MT241P - Finite Mathematics

Assignment #4

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Question 1

Suppose Met Eirann provides the following predictions:
(P1) There is a 60 percent chance that it will rain today.
(P2) There is a 50 percent chance that it will rain tomorrow.

Part A

Let 'R' indicate a day with rain and 'N' a day with no rain. List an appro- priate sample space Ω.

Solution

$$\Omega = \{RR, RN, NR, NN\}$$

Part B

Let A be the event that it rains today and let B be the event that it rains tomorrow. List the outcomes of the following events:

- A^c
- $A \cup B$
- $A \cap B$
- $A \cap B^c$
- $(A \cup B)^c$

Solution

$$A^c = \{NR, NN\}$$

$$A \cup B = \{RR, RN, NR\}$$

$$A \cap B = \{RR\}$$

$$A \cap B^c = \{RN\}$$

$$(A \cup B)^c = \{NN\}$$

Question 1 continued on next page. . .

Part C

Find the probabilities for the following events:

- It will rain today or tomorrow.
- It will rain today and tomorrow.
- It will rain today but not tomorrow.
- It will rain today or tomorrow, but not both days.

Solution

"It will rain today or tomorrow." is the same as $A \cup B$,and therefore form part A:

$$P(A \cup B) = P(RR) + P(RN) + P(NR) =$$

$$= P(A)P(B) + P(A)P(B^c) + P(A^c)P(B) =$$

$$= 0.6 * 0.5 + 0.6 * 0.5 + 0.4 * 0.5 = 0.8$$

"It will rain today and tomorrow." is the same as $A \cap B$,and therefore form part A:

$$P(A \cap B) = P(RR) =$$

$$= P(A)P(B) =$$

$$= 0.6 * 0.5 = 0.3$$

"It will rain today but not tomorrow." is the same as $A \cap B^c$,and therefore form part A:

$$P(A \cap B^c) = P(RN) =$$

$$= P(A)P(B^c) =$$

$$= 0.6 * 0.5 = 0.3$$

"It will rain today or tomorrow, but not both days." has the following outcomes: RN, NR. Then The probability for that event (Let's call it C) will be:

$$P(C) = P(RN) + P(NR) =$$

$$= P(A)P(B^c) + P(A^c)P(B) =$$

$$= 0.6 * 0.5 + 0.4 * 0.5 = 0.5$$

Question 2

Assume you flip a fair coin with a friend n times, where n ≥ 1

Part A

How many possible outcomes are there?

Solution

Part B

What is the probability of each outcome?

Solution

Part C

If you throw the coin ten times, what is the chance of there being exactly one tail in any three consecutive throws?

Solution

Next you play a game for money. Each time heads comes up you win a Euro, each time tails comes up you lose a Euro. However, as soon as you lose for the first time you claim you have to go home and stop playing.

Part D

Describe the sample space Ω in terms of your possible wins/losses.

Solution

Part E

For each outcome $\omega \in \Omega$ give its probability.

Solution

Part F

What is the probability of you winning at least one, but less than four Euro?

Solution

Part G

What is the probability of you winning more than two Euro?

Solution

Question 3

You are at a party attended by k people, including you. What is the likelihood of somebody else at the party sharing your birthday? (We assume that nobody was born in a leap year). What is the likelihood if k = 23?

Solution

Question 4

Use combinatorial arguments to prove that, for every integer n ≥ 0,

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

Solution

Question 5

Let A and B be finite sets such that A has n elements and B has m elements, where n ≥ m. How many injective functions $f : A \to B$ are there, that is, functions where $f(a_1) \neq f(a_2)$, whenever $a_1 \neq a_2$.

Solution

Question 6

In how many ways can 2n tennis players be paired and assigned to n courts?

Solution

Question 7

How many distinct integer solutions does the equation:

$$x_1 + x_2 + x_3 + x_4 = 100$$

heve, if:

Part A

$x_i \geq 0$, for all i = 1, 2, 3, 4

Solution

Part B

$x_i \geq i$, for all i = 1, 2, 3, 4

Solution

Question 8

In my home town, it rains one third of all days. Traffic is heavy on half of the rainy days and a quarter of the dry days. If it's rainy and the traffic is heavy, then I am bound to be late for work half of all days. A quarter of the days that I'm late, it is not rainy but the traffic is heavy. Whenever there is light traffic, I am twice as likely to be late on a rainy day, compared to dry days. I am late for work one quarter of all days.

Part A

Draw the tree diagram, where the first stage gives rain / no rain, the second stage gives traffic / no traffic and the third stage gives late / not late.

Solution

Part B

What is my chance of being on time on a rainy day with light traffic?

Solution

Part C

What is my chance of being on time on a dry day?

Solution

Part D

Given I was late today, what is the chance of there having been light traffic?

Solution

Part E

Given I was on time today and there was light traffic, what is the chance of there having been rain?

Solution