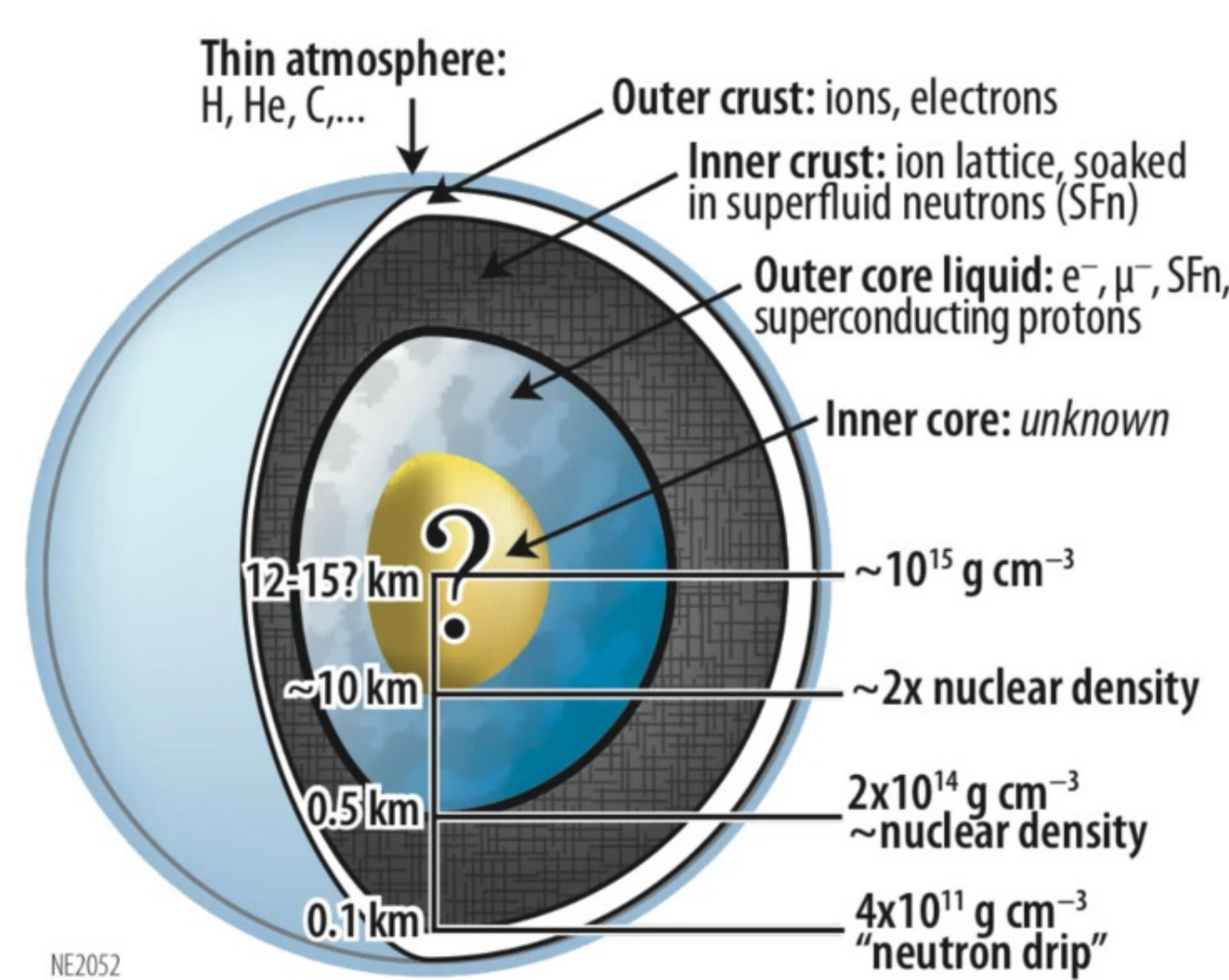


## Abstract

The presence of a neutron superflow in the inner crust and outer core of neutron stars can lead to a regime in which neutrons remain superfluid even though the energy spectrum of quasiparticle excitations exhibits no gap. For such gapless superfluidity, we have shown within the nuclear energy density functional theory that the neutron specific heat is comparable to that in the normal phase. We have studied the implication of gapless superfluidity for the thermal relaxation of transiently accreting neutron stars and compared our calculations with observations.

## I Introduction

With a mass comparable to that of the Sun but a radius of about 12 km, neutron stars are the most compact stars in the Universe. Their **extremely dense matter** is thought to be cold enough to be in various exotic quantum phases [1]. In particular, free neutrons in the crust and in the outer core become **superfluid** at temperatures below  $T_{cn}^{(0)} \sim 10^{10}$  K and can thus flow without resistance.



