Time Variation of Regression Coefficients related to Macroeconomic News affecting Currency Prices

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Contents

abstract here when I'm done with the writing and have all results.

1 Introduction

Some macroeconomic figures used to gauge the health of a nation's economy are released to the public on a predetermined schedule. These include for example the Non-Farm Employment change that is released on the first friday of every month informing economists and investors alike of the status of employment in the United States.

Standard economic theory helps us understand that an increase of interest rates is warranted when economies are performing well and prices are generally increasing. Those who decide to increase national interest rates, the central banks, typically refer to measures of inflation in order to make their decisions. Because of this, investors and traders alike pay close attention to news releases (such as inflation, and also the

Table 1: Summary of the news figures considered in the study

Country	News.Event	Pair.used	GMT.Time	Frequency	Observations	Dates
Single News					**NA**	
Canada	Consumer Price Index	USD/CAD	13:30	Monthly	135	Jan 2008 to Dec 2019
Canada	Core Retail Sales	USD/CAD	12:30	Monthly	144	Oct 2008 to Dec 2019
United States	Consumer Price Index	USD/CHF	13:30	Monthly	135	Oct 2008 to Dec 2019
New Zealand	Consumer Price Index	NZD/USD	21:45	Quarterly	43	Jan 2009 to Oct 2019
Australia	Consumer Price Index	AUD/USD	00:30	Quarterly	48	Jan 2008 to Oct 2019
Australia	Retail Sales	AUD/USD	00:30	Monthly	135	Oct 2008 to Dec 2019
United Kingdom	Consumer Price Index	GBP/USD	09:30	Monthly	135	Oct 2008 to Dec 2019
United Kingdom	Retail Sales	GBP/USD	09:30	Monthly	135	Oct 2008 to Dec 2019
Grouped News					**NA**	
United States	Average Hourly Earnings Change	USD/CHF	13:30	Monthly	135	Oct 2008 to Dec 2019
United States	NonFarm Employment Change	USD/CHF	13:30	Monthly	135	Oct 2008 to Dec 2019
United States	Unemployment Rate	USD/CHF	13:30	Monthly	135	Oct 2008 to Dec 2019
Canada	Employment Change	USD/CAD	13:30	Monthly	135	Oct 2008 to Dec 2019
Canada	Unemployment Rate	USD/CAD	13:30	Monthly	135	Oct 2008 to Dec 2019
Australia	Employment Change	AUD/USD	00:30	Monthly	135	Oct 2008 to Dec 2019
Australia	Unemployment Rate	AUD/USD	00:30	Monthly	135	Oct 2008 to Dec 2019

Non-Farm-Payrolls in the case of the United States) and react according to the results. These news releases are not made public until specific times on specific days and since investors and traders react to the same news the moment it is released, the result is often a violent reaction of price in one direction or another. The common discourse is that the direction and the magnitude of the change of price depends on the difference between the expectation of the market (combined expecation of worldwide investors) and the result of the news release.

In this paper, we decide to use currency pairs to measure the price shocks. As certain news pertaining to a particular country affects its respective currency more than other ones, it makes sense to observe the currency most relevant to the news announcement. As currency prices are typically measured in pairs, the second chosen currency will be another major currency that is known for its high liquidity (EUR, USD or CHF) and does not have any other news announcements at the same time[1]. As an example, we would use the USD/CAD currency pair to measure the effect of the Canadian Consumer Price Index (CPI).

The paper of (Andersen et al. 2003) reveals that over the time period between 1987 and 2002 there has been little time-variation in the reaction to news. Some more recently published literature of (Ben Omrane et al. 2019) involving an analysis on euro-dollar contracts has determined that unlike the the decade(s) encompassing the “Great Moderation” where there was lower relative volatility in the financial markets, the time period between 2004 and 2014 is characterized by evolving reactions to macroeconomic news.

The objective of this paper is two-fold. Firstly, we aim to employ existing methodologies on new data to identify whether or not instability of market reaction is present. When reactions do change over time, we attempt to estimate the path that it takes using different procedures suggested by the literature given that the instability has been identified.

2 Data

The minute-by-minute OHLC Data of 7 currency pairs were collected from the Metatrader5 platform. This represents over 4 million data points for each pair. Only a small fraction of this data is actually used since we consider only the 5 time frame from when each piece of monthly or quarterly news is released until 5 minutes afterwards.

[1] When simultaneous news cannot be avoided, a sequence of stability tests will be applied to ensure time variation is identified on a specific news release

3 Building blocks of the models

3.1 The Stable Linear Model that is Time Invariant

Being consistent with previous literature on the subject, the first step involves estimating the impact that each piece of news has on its respective currency assuming 1.) That the news effects are constant over time. 2.) The surprise element S_t of the regression is evaluated as:

$$S_t = \frac{A_t - E_t}{\sigma_d} \tag{1}$$

A_t is the actual result of the news at time t, E_t is the expected result aggregated from experts and σ_d is the empirical standard deviation of this difference over the entire sample. We use the expectation numbers from the ForexFactory website[2] Thereafter, we use this surprise element in a first simple OLS model.

$$R_t = \beta_0 + \beta_1 S_t + \varepsilon_t \tag{2}$$

With R_t as the 5-minute currency return result and ε_t is the usual assumed normally distributed error. Moreover, because we are working with a dataset where subsequent observations are suspected to be related to one another, one could expect that the errors of the basic model above be autocorrelated. Specifically, adjacent R_t would be more similar to one another than reactions that are separated in time to a greater extent. In this case, the inference on the β_1 would be flawed. Previous researchers have used what is called Heteroskedasticity and Autocorrelation-Consistent (HAC) estimators for the variance of the OLS estimator β_1. Using the Newey-West estimator for this variance from Newey and West (1987), we use modified standard errors of the β_1 in our results. If we are wrong in our assumption in some of the news instances, and there is no underlying autocorrelation of the R_t observations, our estimation of β_1 is less efficient than the original estimator in those cases. Nonetheless, it remains consistent and ensures we avoid type 1 error of rejecting a true null hypothesis suggesting $\beta_1 = 0$.

Table 2 shows the result of the different β_1 coefficients for separate news reports. The construction of the truncation parameter in the Newey-West estimator is such that our monthly news reports consider 2 autocorrelation coefficients whereas the quarterly ones only contain 1. This is due to the difference in the number of observations in our data. A higher estimated autocorrelation between the errors of the regression will result in a stronger correction of the variance of β_1. In all of the news in Table 2, the higher standard error does not affect the significance of the term. While it is not formal evidence, we suspect that the regressions where the HAC standard error is larger than its unedited counterpart contain unknown regressors that may come from any of the findings that are well elaborated in other literature such as in Goldberg and Grisse (2013). Overall, these results and that of the Durbin-Watson test show that there is little evidence for significant autocorrelation in the residuals.

$$R_t = \beta_0 + \beta_{1,t} S_t + \varepsilon_t \tag{3}$$

3.2 The Gaussian Random Walk

The methods discussed in this paper use random walks extensively. This section serves to underline the features of these processes which will later be essential to the understanding of tests and path estimations. Furthermore, it hopefully helps one fully appreciate the assumptions that will be made when using them and how these processes can be used to help decipher the variation of the β.

The Gaussian Random Walk is a sequence of $i.i.d$ random variables where one realization at time t follows the distribution:

[2]The expectations of most online sources such as "Investing.com" or "DailyFX" are very similar. The aggregation methods are not disclosed to the public.

Table 2: Results of regressions tests - HAC standard errors

News.Event	M5.Coefficient	std.error	HAC.std.error
Single News		**NA**	**NA**
UK CPI	12.681***	1.729	2.601
CA CPI	-12.896***	2.254	2.673
CA CRS	-13.773***	2.112	2.871
US CPI	3.968**	1.207	1.230
NZ CPI	24.109***	2.910	4.727
AU CPI	22.293***	4.145	4.226
AU RET	9.647***	1.215	2.656
UK RET	16.574***	1.968	2.572
Grouped News		**NA**	**NA**
US AHE	10.295***	2.577	2.665
US NFP	17.938***	2.572	4.143
US URÂ°	1.975	2.579	2.198
CA EMC	-25.588***	3.940	4.211
CA UR	1.053	3.940	4.262
AU EMC	21.59571***	1.842	2.780
AU UR	-11.93102***	1.842	1.828

$$X_t \sim \mathcal{N}(\mu_t, \sigma_t^2) \tag{4}$$

Using the independence assumption of normally distributed random variables, their sum and the distribution of their sum is then given by 2

$$U = \sum_{t=1}^{T} X_t \quad U \sim \mathcal{N}(\mu_t T, \sigma_t^2 T) \tag{5}$$

In theory the path that the β could take could resemble any of the paths in Figure 1. The process depicted by the Y2 is a baseline case where the parameter would not vary in time. The Y3 process is a random walk with a μ of 0 and a fixed σ which is in reality the same as white noise. The Y1 is a random walk with a constant negative μ. Finally, the Y4 process depicts a scenario with one significant break in the path of β and is the combination of 2 separate random walks with separate constant μ and σ.

