MT251P – Lecture 5

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Example 16.

Use the equivalences in remark 11 to prove that

$$\neg(P \vee Q) \vee (\neg P \wedge Q) \sim \neg P$$

Proof:

$$\neg(P \vee Q) \vee (\neg P \wedge Q) \sim (\neg P \wedge \neg Q) \vee (\neg P \wedge Q), \qquad \text{by De Morgan's Law}$$

$$\sim \neg P \wedge (\neg Q \vee Q), \qquad \text{by using Distributivity}$$

$$\sim \neg P \wedge T, \qquad \text{by using the Complements}$$

$$\sim \neg P \qquad \text{by using the Identity}$$

Chapter 2 – Euclid's Elements.

Section 2.1 – Postulates and Common Notions.

Euclid lived in Alexandria and wrote the Elements around 300BC. His Elements essentially covered all known mathematics at the time. His approach was to accept very few axioms at the beginning and then, using deductive reasoning, he proved many results in mathematics.

A modern online version of the Elements has been made available by David Joyce and I will put a link to the website on moodle. It's important to note that if there are any differences between my notation, definitions etc. and the website above, then you should use my notation, definitions etc.

Postulates.

Note that we don't define what a point or line is but we will accept statements about points and lines. For example, we will accept that between any two different points, we can a draw a unique straight line (Postulate 1 below). We are free to do this in mathematics.

At the beginning of the Elements, Euclid states five postulates. These postulates are like axioms because he accepts them and doesn't prove them.

Postulate 1. – A unique straight line can be drawn between any two points.

Postulate 2. – A given finite straight line can be extended continuously in a straight line.

Postulate 3. – A point (its centre) and a distance (the length of the radius) define a circle.

Postulate 4. – All right angles are equal.

Postulate 5. – If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, then the two lines meet on that side on which the angles are less than two right angles.

Common Notions.

There are five so called common notions in the Elements. These are statements that Euclid accepted.

Common notion 1. – Things which equal the same thing also equal each other.

Common notion 2. – If equals are added to equals, then the sums are equal.

Common notion 3. – If equals are subtracted from equals, then the remainders are equal.

Common notion 4. – Things which coincide with one another equal one another.

Common notion 5. – The whole is greater than the part.

Some notation and conventions.

(i) We will denote points by capital letters like A, B, C.

(ii) The line between two points A, B is denoted by AB and its length is denoted by $|AB|$.

(iii) The angle formed by the two lines AB, AC at the point B is denoted by $\angle ABC$ (or $\angle CBA$).

(iv) Angles are measured in radians and the magnitude of an angle $\angle ABC$ is denoted by $|\angle ABC|$

(v) Denote the triangle with vertices at (non–collinear) points A, B, C by ΔABC.

(vi) We will denote Postulate 1 by P1, Postulate 2 by P2 etc. We will denote Common Notion 1 by CN1 etc.

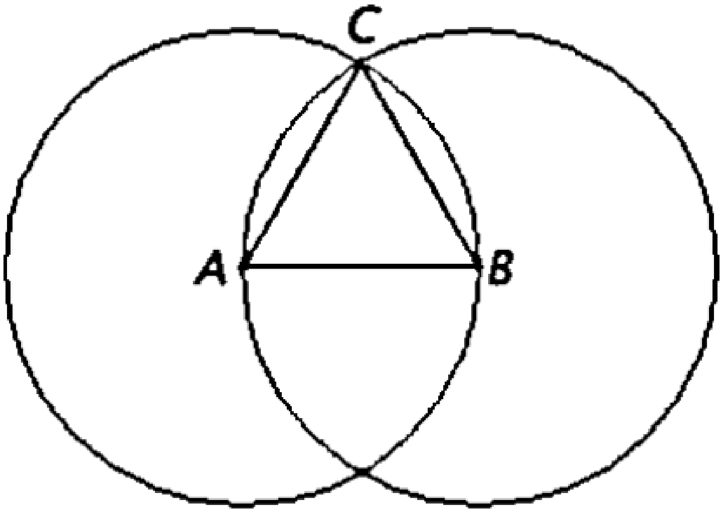
(vii) We will put E in front of proposition numbers to distinguish a result in the Elements from a result here. For example, we will label Proposition I.5 (i.e. Book I, Proposition 5) in the Elements as E.I.5 to distinguish it from Proposition 1.5 here, because Proposition 1.5 here will not be Proposition E.I.5.

Section 2.2. – Fundamental Theorems in Euclidean Geometry.

Proposition 1.1

To construct an equilateral triangle on a given line AB.

Proof.



Draw a circle O_1 with centre A and radius $|AB|$ (by P3) as in the figure above. Draw a circle O_2 with centre B and radius $|AB|$ (by P3). Denote the point of intersection of the two circles by C. Then $|AC| = |AB|$ and $|BC| = |AB|$. By CN1 we then have $|AC| = |BC| = |AB|$ and so the triangle ΔABC is an equilateral triangle and we are done.

Definition 1.

Two triangles ΔABC, ΔDEF are called congruent if

$$|AB| = |DE|, \quad |BC| = |EF|, \quad |AC| = DF|$$

and

$$|\angle ABC| = |\angle DEF|, \quad |\angle BCA| = |\angle EFD|, \quad |\angle CAB| = |\angle FDE|$$

Remark 1.

Propositions E.I.2 and E.I.3 show how we can construct a line at a point A equal to a given line BC and how we can cut from a longer line AB a section equal in length to a given shorter line CD. Proposition E.I.4 states the following first method for proving congruence of triangles.

Proposition 1.2 (SAS)

Suppose we have two triangles ΔABC, ΔDEF. If $|AB| = |DE|$, $|BC| = |EF|$ and $|\angle ABC = |\angle DEF|$, then ΔABC is congruent to ΔDEF.

Remark 2.

Euclid's proof of SAS uses 'superposition'. Nowadays, SAS is typically taken as an axiom and we will do that. We will use two other congruence results.