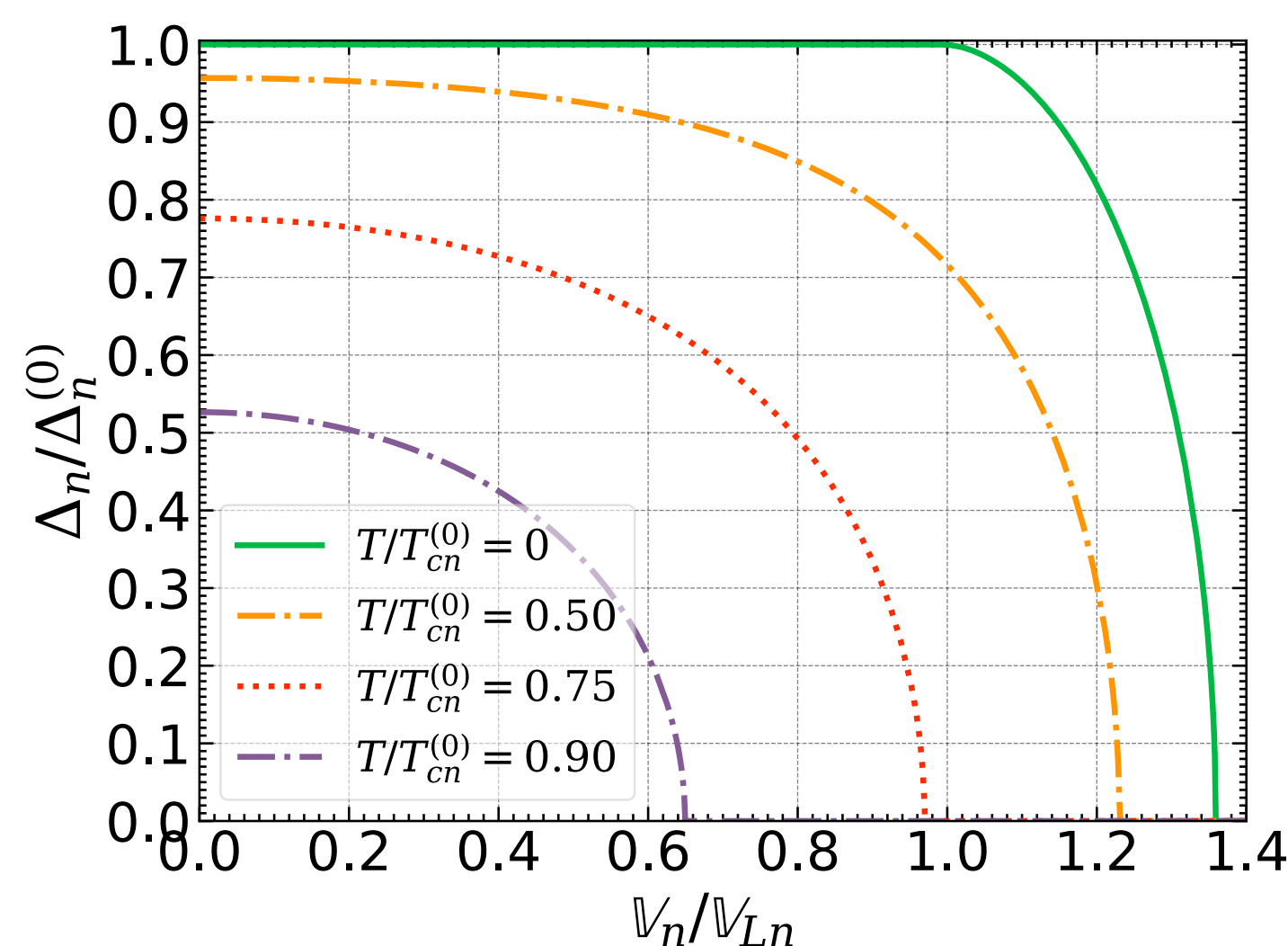
Picture from K.C. Gendreau et al. (2012), SPIE, 8443, 13

This neutron superfluid phase is described by the complex **order parameter**  $\Delta_n$  and is characterized by the **presence of a gap in the energy spectrum of quasiparticle excitations**, which impacts thermal and transport properties.