4 Testing for instability of the news impact parameter

4.1 The quasi-Local-Level Test

There exists many ways to test whether β_t is time dependent or not. We first choose the methodology of the authors of (Elliott and Müller 2006) and briefly replicate their method. The advantage of their test is that it identifies instability no matter whether it comes in the form of a single break, many breaks, or a continuous change (all of which are feasible in our context).

The quasi-Local-Level (qLL) test enables one to test for many different types of persistent processes of the β_t. It is explained that many of these breaking processes can have a “temporary memory” (strictly speaking are strongly mixing) but will be well approximated by a Wiener process.[3]. This is extremely practical in our scenario as there are many possiblities for the possible variation of the β_t. The Null Hypothesis implies there

[3] Theorem 7.30 of (White 2001) can be applied since certain assumptions are made about the process

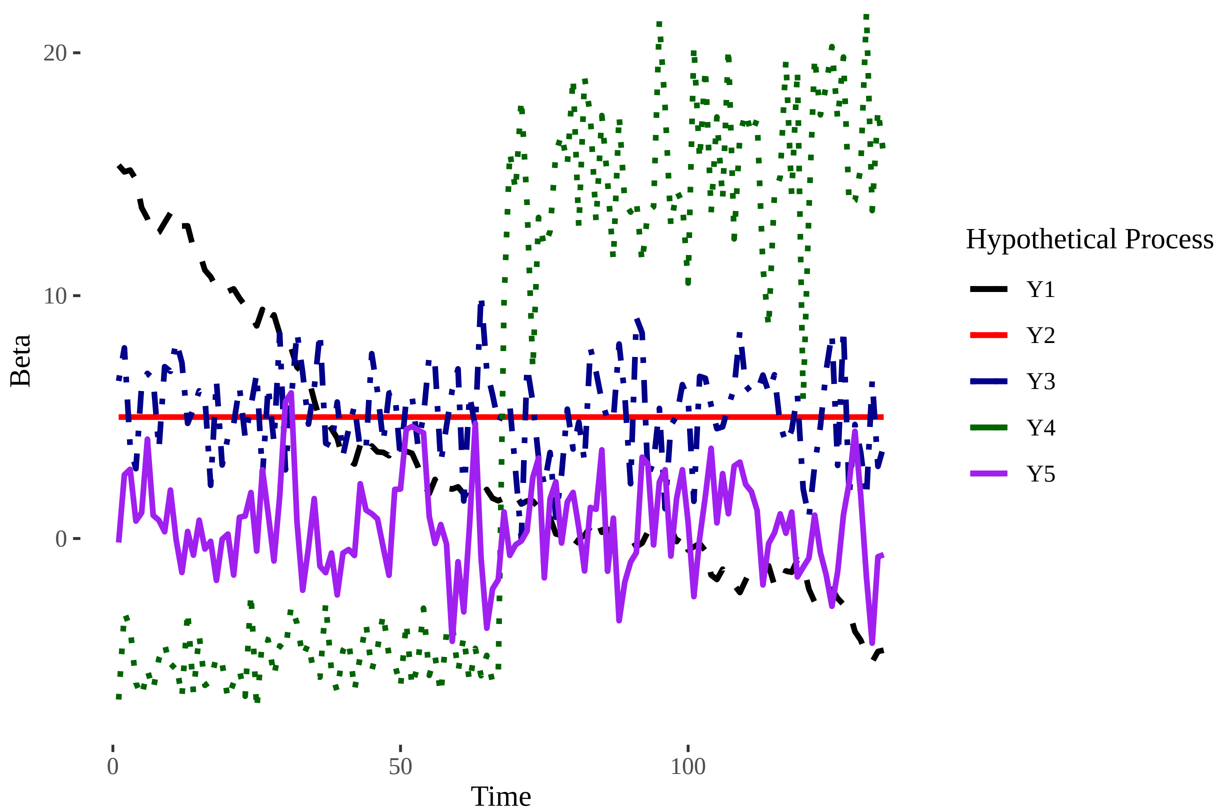


Figure 1: Hypothetical Paths of the Beta

Table 3: Instability Test Results

News.Event	qLL	CUSUM	CUSUM.sq
Single News			
UK CPI	-12.964***	***	**
CA CPI	-18.387***	n.s	***
CA CRS	-14.445***	***	***
US CPI	-21.582***	***	***
NZ CPI	-7.267*	**	**
AU CPI	-9.022**	n.s	***
AU RET	-17.623***	***	n.s
UK RET	-4.503	n.s	***
Grouped News			
US Batch			
Test 1 All News	-28.14***	***	***
Test 2 AHE&NFP	-24.015***		
Test 3 NFP&UR	-20.326***		
Test 4 AHE&UR	-7.078		
Test 5 NFP	-17.643***		
CA Batch			
Test 1 All News	-9.30	**	***
AUD Batch			
Test 1 All News	-22.912***	***	**
Test 2 EMC	-6.85		
Test 3 UR	-5.66		

Table 4: Asymptotic Critical Values of the qLL Statistic

k	1	2	3	4	5
1%	-11.05	-17.57	-23.42	-29.18	-35.09
5%	-8.36	-14.32	-19.84	-25.28	-30.60
10%	-7.14	-12.80	-18.07	-23.37	-28.55

is a stable parameter as in a familiar OLS regression. We obtain the likelihood under the Null assuming that the R_t observations are independently and identically distributed (and therefore so are their first differences):

$$L_{H0}(\beta_0, \beta_1, \sigma^2) = log \prod_{t=1}^{T} p(\Delta R_t | S_t; \beta_0, \beta_1, \sigma^2) \tag{6}$$

$$= -\frac{T}{2} log(2\pi) - T log(\sigma) - \frac{1}{2\sigma^2} \sum_{t=1}^{T} (\Delta R_t - (\Delta\beta_0 + \Delta\beta_1 S_t))^2 \tag{7}$$

Only the last term of (5) is kept as the first constants will cancel out. $\Delta\beta_0 + \Delta\beta_1 S_t$ becomes 0 as the terms do not change with time.

$$L_{H0} = \frac{1}{2\sigma^2} \sum_{t=1}^{T} (\Delta R_t)^2 \tag{8}$$

This contrasts with the alternative where instability is implied. We assume $\beta_t - \beta_0$ is approximated by the Gaussian random walk and ΔR_t is therefore a Gaussian moving average of order 1 MA(1) with the specification: $\Delta R_t \sim \eta_t + \psi_\eta \eta_{t-1}$, $\eta_t \sim iidN(0, \sigma_\eta^2)$, constant $\psi_\eta < 1$. Using the same *i.i.d* assumption we obtain the likelihood of this alternative process:

$$L_{HA} = \frac{1}{2\sigma^2} \sum_{t=1}^{T} \eta^2 \tag{9}$$

The qLL statistic is obtained by subtracting L_{HA} from L_{H0}: $\frac{\sigma^2}{\sigma^2} \sum_{t=1}^{T} \eta^2 - \sum_{t=1}^{T} (\Delta R_t)^2$. The test is therefore a variant of the Likelihood Ratio Test (LR_T), so while it does not follow a chi-square distribution exactly, it does follow a certain related distribution that has its percentiles defined by Elliott and Müller (2006) and reported in their table, reproduced here as Table 4. The general extension to the LR_T can be made where we can reject the model related to the Null Hypothesis (the stable model) if the critical value is sufficiently negative. To obtain the η term it is necessary to follow additional matrix algebra and use the result of regressions (Elliott and Müller 2006). The appropriate steps are available in Appendix Section 9.2.

