MT241P FINITE MATHEMATICS

3. Assignment

Submit your solutions to to following questions by Friday 4pm, 18.11.2022:
Q.3.2, Q.3.4(2)+(7), Q.3.5, Q.3.6(1), Q.3.7(3), Q.3.10, Q.3.12, Q.3.13

Question 3.1. *Determine the canonical prime factorisation of* $55!$.

Question 3.2. *Let* p *be a prime number. Prove that* $\sqrt{p}$ *is not a rational number.*

Question 3.3. *Adapt the approach used in Remark 6.9 to devise and verify a divisibility test for* 3, 5 *and* 7*, respectively.*
Hint: *For* 7*, show that* $7 \mid a$ *if and only if* $7 \mid \left(\frac{a-d_1}{10} - 2d_1\right)$.

Question 3.4. *Use the theory of congruences to answer the following:*

(1) *What is the remainder when* 3^{84} *is divided by* 47*?*
(2) *What are the last two digits of* 67^{165}*?*
(3) *Does* 17 *divide* $19^{18} - 16$*?*
(4) *Is* $\frac{111^{111} - 21^{12}}{11} \in \mathbb{Z}$*?*
(5) *Find the remainder when* 5^{17} *is divided by* 9.
(6) *Prove that* $2^{1999} + 3^{1999} + 4^{1999}$ *is not prime.*
(7) *Today is Saturday. What day of the week will be in* $6^6 + 7^7 + 8^8$ *days?*

Question 3.5. *For each integer* $a \in \{15, 16, \ldots, 24\}$*, find the number of solutions of the linear congruence* $ax \equiv 27 \pmod{36}$.

Question 3.6. *Find the least non-negative solutions, if any, of the following linear congruences:*

(1) $329x \equiv 168 \pmod{889}$,
(2) $4125x \equiv 1486 \pmod{5973}$.

Question 3.7. *Use the Chinese Remainder Theorem to find the least positive integer* x*, if any, such that*

(1) $x \equiv 1 \pmod{10}, \quad x \equiv 6 \pmod{21}, \quad x \equiv 11 \pmod{31}$,
(2) $x \equiv 19 \pmod{36}, \quad x \equiv 35 \pmod{77}$,
(3) $2x \equiv 1 \pmod{5}, \quad 3x \equiv 9 \pmod{6}, \quad 4x \equiv 1 \pmod{7}, \quad 5x \equiv 9 \pmod{11}$,
(4) $x \equiv 1 \pmod{10}, \quad x \equiv 5 \pmod{24}, \quad x \equiv 17 \pmod{18}, \quad x \equiv 3 \pmod{38}$,
(5) $3x \equiv 43 \pmod{50}, \quad 11x \equiv 113 \pmod{120}, \quad 7x \equiv 233 \pmod{270}$.

Question 3.8. *Find all integers that have remainders* $2, 3, 4, 5$ *when divided by* $3, 4, 5, 6$*, respectively.*

Question 3.9. *Find the smallest integer* $a > 2$ *such that*

$$2 \mid a, \quad 3 \mid a+1, \quad 4 \mid a+2, \quad 5 \mid a+3, \quad 6 \mid a+4.$$

Question 3.10. *Obtain three consecutive integers each having a square factor.*
*(***Hint:** *Find an integer* a *such that* $2^2 \mid a$*,* $3^2 \mid a+1$ *and* $5^2 \mid a+2$*.)*

Question 3.11. *A band of* 17 *pirates stole a sack of gold coins. When they tried to divide the fortune into equal portions,* 3 *coins remained. In the ensuing fight over who should get the extra coins, one pirate was killed. The wealth was redistributed, but this time an equal division left* 10 *coins. Again an argument developed in which another pirate was killed. Now the total fortune was evenly distributed among the survivors. What was the least number of coins that could have been stolen?*

Question 3.12. *Let* $z = 4 - 3i$ *and* $w = -2 + \frac{1}{2}i$. *Determine the following*

$$(1)\ |z|, \quad (2)\ z + w, \quad (3)\ zw, \quad (4)\ \frac{z}{w}.$$

Furthermore

(5) *Determine the polar form of* z *and* w, *respectively.*
(6) *Determine the polar form of* $\overline{z}$ *and* $\overline{w}$, *respectively.*

Question 3.13. *Find all complex numbers* z *such that* $z^5 = 32$.
Hint: *First find all complex numbers* z *such that* $z^5 = 1$.