Time Variation of Regression Coefficients related to Macroeconomic News affecting Currency Prices

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Project Motivation and Context

Linear Regression

Testing Stability

Estimating Parameter Paths

Plotting the Paths

Questions / Comments

Project Motivation and Context

Figure 1: Minute-by-minute candlechart of the USD/CAD asset on the 19th of June between 15h00 and 16h00 GMT+2. An example of of the sudden price change that can occur during one of the news releases. Green/Red candles represent an increase/decrease in price. The gray indicator is amount of trading activity taking place (a rough guideline as it only includes transactions from that particular brokerage).

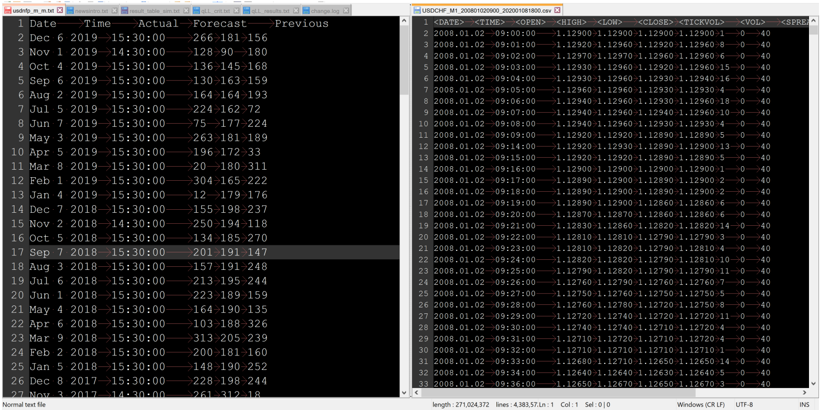
Table 1: Summary of the news figures considered in the study

Country	News.Event	Pair.used	GMT.Time	Frequency	Observations	Dates
Single News					.	
Canada	Consumer Price Index	USD/CAD	13:30	Monthly	103	Jan 2008 to Dec 2019
Canada	Core Retail Sales	USD/CAD	12:30	Monthly	94	Oct 2008 to Dec 2019
United States	Consumer Price Index	USD/CHF	13:30	Monthly	135	Oct 2008 to Dec 2019
New Zealand	Consumer Price Index	NZD/USD	21:45	Quarterly	43	Jan 2009 to Oct 2019
Australia	Consumer Price Index	AUD/USD	00:30	Quarterly	48	Jan 2008 to Oct 2019
Australia	Retail Sales	AUD/USD	00:30	Monthly	135	Oct 2008 to Dec 2019
United Kingdom	Consumer Price Index	GBP/USD	09:30	Monthly	135	Oct 2008 to Dec 2019
United Kingdom	Retail Sales	GBP/USD	09:30	Monthly	135	Oct 2008 to Dec 2019
Grouped News					.	
United States	Average Hourly Earnings Change	USD/CHF	13:30	Monthly	135	Oct 2008 to Dec 2019
United States	NonFarm Employment Change	USD/CHF	13:30	Monthly	135	Oct 2008 to Dec 2019
United States	Unemployment Rate	USD/CHF	13:30	Monthly	135	Oct 2008 to Dec 2019
Canada	Employment Change	USD/CAD	13:30	Monthly	135	Oct 2008 to Dec 2019
Canada	Unemployment Rate	USD/CAD	13:30	Monthly	135	Oct 2008 to Dec 2019
Australia	Employment Change	AUD/USD	00:30	Monthly	135	Oct 2008 to Dec 2019
Australia	Unemployment Rate	AUD/USD	00:30	Monthly	135	Oct 2008 to Dec 2019

Note: The Canadian Consumer Price Index and Core Retail Sales coincided in Date and Time 41 times and these observations were removed.

Data

Left: News figure data. Right: Currency pair price



Date	Time	Actual	Forecast	Previous
Dec 6 2019	15:30:00	266	181	156
Nov 1 2019	14:30:00	128	90	180
Oct 4 2019	15:30:00	136	145	168
Sep 6 2019	15:30:00	130	163	159
Aug 2 2019	15:30:00	164	164	193
Jul 5 2019	15:30:00	224	162	72
Jun 7 2019	15:30:00	75	177	224
May 3 2019	15:30:00	263	181	189
Apr 5 2019	15:30:00	196	172	33
Mar 8 2019	15:30:00	20	180	311
Feb 1 2019	15:30:00	304	165	222
Jan 4 2019	15:30:00	12	179	176
Dec 7 2018	15:30:00	155	198	237
Nov 2 2018	14:30:00	250	194	118
Oct 5 2018	15:30:00	134	185	270
Sep 7 2018	15:30:00	201	191	147
Aug 3 2018	15:30:00	157	191	248
Jul 6 2018	15:30:00	213	195	244
Jun 1 2018	15:30:00	223	189	159
May 4 2018	15:30:00	164	190	135
Apr 6 2018	15:30:00	103	188	326
Mar 9 2018	15:30:00	313	205	239
Feb 2 2018	15:30:00	200	181	160
Jan 5 2018	15:30:00	148	190	252
Dec 8 2017	15:30:00	228	198	244
Nov 3 2017	14:30:00	261	312	18