4.2 CUSUM and CUSUM-squared tests

Separate tests to the one explained previously are considered. We explore the ones elaborated in Brown, Durbin, and Evans (1975) named the CUSUM and CUSUM-squared. These tests use the successive error terms of predictions of a standard Recursive-Least-Squares (RLS) model that assumes stability of the β parameter (Young 2011). In this specific application, we use the result of the standard OLS as a baseline *prior* or initial value for the algorithm (ie. β_0) and we use an empirical $\hat{\sigma}_\varepsilon$ residual error that is based on the entire sample. By examining the prediction errors, one can observe whether or not they violate the $N(0, \sigma^2)$ assumptions. Namely that they are a 1.) zero mean sequence $E(\varepsilon_t) = 0$ and 2.) that they are serially uncorrelated $E(\varepsilon_t \varepsilon_j) = 0$ when $t \neq j$. The first step consists in running a standard RLS algorithm and obtaining one-step-ahead errors from it. These errors are obtained as follows:

$$u_t = R_t - \hat{R}_t | R_{t-1} \tag{10}$$

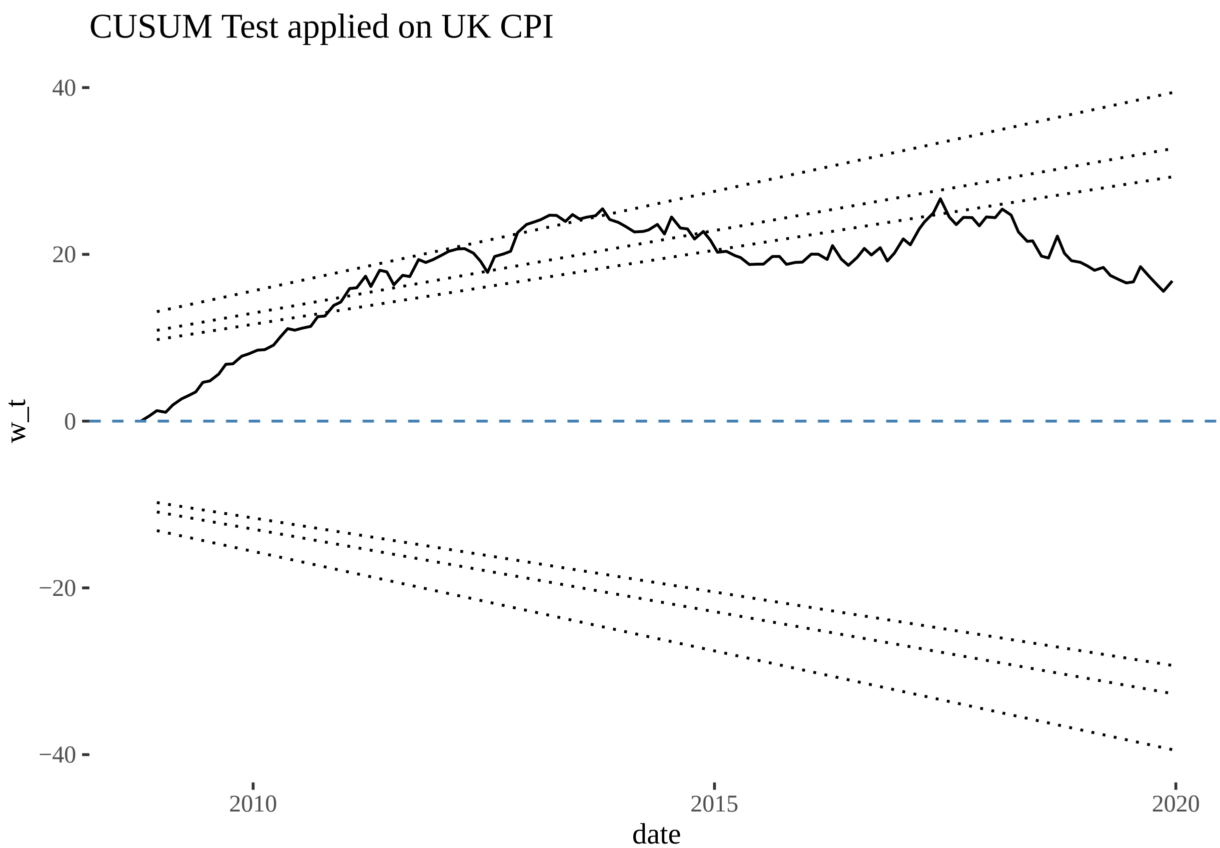
Effectively the realized price shock minus the predicted price shock at each observation. A transformation of these errors enables us to obtain a homoscedastic series.

$$u_{n,t} = \frac{u_t}{(1 + S_t^T P_t S_t)^{0.5}} \tag{11}$$

Where P_t is the covariance (or variance if there is only one news element at a time) of the β_t. A new series of summed up and standardized for use in the CUSUM test.

$$W_t = \frac{1}{\hat{\sigma}_{cs}} \sum_{i=k+1}^{t} u_{n,i} \tag{12}$$

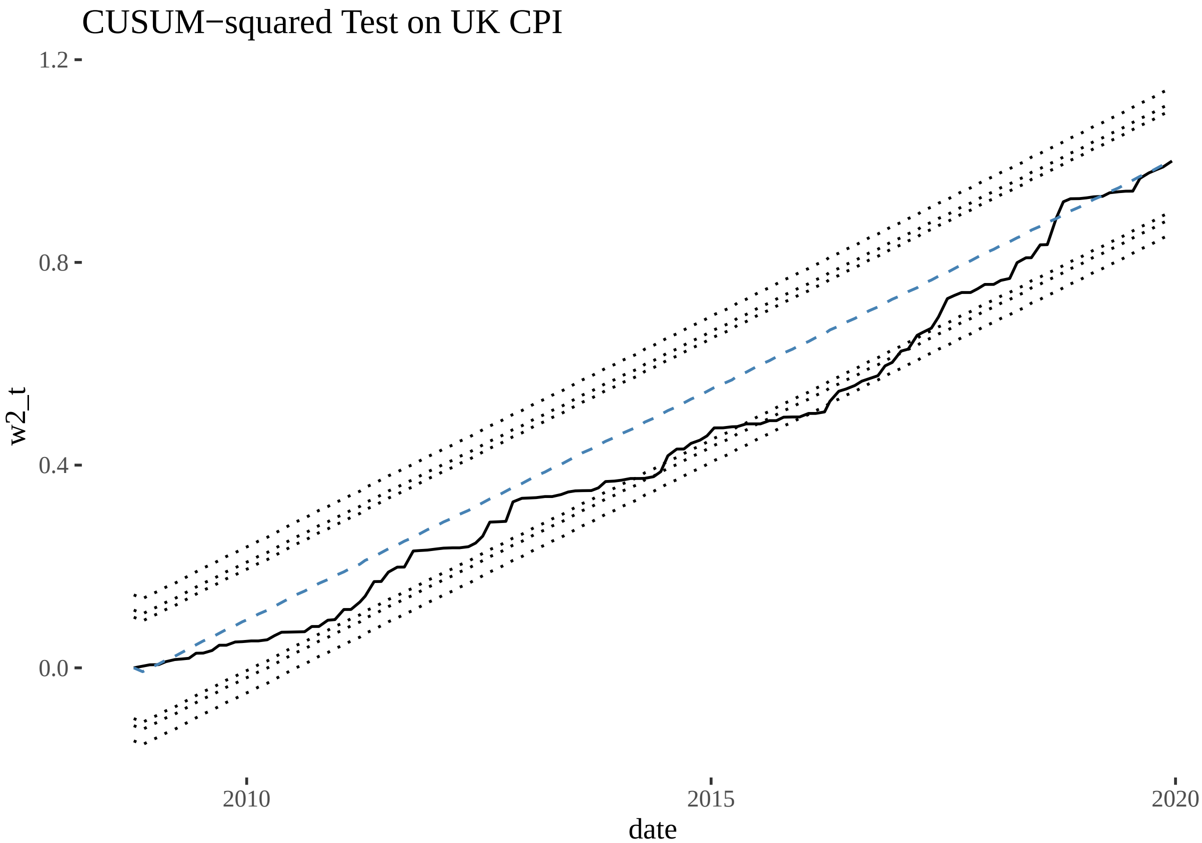
In theory, these successively compounded errors should not stray too far from the zero-line if the true $beta_t$ is constant. We also use the confidence bands suggested by Brown, Durbin, and Evans (1975). They are constructed by constructing pairs of lines starting at time k: $\pm a(T-k)^{0.5}$ and ending at time T: $\pm 3a(T-k)^{0.5}$



The second test, the CUSUM-squared can be run by once more by creating a new series, similar to the W_t from earlier. Here we consider:

$$V_t = \frac{\sum_{i=k+1}^{t} u_{n,i}}{\sum_{i=k+1}^{T} u_{n,i}} \tag{13}$$

With a Null-Hypothesis of a constant parameter(s), these cumulative sum of squares should follow a beta distribution and its mean should be $(k-h)/(N-h)$. As significance levels, we use the bounds set as: $\pm c_o + (k-h)/(N-h)$. The values of c_0 depend on the sample size. Our sample sizes are typically larger than the maximum value given in Brown, Durbin, and Evans (1975). For those cases where



5 Parameter Path Estimations

Having established that there is instability over time in the market reactions to at least some of the news, we take on the task of obtaining a time series that conveys the change over the timeframe considered. Specifically, we would like to obtain a visual representation of the change of the parameter such that a researcher or investor is able to easily gather information from.

5.1 Weighted Average Risk Minimization

Conveniently, there is a natural extension of the qLL test elaborated in Section 4.1. It provides a heuristic means means to approximate a parameter path (Müller and Petalas 2010).

First, we consider our stable model which is analogous to the linear regression model that is presented in the Equation (2). The resulting $\beta_{MLE/OLS}$ parameter obtained either through maximum likelihood estimation or minimization of ordinary least squares (depicted as the dot-dashed line in Figure 2) is representative of a scenario where it is assumed there is no time-related evolution of market reaction to news. As seen before, the likelihood in this case can be developed as Equation (6) or also written more generally as $\sum_{t=1}^{T} l_t(\theta)$ with θ containing the constant parameters.

