MT251P – Lecture 7

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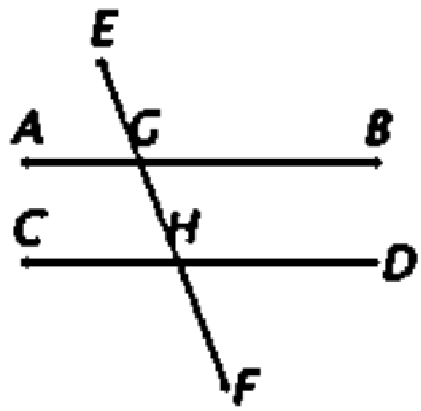
Proposition 1.7

Suppose AB and CD are parallel lines and EF is a third line that intersects AB at G and intersects CD at H. Then, $|\angle AGF| = |\angle DHE|$. The angles $\angle AGF$ and $\angle DHE$ are called alternate angles.

Proof.



We prove it by contradiction. Suppose $|\angle AGF| \neq |\angle DHE|$. Then, either $|\angle AGF| < |\angle DHE|$ or $|\angle AGF| > |\angle DHE|$.

CASE 1. Suppose $|\angle AGF| < |\angle DHE|$. Then

$$|\angle AGF| + |\angle EHC| < |\angle DHE| + |\angle EHC| = \pi$$

By P5, the lines AB and CD meet on the side of A, which is false. So, $|\angle AGF| < |\angle DHE|$ is false.

CASE 2. Suppose $|\angle AGF| > |\angle DHE|$. A similar approach as in CASE 1 will show that $|\angle AGF| > |\angle DHE|$ is false.

So, $|\angle AGF| = |\angle DHE|$.

Proposition 1.8

Suppose ΔABC is a triangle. Then,

$$|\angle ABC| + |\angle BCA| + |\angle CAB| = \pi$$

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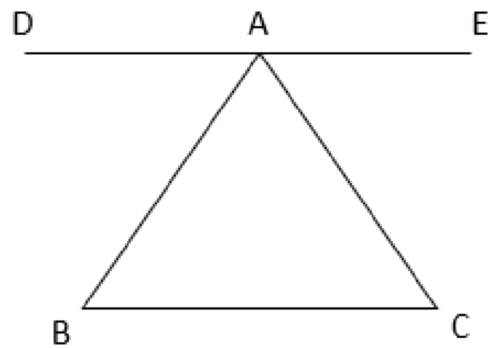
Proof.



Draw a line DE through A that is parallel to BC. Then,

$$|\angle DAB| + |\angle CAB| + |\angle EAC| = \pi$$

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Now,

$$|\angle DAB| = |\angle ABC| \quad \text{and} \quad |\angle EAC| = |\angle BCA|$$

.

So,

$$|\angle ABC| + |\angle CAB| + |\angle BCA| = \pi$$

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Section 2.5 – Areas.

Note the following result:

Suppose ΔABC is a triangle and let AE denote the perpendicular from A onto BC (or an extension of BC if necessary). Then, the area of ΔABC is $\frac{1}{2}|BC||AE|$.

Proposition 1.9

Consider a triangle ΔABC. Choose D on AB and choose E on AC such that DE is parallel to BC. Then,

$$\frac{|AD|}{|DB|} = \frac{|AE|}{|EC|}$$

Definition 2.

Two triangles ΔABC and ΔDEF are called similar if

$$|\angle ABC| = |\angle DEF|, \quad |\angle BCA| = |\angle EFD|, \quad |\angle CAB| = |\angle FDE|$$

Proposition 1.10

Suppose AB and CD are lines and EF is a third line that intersects AB at G and intersects CD at H. If $|\angle EGB| = |\angle DHE|$, then AB and CD are parallel.

Proposition 1.11

Suppose ΔABC and ΔDEF are similar triangles. Then

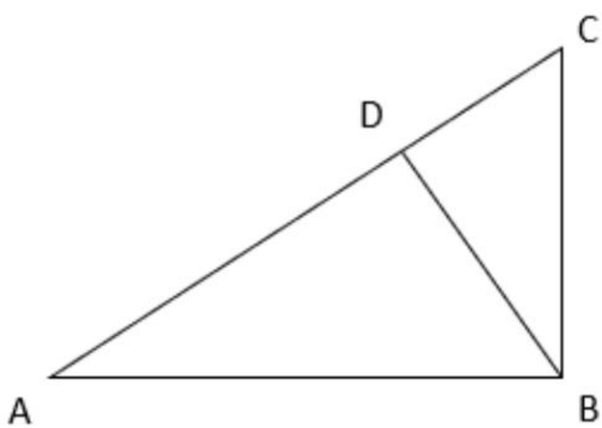
$$\frac{|DE|}{|AB|} = \frac{|EF|}{|BC|} = \frac{|DF|}{|AC|}$$

Proposition 1.12 – Pythagoras' Theorem.

Suppose ΔABC is a right angled triangle with $\angle ABC$ a right angle. Then,

$$|AB|^2 + |BC|^2 = |AC|^2$$

Proof.



Draw a perpendicular from B onto AC at D, The triangles ΔADB and ΔABC are similar and so Proposition 1.11 gives

$$\frac{|AB|}{|AD|} = \frac{|AC|}{|AB|}$$

Hence, $|AB|^2 = |AD||AC|$.

ΔBDC and ΔABC are similar and so

$$\frac{|DC|}{|BC|} = \frac{|BC|}{|AC|}$$

Thus, $|BC|^2 = |DC||AC|$. So,

$$|AB|^2 + |BC|^2 = |AC|(|AD| + |DC|)$$

$$= |AC|^2$$

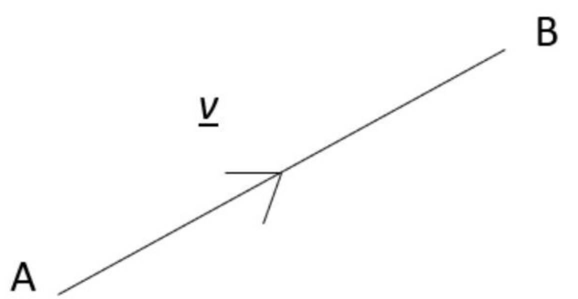
Chapter 3 – Vectors.

Section 3.1 – Introduction.

Remark 1.

Vectors can be used to describe things that require both magnitude and direction. For example, you could say that you are sailing at 30 mph in a south west direction. Here 30 is the magnitude and south west is the direction. If a thing only requires a magnitude, then it can be described by a scalar (or constant). For example, length is a scalar because it only requires a magnitude.

You can think of a vector as being a path (in a particular direction) between two points. Suppose A is your starting point (called the initial point) and B is your finishing point (called the terminal point).



The vector $\underline{v}$ starting at A and finishing at B is also denoted by

$$\underline{v} = \vec{v} = \vec{AB}$$

The magnitude of $\underline{v}$ is the distance from A to B.