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Econometrics

2020480.

(Q2) shoe factory that uses web-based retail platform for marketing & selling their products online. Inputs enjoyed include internet connectivity (bytes of data) and electricity in a matter that the number of pairs sold daily can be represented as

$$S_d = n I_d^{\alpha} E_d^{\beta} e^{\varepsilon_d} \quad -(1)$$

where

S_d = total no. of shoe pairs sold on day d.

I_d = total data bytes used on day d.

E_d = total kilowatt hrs of electricity consumed on day d.

$\varepsilon_d \overset{iid}{\sim} (N)(0, \sigma^2)$, & n, α, β are model parameters.

(a) We know that $S_d = \eta I_d^{\alpha} E_d^{\beta} e^{\epsilon_d}$

Required :- $\ln S_d = \ln \eta + \alpha \ln I_d + \beta \ln E_d + \epsilon_d$

& $\epsilon_d \overset{iid}{\sim} N(0, \sigma^2)$

Define :- $\ln S_d = \tilde{S}_d$, $\ln I_d = \tilde{I}_d$, $\ln E_d = \tilde{E}_d$ &

$\ln \eta = \tilde{\eta}$

See that we must have $(S_d, I_d, E_d) > (0,0,0)$

Transformed model :- $\tilde{S}_d = \tilde{\eta} + \alpha \tilde{I}_d + \beta \tilde{E}_d + \epsilon_d$;

$d = 1, 2, 3 \ldots D$

matrix form :- $\underset{D\times 1}{\tilde{S}} = \underset{D\times 3}{\tilde{X}} \underset{3\times 1}{Y} + \underset{D\times 1}{\epsilon}$

$\underset{1\times 3}{\tilde{X}_d} = [1 \ \tilde{I}_d \ \tilde{E}_d]$ & $\underset{D\times 3}{\tilde{X}} = [\underset{D\times 1}{1} \ \underset{D\times 1}{\tilde{I}} \ \underset{D\times 1}{\tilde{E}}]$

& $\underline{Y} = \begin{bmatrix} \tilde{\eta} \\ \alpha \\ \beta \end{bmatrix}$

$\underset{3\times 1}{Y_{OLS}} = \begin{bmatrix} \hat{\eta}_{OLS} \\ \alpha_{OLS} \\ \beta_{OLS} \end{bmatrix}_{3\times 1} = [\underset{3\times 3}{\tilde{X}' \tilde{X}}]^{-1} \underset{3\times D}{\tilde{X}'} \underset{D\times 1}{\tilde{S}}$

(b) $$\begin{bmatrix} \ln \eta_{OLS} \\ \alpha_{OLS} \\ \beta_{OLS} \end{bmatrix} = \begin{bmatrix} 0.06 & +0.05 & -0.002 \\ -0.05 & 0.1 & -0.07 \\ -0.002 & -0.07 & +0.12 \end{bmatrix}$$

$\eta_{OLS} = \exp(0.714) = 2.04$

$\alpha_{OLS} = 1.09$

$\beta_{OLS} = -27.69$

The least squared estimators are unbaised if

$Cov(\varepsilon_d, \hat{x}_d) = 0$

The least squared estimators when ε_d are normally distributed & homoskedastic, which is are

Per given

→ Unbaised & efficient least squares estimates are consistent

(c) $\hat{\sigma}^2 = \frac{\sum_{i=1}^{D} \hat{\varepsilon}_d^2}{N-k}$

$$= \frac{\sum_{i=1}^{100} \left(\check{S}_d - \tilde{\eta}_{OLS} - \hat{\alpha}_{OLS} \hat{I}_d - \tilde{\beta}_{OLS} \tilde{E}_d \right)^2}{\cancel{Q}}$$

(7a) Random sample of size N denoted as $\{x_i, y_i\}_{i=1}^{100}$ where x & y are R.V.

1. A regression of Y on X i.e., $Y_i = \beta_{0y} + \beta_{xy} x_i + u_i$

2. " " " X on Y i.e., $X_i = \beta_{0x} + \beta_{yx} y_i + e_i$

3. A correlation coefficient i.e., r.

S.T:- $r = \pm \sqrt{\hat{\beta}_{xy}^{OLS}, \hat{\beta}_{yx}^{OLS}}$

regression lines are X on Y $3X - 2Y = 5$

" " " Y on X $= X - 4Y = 5$

We know $B_{yx} = \dfrac{\sum_{y=1} (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$

$$\hat{B}_{xy} \times \hat{B}_{yx} = \frac{\sum_{y=1}^{m} (y_i - \bar{y})(x_i - \bar{x}_i)}{\sum_{y=1}^{m} (x_i - \bar{x})^2} \times \frac{\sum_{y=1}^{m} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{y=1}^{m} (y_i - \bar{y})^2}$$

$$\hat{B}_{xy} \times \hat{B}_{yx} = \frac{(\mathrm{Cov}(x, y))^2}{\mathrm{Var}(x)\,\mathrm{Var}(y)}$$

$$\left[\because \frac{\sum_{i=1}^{m} (x_i - \bar{x})(y - \bar{y})}{(\sum (x_i - \bar{x})^2)} = \frac{\mathrm{Cov}(x,y)}{\mathrm{Var}(x)}\right.$$

$$= r^2$$

$$r = \pm\sqrt{r^2}$$

$$r = r$$

L.H.S = R.H.S

Hence proved

$3x - 2y = 5$

$3x = 5 + 2y$

$x = \frac{5}{3} + \frac{2}{3}y$

$b_{xy} = 2/3$

$x - 4y = 7$

$4y = x - 7$

$y = \frac{x}{4} - 7/4$

$b_{yx} = 1/4$

Coefficient of Correlation $= \sqrt{b_{xy} \times b_{yx}}$

$= \sqrt{2/3 \times \frac{1}{4}}$

$= \frac{1}{\sqrt{6}}$.

(4b)

Covariance measure derived from the data of 2 arbitrary R.V L, m.

$i = 1, 2, \ldots \to N$

$$\mathrm{cov}(L, m)_{i=1}^{N} = \frac{\sum_{i=1}^{N} (l_i - \bar{l})(m_i - \bar{m})}{N}$$

(1) $\hat{\sigma}_x^2 = 2$

(1) $\hat{\beta}_{xy\,OLS} = \hat{\beta}_{xy} + \frac{\mathrm{cov}(x, y)}{\mathrm{var}(x)}$

$\hat{\beta}_{xy}, OLS = 0/2 = 0$ [Bias in OLS]

(2) $\mathrm{cov}(x, y)_{i=1}^{100} = 2$

$\hat{\beta}_{xy}, OLS = 2/2 = 1$

(3) $\mathrm{cov}(x, y)_{i=1}^{50} = 0$ $\mathrm{cov}(x, y)_{i=51}^{100} = -2$

$$\mathrm{cov}(x, y)_{i=1}^{100} = \frac{0 \times 50 + -2 \times 50}{100}$$

$= -1$ $\hat{\beta}_{OLS} = -1/2$

(4) $Cov(x,y)_{i=1}^{50} = 0$ $\quad Cov(x,y)_{i=51}^{75} = -2$

$Cov(x,y)_{i=75}^{100} = 2$

$$Cov(x,y)_{i=1}^{100} = \frac{0 \times 50 + (-2 \times 25) + (2 \times 25)}{100}$$

$= 0$

$\rho_{x,y} = 0/2 = 0$