# EE5111 End-Semester

V SAI KRISHNA EE17B035

July 2020

## Introduction

You are a car manufacturer and own a big plant in the outskirts of a metro city. You employ 1000 people in the plant and manufacture both medium priced and high priced cars.

The number of units you manufacture depends on various factors such as:

- Popularity and demand for a specific model
- Availability of parts
- Ease of transportation of finished goods and raw materials
- Profit margins
- Labour cost and Maintenance.

## Question 1

Think like a Bayesian and assume priors for all these factors and then come up with a plan on how many units you will think of manufacturing and the type of cars you will manufacture so as to maximize your profit.

### Solution

One approach to this problem would be to start manufacturing only after we get orders. This is known as **Just-In-Time manufacturing**. Although this might seem like a reasonable thing to do, it is certainly not the best strategy. The reasons are as follows:

- Maintenance costs of machines aren't going to disappear, and labours (excluding contract workers) need to be paid on a monthly basis even if the factory is closed.

- It is possible that orders are obtained in huge bulks and we might not be able to deliver them in time if we don't have some pre-made cars ready.

Hence, a better approach would be to **estimate the number of cars that will be sold** within a given time frame (For Example: in a month) and try to manufacture those many cars in parts and labour are available. This approach will allow us to **use our resources efficiently** and in-turn maximize the profit obtained.

## Assumptions

- The time frame with which we will work be one week. This means that we will try to predict the number of cars (for each model) that will be purchased in the coming week and try to meet that demand.

- Let the company manufacture 'm' models of medium-priced cars denoted by $M_1, M_2, ...M_m$ and 'h' models of high-priced cars denoted by $H_1 H_2 ...H_h$.

- The popularity and demand for a model is synonymous to the number of orders received for that particular model.

  Since they have asked us to assume a prior for this, let us assume the **number of orders received for the model $\mathbf{M}_i$**[1] ($S_{Mi}$) in a week be Poisson distributed with parameter $\lambda_{Mi}$, i.e:

$$S_{Mi} \sim Poisson(\lambda_{Mi})$$

  where $\lambda_{Mi}$ can be determined using MLE on previously collected data for every model.

- The profit margin **(which is Revenue minus all expenses except for labour and maintenance cost)** will vary for every car that is sold for a variety of reasons (Ex:discounts/offers provided at the time of sale). Instead of trying to represent the Profit margin for every car sold, let us work with the Avg Profit Margin obtained for a particular model in a given week.

  Let us assume that the **Avg Profit Margin (PM)** obtained for the model $M_i$ in a given week is Normal Distributed with parameters $\mu_{Mi}$ and $\sigma^2_{Mi}$, i.e:

$$PM_{Mi} \sim Normal(\mu^{PM}_{M_i}, (\sigma^{PM}_{M_i})^2)$$

  The mean and variance for every model can be determined using MLE on previous Profit Margin data.

- Every factory has a set of permanent workers and contract based workers. Of the 1000 let us assume 'C' of them are contract based workers. For

[1] Can replace $M_i$ with $H_i$ if high-priced model is considered

the permanent workers, we'll be paying them a fixed salary **(FS)** every month irrespective of the production value/working days. This is a fixed expense and we needn't worry about this.

However, the expenses due to contract workers and due to maintenance of machines will vary. Let us assume this cost to be proportional to the number and type of cars produced. Since, the question has asked us to assume a prior, let us assume that the **extra cost incurred (EC) (of labour + maintenance)** for every car produced in a given week follows a Normal Distribution, i.e for every medium-priced car:

$$EC_M \sim Normal(\mu_M^{EC}, (\sigma_M^{EC})^2)$$

and for every high-priced car:

$$EC_H \sim Normal(\mu_H^{EC}, (\sigma_H^{EC})^2)$$

The parameters defining the distributions can once again be determined using MLE on previous data.

- Let us assume that the sales of a particular model is independent of other models available in the market. However, the production/delivery rate of a model will depend upon the production rate of every other model. This is due to constrains on **availability of parts and ease of transportation**. Also, the maximum number of cars that could be delivered depends upon the strength of our labour force.

  Instead of assuming separate priors for each of them we can factor all of them together. The assumptions to achieve this are:

  1) Let us assume that in a given time if you can manufacture and deliver one high-priced car, you can instead manufacture and deliver 'Q' medium-priced cars ('Q' needn't be an integer), i.e we can redirect parts and labours to **manufacture 'Q' medium-priced cars rather than one high-priced car** if desired. This 'Q' will be a deterministic quantity, because we as a manufacturer will know the number of parts and labours it takes to make each model.