<DATE>	<TIME>	<OPEN>	<HIGH>	<LOW>	<CLOSE>	<TICKVOL>	<VOL>	<SPREA
2008.01.02	09:00:00	1.12900	1.12900	1.12900	1.12900	1	0	40
2008.01.02	09:01:00	1.12920	1.12960	1.12920	1.12960	8	0	40
2008.01.02	09:02:00	1.12970	1.12970	1.12960	1.12960	6	0	40
2008.01.02	09:03:00	1.12930	1.12960	1.12920	1.12960	15	0	40
2008.01.02	09:04:00	1.12930	1.12960	1.12930	1.12940	16	0	40
2008.01.02	09:05:00	1.12960	1.12960	1.12930	1.12930	4	0	40
2008.01.02	09:06:00	1.12940	1.12960	1.12930	1.12960	18	0	40
2008.01.02	09:07:00	1.12940	1.12960	1.12940	1.12960	10	0	40
2008.01.02	09:08:00	1.12940	1.12960	1.12930	1.12930	4	0	40
2008.01.02	09:09:00	1.12920	1.12920	1.12890	1.12890	5	0	40
2008.01.02	09:14:00	1.12920	1.12930	1.12890	1.12900	13	0	40
2008.01.02	09:15:00	1.12920	1.12920	1.12890	1.12890	5	0	40
2008.01.02	09:16:00	1.12900	1.12900	1.12900	1.12900	1	0	40
2008.01.02	09:17:00	1.12890	1.12900	1.12890	1.12900	2	0	40
2008.01.02	09:18:00	1.12890	1.12900	1.12890	1.12900	2	0	40
2008.01.02	09:19:00	1.12890	1.12900	1.12860	1.12860	6	0	40
2008.01.02	09:20:00	1.12870	1.12870	1.12860	1.12860	6	0	40
2008.01.02	09:21:00	1.12830	1.12860	1.12820	1.12820	14	0	40
2008.01.02	09:22:00	1.12810	1.12810	1.12790	1.12790	3	0	40
2008.01.02	09:23:00	1.12810	1.12820	1.12790	1.12810	4	0	40
2008.01.02	09:24:00	1.12820	1.12820	1.12790	1.12810	10	0	40
2008.01.02	09:25:00	1.12790	1.12820	1.12790	1.12790	11	0	40
2008.01.02	09:26:00	1.12760	1.12790	1.12760	1.12760	7	0	40
2008.01.02	09:27:00	1.12750	1.12760	1.12750	1.12750	5	0	40
2008.01.02	09:28:00	1.12760	1.12780	1.12720	1.12750	8	0	40
2008.01.02	09:29:00	1.12720	1.12740	1.12720	1.12720	11	0	40
2008.01.02	09:30:00	1.12740	1.12740	1.12710	1.12720	4	0	40
2008.01.02	09:31:00	1.12710	1.12720	1.12710	1.12720	4	0	40
2008.01.02	09:32:00	1.12710	1.12710	1.12710	1.12710	1	0	40
2008.01.02	09:33:00	1.12680	1.12710	1.12650	1.12650	14	0	40
2008.01.02	09:34:00	1.12640	1.12640	1.12630	1.12640	5	0	40
2008.01.02	09:36:00	1.12650	1.12670	1.12650	1.12670	3	0	40

Figure 2: Data observations of our study

Linear Regression

Variables

5 Minute Price Change

$$R_t = \beta_0 + \beta_1 S_t + \varepsilon_t \tag{1}$$

"Suprise" Component of macroeconomic news

$$S_t = \frac{A_t - E_t}{\sigma_d} \tag{2}$$

Results - OLS

News.Event	M5.Coefficient	std.error	HAC.std.error
Single News		.	.
UK CPI	12.681***	1.729	2.601
CA CPI	-8.6286***	1.958	2.502
CA CRS	-10.756***	1.624	2.249
US CPI	3.968**	1.207	1.23
NZ CPI	24.109***	2.910	4.727
AU CPI	22.293***	4.145	4.226
AU RET	9.647***	1.215	2.656
UK RET	16.574***	1.968	2.572
Grouped News		.	.
US AHE	10.295***	2.577	2.665
US NFP	17.938***	2.572	4.143
US URÂ°	1.975	2.579	2.198
CA EMC	-25.588***	3.940	4.211
CA UR	1.053	3.940	4.262
AU EMC	21.59571***	1.842	2.780
AU UR	-11.93102***	1.842	1.828

Note: The result of OLS estimation, referred to as the 'time invariant' or 'stable' case is presented in this table. The standard errors of the estimator as well as the Newey-West corrected standard errors are also included. The *,**,*** are for 10%, 5% and 1% significance levels respectively.

Testing Stability

Three Different Test

- qLL (Elliott and Müller 2006)
- Cusum (Brown, Durbin, and Evans 1975)
- Cusum-Squared (Brown, Durbin, and Evans 1975)

qLL Test

- Likelihood Function under the Null Hypothesis: Stable Assumption with OLS.

$$L_{H0} = -\frac{1}{2\sigma^2}\sum_{t=1}^{T}(\Delta R_t)^2 \tag{3}$$

- Likelihood Function under the Alternative Hypothesis: Unstable Assumption Moving Average order 1 $\Delta R_t \sim \eta_t + \psi_\eta \eta_{t-1}$

$$L_{HA} = -\frac{1}{2\sigma^2}\sum_{t=1}^{T}\eta^2 \tag{4}$$

- Obtain the Test Statistic

$$\frac{\sigma_\epsilon^2}{\sigma_\eta^2}\sum_{t=1}^{T}\eta^2 - \sum_{t=1}^{T}(\Delta R_t)^2 \tag{5}$$

Steps to obtain qLL statistic (Elliott and Müller 2006)

