MT251P – Lecture 3

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Section 1.3 – Propositional Logic.

Elementary and Compound Propositions.

A proposition is a statement that is either true or false, but not both.

Example 9.

Consider the following three statements:

P1: 3 is a prime number.

P2: $2+2=5$

P3: 81 is divisble by 3

The above three statements are all propositions. However the statement – Where are you going? – is not a propostion.

Remark 6.

A proposition takes exactly one of the values, true or false. Typically, we denote true by T or 1 and we denote false by F or 0. For example, P1 and P3 above take the value T or 1 and P2 above takes the value F or 0.

Our three propositions above are called elementary propositions because they cannot be broken down into simpler propositions. However, consider the proposition:

P4: $6>5$ and 110 is even.

Then, P4 is a compound proposition because it's constructed using the connective 'and' from the two simpler propositions, P5, P6 where:

P5: $6>5$

P6: 110 is even.

Remark 7.

We use the following three fundamental operations to construct compound propositions from elementary propositions

1. Negation (NOT).

The negation of a proposition P is denoted by $\neg P$ and is formed by negating the statement in P. So, $\neg P$ takes the value T $\iff$ P takes the value F. This will then mean that $\neg P$ takes the value F $\iff$ P takes the value T.

2. Conjunction (AND).

The conjunction of two propositions P and Q is denoted by $P \wedge Q$ and takes the value T $\iff$ P and Q both take the value T.

3. Disjunction (OR).

The disjunction of two propositions P and Q is denoted by $P \vee Q$ and takes the value T $\iff$ P or Q (or both) take the value T.

Truth Tables.

The expressions $\neg P$, $P \wedge Q$ and $P \vee Q$ are examples of formulas in propositional algebra. Formulas are built from variables (like P, Q etc.), logical operations (like $\neg$, $\wedge$, $\vee$ etc.) and the values T, F. Formulas define propositional functions (for example, $\neg P$ has one variable, $P \wedge Q$ has two variables etc). When we give a value to each variable in a formula, then the value of the formula is determined.

A truth table tells us what value the formula takes for all combinations of the variables. In a truth table, the T, F values in a column give us the value of the formula that appears on the top of the column.

Example 10.

The truth table for negation is

P	$\neg P$
T	F
F	T

Example 11.

The truth table for disjunction is

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Example 12.

The truth table for conjunction is

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Example 13.

Construct the truth table for $\neg(P \wedge (\neg Q))$.

P	Q	$\neg Q$	$P \wedge (\neg Q)$	$\neg(P \wedge (\neg Q))$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

Remark 8.

If formulas G, H have identical truth tables (meaning that we have identical columns under each of the two formulas at the right end of each table), then G and H are called equivalent. This means that G and H define the same propositional function and we write $G \sim H$ to denote that G is equivalent to H.