# **MT241P - Finite Mathematics**

**Assignment #4** 

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## **Question 1**

Suppose Met Eirann provides the following predictions:

- (P1) There is a 60 percent chance that it will rain today.
- (P2) There is a 50 percent chance that it will rain tomorrow.

### Part A

Let 'R' indicate a day with rain and 'N' a day with no rain. List an appro- priate sample space  $\Omega$ .

### **Solution**

$$\Omega = \{RR, RN, NR, NN\}$$

### Part B

Let A be the event that it rains today and let B be the event that it rains tomorrow. List the outcomes of the following events:

- $\bullet$   $A^c$
- $A \cup B$
- $A \cap B$
- $A \cap B^c$
- $(A \cup B)^c$

### **Solution**

$$A^{c} = \{NR, NN\}$$

$$A \cup B = \{RR, RN, NR\}$$

$$A \cap B = \{RR\}$$

$$A \cap B^{c} = \{RN\}$$

$$(A \cup B)^{c} = \{NN\}$$

### Part C

Find the probabilities for the following events:

- It will rain today or tomorrow.
- It will rain today and tomorrow.
- It will rain today but not tomorrow.
- It will rain today or tomorrow, but not both days.

### **Solution**

"It will rain today or tomorrow." is the same as  $A \cup B$ , and therefore form part A:

$$P(A \cup B) = P(RR) + P(RN) + P(NR) =$$

$$= P(A)P(B) + P(A)P(B^{c}) + P(A^{c})P(B) =$$

$$= 0.6 * 0.5 + 0.6 * 0.5 + 0.4 * 0.5 = 0.8$$

"It will rain today and tomorrow." is the same as  $A \cap B$ , and therefore form part A:

$$P(A \cap B) = P(RR) =$$
  
=  $P(A)P(B) =$   
=  $0.6 * 0.5 = 0.3$ 

"It will rain today but not tomorrow." is the same as  $A \cap B^c$ , and therefore form part A:

$$P(A \cap B^c) = P(RN) =$$

$$= P(A)P(B^c) =$$

$$= 0.6 * 0.5 = 0.3$$

"It will rain today or tomorrow, but not both days." has the following outcomes: RN, NR. Then The probability for that event (Let's call it C) will be:

$$P(C) = P(RN) + P(NR) =$$

$$= P(A)P(B^{c}) + P(A^{c})P(B) =$$

$$= 0.6 * 0.5 + 0.4 * 0.5 = 0.5$$

# **Question 2**

Assume you flip a fair coin with a friend n times, where  $n \ge 1$ 

#### Part A

How many possible outcomes are there?

### **Solution**

### Part B

What is the probability of each outcome?

### **Solution**

### Part C

If you throw the coin ten times, what is the chance of there being exactly one tail in any three consecutive throws?

### **Solution**

Next you play a game for money. Each time heads comes up you win a Euro, each time tails comes up you lose a Euro. However, as soon as you lose for the first time you claim you have to go home and stop playing.

### Part D

Describe the sample space  $\Omega$  in terms of your possible wins/losses.

### **Solution**

### Part E

For each outcome  $\omega \in \Omega$  give its probability.

### **Solution**

### Part F

What is the probability of you winning at least one, but less than four Euro?

### **Solution**

### Part G

What is the probability of you winning more than two Euro?

### **Solution**

### **Question 3**

You are at a party attended by k people, including you. What is the likelihood of somebody else at the party sharing your birthday? (We assume that nobody was born in a leap year). What is the likelihood if k = 23?

### **Solution**

# **Question 4**

Use combinatorial arguments to prove that, for every integer  $n \ge 0$ ,

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

### **Solution**

# **Question 5**

Let A and B be finite sets such that A has n elements and B has m elements, where  $n \ge m$ . How many injective functions  $f: A \to B$  are there, that is, functions where  $f(a_1) \ne f(a_2)$ , whenever  $a_1 \ne a_2$ .

### **Solution**

### **Question 6**

In how many ways can 2n tennis players be paired and assigned to n courts?

#### **Solution**

## **Question 7**

How many distinct integer solutions does the equation:

$$x_1 + x_2 + x_3 + x_4 = 100$$

heve, if:

### Part A

 $x_i \ge 0$ , for all i = 1, 2, 3, 4

### **Solution**

### Part B

 $x_i \ge i$ , for all i = 1, 2, 3, 4

#### **Solution**

### **Question 8**

In my home town, it rains one third of all days. Traffic is heavy on half of the rainy days and a quarter of the dry days. If it's rainy and the traffic is heavy, then I am bound to be late for work half of all days. A quarter of the days that I'm late, it is not rainy but the traffic is heavy. Whenever there is light traffic, I am twice as likely to be late on a rainy day, compared to dry days. I am late for work one quarter of all days.

### Part A

Draw the tree diagram, where the first stage gives rain / no rain, the second stage gives traffic / no traffic and the third stage gives late / not late.

### **Solution**

### Part B

What is my chance of being on time on a rainy day with light traffic?

### **Solution**

### Part C

What is my chance of being on time on a dry day?

### **Solution**

### Part D

Given I was late today, what is the chance of there having been light traffic?

### **Solution**

### Part E

Given I was on time today and there was light traffic, what is the chance of there having been rain?

### **Solution**