  2) Let us assume that given the availability of parts and ease of transportation, we can **estimate the total number of medium-priced cars** (or medium + high-priced cars using the previous assumption) that can be delivered in that week.

## Maximizing the profit

In the previous section we have defined distributions for everything from labour costs to the number of orders received. However, all these are Random Variables. As a result the profit obtained will also be a Random Variable. Hence "maximize profit" is a meaningless statement. We will in turn try to **maximize the Expectation of profit (Avg profit)**.

Also, sales of every model is assumed to be independent of other models. Hence, we can find Avg profits for one of them. The final result is going sum across all models.

Given the number of cars manufactured ($N_{Mi}$), number of orders received ($S_{Mi}$), Avg Profit margin ($PM_{Mi}$) obtained during those sales and labour cost incurred ($EC_{Mi}$), we can calculate the profit ($Pr_{Mi}$) as follows:

$$(Pr_{Mi}|PM_{Mi}, S_{Mi}, EC_M, N_{Mi}) = PM_{Mi} * S_{Mi} - EC_M * N_{Mi}$$

Now, let us try to get an estimate for E[Pr|$N_M$], i.e Avg profit that will be obtained if we manufacture $N_M$ cars of a particular model in this month. Since, we assumed $PM_M$ and $EC_M$ to be distributed as Gaussians, we can replace $PM_M$ and $EC_M$ terms with the mean of their distributions, i.e

$$E(Pr_{Mi}|S_{Mi}, N_{Mi}) = \mu_M^{PM}{}_i * S_{Mi} - \mu_M^{EC} * N_{Mi}$$

Now we have to avg over the number of orders received. Since, we assumed it to be a Poisson distribution with parameter $\lambda_M$

$$\begin{aligned} E(Pr_{Mi}|N_{Mi}) = &\sum_{S_{Mi}=0}^{N_{Mi}} (\mu_{M_i}^{PM} * S_{Mi} - \mu_M^{EC} * N_{Mi}) * \frac{\exp^{-\lambda_{Mi}} * \lambda_{M}{}_i^{S_{Mi}}}{S_{Mi}!} \\ &+ \sum_{S_{Mi}=N_{Mi}+1}^{\infty} (\mu_{M_i}^{PM} * N_{Mi} - \mu_M^{EC} * N_{Mi}) * \frac{\exp^{-\lambda_{Mi}} * \lambda_{M}{}_i^{S_{Mi}}}{S_{Mi}!} \end{aligned}$$

The above equation can be explained as follows: If the number of cars ordered is less than or equal to the number of cars manufactured, then the profit is given by the first term in the RHS of the above equation. If the number of orders is greater than the number of cars manufactured, then the profit obtained is given by the second term in the RHS.

Since, $\mu_M^{EC} * N_{Mi}$ is common in both terms and Poisson terms sum to 1 (it's a pdf!) we can take $\mu_M^{EC} * N_{Mi}$ out and re-arrange the terms as follows:

$$\begin{aligned} E(Pr_{Mi}|N_{Mi}) = &\sum_{S_{Mi}=0}^{N_{Mi}} (\mu_{M_i}^{PM} * S_{Mi}) * \frac{\exp^{-\lambda} * \lambda^{S_{Mi}}}{S_{Mi}!} \\ &+ \sum_{S_{Mi}=N_{Mi}+1}^{\infty} (\mu_{M_i}^{PM} * N_{Mi}) * \frac{\exp^{-\lambda} * \lambda^{S_{Mi}}}{S_{Mi}!} - \mu_M^{EC} * N_{Mi} \end{aligned}$$

One can notice that the above terms can be represented using the CDF of the Poisson distribution. If $\mathcal{F}_{poi}$ represents the CDF of Poisson distribution then,

$$\begin{aligned} E(Pr_{M_i}|N_{M_i}) = \mu_{M_i}^{PM}\lambda_{M_i} * \mathcal{F}_{poi}(\lambda_{M_i}, N_{M_i}) \\ +\mu_{M_i}^{PM} * (1 - \mathcal{F}_{poi}(\lambda_{M_i}, N_{Mi})) \\ -\mu_{M}^{EC} * N_{Mi} \end{aligned}$$

