Espaços Vetoriais Reais

Álgebra Linear e Geometria Analítica A **Soluçõs da Folha Prática 2**

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1. (a) Não; (b) Sim; (c) Sim;
2. (a) i. Sim; ii. Não.
   (b) Não.
   (c) i. Sim; ii. Não; iii. Não.
   (d) i. Sim; ii. Não; iii. Não; iv. Sim; v. Não.
   (e) Sim.
4. (a) $(2,-3,-4,3) = -2(1,2,1,0) + (4,1,-2,3)$;
   (b) $(1,1,0) = \frac{1}{3}(2,1,-2) - \frac{1}{3}(1,0,0) + \frac{2}{3}(1,1,1)$;
   (c) e (d) Não é possível.
5. (a) $\mathbb{R}^2$; (b) $\{(x,y) \in \mathbb{R}^2 : x = 0\}$; (c) $\mathbb{R}^3$; (d) $\{(x,y,z) \in \mathbb{R}^3 : y = z\}$; (e) $\mathcal{P}_2$.
6. $\{(-1,-1,1,0),(0,-1,0,1)\}$.
9. (a) Não; (b) Sim; (c) Não; (d) Sim.
10. Sim.
12. (a) Não; (b) Sim; (c) Não; (d) Sim; (e) Sim; (f) Sim.
13. (a) $\{(1,0,0),(0,1,0)\}$, dimensão 2;
    (b) $\{(1,-1,1),(0,2,1)\}$, dimensão 2;
    (c) $\{t^2+1,t\}$, dimensão 2.
    Nota: Em (a) e (c), o conjunto dado também constitui uma base.
14. $a \in \mathbb{R} \setminus \{-1,0,1\}$.
15. $\{(1,0,1,0),(0,1,-1,0),(0,0,1,0),(0,0,0,1)\}$.
16. (b) $\{(1,1,0),(0,3,1)\}$ que é l.i.; (c) 2.
18. (a) $[(-1,2,-6,5)]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -3 \\ -11 \\ 5 \end{bmatrix}$; (b) $[(2,1,0,0)]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$; (c) $[(1,2,3,4)]_{\mathcal{B}} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 4 \end{bmatrix}$.
19. (a) i. $[(2,3,5)]_{\mathcal{B}_1} = \begin{bmatrix} 2 \\ -\frac{1}{2} \\ -3 \end{bmatrix}$ e $[(2,3,5)]_{\mathcal{B}_2} = \begin{bmatrix} -\frac{6}{5} \\ \frac{18}{5} \\ -\frac{1}{5} \end{bmatrix}$;
    ii. $[(-1,2,0)]_{\mathcal{B}_1} = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$ e $[(-1,2,0)]_{\mathcal{B}_2} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$;
    iii. $[(1,1,1)]_{\mathcal{B}_1} = \begin{bmatrix} 1 \\ -\frac{1}{2} \\ 0 \end{bmatrix}$ e $[(1,1,1))]_{\mathcal{B}_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.
    (b) $P = \frac{1}{5}\begin{bmatrix} -3 & -6 & 1 \\ 4 & -2 & -3 \\ 2 & 4 & 1 \end{bmatrix}$.
20. (a) $[X]_T = \begin{bmatrix} -7 \\ 4 \end{bmatrix}$;
    (b) $Z = (-5,-8)$;

(c) $P = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$;

(d) $[X]_S = P[X]_T = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$;

(f) $Q = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$.

21. $T = \{(1,1,1),(0,1,0),(-1,2,2)\}$

22. (a) Verdadeira. (b) Falsa. (c) Falsa. (d) Verdadeira. (e) Falsa. (f) Falsa. (g) Falsa. (h) Falsa. (i) Verdadeira.

23. (a) $\{(-1,1,0,0),(-2,0,1,1)\}$ e $\operatorname{nul} A = 2$.

    (b) $\{(a,b,c) \in \mathbb{R}^3 : \ c = a + 2b\}$.

24. i. (a), (d), (f), (g), (h) $\emptyset$;

    (b) $\{(-8,7,4,0),(-4,5,0,4)\}$;

    (c) $\{(5,-2,-9,13,0),(1,2,0,-1,-3)\}$;

    (e) $\{(0,0,1,0)\}$.

    ii. (a) $B_{\mathcal{L}(A)} = \{(1,0,0),(0,1,0),(0,0,1)\}$;
    $B_{\mathcal{C}(A)} = \{(1,2,-3,1),(0,1,2,2),(0,0,1,2/3)\}$;

    (b) $B_{\mathcal{L}(A)} = \{(1,0,2,1),(0,1,-7/4,-5/4)\}$;
    $B_{\mathcal{C}(A)} = \{(1,0),(0,1)\}$;

    (c) $B_{\mathcal{L}(A)} = \{(1,2,3,2,1),(0,1,9/5,7/5,1/5),(0,0,1,9/13,3/13)\}$,
    $B_{\mathcal{C}(A)} = \{(1,0,0),(0,1,0),(0,0,1)\}$;

    (d), (f) $B_{\mathcal{L}(A)} = B_{\mathcal{C}(A)} = \{(1,0,0),(0,1,0),(0,0,1)\}$;

    (e) $B_{\mathcal{L}(A)} = B_{\mathcal{C}(A)} = \{(1,0,0,0),(0,1,0,0),(0,0,0,1)\}$;

    (g) $B_{\mathcal{L}(A)} = \{(1,-2,-1),(0,1,5/3),(0,0,1)\}$;
    $B_{\mathcal{C}(A)} = \{(1,2,1,1),(0,1,2/3,1),(0,0,1,-9/2)\}$;

    (h) $B_{\mathcal{L}(A)} = B_{\mathcal{C}(A)} = \{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$.

    Nota: Em (a) e (g), as colunas da matriz dada também constituem uma base.
    Em (b) e (c), as linhas da matriz dada também constituem uma base.
    Em (d), (f) e (h), as linhas/colunas da matriz dada também constituem bases.

    iii. (a) $\operatorname{car} A = 3$, $\operatorname{nul} A = 0$; (b) $\operatorname{car} A = 2$, $\operatorname{nul} A = 2$; (c) $\operatorname{car} A = 3$, $\operatorname{nul} A = 2$;
    (d) $\operatorname{car} A = 3$, $\operatorname{nul} A = 0$; (e) $\operatorname{car} A = 3$, $\operatorname{nul} A = 1$; (f) $\operatorname{car} A = 3$, $\operatorname{nul} A = 0$;
    (g) $\operatorname{car} A = 3$, $\operatorname{nul} A = 0$; (h) $\operatorname{car} A = 4$, $\operatorname{nul} A = 0$.

    iv. (b), (c), (d), (f), (h) Sim.