## Problem 9.1:

a-) The numbers we should represent are from 0 to 6. Hence, we will need three inputs to represent it in binary.

Let's take $x_0, x_1, x_2$ such as $x_0, x_1$ and $x_2$ base 7 representation.

⟹ base 7 numerals can be converted to binary using three binary digits and the following table.

| $x_0$ | $x_1$ | $x_2$ | base 7 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 2 |
| 0 | 1 | 1 | 3 |
| 1 | 0 | 0 | 4 |
| 1 | 0 | 1 | 5 |
| 1 | 1 | 0 | 6 |

⟹ We will then use three input lines $x_0, x_1, x_2$ representing the three binary digits to generate 7 binary patterns representing the base 7 digits from 0 to 6.

| | | | | | | |
|---|---|---|---|---|---|---|
| base 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Binary | 000 | 001 | 010 | 011 | 100 | 101 | 110 |

⟹ We will then use logic gates circuits to control each of the 6 LED based on the three inputs $x_0, x_1, x_2$.

⟹ We will need 7 outputs one for each LED, So let's investigate each LED one at a time.

\* From the previous table we can deduct the following:

LED A is ON : 001; 011; 100; 101; 110
        OFF : 000; 010

LED B, C is ON : 010; 011; 100; 101; 110
         OFF : 000; 001

LED D, F is ON : 101; 110
         OFF : 000; 001; 010; 011; 100

LED E is ON : 100; 110
       OFF : 000; 001; 010; 011; 101

Truth tables:

LED A:

| Input | | | Output |
|---|---|---|---|
| $x_0$ | $x_1$ | $x_2$ | LED A |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |

LED B and C:

| Input | | | Output |
|---|---|---|---|
| $x_0$ | $x_1$ | $x_2$ | LED B and C |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |

## LED D and F:

| Input | | | Output |
|---|---|---|---|
| $x_0$ | $x_1$ | $x_2$ | LED D and F |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |

## LED E:

| Input | | | Output |
|---|---|---|---|
| $x_0$ | $x_1$ | $x_2$ | LED E |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

b-) The previous truth tables can be each converted to a K-map which will help us define the boolean expressions.

# Karnaugh Map:

## LED A:

| $x_2$ \ $x_0 x_1$ | 00 | 01 | 10 | 11 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | - |

$$\implies x_0 + x_2$$

## LED B and C:

| $x_2$ \ $x_0 x_1$ | 00 | 01 | 10 | 11 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | - |

$$\implies x_0 + x_1$$

## LED D and F:

| $x_2$ \ $x_0 x_1$ | 00 | 01 | 10 | 11 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | - |

$$\implies x_0 x_1 + x_0 x_2$$

## LED E:

| $x_2$ \ $x_0 x_1$ | 00 | 01 | 10 | 11 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | - |

$$\implies x_0 x_2$$