1. Compute the OLS residuals $\hat{\varepsilon}_t$ by regressing R_t on S_t, Z_t;
2. Construct a consistent estimator $\hat{V}_X$ of the $k * k$ long-run covariance matrix of $S_t\varepsilon_t$. When ε_t can be assumed uncorrelated, a natural choice is the heteroscedasticity robust estimator $\hat{V}_X = T^{-1}\sum_{t=1}^{T} X_t X_t' \varepsilon_t^2$
3. Compute $\hat{U}_t = \hat{V}_X^{-1/2} X_t \hat{\varepsilon}_t$ and denote the k elements of $\hat{U}_t$ by $\hat{U}_{t,i}$, $i = 1, ..., k$.
4. For each series $\hat{U}_{t,i}$, compute a new series, $\hat{w}_{t,i}$ via $w_{t,i} = \bar{r}\hat{w}_{t-1,i} + \Delta\hat{U}_{t,i}$, and $\hat{w}_{1,i} = \hat{U}_{1,i}$, where $\bar{r} = 1 - 10/T$.
5. Compute the squared residuals from OLS regressions of $\hat{w}_{t,i}$ on $\bar{r}^t$ individually, and sum all of those over $i = 1, ..., k$.
6. Multiply this sum of sum of squared residuals by $\bar{r}$, and subtract $\sum_{i=1}^{k}\sum_{t=1}^{T}(\hat{U}_{t,i})^2$

qLL Statistic

- Monotone transformation of the LRT test.
- $\beta_t - \beta_0$ following a Gaussian Random Walk assumption
- Leads to: First Differences follow Gaussain Moving Average

Table 2: Asymptotic Critical Values of the qLL Statistic

k	1	2	3	4	5
1%	-11.05	-17.57	-23.42	-29.18	-35.09
5%	-8.36	-14.32	-19.84	-25.28	-30.60
10%	-7.14	-12.80	-18.07	-23.37	-28.55

Note:
Extract of the critical values of the qLL Statistic. k represents the number of potential unstable coefficients (number of parameters in the model) whereas 1%, 5% and 10% are the significance levels where a lower value is stronger evidence that instability is present.

Cusum Test

- Use of the Recursive Least Squares Algorithm (to obtain the errors!)
- OLS but adding additional observations sequentially.

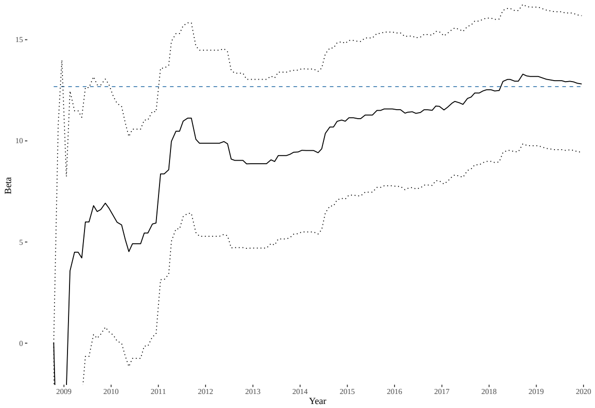


Figure 4: RLS applied to the UK CPI

RLS Algorithm

1. $\hat{\beta}_t = \hat{\beta}_{t-1} + g_t(R_t - S_t^T \hat{\beta}_{t-1})$
2. $g_t = P^*_{t-1} S_t (\hat{\sigma}^2 + S_t^T P^*_{t-1} S_t)^{-1}$
3. $P^*_t = P^*_{t-1} - g_t S_t^T P^*_{t-1}$

(Young 2011)

$$u_t = R_t - E(\hat{R}_t | R_{t-1}) \tag{6}$$

Normalized:

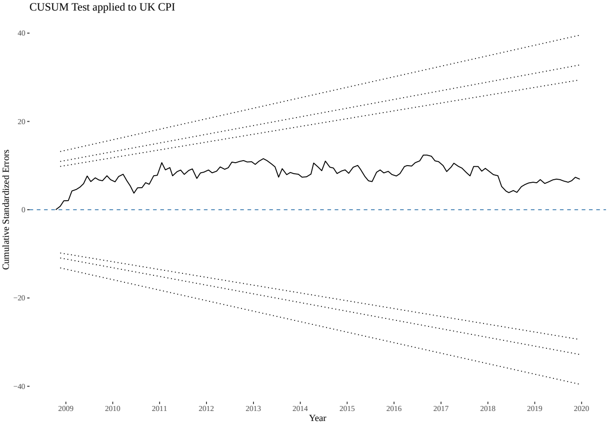
$$u_{n,t} = \frac{u_t}{(1 + S_t^T P_t S_t)^{0.5}} \tag{7}$$

Compounded:

$$W_t = \frac{1}{\hat{\sigma}_{cs}} \sum_{i=k+1}^{t} u_{n,i} \tag{8}$$

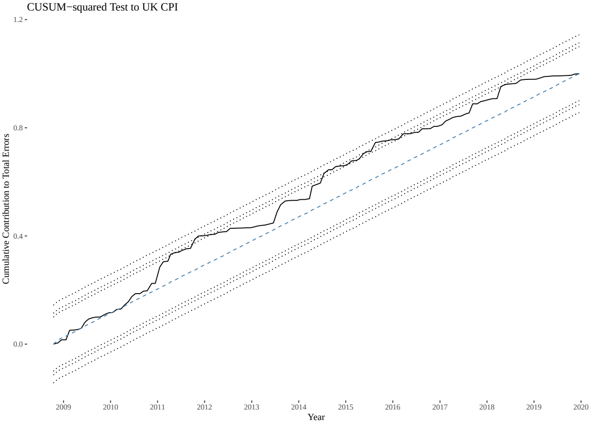
Cusum Plot

Use the confidence bands suggested by Brown, Durbin, and Evans (1975) starting at time k: $\pm a(T-k)^{0.5}$ and ending at time T: $\pm 3a(T-k)^{0.5}$.