Now, this is the average profit obtained from one model. Since this is a general equation, we can replicate it for other models too. **Let 'M' be the vector of length 'm' containing the number of cars manufactured for each medium-priced model and 'H' be a vector of length 'h' containing the number of cars manufactured for each high-priced model**. Hence, total avg profit given M and H is:

$$E(Profit|M, H) = \sum_{i=0}^{m} E(Pr_{Mi}|M(i)) + \sum_{j=0}^{h} E(Pr_{Hj}|H(j))$$

Hence, given M and H, we can find the Expected profit. Now to maximize this profit, we can try and find the value of $N_{Mi}$ and $N_{Hj}$ for each model individually as every model is independent of the other. We can't come up with an elegant equation to find the optimal $N_{Mi}$'s and $N_{Hj}$'s, and the only way is to try all possible values in a computer.

However, there is a big catch. The above method is applicable only if we are able to produce the optimal number of cars for each model. This depends upon our labour strength and availability of parts. If these factors are not sufficient then we can't meet the market demand. But nevertheless we can try and maximize our expected profit.

Unfortunately there is no easy way to do this by hand and we would need a computer to try all possible values and come up with the best M and H vectors. Things we need to proceed are:

- Two matrices, one for medium-priced cars and one for high-priced cars. The rows and columns are the number of cars produced (0 to max possible cars that we can produce in the time frame) for each model and expected profits if those many cars of that particular models are produced respectively.

- A program that tries all possible values for vectors M and H, given the total number of medium-priced cars that can be manufactured (this in-turn sets the bound for total number of medium+high-priced cars, refer to the 6th Assumption). **Refer to the Appendix for the Matlab code which finds the optimal number of cars to produce for each model given the total number of cars we can manufacture**.

## The Strategy

Our strategy shouldn't be a blind application of our Estimation Model. It should be flexible so that even if we receive a very big order or receive no orders at all, we should be able to meet that demand at least by the end of next time frame (next week). Here's the strategy to achieve it:

- Step 1: Before the next week begins, update the Profit Margin, Labour+Maintenance cost and Popularity+Demand parameters for every car model.
- Step 2: Obtain the availability of parts and ease of transportation data. Use it to identify the maximum number of cars we can deliver in the next week.
- Step 3: Identify the number of cars that are ordered and not yet delivered. If the number turns out to be positive for any model, we will redirect our materials and labour to finish those orders first. Let's call these vectors containing the number of medium and high priced cars that we must manufacture as $M_{must}$ and $H_{must}$.
- If we still have parts/labour to manufacture more cars
  - Step 4: Identify the optimal number of cars to manufacture for each model to maximize the profit (using the code provided). Now we have $M_{maxprofit}$ and $H_{maxprofit}$.
  - Step 5: Subtract the number of cars produced during the last week which aren't sold yet from $M_{maxprofit}$ and $H_{maxprofit}$ vectors. Add $M_{must}$ and $H_{must}$ to this. These will be our target vectors $M_{target}$ and $H_{target}$.
  - If we have enough parts/raw materials to meet $M_{target}$ and $H_{target}$:
    * Step 6: Divide $M_{target}$ and $H_{target}$ by the total number of working days in that week. This will be our per-day target.
  - If we have don't enough parts to meet $M_{target}$ and $H_{target}$:
    * Step 6: Given the total number of cars that can be manufactured, construct two matrices, one for medium-priced cars and one for high-priced cars. The rows and columns are the number of cars produced (0 to max possible cars that we can produce in the time frame) for each model and expected profits if those many models are produced respectively.
    * Step 7: Use the code provided to find the optimal number of cars that we need to produce for each model. Add $M_{must}$ and $H_{must}$ to this. Now this is our $M_{target}$ and $H_{target}$.
- Final Step: Divide $M_{target}$ and $H_{target}$ by the total number of working days in that week. This will be our per-day target.

# Question 2

Now suddenly there is an unexpected shutdown, people are going to lose jobs, labour may not be available, there are multiple restrictions on the number of people you can employ, banks are wary of lending loans to buyers, all the components you need may not be available since all those companies are also suffering from shutdown.

- Is all your prior knowledge based on which you came up with a plan now not useful? What will you do now to try to salvage the situation?
- Also note since public transport is not functioning and people want to maintain social distancing, the market for cars is not completely gone, there is demand but people do not have liquidity to buy, how can you **incentivize** the buyers and still make a profit?

