PROBLEM SHEET 3

DEADLINE: 4pm Monday 29 November

1. Give a rigorous proof of the following:

$$\lim_{n\to\infty} \frac{n^2}{3n^2 - 2} = \frac{1}{3}.$$

2. Prove that the following sequences $\{a_n\}_1^\infty$ and $\{b_n\}_1^\infty$ both converge to 0:

   (a) $a_n = \dfrac{1}{n} + \dfrac{\sin(n)}{n+1}$;

   (b) $b_n = \sqrt{n+1} - \sqrt{n}$.

   [*Hint for (b): multiply b_n by* $\dfrac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}}$.]

3. Suppose that $\{a_n\}_1^\infty$ is a bounded sequence but is not necessarily convergent. Suppose the sequence $\{b_n\}_1^\infty$ converges to zero. Prove that $\lim_{n\to\infty} (a_n b_n) = 0$.