MT241P - Finite Mathematics

Assignment #4

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Question 1

Suppose Met Eirann provides the following predictions:
(P1) There is a 60 percent chance that it will rain today.
(P2) There is a 50 percent chance that it will rain tomorrow.

Part A

Let 'R' indicate a day with rain and 'N' a day with no rain. List an appro- priate sample space Ω.

Solution

$$\Omega = \{RR, RN, NR, NN\}$$

Part B

Let A be the event that it rains today and let B be the event that it rains tomorrow. List the outcomes of the following events:

- A^c
- $A \cup B$
- $A \cap B$
- $A \cap B^c$
- $(A \cup B)^c$

Solution

$$A^c = \{NR, NN\}$$
$$A \cup B = \{RR, RN, NR\}$$
$$A \cap B = \{RR\}$$
$$A \cap B^c = \{RN\}$$
$$(A \cup B)^c = \{NN\}$$

Question 1 continued on next page...

Part C

Find the probabilities for the following events:

- It will rain today or tomorrow.
- It will rain today and tomorrow.
- It will rain today but not tomorrow.
- It will rain today or tomorrow, but not both days.

Solution

"It will rain today or tomorrow." is the same as $A \cup B$,and therefore form part A:

$$P(A \cup B) = P(RR) + P(RN) + P(NR) =$$

$$= P(A)P(B) + P(A)P(B^c) + P(A^c)P(B) =$$

$$= 0.6 * 0.5 + 0.6 * 0.5 + 0.4 * 0.5 = 0.8$$

"It will rain today and tomorrow." is the same as $A \cap B$,and therefore form part A:

$$P(A \cap B) = P(RR) =$$

$$= P(A)P(B) =$$

$$= 0.6 * 0.5 = 0.3$$

"It will rain today but not tomorrow." is the same as $A \cap B^c$,and therefore form part A:

$$P(A \cap B^c) = P(RN) =$$

$$= P(A)P(B^c) =$$

$$= 0.6 * 0.5 = 0.3$$

"It will rain today or tomorrow, but not both days." has the following outcomes: RN, NR. Then The probability for that event (Let's call it C) will be:

$$P(C) = P(RN) + P(NR) =$$

$$= P(A)P(B^c) + P(A^c)P(B) =$$

$$= 0.6 * 0.5 + 0.4 * 0.5 = 0.5$$

Question 2

Assume you flip a fair coin with a friend n times, where n ≥ 1

Part A

Question 2 continued on next page. . .

How many possible outcomes are there?

Solution

There are 2^n solutions since on each throw only one of two things can happen. The coin will either fall on heads or on tails.

Part B

What is the probability of each outcome?

Solution

Since the coin is a fair coin, then the probability of heads is the same as the probability of tails, which is $(\frac{1}{2})^n$, where n is the number of throws.

Part C

If you throw the coin ten times, what is the chance of there being exactly one tail in any three consecutive throws?

Solution

If we represent a tail with a T and a head with an H, then we can have the following configurations for three consecutive throws that satisfy the prompt:

- HHT
- HTH
- THH

Next you play a game for money. Each time heads comes up you win a Euro, each time tails comes up you lose a Euro. However, as soon as you lose for the first time you claim you have to go home and stop playing.

Part D

Describe the sample space Ω in terms of your possible wins/losses.

Solution

$$\Omega = \{T, HT, HHT, HHHT, HHHHT, HHHHHT, \dots\}$$

Part E

Question 2 continued on next page. . .

For each outcome $\omega \in \Omega$ give its probability.

Solution

$\omega = (\frac{1}{2})^n$, where n is the index of the element in Ω

Part F

What is the probability of you winning at least one, but less than four Euro?