We then turn to the case of varying β_t. Here, the general likelihood is the same but with time varying parameter(s) contained in $\theta_t = \theta + \delta_t$ in $t = 1, ..., T$. The δ_t can be imagined as the vertical distances in Figure 2) between the constant β case and the *true* parameter path at time t that is unknown and that we are attempting to approximate. The main argument for the method is that this general likelihood function can be approximated by a second-order Taylor expansion of the likelihood function around β_{MLE} and as a result an approximate estimate of the δ_t term mentioned earlier is obtainable. Effectively, the approximation of the log-likelihood function of the parameter path is re-arranged until a log-likelihood function of a gaussian random variable is recognizable and results in a "pseudo model" as:

$$\beta_{MLE} = \beta_t + T^{-1/2}\hat{H}^{-1}v_0 \tag{14}$$

$$s_t(\beta) = \hat{H}\delta_t + v_t, t = 1, ..., T \tag{15}$$

$\hat{H}$ is the Hessian of the Taylor approximation divided by the sample size T, s_t is the score function for the stable model. The exact derivations to obtain (15) are elaborated in Müller and Petalas (2010) and in Section 9.3.

Essentially, the score function s_t and Hessian $\hat{H}$ can be extracted from the stable linear regression from section Section 2 and when the likelihood function is therefore the one seen in equation (7).

$$s_t = \frac{dl}{d\beta_1} = -\sigma^{-2}\sum_{t=1}^{T}(R_t - \hat{\beta}_0 - \hat{\beta}_1 S_t)S_t \tag{16}$$

$$\hat{H} = \frac{dl^2}{d^2\beta_1^T} = -\sigma^{-2}\sum_{t=1}^{T} S_t^2 \tag{17}$$

The weights in step (e) of the Appendix Section 9.3 depend on the sample size and the qLL statistic that are obtained using the qLL test statistic from Section 4.1. It can be observed that the more negative the statistic becomes for the associated random walk function, the more importance is placed on that particular weight. The random walk that contributes the most weight for the UK CPI case is the 4th one in our example.

5.2 Standard Recursive Time Variable Parameter Algorithm (STVP)

Until now, the WAR minimization method provided a means to analyze the price reactions to the news over an entire sample period. In practice, it may be more useful to have a self-updating method that iteratively uses every new observation to improve the estimation progressively. It can also be argued that more importance can be placed on gauging the shock that could occur today rather than conducting an analysis of reactions years ago. This is why this particular method is also included. Starting once more from the baseline case introduced in section Section 2, a time-dependent measure of market reaction to news β_t can be constructed by 1.) Transforming the problem of Ordinary Least Squares so that it can be recursively solved and 2.) Allowing for the parameter(s) to change over time sequentially.

To obtain the STVP algorithm presented in Section 9.3 the first step consists of replacing the OLS equations by their recursive counterparts and achieve the intermediate SLRS algorithm. The following equations are similar to the familiar OLS equations setup.

$$R_t = S_t^T\beta_1 + e_t \tag{18}$$

The error vector contains a series of random variables e_t that are assumed to be normally distributed with 0 mean and serially uncorrelated. We omit the constant intercept; the reasoning is elaborated in Section 9.3.

$$\hat{\beta}_{OLS} = (S^TS)^{-1}S^TR \tag{19}$$

$$\beta_t^{OLS} = \beta_{t-1}^{OLS}\forall t \tag{20}$$

When an observation y_j is added to the samples $i = 1, ..., N$, an update to the $\hat{\beta}$ is the same as re-estimating the entire sample with the new observation. Effectively, the "intermediate model" or Stochastic Recursive

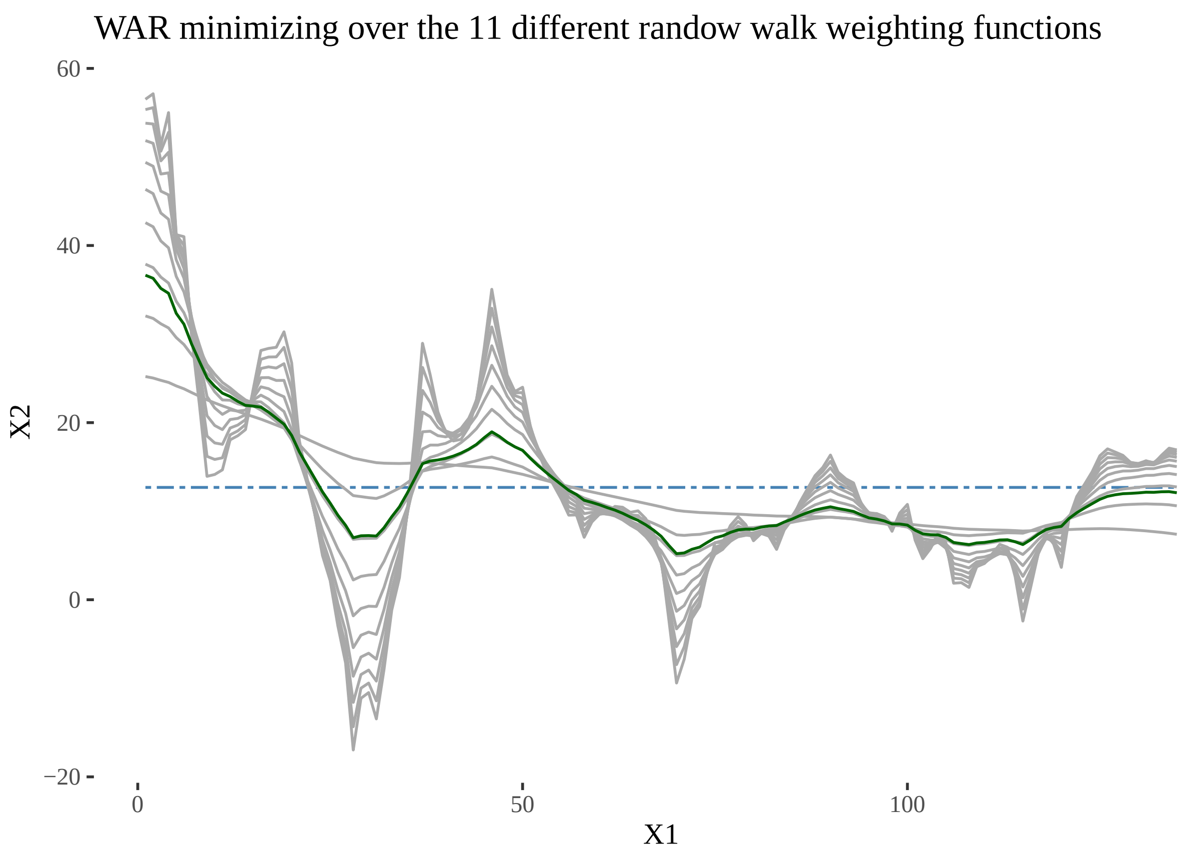


Figure 2: WAR minimizing over the 11 different randow walk weighting functions

Least Squares Algorithm (SLRS) is thereafter obtained (the exact step-by-step is in Section 9.3). While this is a powerful tool, it does not necessarily help estimate y_j effectively. It is giving just as much weight to realizations of y early in the sample than those that immediately precede the new extra observation j. The contribution of every marginal observation is smaller and smaller as the sample grows. The $\hat{\beta}$ converges to the full sample parameter estimator $\hat{\beta}_{OLS}$ as seen in Figure 3.