## II Gapless superfluidity

### II.a Order parameter

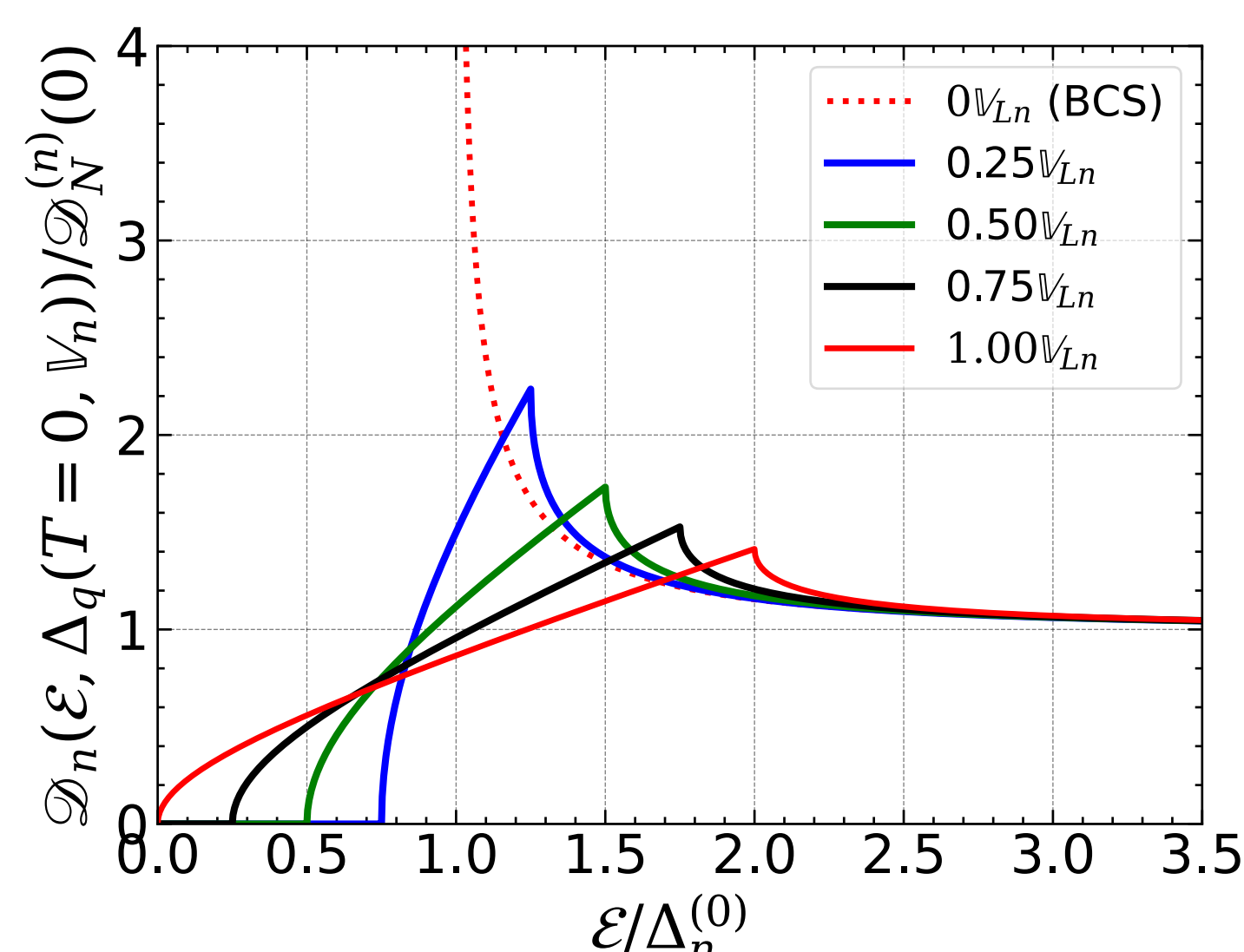
We have studied the **effects of finite temperature and superflow** using the self-consistent nuclear energy density functional theory [2].



The behavior of  $\Delta_n$  is found to be **universal** after introducing some **effective superfluid velocity**  $\mathbb{V}_n$  and after proper rescaling:  $\Delta_n^{(0)}$  is the order parameter at zero temperature and in the absence of superflows,  $\mathbb{V}_{Ln} = \Delta_n^{(0)} / (\hbar k_{Fn})$  (with  $k_{Fn}$ , the Fermi wave vector of free neutrons) and  $T_{cn}^{(0)} = e^{\gamma} \Delta_n^{(0)} / \pi$  is the **critical temperature**. Numerical interpolations are available in [3].

### II.b Gapless regime and specific heat

Focusing on low temperatures, the energy gap in the quasiparticle density of state  $\mathcal{N}_n$  shrinks with increasing  $\mathbb{V}_n$  and disappears at a velocity  $\mathbb{V}_{Ln}$  corresponding to Landau’s criterion [4]. However, the **order parameter**  $\Delta_n$  **remains finite: superfluidity becomes gapless**.



