# MT241P FINITE MATHEMATICS

## 2. Assignment

Upload your solutions to the following questions by Friday 4pm, 28.10.2022:
Q.2.1, Q.2.3(1), Q.2.4(1),(2), Q.2.5, Q.2.6(1), Q.2.7, Q.2.8

**Question 2.1.** *Give a proof for Euclid's Lemma.*

**Question 2.2.**

(1) *Let* $p, q \in \mathbb{Z}$. *Use the Division algorithm to show that if* $pq = 1$, *then* $p, q \in \{\pm 1\}$.
(2) *Let* $a, b \in \mathbb{Z}$. *Show* $a \mid b$ *and* $b \mid a$ *if and only if* $b = \pm a$

**Question 2.3.** *Use Euclid's Algorithm to find the greatest common divisor of the two given integers* $a$ *and* $b$. *Furthermore express* $\gcd(a, b)$ *as a linear combination of* $a$ *and* $b$, *that is, find integers* $x$ *and* $y$ *such that* $\gcd(a, b) = ax + by$.

(1) $a = 1841,\ b = -392$
(2) $a = 23427,\ b = 26049$
(3) $a = -1931,\ b = 4722$

**Question 2.4.** *Determine all positive integer solutions (if any exist) of the following Linear Diophantine Equations:*

(1) $172x + 20y = 1000$
(2) $123x + 360y = 91$
(3) $158x - 57y = 7$

**Question 2.5.** *A farmer bought* 100 *livestock for a total cost of 4000 Euro. Calves cost* 120 *Euro each, Lambs* 50 *each and Piglets* 25 *Euro each. If the farmer obtained an even number of animals of each type, how many did he buy?*

**Question 2.6.** *(1) Alcuin of York (*775 *ad): A hundred bushels of grain are distributed among* 100 *persons in such a way that each man receives* 3 *bushels, each woman* 2 *bushels, and each child* $\frac{1}{2}$ *a bushel. How many men, women and children are there?*
*(2) Mahaviracarya (*850 *ad): There were* 63 *equal piles of plantain fruit put together and* 7 *single fruit. They were divided equally amongst* 23 *travelers. What is the minimal number of fruits in each pile?*
*(3) Christoff Rudolff (*1526 *ad): Find the number of men, women and children in a company of* 20 *persons if together they pay* 20 *coins, each man paying* 3, *each woman* 2 *and each child* $\frac{1}{2}$.
*(4) Leonhard Euler (*1770 *ad): Divide* 100 *into two positive summands such that one is divisible by* 7 *and the other by* 11.

**Question 2.7.** *Find an example for integers* $a, b, c$ *and an integer* $n \geq 1$ *such that* $ac \equiv bc \mod n$, *but* $a \not\equiv b \mod n$.

**Question 2.8.** *Let* $a, b, c, d, n$ *be integers, with* $n \geq 1$. *Suppose* $a \equiv b \mod n$ *and* $c \equiv d \mod n$. *Show that*

(1) $ac \equiv bd \mod n$
(2) $a^k \equiv b^k \mod n$, *for all integers* $k \geq 0$