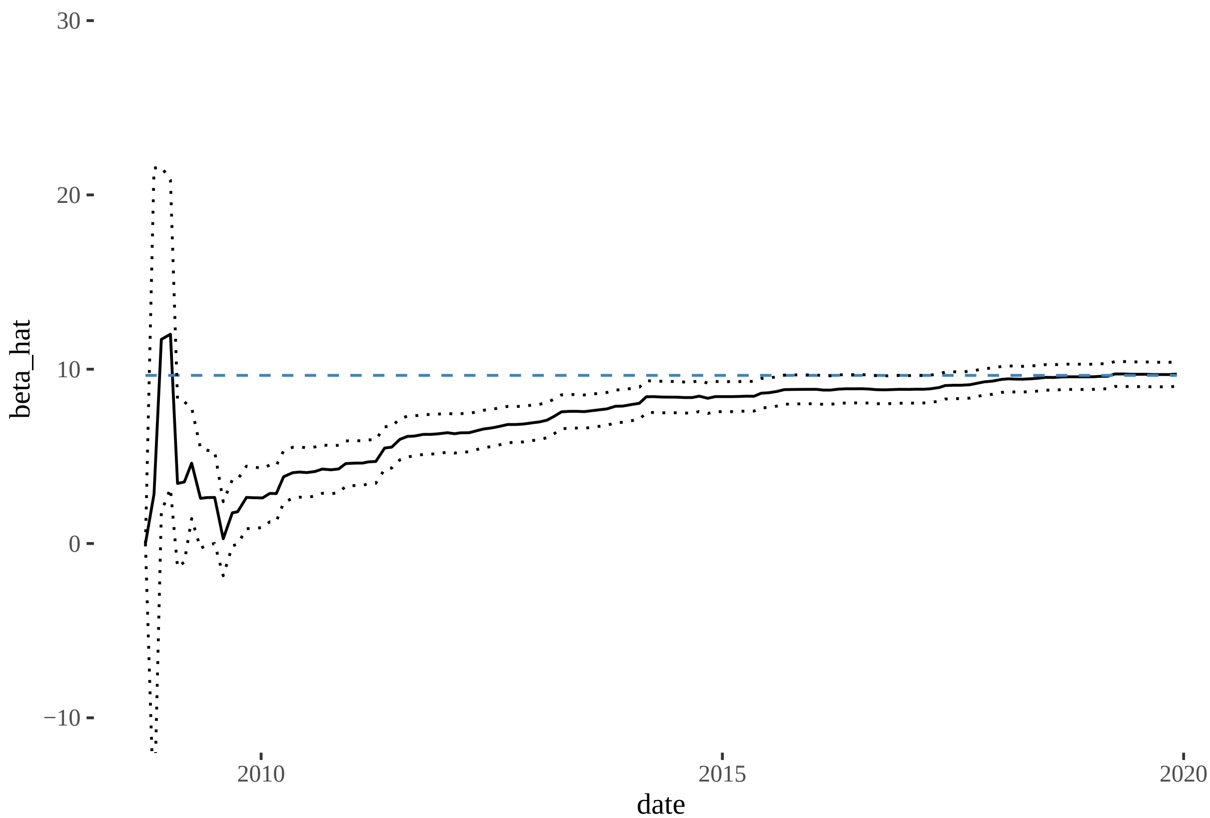


Figure 3: SLRS applied to the UK CPI

Additional data points in the aformentioned methodology add information to the model and improve the accuracy of the $\hat{\beta}$ estimator but it is always assumed that there is only one true β regardless of time t. In order to attenuate this assumption and include the flexibility that allows for a time dependence in a β_t, we introduce the random walk disturbance term mentioned in Section 2. The error term is now characterized by the incremental steps of a random walk. In bayesian terms, the initial estimation of β_t using the random walk becomes the *prior*.

$$R_t = S_t^T \beta_t + e_t \quad e_t \sim \mathcal{N}(0, \sigma^2) \tag{21}$$

$$\hat{\beta}_t = \breve{\beta}_t | \hat{\beta}_{t-1} \tag{22}$$

$$\beta_t = \beta_{t-1} + \eta_{t-1} \quad \eta_t \sim \mathcal{N}(0, \sigma_\beta^2) \tag{23}$$