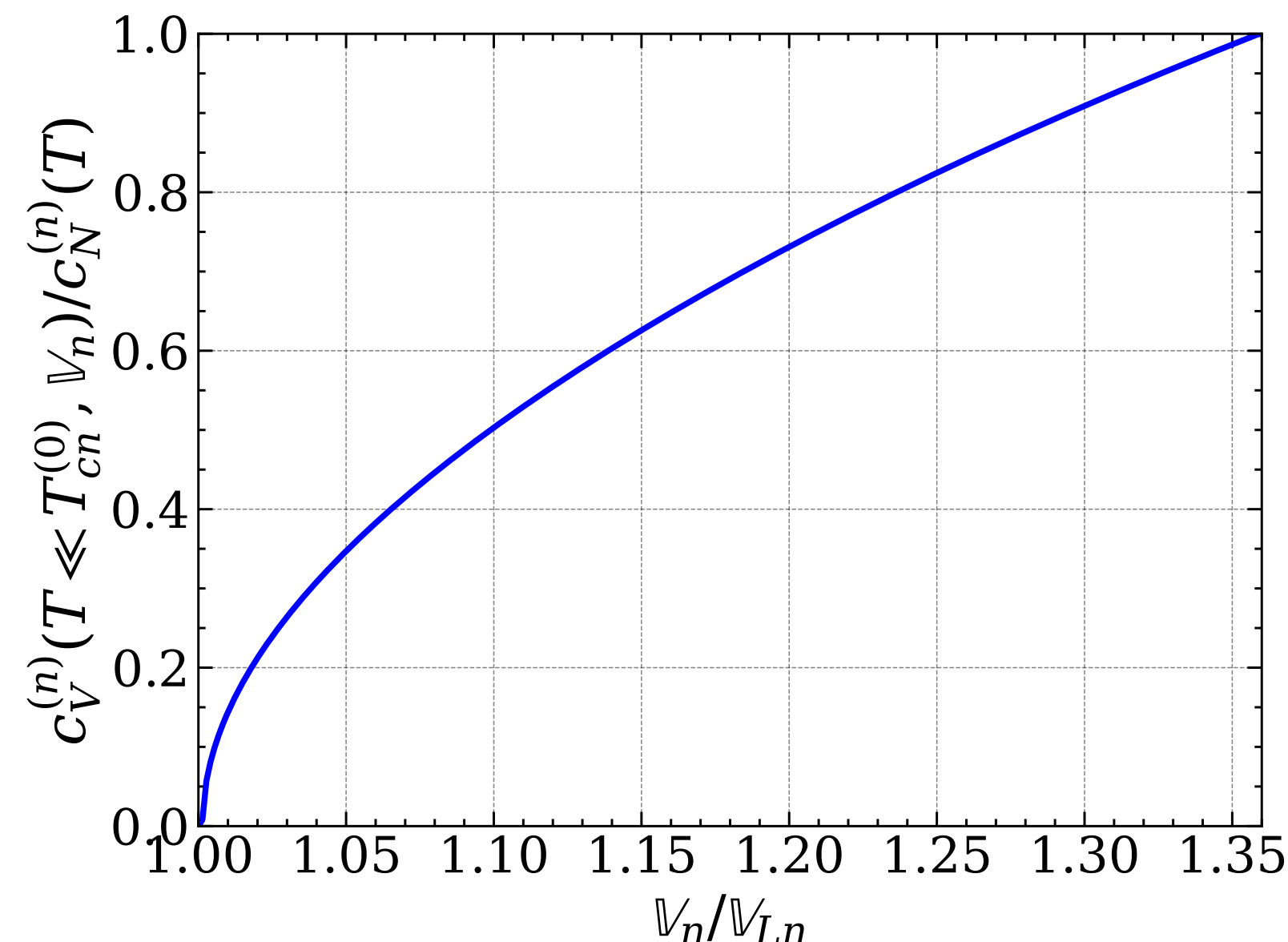
Superfluidity is destroyed at the **critical velocity**  $\mathbb{V}_{cn}^{(0)} = e \mathbb{V}_{Ln} / 2 \simeq 1.36 \mathbb{V}_{Ln}$ .

One immediate consequence of the gapless phase is the modification of the neutron specific heat  $c_V^{(n)}$ :

$$c_V^{(n)}(T \ll T_{cn}^{(0)}, \mathbb{V}_n > \mathbb{V}_{Ln}) \approx \sqrt{1 - \left( \frac{\Delta_n \mathbb{V}_{Ln}}{\Delta_n^{(0)} \mathbb{V}_n} \right)^2} c_N^{(n)}(T). \quad (1)$$

with  $c_N^{(n)}$  the neutron specific heat in the normal phase.

The neutron specific heat  $c_V^{(n)}$  is no longer exponentially suppressed as in the BCS phase but is comparable to  $c_N^{(n)}$ . Their ratio can be expressed as an increasing universal function of  $\mathbb{V}_n / \mathbb{V}_{Ln}$ :



Gapless superfluidity can thus leave an imprint on neutron star cooling.

