

MT241P FINITE MATHEMATICS

4. ASSIGNMENT

Submit your solutions to all questions by Friday 4pm, 9.12.2022

Question 4.1. Suppose Met Éirann provides the following predictions:

- (P1) There is a 60 percent chance that it will rain today.
 - (P2) There is a 50 percent chance that it will rain tomorrow.
 - (P3) There is a 30 percent chance that it will not rain either day.
- (1) Let 'R' indicate a day with rain and 'N' a day with no rain. List an appropriate sample space Ω .
 - (2) Let A be the event that it rains today and let B be the event that it rains tomorrow. List the outcomes of the following events:
 - (i) A^c , (ii) $A \cup B$, (iii) $A \cap B$, (iv) $A \cap B^c$, (v) $(A \cup B)^c$
 - (3) Find the probabilities for the following events:
 - (i) It will rain today or tomorrow.
 - (ii) It will rain today and tomorrow.
 - (iii) It will rain today but not tomorrow.
 - (iv) It will rain today or tomorrow, but not both days.

Question 4.2. Assume you flip a fair coin with a friend n times, where $n \geq 1$.

- (1) How many possible outcomes are there?
- (2) What is the probability of each outcome?
- (3) If you throw the coin ten times, what is the chance of there being exactly one tail in any three consecutive throws?

Next you play a game for money. Each time heads comes up you win a Euro, each time tails comes up you lose a Euro. However, as soon as you lose for the first time you claim you have to go home and stop playing.

- (4) Describe the sample space Ω in terms of your possible wins/losses.
- (5) For each outcome $\omega \in \Omega$ give its probability.
- (6) What is the probability of you winning at least one, but less than four Euro?
- (7) What is the probability of you winning more than two Euro?

Question 4.3. You are at a party attended by k people, including you. What is the likelihood of somebody else at the party sharing your birthday? (We assume that nobody was born in a leap year). What is the likelihood if $k = 23$?

Question 4.4. Use combinatorial arguments to prove that, for every integer $n \geq 0$,

$$\sum_{k=0}^n \binom{n}{k} = 2^n$$

Question 4.5. Let A and B be finite sets such that A has n elements and B has m elements, where $n \geq m$. How many injective functions $f : A \rightarrow B$ are there, that is, functions where $f(a_1) \neq f(a_2)$, whenever $a_1 \neq a_2$.

Question 4.6. *In how many ways can $2n$ tennis players be paired and assigned to n courts?*

Question 4.7. *How many distinct integer solutions does the equation*

$$x_1 + x_2 + x_3 + x_4 = 100$$

have, if

- (1) $x_i \geq 0$, for all $i = 1, 2, 3, 4$,
- (2) $x_i \geq i$, for all $i = 1, 2, 3, 4$.

Question 4.8. *In my home town, it rains one third of all days. Traffic is heavy on half of the rainy days and a quarter of the dry days. If it's rainy and the traffic is heavy, then I am bound to be late for work half of all days. A quarter of the days that I'm late, it is not rainy but the traffic is heavy. Whenever there is light traffic, I am twice as likely to be late on a rainy day, compared to dry days. I am late for work one quarter of all days.*

- (1) *Draw the tree diagram, where the first stage gives rain / no rain, the second stage gives traffic / no traffic and the third stage gives late / not late.*
- (2) *What is my chance of being on time on a rainy day with light traffic?*
- (3) *What is my chance of being on time on a dry day?*
- (4) *Given I was late today, what is the chance of there having been light traffic?*
- (5) *Given I was on time today and there was light traffic, what is the chance of there having been rain?*