MT251P - Foundations of Euclidean Geometry

Assignment #4

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Question 1

In each case below, state whether the statement is true or false. Justify your answer in each case.

Part A

There are infinitely many 4×4 matrices that are not invertible.

Solution

Since a square matrix is invertible if and only if its determinant is non-zero. Since there are infinitely many possible values for the elements of a 4×4 matrix, there are also infinitely many matrices that have a determinant of zero and are therefore not invertible. Therefore the statement is True

Part B

There is a 4×4 invertible matrix A such that $A^3 = 2A^2$ and $\det A = 2$.

Solution

The equation $A^3 = 2A^2$ can be rewritten as $A^3 - 2A^2 = 0$. Since the determinant of a matrix must be nonzero in order for it to be invertible, we know that the determinant of A is nonzero. Therefore, the determinant of $A^3 - 2A^2$ must also be nonzero, so the equation $A^3 - 2A^2 = 0$ must have only the trivial solution $A = 0$. However we are given that $A = 2$. Therefore the statement is False

Part C

There is a 4×4 matrix A such that $A^2 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Solution

We see that the top left entry of A^2 is -1. Since the square of a number is always nonnegative and the entries have to be Real we have a contradiction. Therefore the statement is False.

Question 2

Part A

Question 2 continued on next page. . .

Prove that $\det(A^{-1}BA) = \det(B)$, for all n × n matrices A, B, where A is invertible and n > 1.

Solution

Let A be an invertible $n \times n$ matrix, and let B be any $n \times n$ matrix. We have $\det(A^{-1}BA) = \det(A^{-1})\det(B)\det(A) = \frac{1}{\det(A)}\det(B)\det(A) = \det(B)$, so $\det(A^{-1}BA) = \det(B)$

Part B

Suppose $\underline{a} = i + 2j - k$, $\underline{b} = i + 3j + k$ and $\underline{c} = 3i + 8j + 4k$. Find $||\underline{w}||^2$ if $\underline{w} \in \mathbb{R}^3$ such that $\underline{w}.\underline{a} = 3$, $\underline{w}.\underline{b} = 5$ and $\underline{w}.\underline{c} = 17$.

Solution

matrix 103/3

Question 3

Find the solution set of the following system of linear equations:

$$x_1 - 4x_2 + 3x_3 = 0$$

$$2x_1 - 6x_2 + 10x_3 = 6$$

$$x_1 - 2x_2 + 7x_3 = 5$$

Solution

no solution

Question 4

Find the solution set of the following system of linear equations:

$$4x - 6y + 8z = 8$$

$$x + 2y - 5z = 2$$

$$y + 4x - 6z = 8$$

Solution

idk but there isa solution

Question 5

Find A^{-1} if $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}$

Solution

-9 devided by 7....