## Solution for Part 1

The following are the consequences one can expect if a lock down is imposed:

- The number of people who are willing to buy personal vehicles will increase. This has already been reported by numerous companies across the world. This is because personal mobility will gain much more significance in the minds of the public than shared mobility or public transport. The shift is although expected towards smaller cars and first-time buyers. However, this doesn't imply that the demand for cars would increase as people still might not have the liquidity to afford them.
- The availability of parts and raw materials decreases drastically. This is due to the drop in labour force across every production sector.
- Since the demand for raw materials has increased, the production prices surge up. Hence, the Profit margin is going to decrease unless we increase our Selling Price or take alternative measures to decrease our production costs such as cutting down on labour costs.

As you can see, all the parameters based on which we would decide how many cars to manufacture has changed. But we haven't assumed any numerical values in our model. In fact the only things we assumed are the distributions our parameters follow, and even those assumptions are pretty solid.

For example: The number of cars sold in a given week (or within any time frame) can be described very accurately by a Poisson distribution no matter how good/bad the sales are. The labour costs and Profit margins generally don't vary a lot within a given time frame and a Normal distribution with a very small variance is a good distribution to describe them. Hence, **all of our prior knowledge based on which we have come up with the plan is not a complete waste**. We can still use the model we build.

With that being said, we still need to re-estimate the parameters. Immediately after the lock down is partially lifted we can implement **Just-In-Time manufacturing** for sometime. During this time, we can collect enough data to estimate our parameters using MLE.

## Solution for Part 2

The following plans can be adopted to incentivize buyers and still make a profit.

- Early booking: Instead of trying to sell cars right now, we can ask customers to pre-book their order. We can give them an attractive discount but the delivery will be after 3-6 months. They can pay for the car during delivery. This benefits of this strategy are as follows:
  - Customers who have plans of buying a car but aren't really in an urgent to do so will find the discount attractive. This will make them opt for us over our competitors.
  - Delay in the delivery period is useful for us as a manufacturer during the lock down as it is difficult to get raw materials and labour.
  - Soon after the lock down is lifted, we can continue working at full swing without much concern cause we have essentially created demand for the next 3-6 months.
- Car flipping: Customers can bring in their old cars and exchange them to buy new cars at a lesser cost. We can use these old cars to obtain parts, which are difficult to obtain due to lock down.

  One more thing we can do is to renovate these old cars and sell them to new customers. This is known as Car Flipping. Renovation costs will generally be lesser than manufacturing a new one. Hence, we can sell them to customers at a cheaper price while maintaining the same profit margin.
- Introducing new low-priced models: As mentioned before, the number of people willing to buy personal vehicles will increase. However, the shift is expected towards smaller and cheaper cars. If at all we had plans to manufacture low-priced cars, now is the right time to implement those plans.
- Payment as installments: Current EMI's require the customer to pay a fixed amount every month. However many people won't receive salary on a regular basis during lock down. Hence, we can offer 2 or 3 month installment plan, until the lock downs are lifted. Once situation returns to normal, we can change these plans to normal EMI scheme. To attract more customers, we can promise them services like free yearly maintenance.

## Question 3

What is the key take away you learnt from this problem and its solution?

- Real life problems are complex and contain too many factors. Blindly trying to represent these problems as mathematical models will not get us any closer to the solution.
- One needs to have good mathematical and practical knowledge to approximate complex scenarios into solvable models.
- Once these complex models are smartly approximated, one can use simple Estimation Theory concepts to come up with good interpretable solutions.

## References


- https://economictimes.indiatimes.com/industry/auto/auto-news/covid-19-impact-major-automakers-see-demand-for-personal-vehicles-going-up/articleshow/75938828.cms?from=mdr
- https://en.wikipedia.org/wiki/List_of_probability_distributions
- https://www.moneycrashers.com/make-money-flipping-cars-fix-sell-profit/
- https://towardsdatascience.com/poisson-distribution-intuition-and-derivation
- https://in.mathworks.com/matlabcentral/fileexchange/12009-partitions-of-an-integer


## Appendix

### Matlab Code to Compute optimal M and H vectors

There are two functions: Optcars and partitions. Function Optcars outputs the most optimal M and H vectors. The inputs are:

- Q: The number of Medium-Priced cars that can be manufactured instead of one High-Priced car.
- Avai_Parts: The maximum number of Medium-Priced cars one can manufacture.
- N_M and N_H: No of Medium and High-Priced models.
- m_matrix and h_matrix: These are the Profit vs No of cars manufactured for each and every model. m_matrix contains all medium-priced models and h_matrix all the high-priced models.

