DEPARTMENT OF MATHEMATICS: COURSE MT232P

PROBLEM SHEET 1

DEADLINE: 4pm Friday 19 February

1. Carefully prove that for any sets A and B the following equation holds:

$$A = (A \cap B) \cup (A \setminus B).$$

   (Hint: argue that the subset relations $A \subset (A \cap B) \cup (A \setminus B)$ and $A \supset (A \cap B) \cup (A \setminus B)$ both hold. Note that a Venn diagram does not constitute a valid argument.)

2. Let $a,\ b \in \mathbb{R}$ and $a < b$. Find a bijection from (a, b) to $(0, 1)$. Justify your answer.

3. Prove that a function $f : A \to B$ which possesses an inverse must be a bijection.

4. (a) Consider a function $f : A \to B$. Show that setting $a_1 \sim a_2$ if $f(a_1) = f(a_2)$ defines an equivalence relation on A.

   (b) Identify the equivalence classes under this equivalence relation if $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = x^2$.

   (c) In the special case of the 'floor' function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \lfloor x \rfloor$, where $\lfloor x \rfloor$ indicates the greatest integer less than or equal to x, identify the equivalence classes under this equivalence relation.

5. Let C be the set of counties in Ireland.

   (a) Give an example of an equivalence relation on C. What are the equivalence classes of this relation?

   (b) Give another example of an equivalence relation on C.

   (c) Give an example of a relation on C which is not an equivalence relation.