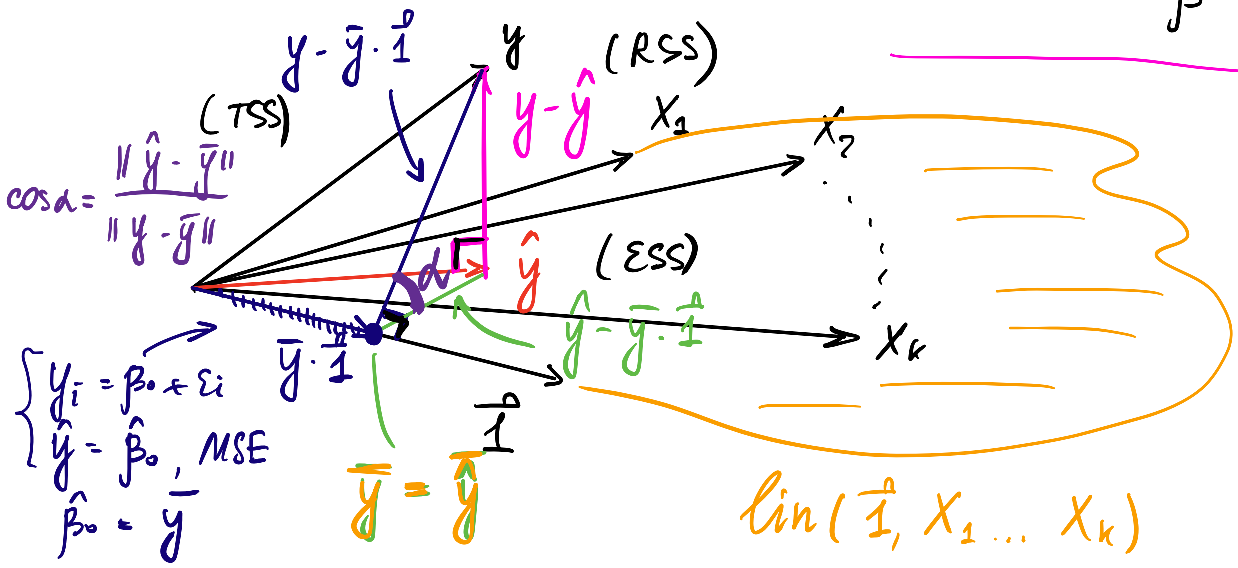
$$\underset{N\times 1}{y} = \underset{N\times(K+1)}{X}\ \underset{(K+1)\times 1}{\beta} + \underset{N\times 1}{\varepsilon} \qquad X = \begin{bmatrix} | & | & | & & | \\ 1 & X_1 & X_2 & \dots & X_K \\ | & | & | & & | \end{bmatrix}$$

$\hat{y} = X\hat{\beta}$ при пом. МНК : $MSE = \|y - \hat{y}\|_2^2 \to \min\limits_{\hat{\beta}}$



$$\hat{y} = X\hat{\beta} = \hat{\beta}_0 \begin{bmatrix} | \\ 1 \\ | \end{bmatrix} + \hat{\beta}_1 \begin{bmatrix} | \\ X_1 \\ | \end{bmatrix} + \hat{\beta}_2 \begin{bmatrix} | \\ X_2 \\ | \end{bmatrix} + \dots + \beta_K \begin{bmatrix} | \\ X_K \\ | \end{bmatrix}$$

$$\Rightarrow \hat{y} \in lin(\vec{1}, X_1 \dots X_k)$$

$$RSS = \sum_i (y_i - \hat{y}_i)^2 = \|y - \hat{y}\|_2^2$$

residual sum of squares

$$TSS = \sum_i (y_i - \bar{y})^2 = \|y - \bar{y}\cdot\vec{1}\|_2^2$$

\ total sum of squares

$$ESS = \sum_i (\hat{y}_i - \bar{\hat{y}})^2 = \|\hat{y} - \bar{y} \cdot \vec{1}\|_2^2$$

$\bar{\hat{y}} = \bar{y}$

estimated sum of squares

$$TSS = ESS + RSS$$

$$R^2 = \frac{\sum_i (\hat{y}_i - \bar{\hat{y}})^2}{\sum_i (y_i - \bar{y})^2} = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} =$$

$= \cos^2 \alpha$ (TSS, ESS)

$$y_i = \beta_0 + \beta_1 x_{1i} + \ldots + \beta_k x_{ki} + \varepsilon_i$$ UR (unrestricted model)

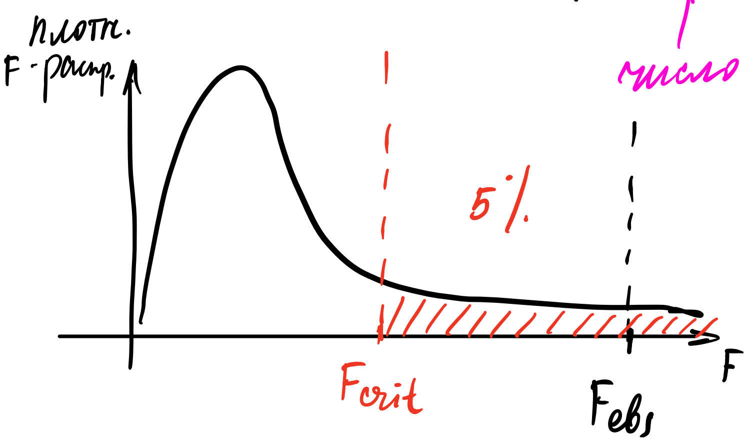
+ огранич. $\longrightarrow$ R (restricted model)