Optcars uses another function called partitions [2] to compute every possible values of M and H. All these values are tried by our Optcars function for finding the best M and H vectors.

[2]Obtained from Internet. Refer to point number 5 in References

```
function [M,H] = Optcars(Q,Avai_Parts,N_M,N_H,m_matrix,h_matrix)

    nm=ones(1,N_M);
    nh=ones(1,N_H);
    max_profit=0;
    M = zeros(1,N_M);
    H = zeros(1,N_H);

    for i=1:floor(Avai_Parts/Q)+1
        mprofit=0;
        hprofit=0;
        tempM=zeros(1,N_M);
        tempH=zeros(1,N_H);
        medium_quantity=floor(Avai_Parts-Q*(i-1));
        high_quantity=floor((i-1));
        m_split=partitions(medium_quantity,nm);
        h_split=partitions(high_quantity,nh);
        idx=(1:N_M);
        for j=1:length(m_split)
            profit=m_matrix(m_split(j)+1,idx);
            if(profit>mprofit)
                mprofit=profit;
                tempM=m_split(j,:);
                disp(tempM);
            end
        end
        idx=(1:N_H);
        for j=1:length(h_split)

            profit=h_matrix(h_split(j)+1,idx);
            if(profit>hprofit)
                hprofit=profit;
                tempH=h_split(j,:);
            end
        end
        if(mprofit+hprofit>max_profit)
            max_profit=mprofit+hprofit;
            M=tempM;
            H=tempH;
        end
    end
end
```

```
function plist = partitions(total_sum,candidate_set,max_count,fixed_count)

  % default for candidate_set
  if (nargin<2) || isempty(candidate_set)
    candidate_set = 1:total_sum;
  end

  % how many candidates are there
  n = length(candidate_set);

  % error checks
  if any(candidate_set<0)
    error('All members of candidate_set must be >= 0')
  end
  % candidates must be sorted in increasng order
  if any(diff(candidate_set)<0)
    error('Efficiency requires that candidate_set be sorted')
  end

  % check for a max_count. do we supply a default?
  if (nargin<3) || isempty(max_count)
    % how high do we need look?
    max_count = floor(total_sum./candidate_set);
  elseif length(max_count)==1
    % if a scalar was provided, then turn it into a vector
    max_count = repmat(max_count,1,n);
  end

  % check for a fixed_count
  if (nargin<4) || isempty(fixed_count)
    fixed_count = [];
  elseif (fixed_count<0) || (fixed_count~=round(fixed_count))
    error('fixed_count must be a positive integer if supplied')
  end

  % check for degenerate cases
  if isempty(fixed_count)
    if total_sum == 0
      plist = zeros(1,n);
      return
    elseif (n == 0)
      plist = [];
      return
    elseif (n == 1)
      % only one element in the set. can we form
      % total_sum from it as an integer multiple?
```

```
      p = total_sum/candidate_set;
      if (p==fix(p)) && (p<=max_count)
        plist = p;
      else
        plist = [];
      end
      return
   end
  else
    % there was a fixed_count supplied
    if (total_sum == 0) && (fixed_count == 0)
      plist = zeros(1,n);
      return
    elseif (n == 0) || (fixed_count <= 0)
      plist = [];
      return
    elseif (n==1)
      % there must be a non-zero fixed_count, since
      % we did not trip the last test. since there
      % is only one candidate in the set, will it work?
      if ((fixed_count*candidate_set) == total_sum) &&
                                      (fixed_count <= max_count)
        plist = fixed_count;
      else
        plist = [];
      end
      return
    end
  end

  % finally, we can do some work. start with the
  % largest element and work backwards
  m = max_count(end);
  % do we need to back off on m?
  c = candidate_set(end);
  m = min([m,floor(total_sum/c),fixed_count]);

  plist = zeros(0,n);
  for i = 0:m
    temp = partitions(total_sum - i*c, ...
        candidate_set(1:(end-1)), ...
        max_count(1:(end-1)),fixed_count-i);
    plist = [plist;[temp,repmat(i,size(temp,1),1)]];   %#ok
end
```