + (1-p) \log_{\frac{1}{2}} (1-p)$$



5

$Z = \{1, 2, 3\}$

γ_1 – набл. $\in$ 1 кл.

γ_2 – 2 кл.

$1-\gamma_1-\gamma_2$ – 3 кл.

E 1:

$$\underset{g_1}{p(z_i = 1 \mid x_i, \Theta_{old})} = \frac{\gamma_1 \, p(x_i \mid z_i = 1, \Theta_{old})}{\gamma_1 \, p(x_i \mid z_i = 1, \Theta_{old}) + \gamma_2 p(x_i \mid z_i = 2, \Theta_{old}) + (1-\gamma_1-\gamma_2) \, p(x_i \mid z_i = 3, \Theta_{old})}$$

$$g_2 = p(z_i = 2 \mid x_i, \Theta_{old}) = \frac{\gamma_2 \, p(x_i \mid z_i = 2, \Theta_{old})}{— \text{''} —}$$

E_2, M:

$$Q = \sum_{i=1}^{n} g_1 \left(\ln p(x \mid \Theta) + \ln \gamma_1 \right) +$$

$$+ g_2 \left(\ln p(x \mid \Theta) + \ln \gamma_2 \right) +$$

$$+ (1 - g_1 - g_2)\left(\ln p(x \mid \Theta) + \ln(1 - \gamma_1 - \gamma_2) \right)$$

$$\to \max_{\Theta, \gamma_1, \gamma_2}$$

6

$$X \sim \chi^2_5 \qquad X \perp\!\!\!\perp Y$$

$$Y \sim \chi^2_{10}$$

$$F_{k,m} = \frac{\chi^2_k / k}{\chi^2_m / m}$$

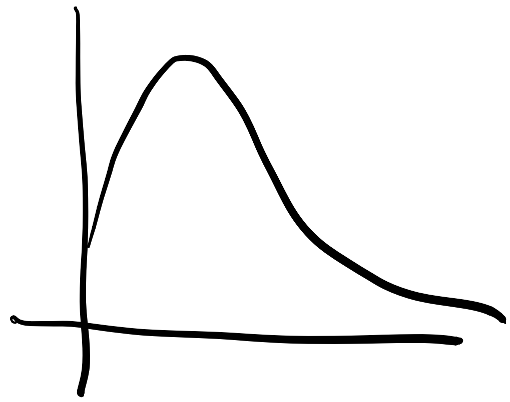
$$Z = \frac{X + Y}{X}$$

$$\underline{\mathbb{P}\{ Z > z^* \} = 0.05}$$

$$\mathbb{P}\left\{ 1 + \frac{Y}{X} > z^* \right\} = 1 - \mathbb{P}\left\{ 1 + \frac{Y}{X} \leq z^* \right\} =$$

$$= 1 - \mathbb{P}\left\{ y/x \le z^* - 1 \right\} =$$

$$= 1 - \mathbb{P}\left\{ \underbrace{\frac{y/10}{x/5}}_{Q \sim F_{10,5}} \le (z^* - 1)\frac{5}{10} \right\}$$



$$\mathbb{P}\{ z > z^* \} = 0.05$$

$$1 - \mathbb{P}\left\{ Q \le (z^* - 1)\frac{5}{10} \right\} = 0.05$$

$$\mathbb{P}\left\{ Q \le (z^* - 1)\frac{5}{10} \right\} = 0.95$$

$$(z^* - 1)\frac{5}{10} = 4.74$$

$$z^* = 10.48$$

7 $\mathbb{R}^3$ $W = \{ (x_1\ x_2\ x_3) \mid x_1 + 2x_2 + x_3 = 0 \}$

$V = \text{lin}\left[(1,2,3)^T \right]$

$\dim V = 1 - \|\hat{u}\|^2 \sim \chi^2_1$ $\quad \mathbb{R}^n$

$\dim W = 2 - \|\hat{u}\|^2 \sim \chi^2_2$ $\quad V,\ \dim V = k$

$\dim(V \cap W) = 0$ $\qquad \dim V^{\perp} = n-k$

$\dim V^{\perp} = 3-1 = 2 - \|\hat{u}\|^2 \sim \chi^2_2$

$\dim W^{\perp} = 3-2 = 1$

T. $\underline{\underline{u}}$ имеет многомерн. станд. норм. распр.
проекц. u на V_k — $\hat{u}$ - проекц.

$$\|\hat{u}\|^2 \sim \chi^2_{\underline{\underline{k}}}$$

2) М-ца проектора: $\underbrace{X(X^T X)^{-1} X^T}_{\text{м-ца проектора вектора } u \text{ на подпр-о, обр. ст-цами } X} \cdot u = \hat{u}$

$u \to V$, $\dim V = 1$

$$V = \operatorname{lin}\begin{pmatrix}1\\2\\3\end{pmatrix} \qquad X = \begin{pmatrix}1\\2\\3\end{pmatrix}$$

