

Department of Mathematics and Statistics: MT232P

Assignment 4

Deadline: 4pm Monday 12 December.

Please upload your solutions as a pdf file to the MT232P Moodle page.

1. Let $x_1 = 1$, and set $x_{n+1} = \sqrt{2 + x_n}$ for all $n \in \mathbb{N}$. Use the Monotone Convergence Theorem to show that $\{x_n\}_1^\infty$ converges, and find $\lim_{n \rightarrow \infty} x_n$.
2. Show that if every subsequence of $\{a_n\}_1^\infty$ has itself a subsequence which converges to 0, then $\{a_n\}_1^\infty$ converges to 0.
3. Assume $\{a_n\}_1^\infty$ and $\{b_n\}_1^\infty$ are Cauchy sequences. Use a triangle inequality argument to prove directly from the definition of a Cauchy sequence that $\{c_n\}_1^\infty$, where $c_n = |a_n - b_n|$, is also a Cauchy sequence.
4. What is the value of $\lim_{x \rightarrow 1} \frac{1}{1 + \sqrt{x}}$? Prove your assertion using an $\epsilon - \delta$ argument.