MT241P FINITE MATHEMATICS

1. Assignment

Attempt all questions and submit your solutions to the indicated problems by 4pm, Friday, 14.10.2022: Q.1.1(1), Q.1.2, Q.1.4, Q.1.5(1), Q.1.6

Question 1.1. *Use induction to show that*

(1) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$, *for all integers* $n \geq 1$

(2) $n^2 < 2^n$, *for all integers* $n \geq 5$.

Question 1.2. *Let* f *be a function in two variables such that for all positive integers* m, n

(1) $f(1,1) = 2$

(2) $f(m+1, n) = f(m,n) + 2(m+n)$

(3) $f(m, n+1) = f(m,n) + 2(m+n-1)$

Prove that for all positive integers m, n

$$(\star)\ f(m,n) = (m+n)^2 - (m+n) - 2n + 2$$

Steps: (1) Verify $(\star)$ for $(m,n) = (1,1)$. (2) Use induction on m to show that $(\star)$ holds for all pairs $(m,1)$, where $m \in \mathbb{N}$. (3) Given a fixed $m \in \mathbb{N}$, use induction on n to verify show that $(\star)$ holds for all pairs (m,n), where $n \in \mathbb{N}$.

Question 1.3. *Recall the Lucas Sequence from the lectures given by* $T_1 = 1, T_2 = 3$ *and* $T_n = T_{n-1} + T_{n-2}$, *for* $n \geq 3$. *In class we have shown that*

$$T_n < \left(\frac{7}{4}\right)^n, \quad \text{for all } n \geq 1.$$

Find a rational number $x < 7/4$ *such that* $T_n < x^n$, *for all* $n \geq 1$.

Question 1.4. *Let* $a, b, c \in \mathbb{Z}$. *Prove that*

(1) *If* $a \mid b$ *and* $b \mid c$, *then* $a \mid c$.

(2) *If* $a \mid b$ *and* $a \mid c$ *and* $u, v \in \mathbb{Z}$, *then* $a \mid (ub + vc)$.

Question 1.5.

(1) *Show that* 2003 *divides* $4007^n - 1$, *for all integers* $n \geq 0$.

(2) *Show that* $2002^n + 2003^n$ *is divisible by* 4005, *for all odd integers* $n \geq 1$.

Question 1.6. *Show that* 5 *divides* $n^5 - n$, *for any* $n \in \mathbb{Z}$.