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Problem Sheet #07

Problem 7.1:

Let's prove that "not-or" is a universal boolean function:

* Implementing AND:

In order to implement the AND function using Not-OR, we can apply the NOT-OR operation twice on the inputs: $AND(P, Q) = Not(Not(P) \text{ OR } Not(Q))$

* Implementing OR:

To implement the OR function using NOT-OR, we can apply the NOT-OR operation to the negation of inputs and then negate the results: $OR(P, Q) = NOT(Not(P)\ Not(Q))$

* Implementing Not:

To implement the Not function using Not-OR, we can apply the Not-OR operation to one of the inputs while maintaining the other input constant at a logical 1 (True). $Not(P) = Not\text{-}OR(P, 1)$

⟹ All in all, we showed how Not-OR can be used to implement AND, OR and Not. Hence, Not-OR is a universal Boolean function.

Problem 7.2:

⟹ We have $F(X,Y,Z) = (((X \land Y) \lor (X \land \neg Z)) \lor (Z \land \neg 0))$

Let's start our derivation:

$F(X,Y,Z) = ((X \land (Y \lor \neg Z)) \lor (Z \land \neg 0))$ Distributivity

$F(X,Y,Z) = ((X \land (Y \lor \neg Z)) \lor 0)$ De Morgan's Law

$F(X,Y,Z) = (X \land (Y \lor \neg Z))$ Indentity

$F(X,Y,Z) = ((X \land Y) \lor (X \land \neg Z))$ Distributivity

$F(X,Y,Z) = (X \lor (X \land \neg Z))$ Absorption

$F(X,Y,Z) = (X \lor 0)$ Distributivity

$F(X,Y,Z) = X$ Indentity

⟹ We have shown that $F(X,Y,Z)$ simplifies to $G(X,Y,Z) = X$, which is equivalent to $G(X,Y,Z) = (X \lor Z)$. Hence, F and G are indeed equivalent.