Cusum-Squared

$$V_t = \frac{\sum_{i=k+1}^{t} u_{n,i}^2}{\sum_{i=k+1}^{T} u_{n,i}^2} \tag{9}$$



Under the null, the cumulative sum of squares follow a beta distribution and the mean is $(k-h)/(N-h)$. As significance levels:

Test Results

Table 3: Instability Test Results

News.Event	qLL	CUSUM	CUSUM.sq
Single News			
UK CPI	-12.964***	n.s	**
CA CPI	-13.015***	n.s	***
CA CRS	-9.713**	n.s	**
US CPI	-21.582***	n.s	***
NZ CPI	-7.267*	n.s	***
AU CPI	-9.022**	n.s	***
AU RET	-17.623***	**	***
UK RET	-4.503	n.s	***
Grouped News			
US Batch			
Test 1 All News	-28.14***	*	***
Test 2 AHE&NFP	-24.015***		
Test 3 NFP&UR	-20.326***		
Test 4 AHE&UR	-7.078		
Test 5 NFP	-17.643***		
CA Batch			
Test 1 All News	-9.30	n.s	***
AUD Batch			
Test 1 All News	-22.912***	n.s	***
Test 2 EMC	-6.85		
Test 3 UR	-5.66		

Note: Results of the three instability tests performed for each piece of macroeconomic news.

Estimating Parameter Paths

Two Different Methods

- WAR Minimizing (Elliott and Müller 2006)
- Stochastic Time Varying Parameter (Young 2011)

WAR Minimizing

- Start from a likelihood function in its general form.
- From a “stable” case to unknown “unstable” model where:

Likelihood: $\sum_{t=1}^{T} \ell_t(\beta)$ to: $\sum_{t=1}^{T} \ell_t(\beta + \delta_t)$

- 2nd order Taylor Approximation in its classical form:

$$f(x) = f(a) + f'(x-a) + \frac{f''}{2}(x-a)^2 \tag{10}$$

- Taylor Expansion of ℓ_t around $\hat{\beta}_{OLS}$ the unstable model (Univariate):

$$\ell_t(\beta+\delta_t) = \ell_t(\hat{\beta}) + s_t(\hat{\beta})(\beta+\delta_t-\hat{\beta}) - \frac{1}{2}(h_t(\hat{\beta})(\beta+\delta_t-\hat{\beta})^2 \tag{11}$$

- It is possible to find $s_t(\hat{\beta})$ and $h_t(\hat{\beta})$!

WAR Minimizing (continued) (1)

- An average $\hat{H}$ is used instead of individual hessians at each observation.

$$\ell_t(\beta+\delta_t) = \ell_t(\hat{\beta}) + s_t(\hat{\beta})(\beta+\delta_t-\hat{\beta}) - \frac{1}{2}(\hat{H}(\hat{\beta})(\beta+\delta_t-\hat{\beta}) \quad (12)$$

- Rearranging and subtracting $\frac{1}{2}s_t(\hat{\beta})\hat{H}^{-1}$ from each side.

$$\ell_t(\beta+\delta_t) - \ell_t(\hat{\beta}) - \frac{1}{2}s_t(\beta)\hat{H}^{-1} \approx -\frac{1}{2}(s_t(\hat{\beta}) - \hat{H}(\beta+\delta_t-\hat{\beta})\hat{H}^{-1} \quad (13)$$

- Comparing to the log-likelihood of an arbitrary Gaussian Random Variable X_n (standard nomenclature)

$$-\frac{N}{2}\log(2\pi) - N\log(\sigma^2) - \frac{1}{2\sigma^2}\sum_{n=1}^{N}(X_n-\mu)^2 \quad (14)$$

WAR Minimizing (continued) (2)

- Arbitrary Gaussian Random Variable X_i: (standard nomenclature)

$$X_i \sim \mathcal{N}(\mu, \sigma^2)$$
$$X_i = \mu + \epsilon_i \quad \epsilon_i \sim \mathcal{N}(0, \sigma^2) \tag{15}$$

- Scenario at hand:

$$s_t(\hat{\beta}) + \hat{H}\hat{\beta} \sim \mathcal{N}(\hat{H}(\beta + \delta_t), \hat{H})$$
$$s_t(\hat{\beta}) + \hat{H}\hat{\beta} = \hat{H}(\beta + \delta_t) + v_t \quad v_t \sim \mathcal{N}(0, \hat{H}) \tag{16}$$

WAR Minimizing (continued) (3)

$$\beta_{OLS} = \beta_t + T^{-1/2}\hat{H}^{-1}v_0 \tag{17}$$

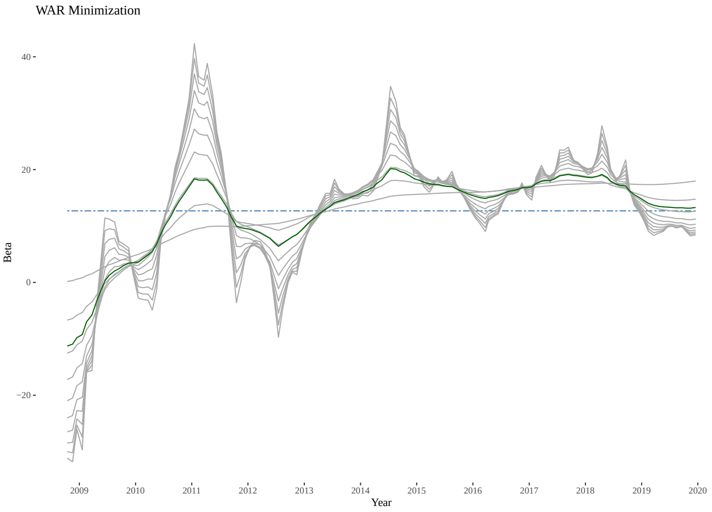
$$s_t(\beta) = \hat{H}\delta_t + v_t, t = 1, ..., T \tag{18}$$

