MT251P – Lecture 6

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ASA.

Suppose we have two triangles ΔABC, ΔDEF. If $|\angle ABC| = |\angle DEF|$, $|BC| = |EF|$ and $|\angle BCA| = |\angle EFD|$, then ΔABC is congruent to ΔDEF.

SSS.

Suppose we have two triangles ΔABC, ΔDEF. If $|AB| = |DE|$, $|BC| = |EF|$ and $|AC| = |DF|$, then ΔABC is congruent to ΔDEF.

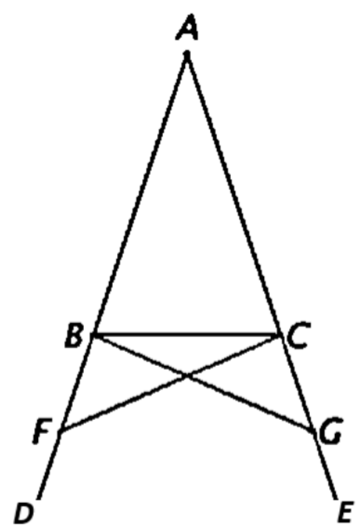
Proposition 1.3

Suppose we have an isosceles triangle ΔABC with $|AB| = |AC|$. Then,

$$|\angle ABC| = |\angle BCA|$$

.

Proof.



Choose F on AD and G on AE such that $|AG| = |AF|$ (by Proposition E.I.3). Join F to C and join B to G. Consider the triangles ΔABG, ΔACF. We have $|AC| = |AB|$, $|AF| = |AG|$, $|\angle FAC| = |\angle BAG|$ and so SAS implies that ΔABG is congruent to ΔACF. So, $|FC| = |BG|$, $|\angle AFC| = |\angle AGB|$.

Hence, $|\angle BFC| = |\angle CGB|$ and $|FC| = |BG|$. Also, since $|AF| = |AG|$ and $|AB| = |AC|$, then we get $|BF| = |CG|$ (by CN3). So, SAS implies that ΔBFC is congruent to ΔCGB.

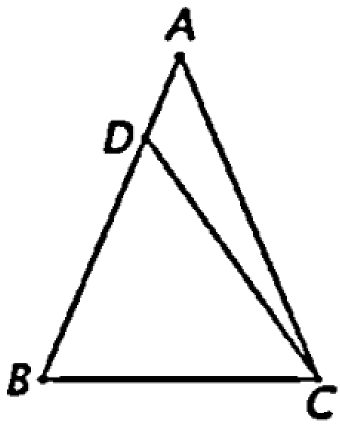
Then,

$$|\angle ABC| = |\angle ABG| - |\angle CBG| = |\angle ACF| - |\angle FCB| = |\angle BCA|$$

Proposition 1.4

Suppose ΔABC is a triangle with $|\angle ABC| = |\angle BCA|$. Then, $|AB| = |AC|$.

Proof.



We will prove this by contradiction. Suppose $|AB| \neq |AC|$. Then , we either have $|AB| > |AC|$ or $|AB| < |AC|$.

CASE 1. Suppose $|AB| > |AC|$.

Choose D on AB with $|BD| = |AC|$. Join C to D and use SAS to get that ΔABC is congruent to ΔDCB which contradicts CN5. Thus, $|AB| > |AC|$ is false.

CASE 2. Suppose $|AB| < |AC|$.

Use a similar approach as in CASE 1 to get thet $|AB| < |AC|$ is false.

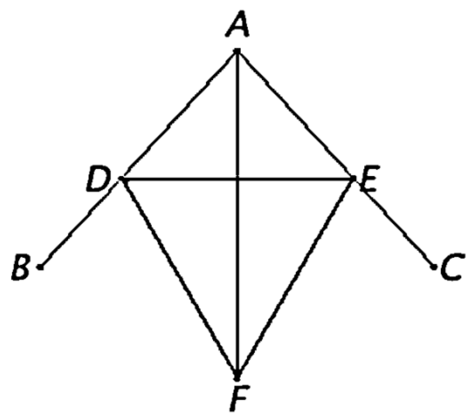
Overall then, we have $|AB| = |AC|$ and we are done.

Section 2.3 – Some elementary constructions.

Proposition 1.5

To construct the bisector of a given angle $\angle BAC$.

Proof.



Choose D on AB and E on AC with $|AE| = |AD|$. Join D to E and construct the equilateral triangle ΔDEF on DE. Join A to F. By SSS we get that ΔADF is congruent to ΔAEF and so $|\angle DAF| = |\angle EAF|$. Hence, AF bisects $\angle DAE$ which equals $\angle BAC$ and we are done.

Section 2.4 – Angles and Parallels.

Remark 3.

We will use the following results:

1. Suppose a line CD is drawn from a point C on the line AB between A and B. Then, $|\angle ACD| + |\angle DCB| = \pi$

2. Suppose C is a point not on a line AB. Then, there exists a unique line through C that is parallel to AB.

Remark 4.

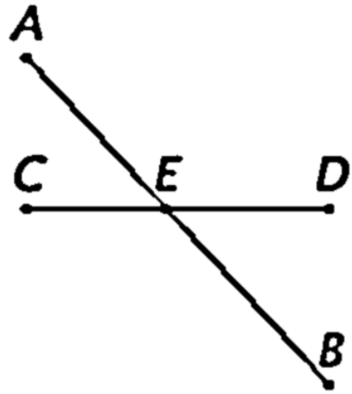
Result 2 in remark 3 above is equivalent to P5. It is a simpler version of P5.

Proposition 1.6

Suppose the line AB and CD intersect at the point E. Then $|\angle AEC| = |\angle BED|$.

The angles $\angle AEC$ and $\angle BED$ are called opposite angles.

Proof.



We have $|\angle AEC| + |\angle CEB| = \pi$ and $|\angle CEB| + |\angle BED| = \pi$. By CN1 we then get $|\angle AEC| + |\angle CEB| = |\angle CEB| + |\angle BED|$ Hence $|\angle AEC| = |\angle BED|$.