## III Cooling of MXB 1659-29 & KS 1731-260

### III.a Quasipersistent soft X-ray transients

Quasipersistent soft X-ray transients are made of a neutron star and a low-mass stellar companion [5]. The crust of the neutron star is heated due to mass transfer for a long period of time (from years to decades) before cooling when accretion stops. The heat diffuses over a typical timescale [6]

$$\tau \propto \frac{c_V^{\text{tot}}}{\kappa} (\Delta R)^2, \quad (2)$$

with  $\Delta R$  the thickness of the crust,  $\kappa$  its thermal conductivity and  $c_V^{\text{tot}}$  its *total* specific heat.

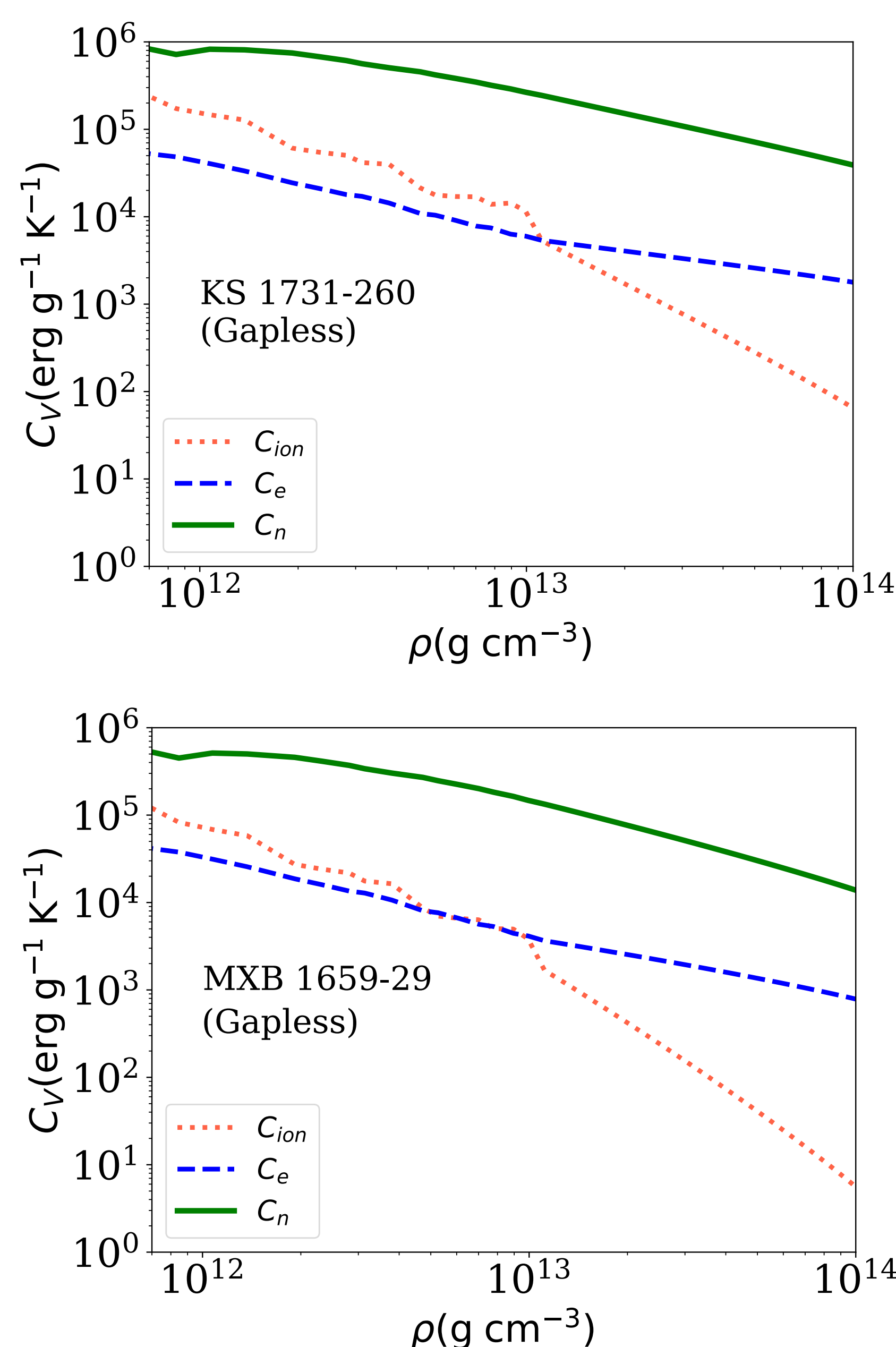
In the following, we focus on two particular systems:

- **KS 1731–260**: entered into quiescence in 2001 after 12.5 years of accretion [7]. Further observations by Cackett et al. [8] suggested the source might still be cooling, requiring a higher thermal timescale (2) than previously thought. Turlione et al. [9] explained this late time cooling by adjusting artificially the order parameter (“deep gap”) to avoid the exponential suppression of  $c_V^{(n)}$  in the BCS phase thus increasing  $\tau$ .
- **MXB 1659–29**: entered into quiescence in 2001 after 2.5 years of accretion [10]. The unexpected temperature drop reported in 2013 [11] lead Deibel et al. [12] to suggest that neutrons are not superfluid at the crust bottom, at variance with predictions from microscopic calculations [13, 14]. Alternatively, Horowitz et al. [15] proposed the presence of a low thermal conductivity crustal layer due to nuclear “pastas”. But Nandi&Schramm [16] performed quantum molecular dynamics simulations and did not find any appreciable change of  $\kappa$ .

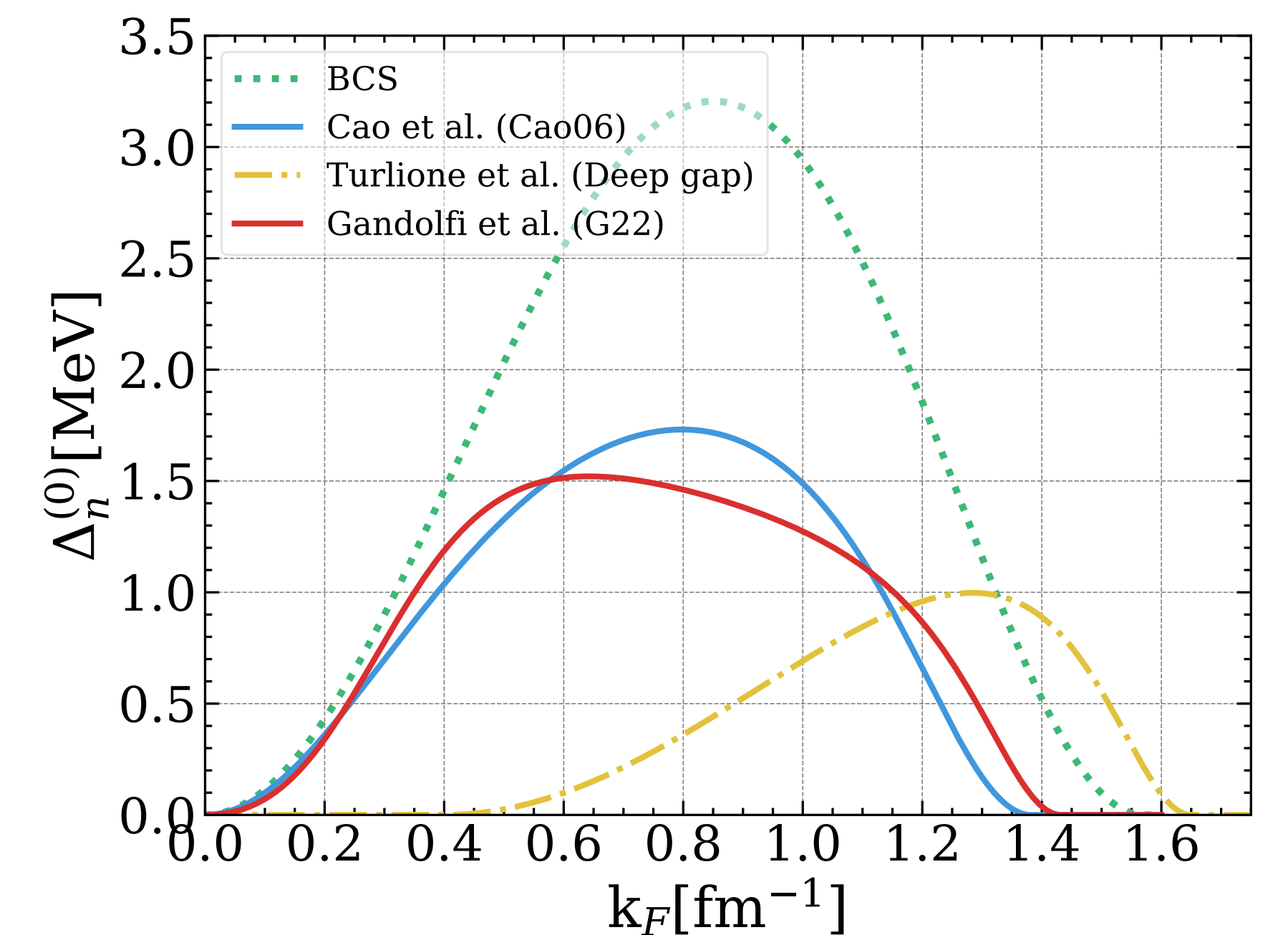
### III.b Cooling of MXB 1659–29 & KS 1731–260

We have revisited the cooling of transiently accreting neutron stars taking into account gapless superfluidity. Simulations were carried out based on the CRUSTPY Python code [17].