- Obtain 11 Different Random Walk weighting functions (paths).
- They all use the model above but have a different variance.
- $s_t(\hat{\beta})$ and $\hat{H}$ as:

$$s_t = \frac{\partial l}{\partial \beta_1} = \sigma^{-2}(R_t - \hat{\beta}_0 - \hat{\beta}_1 S_t)S_t \tag{19}$$

$$\hat{H} = \frac{1}{T}\sum_{t=1}^{T} h_t(\hat{\beta}) = -\frac{\partial l^2}{\partial^2 \beta_1^T} = \sigma^{-2}\sum_{t=1}^{T} S_t^2 \tag{20}$$

WAR Plot



WAR Minimization
40
20
0
−20
Beta
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
Year

Steps to obtain WAR minimization path

1. For t = 1,...,T, let a_t and b_t be the first p elements of $\hat{H}^{-1}s_t(\hat{\theta})$ and $\hat{H}\hat{V}^{-1}s_t(\hat{\theta})$ respectively.
2. For $c_i \in C = 0, 5, 10, ..., 50$, $i = 1, ..., 11$ compute (a) $r_i = 1 - c_i/T$, $z_{i,1} = x_1$ and $z_{i,t} = r_i z_{i,t-1} + x_t - x_{t-1}$, $t = 2, ..., T$;
(b) the residuals $\{\tilde{z}_{i,t}\}_{t=1}^T$ of a linear regression of $\{z_{i,t}\}_{t=1}^T$ on $\{r_i^{t-1} I_p\}_{t=1}^T$
(c) $\bar{z}_{i,T} = \tilde{z}_{i,T}$, and $\bar{z}_{i,t} = r_i \bar{z}_{i,t+1} + \tilde{z}_{i,t} - \tilde{z}_{i,t+1}$, $t = 1, ..., T-1$;
(d) $\{\hat{\beta}_{i,t}\}_{t=1}^T = \{\hat{\theta} + a_t - r_i \bar{z}_{i,t}\}_{t=1}^T$;
(e) $qLL(c_i) = \sum_{t=1}^T (r_i)\bar{z}_{i,t} - a_t)'\tilde{b}_t$ and
$\tilde{w}_i = \sqrt{T(1-r_i^2)r_i^{T-1}/(1-r_i^{2T})} e^{-\frac{1}{2}qLL(c_i)}$ (set $\tilde{w}_0 = 1$) 3. Compute $w_i = \tilde{w}_i / \sum_{j=1}^{11} \tilde{w}_j$.
4. The parameter path estimator is given by $\{\hat{\beta}_t\}_{t=1}^T = \{\sum_{i=1}^{11} w_i \hat{\beta}_{i,t}\}_{t=1}^T$.
5. The statistic qLL(10) tests the null hypothesis of stability of β and rejects for small values.

(Müller and Petalas 2010)

STVP Algorithm

- RLS (used for the tests earlier)

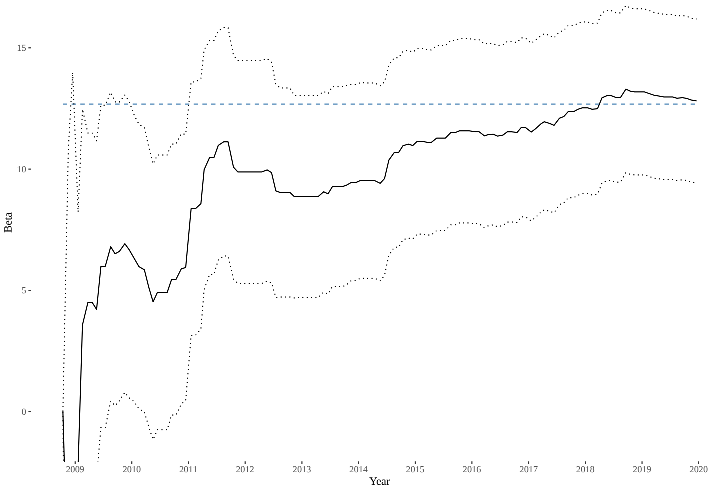


Figure 5: SRLS applied to the UK CPI

STVP Algorithm (continued)

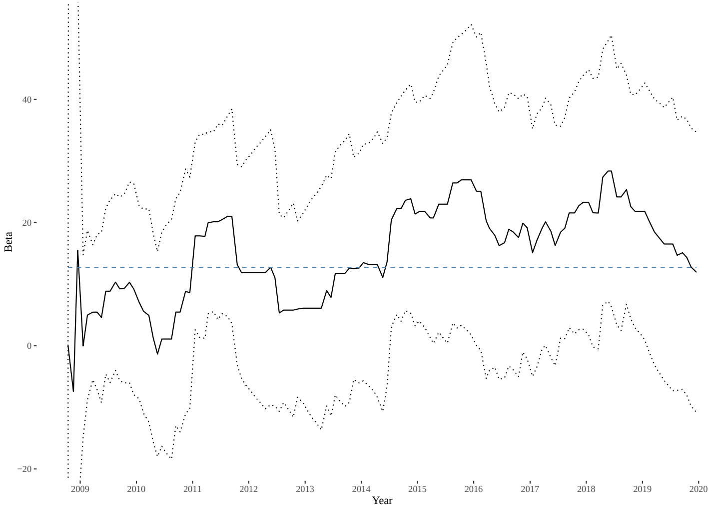


Figure 6: STVP applied to the UK CPI

Standard Recursive Time Variable Parameter Algorithm (STVP)