R: $y_i = \beta_0 + \beta_1 x_{1i} + \varepsilon_i$ Огранич: $\beta_2 = \ldots = \beta_k = 0$

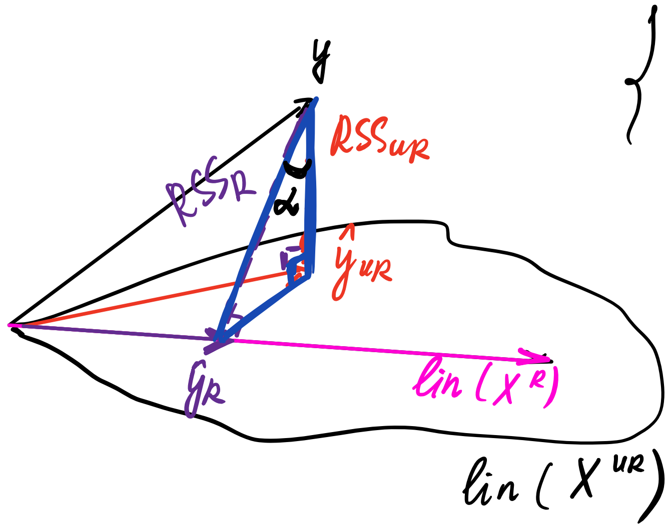
$$\begin{cases} H_0: \beta_2 = \ldots = \beta_k = 0 \\ H_1: \beta_2^2 + \ldots + \beta_k^2 > 0 \end{cases} \Rightarrow$$ Тест. зн. о коэф. $\equiv$ UR vs R

2) $\begin{cases} H_0: \text{верны и } UR, \text{ и } R \text{ модель} \\ H_1: \text{верна } UR, \text{ а } R \text{ модель неверна} \end{cases}$

$$F = \frac{(RSS_R - RSS_{UR}) \,/\, (\overset{\text{число регр. в } UR}{k_{UR}} - \overset{\text{число регр. в } R}{k_R})}{RSS_{UR} \,/\, (\underset{\text{число набл.}}{N} - k_{UR})} \overset{H_0}{\sim} F_{(k_{UR} - k_R;\ N - k_{UR})}$$



$\begin{cases} H_0: UR \text{ верна}, R \text{ вер.} \\ H_1: UR \text{ верна}, R \text{ невер.} \end{cases}$



$$tg^2\alpha = \frac{\|\hat{y}_{UR} - \hat{y}_R\|^2}{\|y - \hat{y}_{UR}\|^2} = \frac{RSS^R - RSS^{UR}}{RSS_{UR}}$$

Пример:

UR: $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i$

$\varepsilon_i \sim N(0, \sigma^2 I)$

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \qquad y = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$$

$$\hat{\beta} = (X^T X)^{-1} X^T y = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \qquad \hat{y} = X\hat{\beta} = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 4 \\ 5 \end{pmatrix}$$

$$RSS_{UR} = \sum_i (y_i - \hat{y}_i)^2 = 1 + 0 + 1 + 0 + 0 = 2$$

Тест. регр. на знач. в целом

$$\begin{cases} \boxed{H_0: \beta_1 = \beta_2 = 0} \\ H_1: \beta_1^2 + \beta_2^2 > 0 \end{cases}$$

$$\boxed{R: y_i = \beta_0 + \varepsilon_i}$$

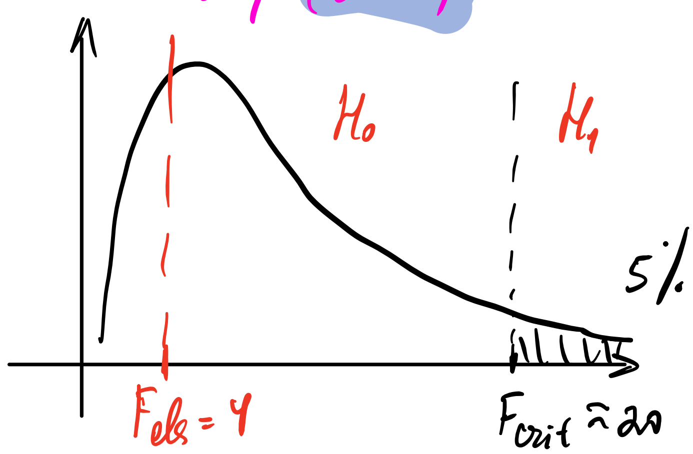
$$\hat{\beta}_0 = \bar{y} = \frac{15}{5} = 3$$

$$y = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \qquad \hat{y}_R = \begin{pmatrix} 3 \\ 3 \\ 3 \\ 3 \\ 3 \end{pmatrix}$$

$$RSS_R = \sum_i (y_i - \hat{y}_R)^2 = 4 + 1 + 1 + 4 = 10$$

$$F = \frac{(RSS_R - RSS_{UR}) / (k_{UR} - k_R)}{RSS_{UR} / (N - k_{UR})} =$$

$$= \frac{(10-2) / (3-1)}{2 / (5-3)} = \frac{8/2}{2/2} = 4 \overset{H_0}{\sim} F_{(2,2)}$$



5% H_0 не отверг.