

DEPARTMENT OF MATHEMATICS & STATISTICS

MT 251P

Homework 1

Due by 4 p.m. on October 21, 2022

1. State whether or not the following statements are true or false. Justify your answers.
  - (a)  $\sqrt[3]{2}$  is irrational.
  - (b)  $\frac{x}{y}$  is irrational for all irrational numbers  $x, y$  with  $x \neq y$ .
  - (c) There exist rational numbers  $m, t, w$  such that  $m - 2w + t\sqrt{2} = 0$
2. Prove that  $\sqrt{5}$  is irrational.
3. Prove that  $\neg(\neg P \vee Q) \vee (P \wedge Q) \sim P$ .
4. Is  $(\neg P \wedge Q) \Rightarrow (Q \vee \neg P)$  a tautology? Justify your answer.
5. Are there integers  $x, y$  such that  $9y - x^2 + 3 = 0$ ? Justify your answer.