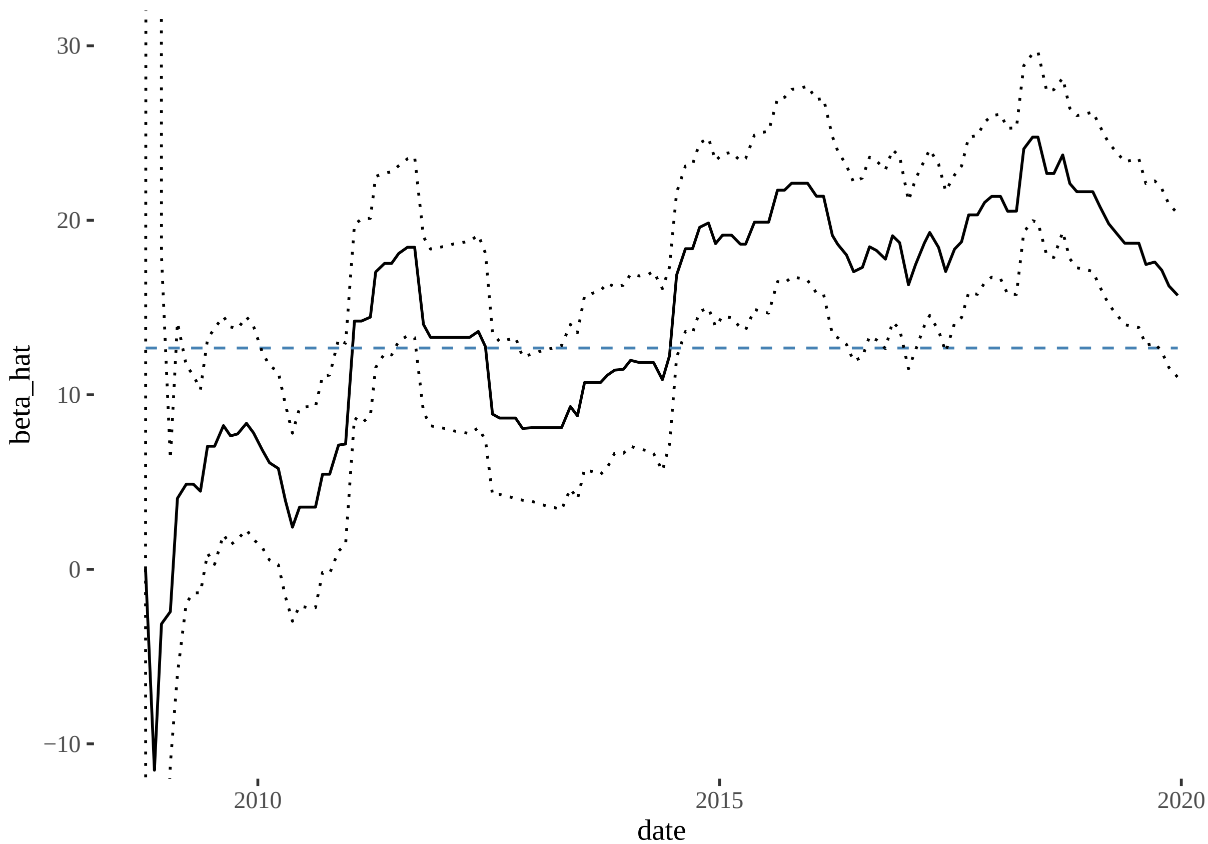


Figure 4: STVP applied to the UK CPI

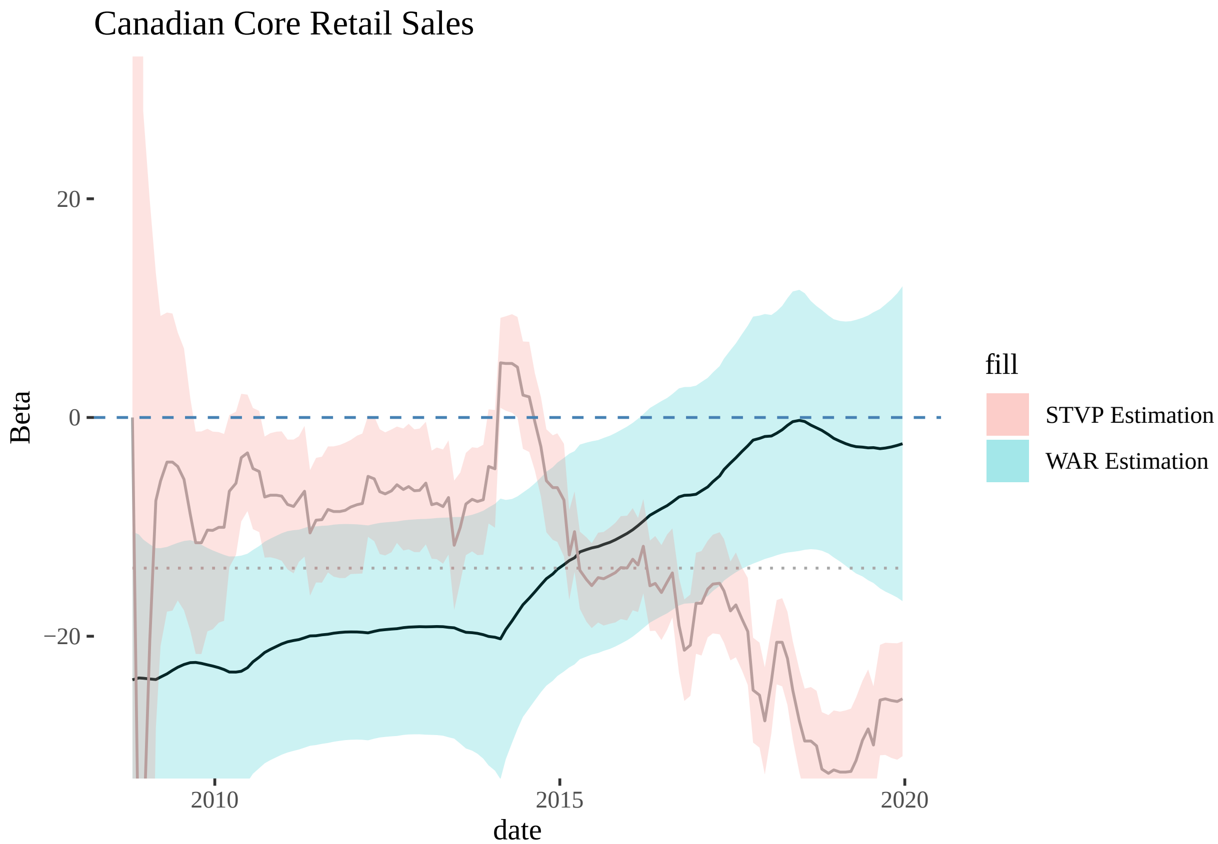
6 Discussion - A comparison of methods

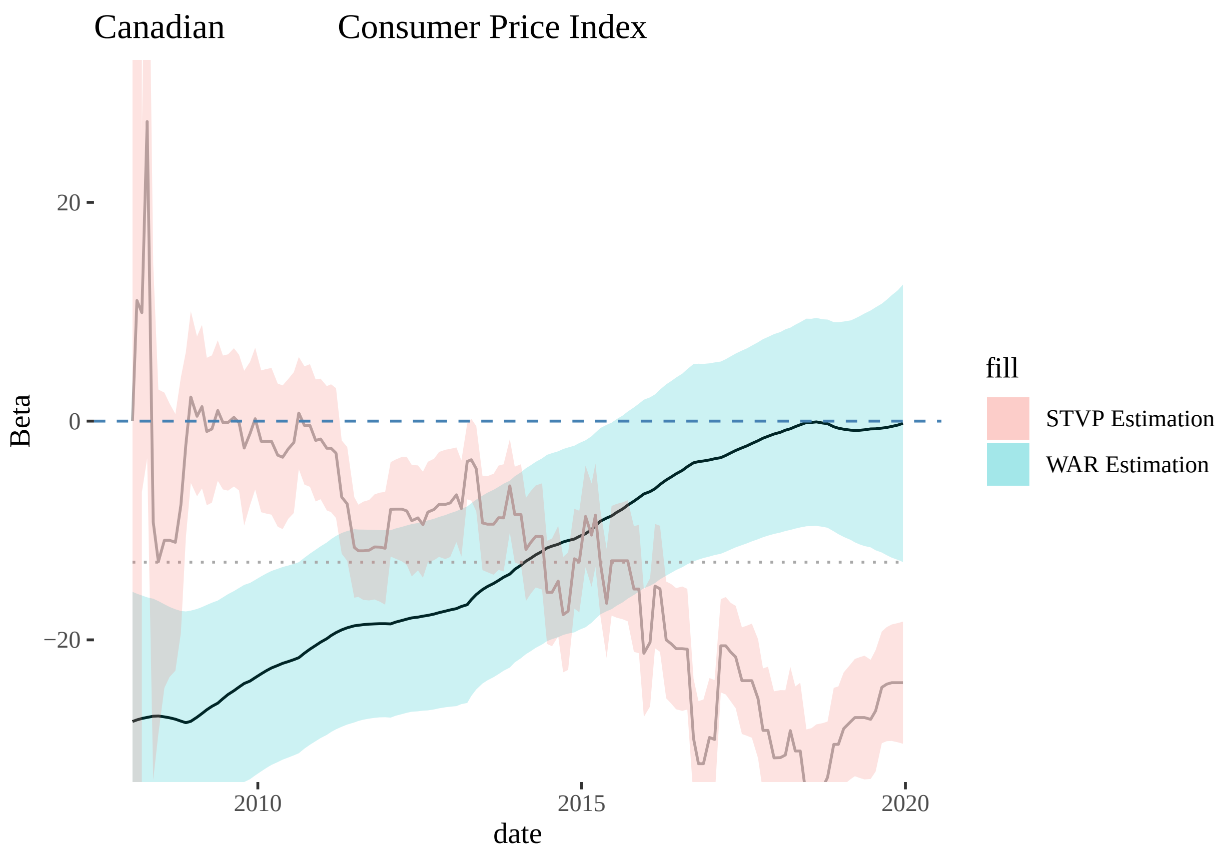
7 Conclusion

8 Appendix

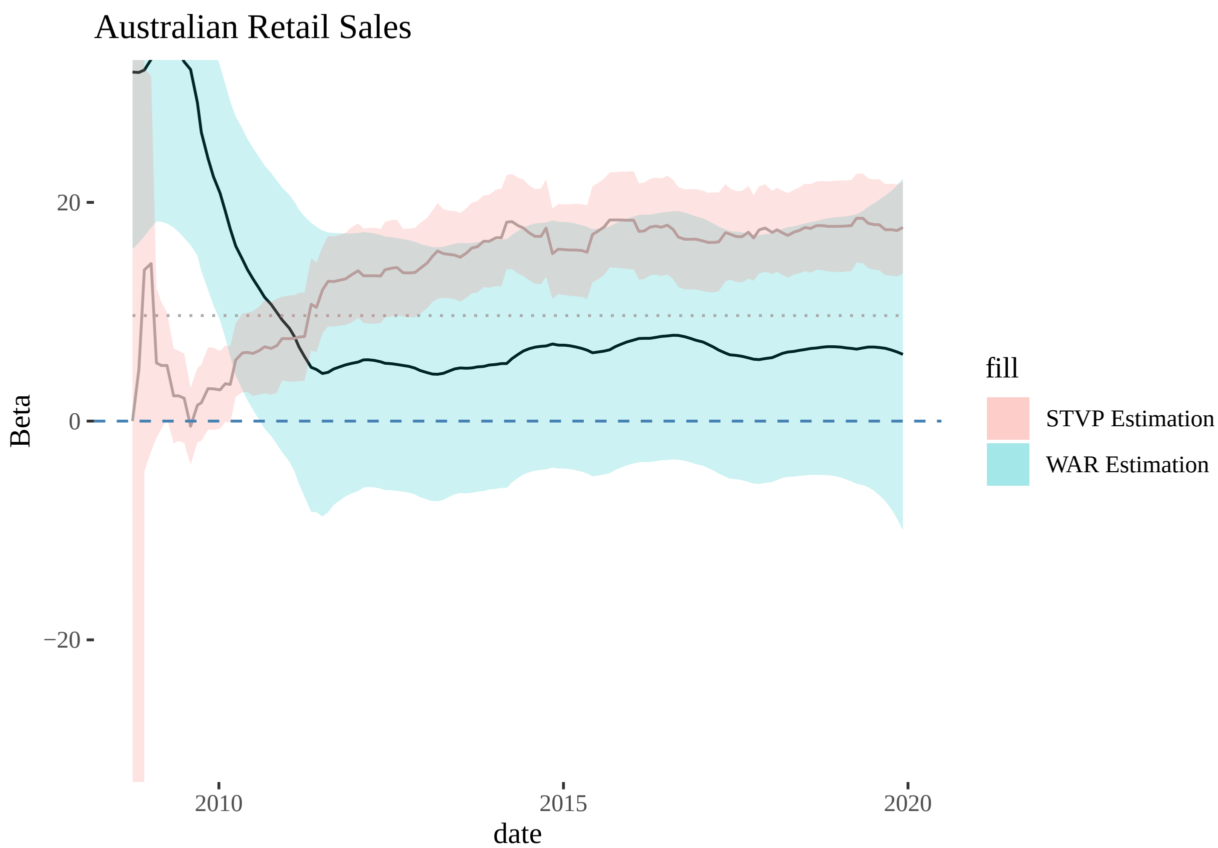
8.1 Parameter Paths

The following path estimations should be interpreted as follows: a one standard deviation unanticipated positive change in the economic variable S_t results in an appreciation or depreciation in the exchange rate of the relevant currency pair by R_t amount of pips or basis points.





