MT251P – Lecture 8

Fiacre Ó Cairbre

Remark 2.

Once we know the intial point A and the terminal point B of a vector $\underline{v}$, then we can think of $\underline{v}$ in terms of the coordinates of A and B.

For example, denote the usual two–dimensional xy–plane by

$$\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}\}$$

and suppose

$$A = (x_1, y_1),\ B = (x_2, y_2) \in \mathbb{R}^2$$

Then the vector $\underline{v} = \vec{AB}$ can be written in many ways including:

$$\underline{v} = (x_2 - x_1, y_2 - y_1)$$

$$\underline{v} = v_1 i + v_2 j, \quad \text{where} \quad v_1 = x_2 - x_1,\ v_2 = y_2 - y_1$$

$\underline{v}$ is called the displacement vector.

Example 1.

Suppose $\underline{v} = \vec{AB}$, where $A = (2, 1),\ B = (4, -6)$. Then,

$$\underline{v} = (4 - 2)i + (-6 - 1)j = 2i - 7j$$

Remark 3.

$0i + 0j$ is called the zero vector.

Theorem 1.

Suppose $\underline{w} = w_1 i + w_2 j$, $\underline{v} = v_1 i + v_2 j$ and $t \in \mathbb{R}$. Then

(a) $\underline{w} + \underline{v} = (w_1 + v_1)i + (w_2 + v_2)j$.

(b) $t\underline{w} = tw_1 i + tw_2 j$.

(c) The magnitude (or length) of $\underline{v}$ is denoted by $||\underline{v}||$ and satisfes

$$||\underline{v}|| = \sqrt{{v_1}^2 + {v_2}^2}$$

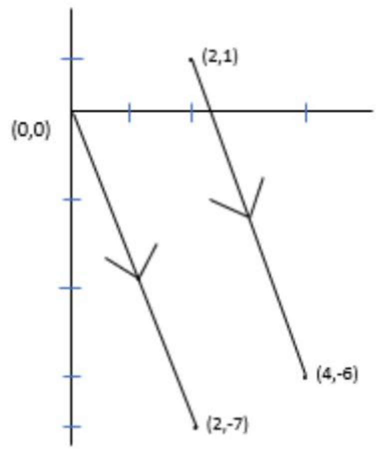
(d) The dot (or scalar) product of $\underline{w}$ and $\underline{v}$ is denoted by $\underline{w}.\underline{v}$ and is defined by

$$\underline{w}.\underline{v} = w_1v_1 + w_2v_2$$

(e) Suppose neither $\underline{w}$ nor $\underline{v}$ is the zero vector. Then, $\underline{w}.\underline{v} = ||\underline{w}||||\underline{v}|| \cos\theta$, where θ is the angle between $\underline{w}$ and $\underline{v}$ and $0 \leq \theta \leq \pi$.

Example 2.

Suppose $E = (0,0)$, $F = (2,-7)$ and $\underline{w} = \vec{EF}$. Then, $\underline{w} = \underline{v}$ from example 1 because $\underline{w} = 2i - 7j$. This is an example of the fact that $\vec{AB}$ (from example 1) may equal $\vec{EF}$ even though $A \neq E$ and $B \neq F$. The reason that $\vec{AB}$ and $\vec{EF}$ are actually the same vector is because $\vec{AB}$ and $\vec{EF}$ both have the same magnitude and direction.
The following picture shows $\vec{AB}$ and $\vec{EF}$



Example 3.

Suppose $\underline{w} = 2i - 3j$, $\underline{v} = 4i + 2j$. Then

$$\underline{w} + \underline{v} = 6i - j$$

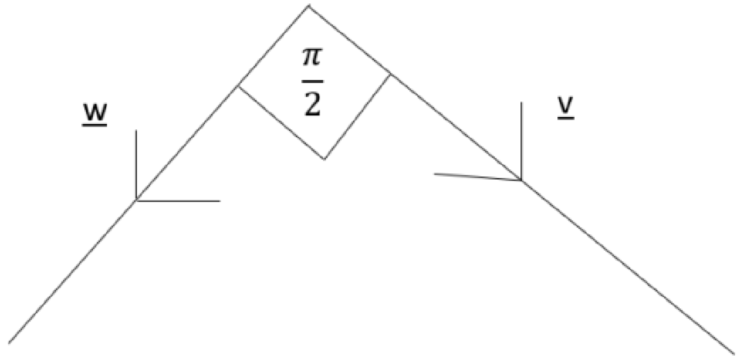
$$2\underline{v} = 8i + 4j$$

$$||\underline{w}|| = \sqrt{4+9} = \sqrt{13}$$

$$\underline{w}.\underline{v} = 2(4) + (-3)2 = 2$$

Definition 1.

The non–zero vectors $\underline{v}, \underline{w}$ are said to be perpendicular (or orthogonal) if the angle between $\underline{v}$ and $\underline{w}$ is $\frac{\pi}{2}$.



We also define the zero vector to be perpendicular to any vector.

Remark 4.

Two vectors $\underline{v}$, $\underline{w}$ in $\mathbb{R}^2$ are perpendicular $\iff$ $\underline{v}.\underline{w} = 0$

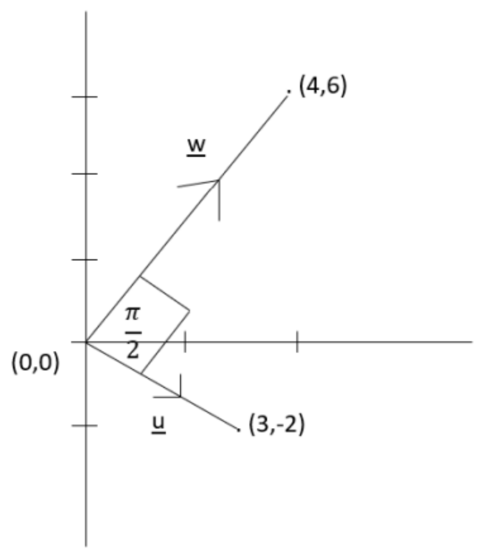
Example 4.

Supppose $\underline{u} = 3i - 2j$ and $\underline{w} = 4i + 6j$. Then, find the angle between $\underline{u}$ and $\underline{w}$.

Solution.

Note that $\underline{u}.\underline{w} = 0$ and so $\underline{u}$ and $\underline{w}$ are perpendicular. Hence, the angle between $\underline{u}$ and $\underline{w}$ is $\frac{\pi}{2}$.

Here is a picture of this example:



Remark 5.

Suppose $A = (0,0)$, $B = (x_1, x_2)$. Then the vector $\underline{v} = \vec{AB}$ is called a position vector. So, a position vector is a vector that has the origin $(0,0)$ as initial point.