$y = X\beta + u$

$\hat{y} = X\hat{\beta}$, $\hat{\beta} = (X^T X)^{-1} X^T y$

$$\hat{u} = \underline{X(X^T X)^{-1} X^T} u$$

$$\hat{u} = \underset{3\times 1}{\begin{pmatrix}1\\2\\3\end{pmatrix}} \Bigg[\underbrace{(1\ 2\ 3)\begin{pmatrix}1\\2\\3\end{pmatrix}}_{1\times 1}\Bigg]^{-1} \underset{1\times 3}{(1\ 2\ 3)} \underset{3\times 1}{\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}}$$

$$W: \quad \underline{x_1 + 2x_2 + x_3 = 0}$$

$$_3 = -x_1 - 2x_2$$

$\begin{pmatrix}1\\0\\-1\end{pmatrix} \begin{pmatrix}0\\1\\-2\end{pmatrix}$ – один из базисов W

$$X = \begin{pmatrix}1 & 0\\0 & 1\\-1 & -2\end{pmatrix}$$

3) $u \sim N(0, I)$

$\underbrace{Au} \sim N(0, I)$

u – сл. вектор
A – матрица consts
$\mathbb{E}(A \cdot u) = A\,\mathbb{E}(u)$
$Var(Au) = A \cdot Var(u) \cdot A^T$

$$Var(Au) = I$$

$$A \underbrace{Var(u)}_{I} A^T = I$$

$AA^T = I \Rightarrow A^T = A^{-1} \Rightarrow A$ – ортогональная.

2) $\det(A) = \pm 1$

3) $\langle c_1, c_2 \rangle = c_1^T \cdot c_2 = 0$

4) Распр $\|u\|^2 \sim \chi^2_n$

размерность простр-а, в к-м лежит u.

1. i-ый друг:

$Б_1 \; ... \; Б_{10}$

$\begin{cases} \text{выигр}, & p \\ \text{проигр}, & 1-p \end{cases}$

1) $X_i \sim Bin(10, p)$

2) $x_1 \dots x_n$ – выборка

$$L = \mathbb{P}\{X_1 = x_1 \dots X_n = x_n\} = \underbrace{\mathbb{P}\{X_1 = x_1\}}\cdot \dots \mathbb{P}\{X_n = x_n\} =$$

$$= \underbrace{C_{10}^{x_1} p^{x_1} (1-p)^{10-x_1}}_{\mathbb{P}\{X_1 = x_1\}} \cdot C_{10}^{x_2} p^{x_2} (1-p)^{10-x_2} \cdot \dots$$

$$\cdot \ldots \cdot C_{10}^{x_n} p^{x_n} (1-p)^{10-x_n} =$$

$$= \cancel{C_{10}^{x_1} C_{10}^{x_2} \ldots C_{10}^{x_n}} \, p^{\sum_{i=1}^{n} x_i} (1-p)^{\sum_{i=1}^{n}(10-x_i)}$$

$$\ell = \sum_{i=1}^{n} x_i \ln p + \sum_{i=1}^{n} (10-x_i) \ln (1-p)$$

$$\ell'_p = \frac{\sum x_i}{p} - \frac{\sum (10-x_i)}{1-p}$$

$$\frac{\sum x_i}{\hat{p}} - \frac{10n - \sum x_i}{1-\hat{p}} = 0$$

3) $$\hat{p}_{ML} = \frac{\sum_{i=1}^{n} x_i}{10n}$$

4) $$\hat{p} \overset{ac.}{\sim} N(p, I^{-1}(p))$$

$$\ell''_p = -\frac{\sum x_i}{p^2} - \frac{\sum (10-x_i)}{(1-p)^2}$$

$$I = \mathbb{E}(-H) = \mathbb{E}\left(\frac{\sum x_i}{p^2} + \frac{\sum (10-x_i)}{(1-p)^2}\right) =$$

$$= \frac{n\cancel{p}}{p^{\cancel{2}}} + \frac{10n - np}{(1-p)^2} = \frac{n}{p} + \frac{10n - np}{(1-p)^2}$$

$$\hat{I} = \frac{n}{\hat{p}} + \frac{10n - n\hat{p}}{(1-\hat{p})^2}$$

$$\hat{Var}(\hat{p}) = \hat{I}^{-1} = \frac{1}{\hat{I}}$$

$$p \in \left[\hat{p} - 1.96\sqrt{\hat{Var}(\hat{p})}\ ;\ \hat{p} + 1.96\sqrt{\hat{Var}(\hat{p})}\right]$$

5) $\widehat{\mathbb{E}(X_i)}$

$\mathbb{E}(X_i) = 10p$

$\widehat{\mathbb{E}(X_i)} = 10\hat{p}$

$Var(X_i) = 10p(1-p)$

$\hat{Var}(X_i) = 10\hat{p}(1-\hat{p})$

6) $Var(10\hat{p}) = 100\, Var(\hat{p})$

$$Var(10\hat{p}(1-\hat{p})) = 100\, Var(\hat{p}-\hat{p}^2) =$$

$$= 100\left[\underbrace{Var(\hat{p})}_{\substack{\shortparallel \\ \text{знаем}}} + Var(\hat{p}^2) - 2cov(\hat{p},\hat{p}^2)\right]$$

$$\hat{p} \sim N(p, \underbrace{I^{-1}(p)}_{\sigma^2})$$

$$Var(\hat{p}^2) = \underbrace{\mathbb{E}(\hat{p}^4)}_{p^4+6p^2\sigma^2+3\sigma^4} - [\mathbb{E}(\hat{p}^2)]^2 =$$

$$p^2+\sigma^2$$

$$= p^4 + 6p^2\sigma^2 + 3\sigma^4 - (p^2+\sigma^2)^2$$

$$cov(\hat{p},\hat{p}^2) = \underset{\substack{\shortparallel \\ p^3+3p\sigma^2}}{\mathbb{E}(\hat{p}^3)} - \underset{\substack{\shortparallel \\ p}}{\mathbb{E}(\hat{p})}\,\underset{\substack{\shortparallel \\ (p^2+\sigma^2)}}{\mathbb{E}(\hat{p}^2)} =$$

$$= p^3 + 3p\sigma^2 - p^3 - p\sigma^2 = 2p\sigma^2.$$