- Additional assumption: β_t is following a Gaussian Random Walk $\mathcal{N}(0, q_a)$ so that: $\beta_t = \beta_{t-1} + \eta_{t-1}$
- Diagonals of Q_a are 25% of the OLS Coefficient
- $A = D = I$. (with I as the identity matrix)

Prediction (Prior)

1. $\hat{\beta}_t | \hat{\beta}_{t-1} = A\beta_{t-1}$
2. $P_t^* | P_{t-1}^* = AP_{t-1}^* A^T + DQ_a D^T$

Correction (Posterior, same as the RLS seen earlier)

3. $\hat{\beta}_t = \hat{\beta}_t | \hat{\beta}_{t-1} + g_t(R_t - S_t^T(\hat{\beta}_t | \hat{\beta}_{t-1}))$
4. $g_t = (P_t^* | P_{t-1}^*) S_t (\hat{\sigma}^2 + S_t^T (P_t^* | P_{t-1}^*) S_t)^{-1}$
5. $P_t^* = P_t^* | P_{t-1}^* - g_t S_t^T (P_t^* | P_{t-1}^*)$

(Young 2011)

Plotting the Paths

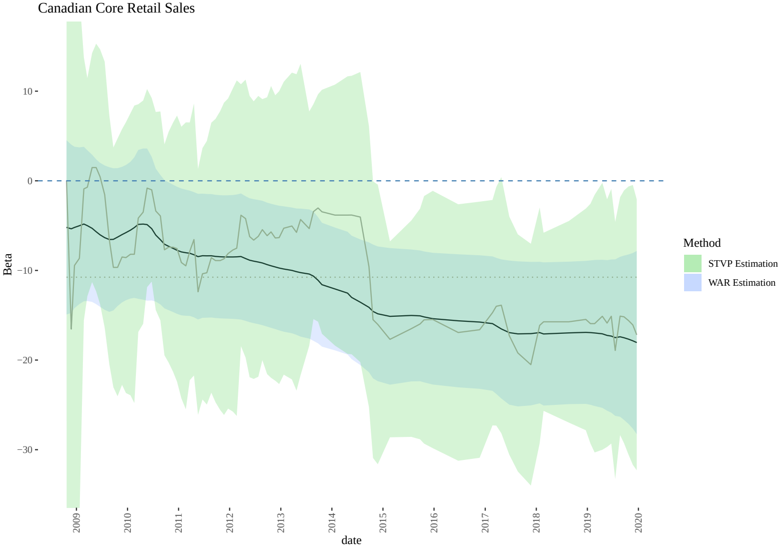


Figure 7: Note: 1 Std Dev is +0.605% greater than expected. Currency pair: USD/CAD.

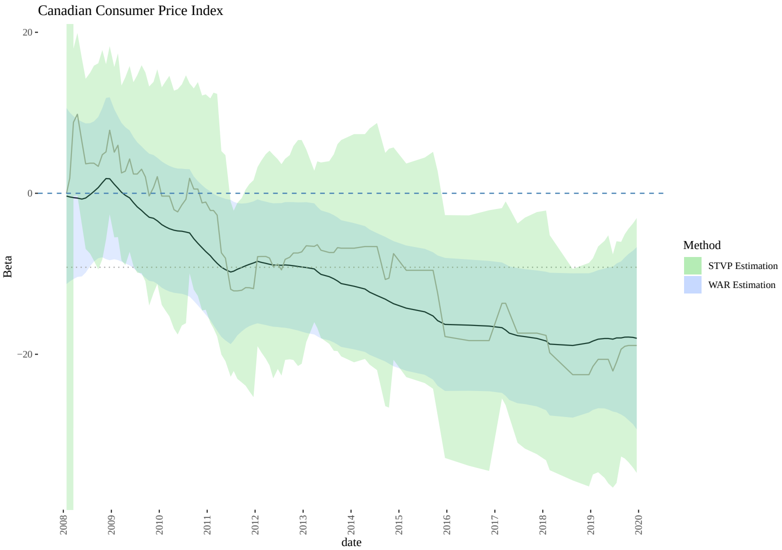


Figure 8: Note: 1 Std Dev is +0.229% greater than expected. Currency pair: USD/CAD.

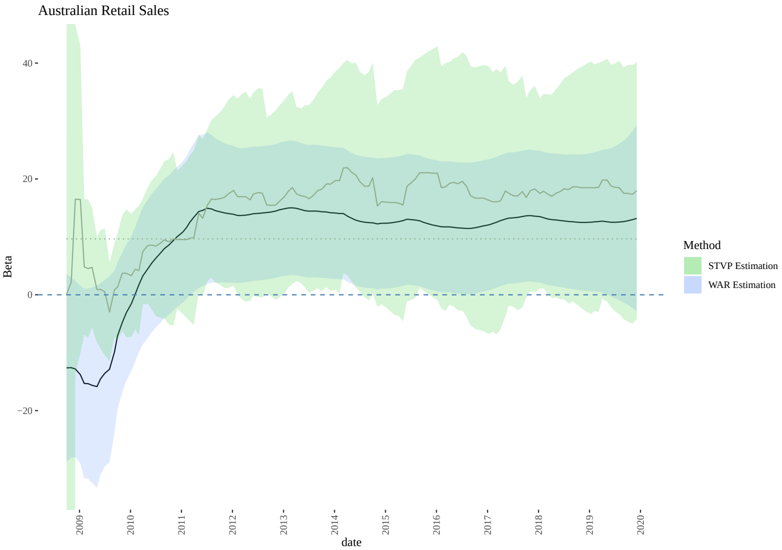


Figure 9: Note: 1 Std Dev is +0.569% greater than expected. Currency pair: AUD/USD.

