MT241P FINITE MATHEMATICS

4. ASSIGNMENT

Submit your solutions to all questions by Friday 4pm, 9.12.2022

Question 4.1. *Suppose Met Éirann provides the following predictions:*

(P1) *There is a* 60 *percent chance that it will rain today.*

(P2) *There is a* 50 *percent chance that it will rain tomorrow.*

(1) *Let 'R' indicate a day with rain and 'N' a day with no rain. List an appropriate sample space* Ω.

(2) *Let A be the event that it rains today and let B be the event that it rains tomorrow. List the outcomes of the following events:*

$$(i)\ A^c,\ (ii)\ A \cup B,\ (iii)\ A \cap B,\ (iv)\ A \cap B^c,\ (v)\ (A \cup B)^c$$

(3) *Find the probabilities for the following events:*
  (i) *It will rain today or tomorrow.*
  (ii) *It will rain today and tomorrow.*
  (iii) *It will rain today but not tomorrow.*
  (iv) *It will rain today or tomorrow, but not both days.*

Question 4.2. *Assume you flip a fair coin with a friend* n *times, where* $n \geq 1$.

(1) *How many possible outcomes are there?*

(2) *What is the probability of each outcome?*

(3) *If you throw the coin ten times, what is the chance of there being exactly one tail in any three consecutive throws?*

Next you play a game for money. Each time heads comes up you win a Euro, each time tails comes up you lose a Euro. However, as soon as you lose for the first time you claim you have to go home and stop playing.

(4) *Describe the sample space* Ω *in terms of your possible wins/losses.*

(5) *For each outcome* $\omega \in \Omega$ *give its probability.*

(6) *What is the probability of you winning at least one, but less than four Euro?*

(7) *What is the probability of you winning more than two Euro?*

Question 4.3. *You are at a party attended by* k *people, including you. What is the likelihood of somebody else at the party sharing your birthday? (We assume that nobody was born in a leap year). What is the likelihood if* $k = 23$?

Question 4.4. *Use combinatorial arguments to prove that, for every integer* $n \geq 0$,

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

Question 4.5. *Let A and B be finite sets such that A has* n *elements and B has* m *elements, where* $n \geq m$. *How many injective functions* $f : A \to B$ *are there, that is, functions where* $f(a_1) \neq f(a_2)$, *whenever* $a_1 \neq a_2$.

Question 4.6. *In how many ways can $2n$ tennis players be paired and assigned to n courts?*

Question 4.7. *How many distinct integer solutions does the equation*

$$x_1 + x_2 + x_3 + x_4 = 100$$

have, if

    (1) $x_i \geq 0$, for all $i = 1, 2, 3, 4$,
    (2) $x_i \geq i$, for all $i = 1, 2, 3, 4$.

Question 4.8. *In my home town, it rains one third of all days. Traffic is heavy on half of the rainy days and a quarter of the dry days. If it's rainy and the traffic is heavy, then I am bound to be late for work half of all days. A quarter of the days that I'm late, it is not rainy but the traffic is heavy. Whenever there is light traffic, I am twice as likely to be late on a rainy day, compared to dry days. I am late for work one quarter of all days.*

    (1) Draw the tree diagram, where the first stage gives rain / no rain, the second stage gives traffic / no traffic and the third stage gives late / not late.
    (2) What is my chance of being on time on a rainy day with light traffic?
    (3) What is my chance of being on time on a dry day?
    (4) Given I was late today, what is the chance of there having been light traffic?
    (5) Given I was on time today and there was light traffic, what is the chance of there having been rain?