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Problem Sheet #03

Problem 3.1:

We have $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

1

* Let x be an arbitrary element in $A \cap (B \cup C)$ $(\forall x \in A \cap (B \cup C))$.

$\Longrightarrow x \in A$ and $x \in (B \cup C)$

Since $x \in (B \cup C)$

Then, either $x \in B$ or $x \in C$ or both.

* if $x \in B \Longrightarrow x \in (A \cap B)$
* if $x \in C \Longrightarrow x \in (A \cap C)$

(we know that $x \in A$)

In both cases, x is in either $(A \cap B)$ or $(A \cap C)$, Hence, x is the union of these 2 sets which is $(A \cap B) \cup (A \cap C)$

Hence, if x is in $A \cap (B \cup C)$ Then x is in $(A \cap B) \cup (A \cap C)$

Therefore, LHS is a subset of RHS 1

2

* Let y be an arbitrary element in $(A \cap B) \cup (A \cap C)$

$\Rightarrow y$ is in either $(A \cap B)$ or $(A \cap C)$

* if $y \in (A \cap B) \Rightarrow y \in A ; y \in B$

Since, $y \in A$ and $A \subset (B \cup C)$

Also, $y \in (B \cup C)$

* if $y \in (A \cap C) \Rightarrow y \in A ; y \in C$

we have $A \subset (B \cup C)$

Hence, $y \in (B \cup C)$

in both cases $y \in (B \cup C)$

2 which means RHS is subset of LHS

from (1) and (2) we cann deduct that

$$\boxed{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$$

Problem 3.2:

a-) This proposition is true.

$(x,y) \in (A \cap B) \times (C \cap D) \Leftrightarrow x \in (A \cap B) \wedge y \in (C \cap D)$

(prove by equivalences)

$\Leftrightarrow x \in A \wedge x \in B \wedge y \in C \wedge y \in D$

$\Leftrightarrow (x,y) \in (A \times C) \wedge (x,y) \in (B \times D)$

$\Leftrightarrow (x,y) \in (A \times C) \cap (B \times D)$

b-) This proposition is not true.

(prove by counter example)

$A = \{1\}$; $B = \{2\}$; $C = \{a\}$; $D = \{b\}$

$A \cup B = \{1, 2\}$ $\qquad$ $A \times C = \{(1, a)\}$

$C \cup D = \{a, b\}$ $\qquad$ $B \times D = \{(2, b)\}$

$(A \cup B) \times (C \cup D) = \{(1,a), (1,b), (2,a), (2,b)\} \neq \{(1,a), (2,b)\}$

$\neq (A \times C) \cup (B \times D)$

Problem 3.3:

a-) $R = \{(a,b) \mid a, b \in \mathbb{Z} \wedge |a-b| \leqslant 3\}$

* reflexive, since $|a-a| = 0 \leqslant 3$ $(\forall a \in \mathbb{Z})$
* symmetric, $|a-b| = |b-a| \Longrightarrow |a-b| \leqslant 3$ implies $|b-a| \leqslant 3$.
* not transitive, because $|2-5| \leqslant 3$ and $|5-8| \leqslant 3$ does not imply that $|2-8| \leqslant 3$

b-) $R = \{(a,b) \mid a, b \in \mathbb{Z} \wedge (a \bmod 10) = (b \bmod 10)\}$

* reflexive, $(\forall a \in \mathbb{Z})$ $(a \bmod 10 = a \bmod 10)$
* symmetric, $(a \bmod 10) = (b \bmod 10) \Longleftrightarrow (b \bmod 10) = (a \bmod 10)$
* transitive, $(a \bmod 10) = (b \bmod 10)$ and $(b \bmod 10) = (c \bmod 10)$ implies $(a \bmod 10) = (c \bmod 10)$.

(if a and b have the same last digit and b and c have the same last digit, then a and c have the same last digit as well.

Problem 3.4:

a-) zip :: [a] -> [b] -> [(a,b)]

* there's two variables a and b, they represent the types of elements in the two input lists. the "zip" function combines elements from the two input lists into pairs and return a list of those pairs.

zip fct * No, the number of variables can not be more or less, because precisely specifies that it takes only two lists with potentially different element types and returns a list of pairs.