MT251P

Homework 4

Due by 4 p.m. on December 9, 2022.

1. In each case below, state whether the statement is true or false. Justify your answer in each case.

(i) There are infinitely many 4×4 matrices that are not invertible.

(ii) There is a 4×4 invertible matrix A such that $A^3 = 2A^2$ and $\det A = 2$.

(iii) There is a 4×4 matrix A such that $A^2 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2. (a) Prove that $\det(A^{-1}BA) = \det(B)$, for all $n \times n$ matrices A, B where A is invertible and $n > 1$.

(b) Suppose $\underline{a} = i + 2j - k$, $\underline{b} = i + 3j + k$ and $\underline{c} = 3i + 8j + 4k$. Find $||\underline{w}||^2$ if $\underline{w} \in \mathbb{R}^3$ such that $\underline{w}.\underline{a} = 3$, $\underline{w}.\underline{b} = 5$ and $\underline{w}.\underline{c} = 17$.

3. Find the solution set of the following system of linear equations;
$x_1 - 4x_2 + 3x_3 = 0$
$2x_1 - 6x_2 + 10x_3 = 6$
$x_1 - 2x_2 + 7x_3 = 5$

4. Find the solution set of the following system of linear equations:
$4x - 6y + 8z = 8$
$x + 2y - 5z = 2$
$y + 4x - 6z = 8$

5. Find A^{-1} if $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}$