### MT251P – Lecture 3

## Fiacre Ó Cairbre

## Section 1.3 – Propositional Logic.

## Elementary and Compound Propositions.

A proposition is a statement that is either true or false, but not both.

## Example 9.

Consider the following three statements:

P1: 3 is a prime number.

P2: 2 + 2 = 5

P3: 81 is divisble by 3

The above three statements are all propositions. However the statement – Where are you going? – is not a proposition.

### Remark 6.

A proposition takes exactly one of the values, true or false. Typically, we denote true by T or 1 and we denote false by F or 0. For example, P1 and P3 above take the value T or 1 and P2 above takes the value F or 0.

Our three propositions above are called elementary propositions because they cannot be broken down into simpler propositions. However, consider the proposition:

P4: 6 > 5 and 110 is even.

Then, P4 is a compound proposition because it's constructed using the connective 'and' from the two simpler propositions, P5, P6 where:

P5: 6 > 5

P6: 110 is even.

### Remark 7.

We use the following three fundamental operations to construct compound propositions from elementary propositions

# 1. Negation (NOT).

The negation of a proposition P is denoted by  $\neg P$  and is formed by negating the statement in P. So,  $\neg P$  takes the value T  $\iff$  P takes the value F. This will then mean that  $\neg P$  takes the value F  $\iff$  P takes the value T.

# 2. Conjunction (AND).

The conjunction of two propositions P and Q is denoted by  $P \wedge Q$  and takes the value T  $\iff$  P and Q both take the value T.

## 3. Disjunction (OR).

The disjunction of two propositions P and Q is denoted by  $P \vee Q$  and takes the value T  $\iff$  P or Q (or both) take the value T.

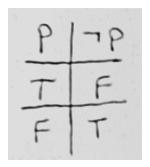
### Truth Tables.

The expressions  $\neg P$ ,  $P \land Q$  and  $P \lor Q$  are examples of formulas in propositional algebra. Formulas are built from variables (like P, Q etc.), logical operations (like  $\neg$ ,  $\land$ ,  $\lor$  etc.) and the values T, F. Formulas define propositional functions (for example,  $\neg P$  has one variable,  $P \land Q$  has two variables etc). When we give a value to each variable in a formula, then the value of the formula is determined.

A truth table tells us what value the formula takes for all combinations of the variables. In a truth table, the T, F values in a column give us the value of the formula that appears on the top of the column.

# Example 10.

The truth table for negation is



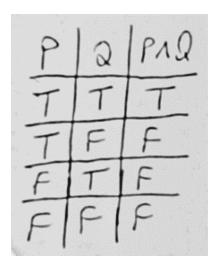
# Example 11.

The truth table for disjunction is



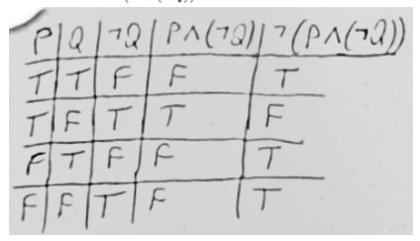
# Example 12.

The truth table for conjunction is



## Example 13.

Construct the truth table for  $\neg (P \land (\neg Q))$ .



## Remark 8.

If formulas G, H have identical truth tables (meaning that we have identical columns under each of the two formulas at the right end of each table), then G and H are called equivalent. This means that G and H define the same propositional function and we write  $G \sim H$  to denote that G is equivalent to H.