Solution

The probability of winning at least 1, but less than 4 euro is the same as the probability of getting 1, 2, or 3 Euro, which is the same as the probability of getting 1 Euro + the probability of getting 2 Euro + the probability of getting 3 Euro, which is the same as:

$$P(HHT) + P(HHHT) + P(HHHHT) =$$
$$= P(H)P(H)P(T) + P(H)P(H)P(H)P(T) + P(H)P(H)P(H)P(H)P(T) =$$
$$= \frac{1}{2}\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2} =$$
$$= \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} =$$
$$= \frac{2^2}{2^5} + \frac{2}{2^5} + \frac{1}{2^5} =$$
$$= \frac{7}{2^5}$$

Part G

What is the probability of you winning more than two Euro?

Solution

The probability of winning more than two Euro, is the same as winning 3 or more Euro, which is the same as getting at least 4 heads, allowing for the next throw to leave us with at least 3 euro even if the throw gives a tail. The probability of that is:

$$P(HHHH) =$$
$$= P(H)P(H)P(H)P(H) =$$
$$= \frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2} =$$
$$= \frac{1}{16}$$

Question 3

You are at a party attended by k people, including you. What is the likelihood of somebody else at the party sharing your birthday? (We assume that nobody was born in a leap year). What is the likelihood if k = 23?

Solution

k = 23 = 22 + 1(you)

The chance for each person to have the same birthday as you is: $\frac{1}{365}$ The chance for each person not to have the same birthday as you is: $\frac{364}{365}$ Then the chance for no one to have the same birthday as you is: $(\frac{364}{365})^{22}$

Question 4

Use combinatorial arguments to prove that, for every integer n ≥ 0,

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

Solution

If we say n is a number of balls, the left side of the equation gives the possible subsets of n balls. As each ball can either be in the subset or not, then there must be 2^n possible subsets for n balls. Therefore we get: $\sum_{k=0}^{n} \binom{n}{k} = 2^n$

Question 5

Let A and B be finite sets such that A has n elements and B has m elements, where n ≥ m. How many injective functions $f : A \longrightarrow B$ are there, that is, functions where $f(a_1) \neq f(a_2)$, whenever $a_1 \neq a_2$.

Solution

If A has n elements, and B has m elements, and the function is injunctive, then:

$\binom{n}{m}$ is the number of sets of size m possible to make with n elements

$m!$ is the number of orientations you can have m elements in

Therefore the amount of injective functions is $\binom{n}{m}m!$

Question 6

In how many ways can 2n tennis players be paired and assigned to n courts?

Solution

Since there are 2n players and n courts with 2 players per court, then the possible pairs for the first court are $\binom{2n}{2}$. Since we already picked 2 people, then the possible pairs for the second court are $\binom{2(n-1)}{2}$. With that knowledge we can deduce that the combinations are:

$$\binom{2n!}{(2!)^n}$$

Question 6 continued on next page. . .

Question 7

How many distinct integer solutions does the equation:

$$x_1 + x_2 + x_3 + x_4 = 100$$

heve, if:

Part A

$x_i \geq 0$, for all i = 1, 2, 3, 4

Solution

Part B

$x_i \geq i$, for all i = 1, 2, 3, 4

Solution

Question 8

In my home town, it rains one third of all days. Traffic is heavy on half of the rainy days and a quarter of the dry days. If it's rainy and the traffic is heavy, then I am bound to be late for work half of all days. A quarter of the days that I'm late, it is not rainy but the traffic is heavy. Whenever there is light traffic, I am twice as likely to be late on a rainy day, compared to dry days. I am late for work one quarter of all days.

Part A

Draw the tree diagram, where the first stage gives rain / no rain, the second stage gives traffic / no traffic and the third stage gives late / not late.

Solution

Question 8 continued on next page. . .

Part B

What is my chance of being on time on a rainy day with light traffic?

Solution

Part C

What is my chance of being on time on a dry day?

Solution

Part D

Given I was late today, what is the chance of there having been light traffic?

Solution

Part E

Given I was on time today and there was light traffic, what is the chance of there having been rain?

Solution