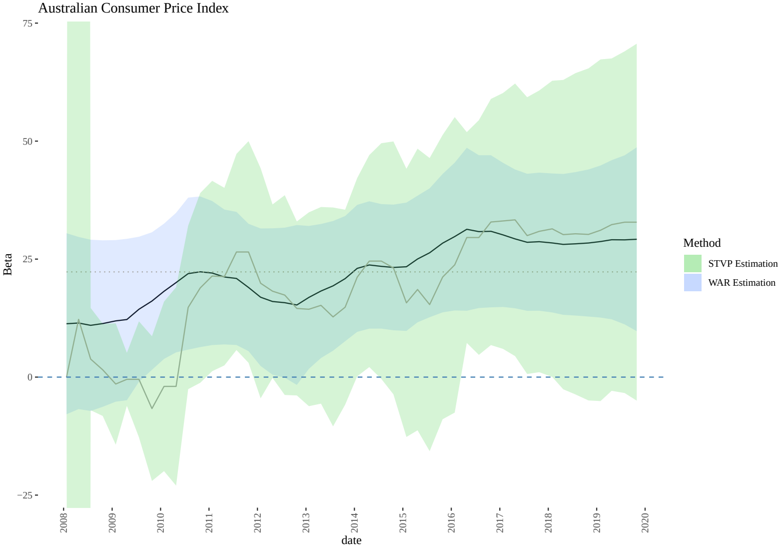


Figure 10: Note: 1 Std Dev is +0.229% greater than expected. Currency pair: AUD/USD.

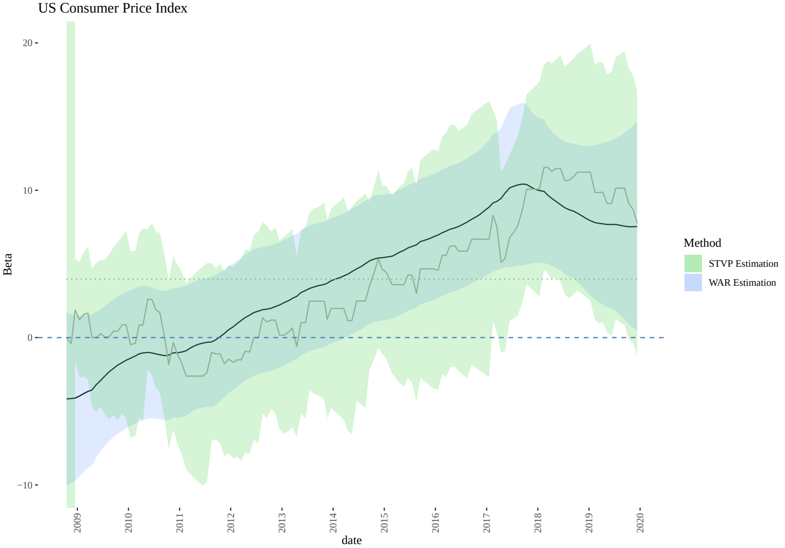


Figure 11: Note: 1 Std Dev is +0.117% greater than expected. Currency pair: USD/CHF.

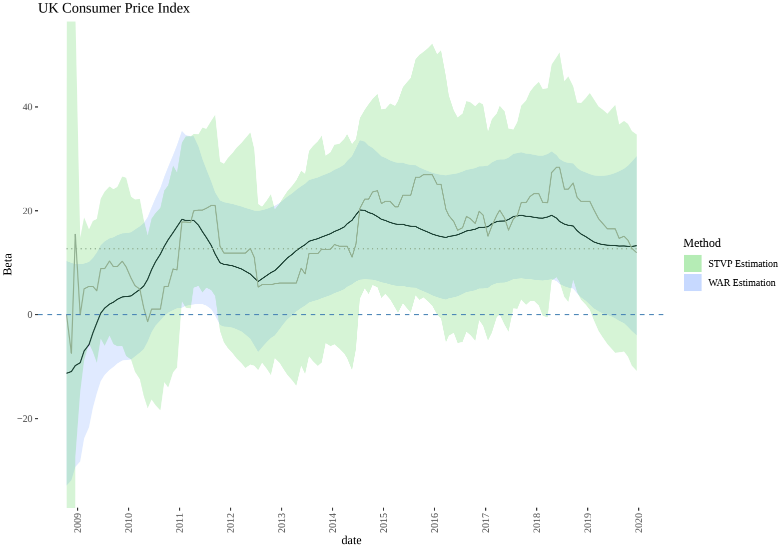


Figure 12: Note: 1 Std Dev is +0.173% greater than expected. Currency pair: GBP/USD.