Canadian
Consumer Price Index
20
0
−20
Beta
2010
2015
2020
date
fill
STVP Estimation
WAR Estimation



Australian Retail Sales
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0
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Beta
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2020
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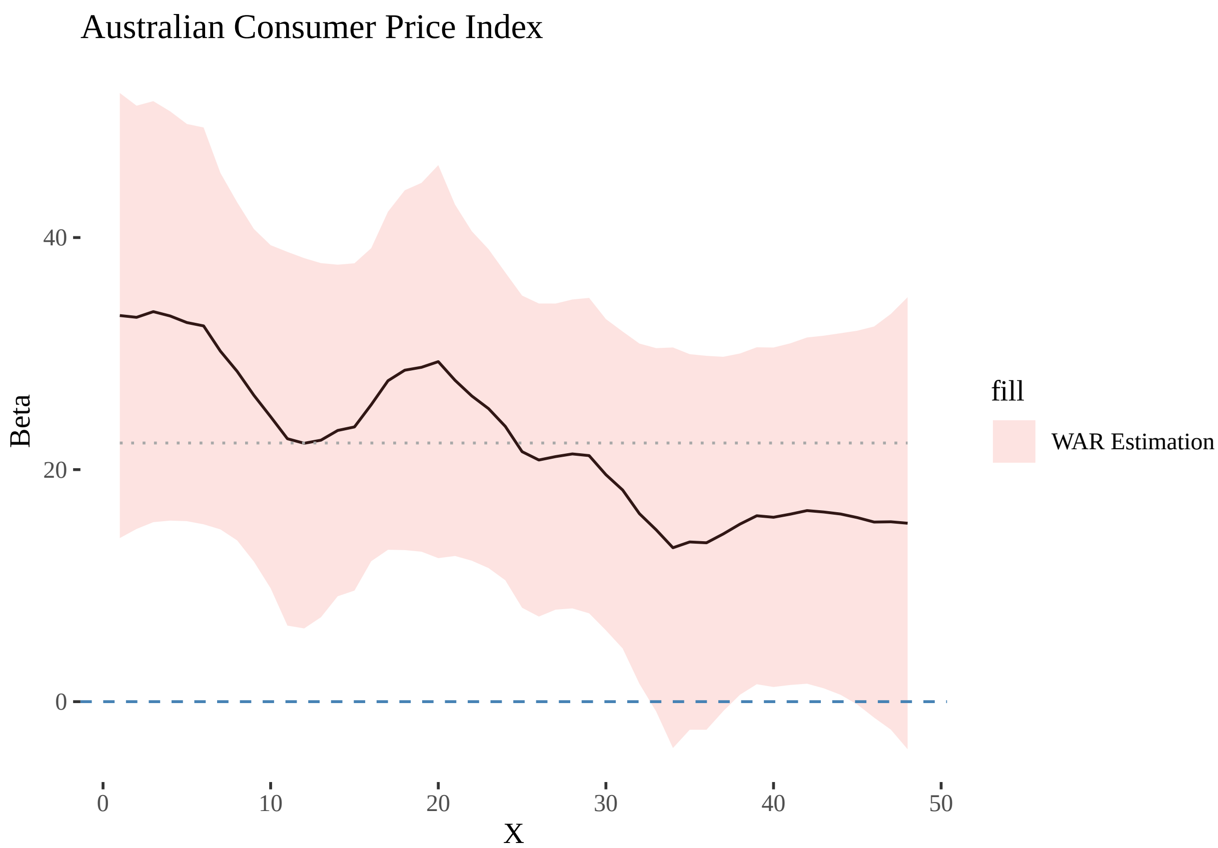
STVP Estimation
WAR Estimation


Australian Consumer Price Index
Beta
40
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0
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30
40
50
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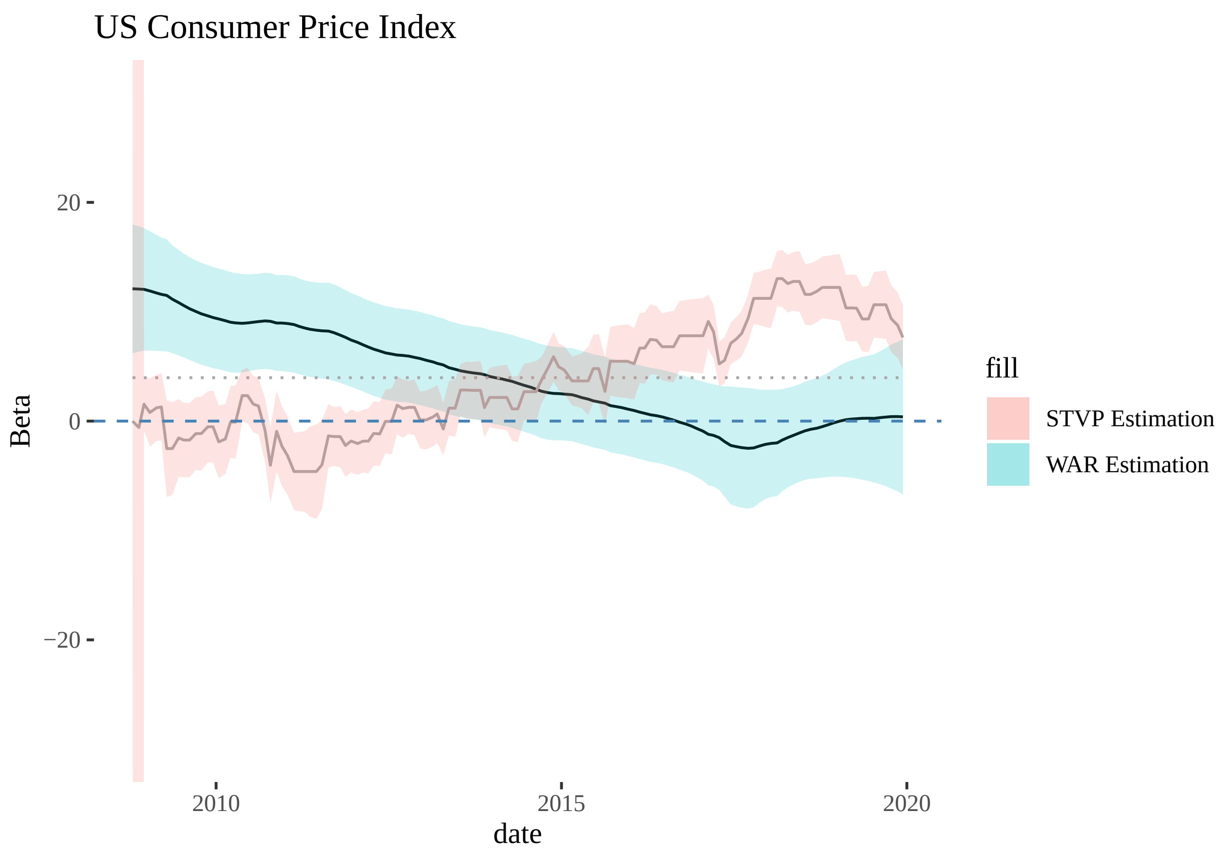
WAR Estimation


US Consumer Price Index
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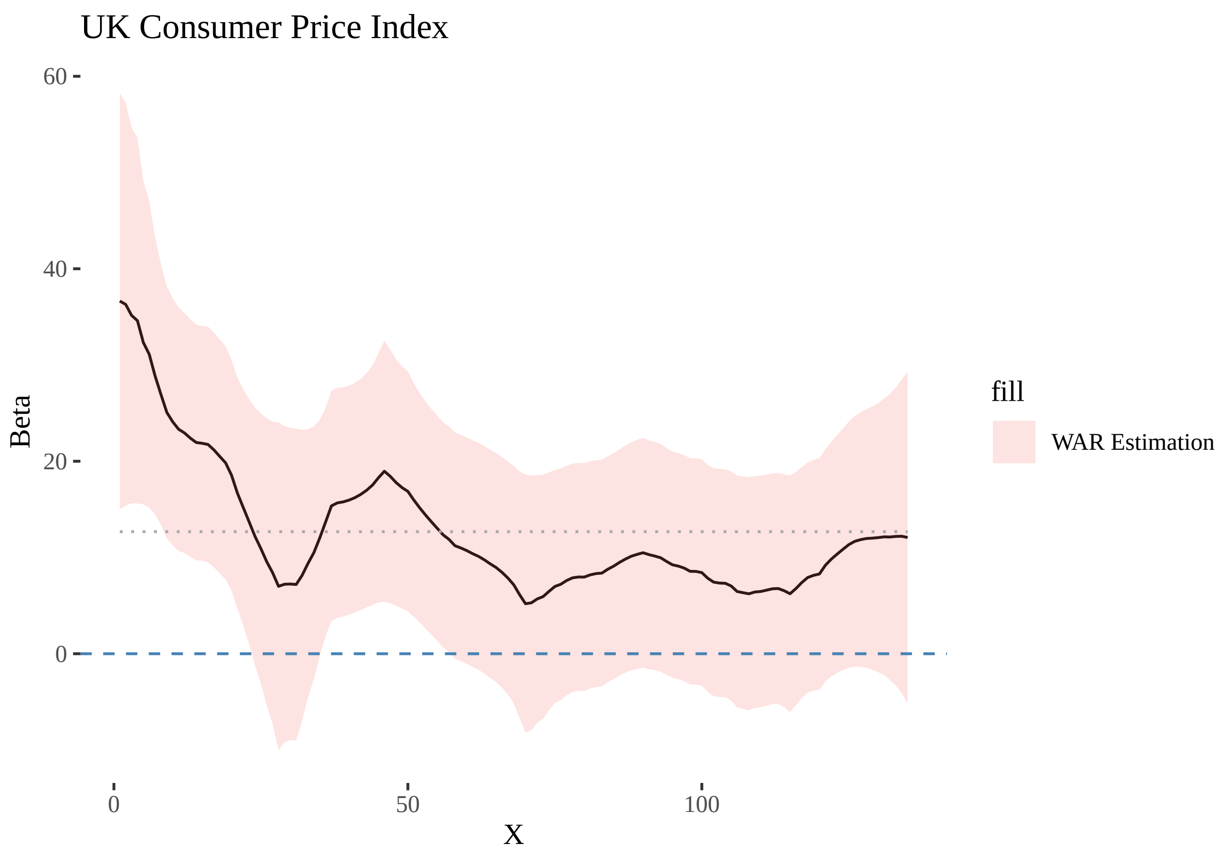
STVP Estimation
WAR Estimation