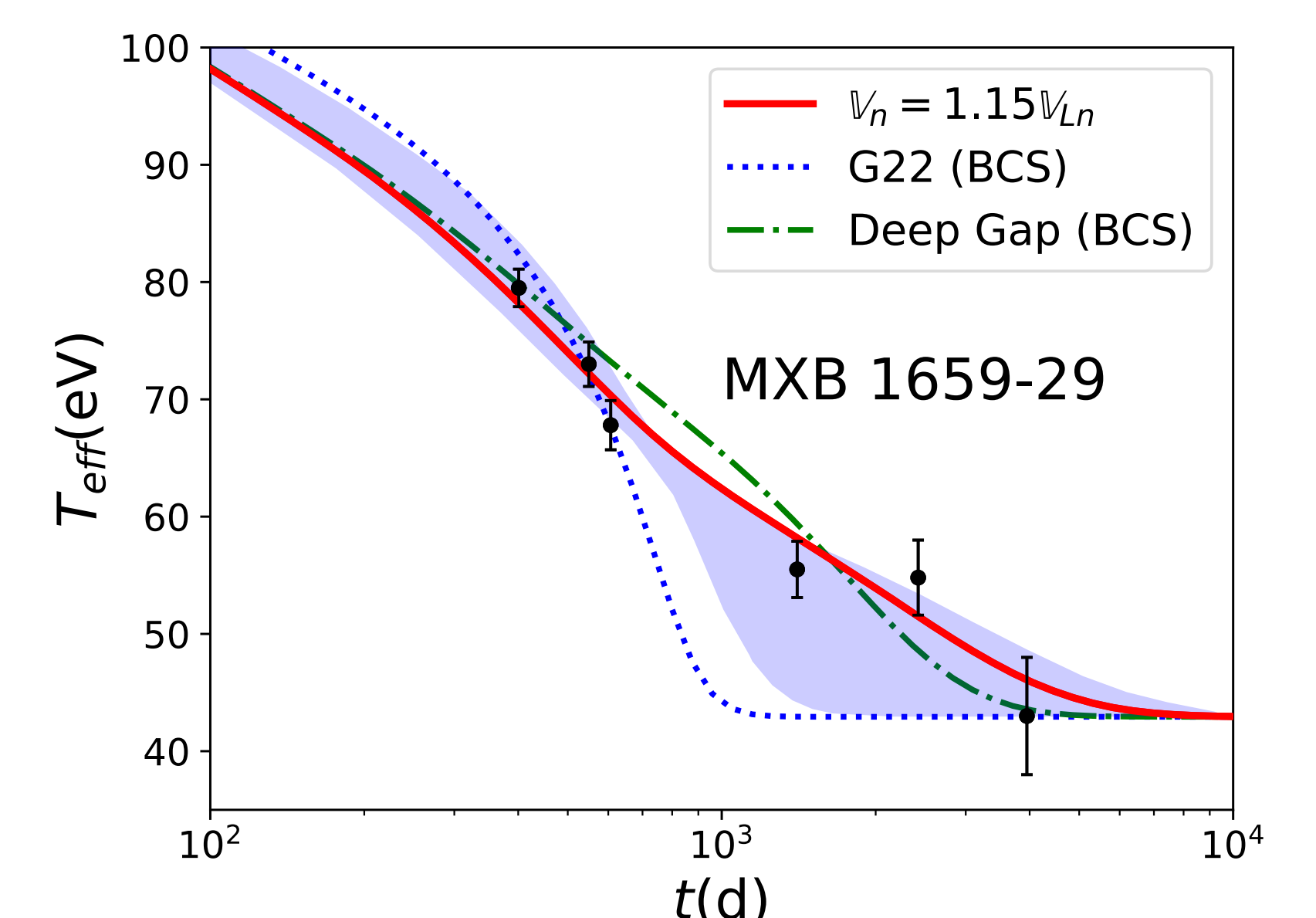
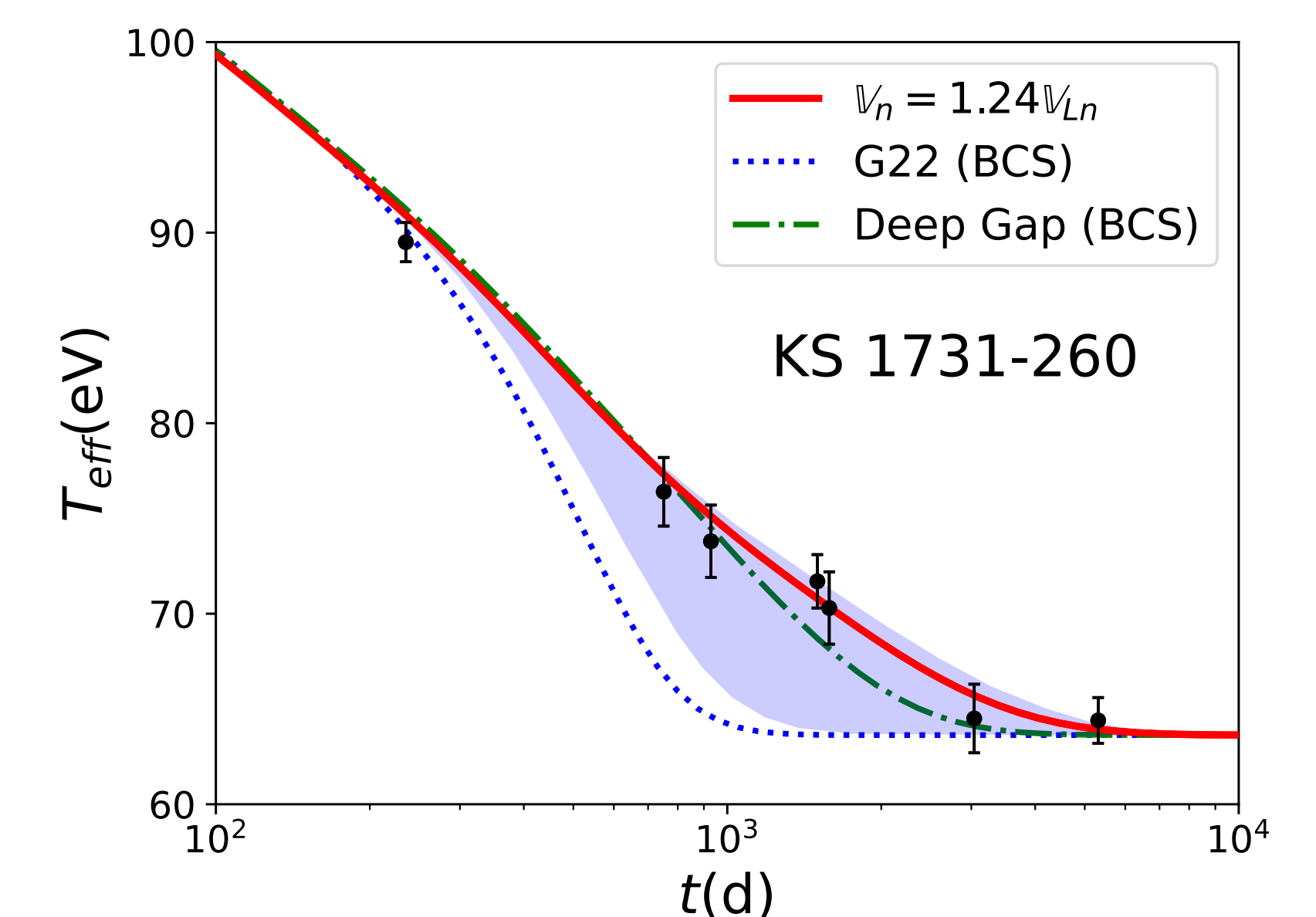
The neutron star crust is made of ions, electrons and free neutrons. **In the gapless phase, neutrons give the main contribution to the total specific heat of the crust  $c_V^{\text{tot}}$  whereas it is negligible in the BCS phase:**



Realistic order parameters from a diagrammatic many-body approach [13] and quantum Monte Carlo calculations [14] are compared to the BCS prediction (no medium effects) and the empirical “deep gap” employed previously in neutron-star cooling simulations:



Whereas the usual BCS superfluidity requires unrealistic order parameters (“deep gap”) to fit the cooling curves, **microscopic calculations can be reconciled with observations by considering gapless superfluidity (in red)**:



## IV Conclusions

We have shown within the nuclear energy density functional theory that the presence of neutron superflow in the inner crust and in the outer core of a neutron star can lead to a regime in which neutrons are superfluid even though the energy spectrum of quasiparticle excitations exhibits no gap. In this gapless phase, the neutron specific heat  $c_V^{(n)}$  is not exponentially suppressed at low temperatures (as it is in the BCS phase) but is comparable to that in the normal phase  $c_N^{(n)}$ . We have shown that gapless superfluidity can naturally explain the late time cooling observed in MXB 1659–29 and KS 1731–29 without invoking unrealistic pairing as in previous studies based on the BCS phase. Gapless superfluidity is likely to play a role in other observed astrophysical phenomena.

### Acknowledgments

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