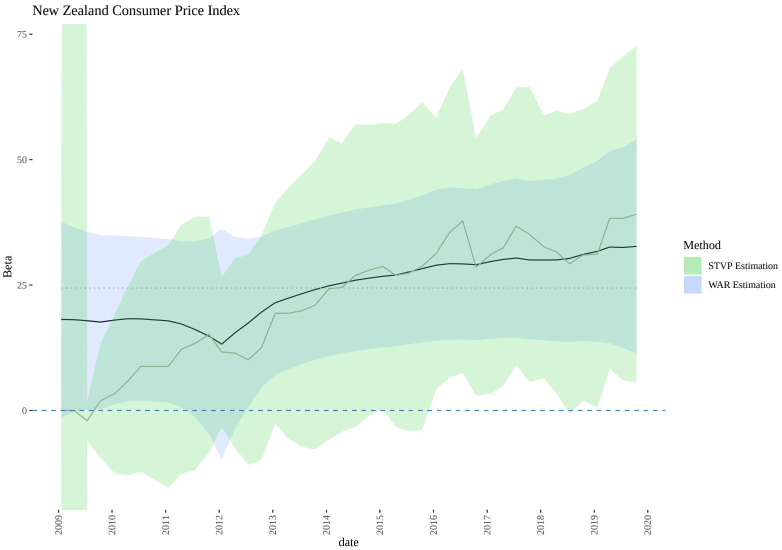


Figure 13: Note: 1 Std Dev is +0.206% greater than expected. Currency pair: NZD/USD.

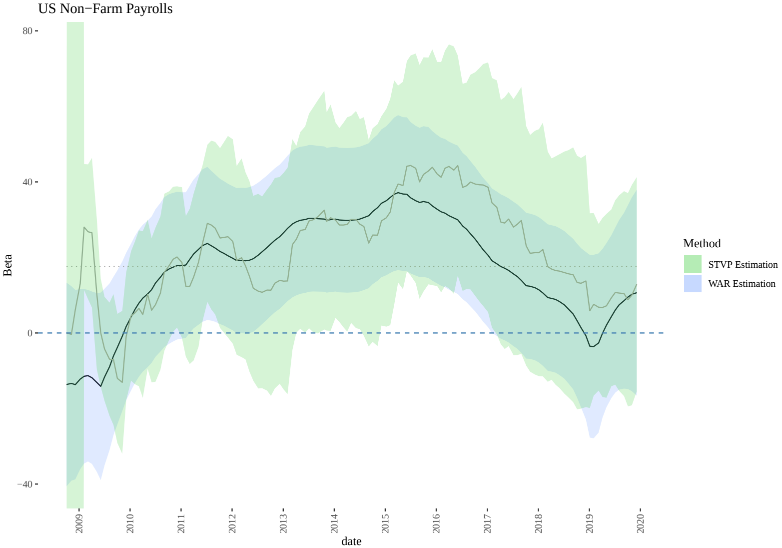


Figure 14: Note: 1 Std Dev is +66.5 thousand more people than expected. Currency pair: USDCHF.

Prediction

Calculating the Prediction Intervals

$$E(\hat{R}_{t+1}|\mathcal{F}_t, S_{t+1}) = \hat{\beta}_t S_{t+1} \tag{21}$$

$$Var(\hat{R}_{t+1}|\mathcal{F}_t, S_{t+1}) = S_{t+1}^2 \sigma_\beta^2 + \sigma_\epsilon^2 \tag{22}$$

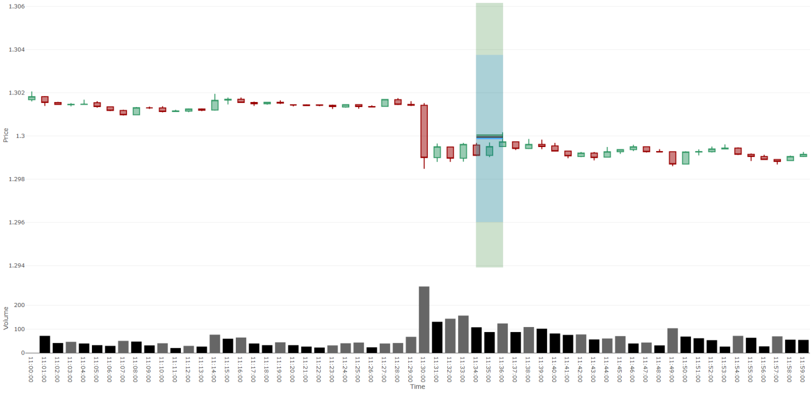


Figure 15: GBP/USD on the 15th of January between 14h00 and 15h00 GMT+2. UK CPI released with a figure of 1.3% versus the expected 1.5% (equivalently a -1.1579697 Std. Dev shock). Point-Estimates and 95% prediction bounds are colored in blue and green for the WAR and STVP estimations respectively. The time-invariant OLS case is also added in black for reference.

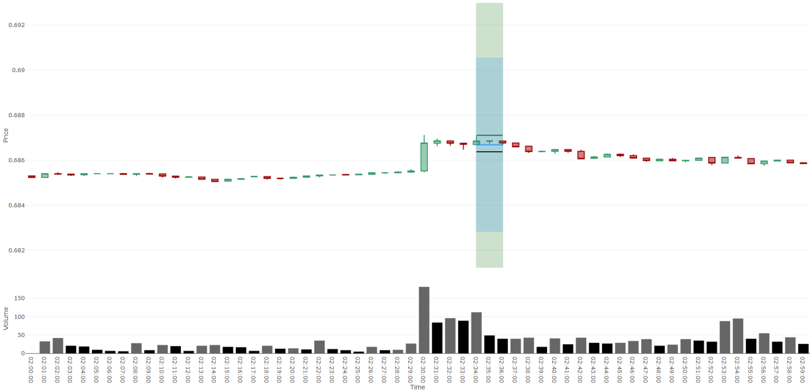


Figure 16: AUD/USD on the 10th of January between 02h00 and 03h00 GMT+2. Retail Sales announced at 0.9% versus the expected 0.4% that was expected (equivalently a +0.8801813 Std. Dev shock). Point-Estimates and 95% prediction bounds are colored in blue and green for the WAR and STVP estimations respectively. The time-invariant OLS estimate is also added in black for reference.

Questions / Comments

References

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