UK Consumer Price Index
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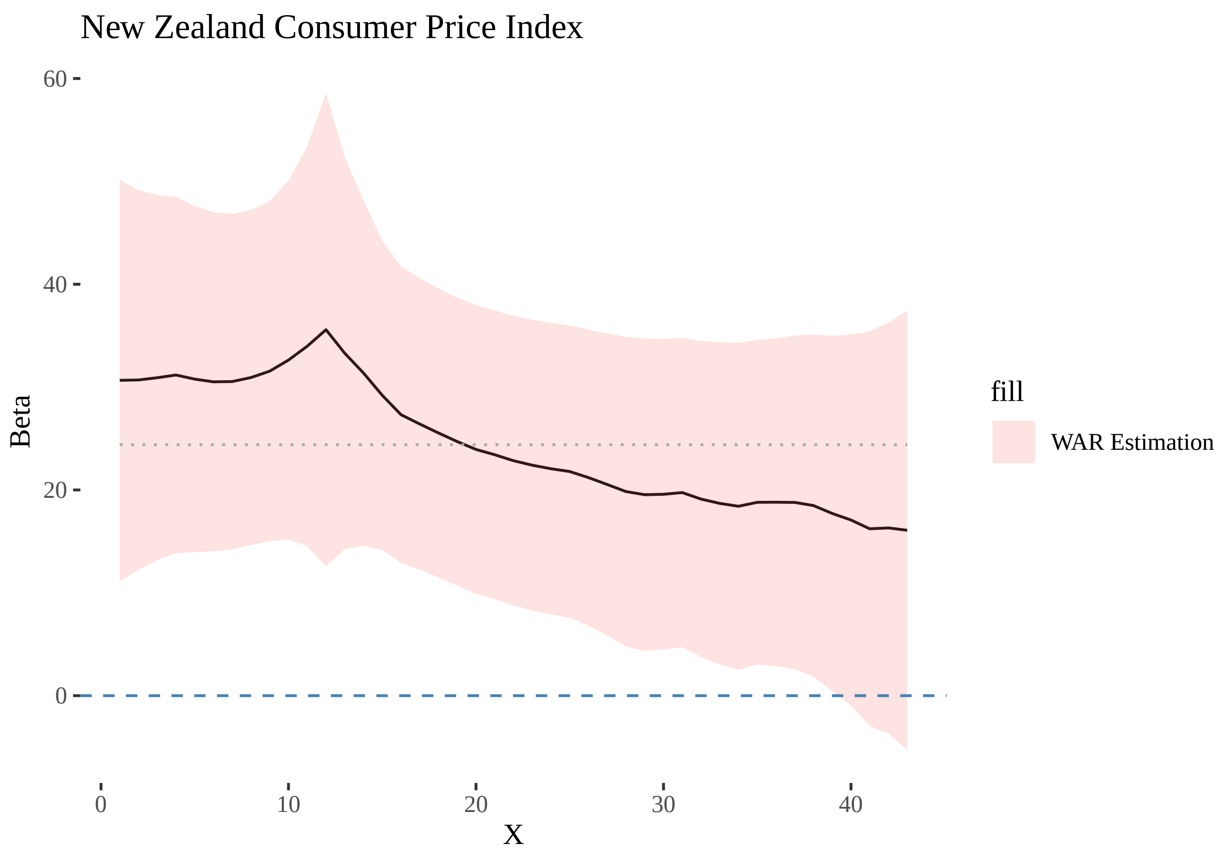
40
20
0
Beta
0
50
100
X
fill
WAR Estimation



8.2 Stochastic Recursive Least Squares Algorithm (SLRS)

1. $\hat{\alpha}_t = \hat{\alpha}_{t-1} + g_t(R_t - S_t^T \hat{\alpha}_{t-1})$
2. $g_t = P_{t-1}^* S_t(\hat{\sigma}^2 + S_t^T P_{t-1}^* S_t)^{-1}$
3. $P_t^* = P_{t-1}^* - g_t S_t^T P_{t-1}^*$

8.3 Standard Recursive Time Variable Parameter Algorithm (STVP)

Prediction (Prior)
1. $\hat{\alpha}_t | \hat{\alpha}_{t-1} = A\alpha_{t-1}$
2. $P_t^* | P_{t-1}^* = AP_{t-1}^* A^T + DQ_a D^T$

Correction (Posterior)
3. $\hat{\alpha}_t = \hat{\alpha}_t | \hat{\alpha}_{t-1} + g_t(R_t - S_t^T(\hat{\alpha}_t | \hat{\alpha}_{t-1}))$
4. $g_t = (P_t^* | P_{t-1}^*) S_t(\hat{\sigma}^2 + S_t^T(P_t^* | P_{t-1}^*)$ 5. $P_t^* = P_t^* | P_{t-1}^* - g_t S_t^T(P_t^* | P_{t-1}^*)$

8.4 Steps to obtain qLL statistic

1. Compute the OLS residuals $\hat{\varepsilon}_t$ by regressing R_t on S_t, Z_t;

2. Construct a consistent estimator $\hat{V}_X$ of the $k * k$ long-run covariance matrix of $S_t \varepsilon_t$. When ε_t can be assumed uncorrelated, a natural choice is the heteroscedasticity robust estimator $\hat{V}_X = T^{-1} \sum_{t=1}^{T} X_t X_t' \varepsilon_t^2$

3. Compute $\hat{U}_t = \hat{V}_X^{-1/2} X_t \hat{\varepsilon}_t$ and denote the k elements of $\hat{U}_t$ by $\hat{U}_{t,i}$, $i = 1, ..., k$.

4. For each series $\hat{U}_{t,i}$, compute a new series, $\hat{w}_{t,i}$ via $w_{t,i} = \bar{r}\hat{w}_{t-1,i} + \Delta \hat{U}_{t,i}$, and $\hat{w}_{1,i} = \hat{U}_{1,i}$, where $\bar{r} = 1 - 10/T$.

5. Compute the squared residuals from OLS regressions of $\hat{w}_{t,i}$ on $\bar{r}^t$ individually, and sum all of those over $i = 1, ..., k$.

6. Multiply this sum of sum of squared residuals by $\bar{r}$, and subtract $\sum_{i=1}^{k} \sum_{t=1}^{T} (\hat{U}_{t,i})^2$

8.5 Steps to obtain WAR minimization path

1. For t = 1,...,T, let a_t and b_t be the first p elements of $\hat{H}^{-1} s_t(\hat{\theta})$ and $\hat{H}\hat{V}^{-1} s_t(\hat{\theta})$ respectively.

2. For $c_i \in C = 0, 5, 10, ..., 50, i = 1, ..., 11$ compute

(a) $r_i = 1 - c_i/T$, $z_{i,1} = x_1$ and $z_{i,t} = r_i z_{i,t-1} + x_t - x_{t-1}, t = 2, ..., T$;

(b) the residuals $\{\tilde{z}_{i,t}\}_{t=1}^{T}$ of a linear regression of $\{z_{i,t}\}_{t=1}^{T}$ on $\{r_i^{t-1} I_p\}_{t=1}^{T}$

(c) $\bar{z}_{i,T} = \tilde{z}_{i,T}$, and $\bar{z}_{i,t} = r_i \bar{z}_{i,t+1} + \tilde{z}_{i,t} - \tilde{z}_{i,t+1}, t = 1, ..., T - 1$;

(d) $\{\hat{\beta}_{i,t}\}_{t=1}^{T} = \{\hat{\theta}\ + a_t - r_i \bar{z}_{i,t}\}_{t=1}^{T}$;

(e) $qLL(c_i) = \sum_{t=1}^{T} (r_i) \bar{z}_{i,t} - a_t)' \tilde{b}_t$ and $\tilde{w}_i = \sqrt{T(1 - r_i^2) r_i^{T-1} / (1 - r_i^{2T})} e^{-\frac{1}{2} qLL(c_i)}$ (set $\tilde{w}_0 = 1$)

3. Compute $w_i = \tilde{w}_i / \sum_{j=1}^{11} \tilde{w}_j$.

4. The parameter path estimator is given by $\{\hat{\beta}_t\}_{t=1}^{T} = \{\sum_{i=1}^{11} w_i \hat{\beta}_{i,t}\}_{t=1}^{T}$.

5. The statistic qLL(10) tests the null hypothesis of stability of β and rejects for small values. Critical values depend on p and are tabulated in Table 1 of Elliott and Müller (2006) and 4.

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