University of Waterloo
CS 486, Winter 2024
Assignment 1

Problem 1

a)

```
from math import log2
from queue import PriorityQueue
import matplotlib.pyplot as plt
from multiprocessing import Pool

##### Variables ######
NUM_FEATURES = 0 # Define later
NUM_SAMPLE = 1500
ATHEISM_ID = 1
BOOKS_ID = 2
LABEL_STR = {ATHEISM_ID: "Atheism", BOOKS_ID: "Books" }

# We define P(x) as # belongs to atheism over total.

##### READING FILE ######

# Function to read label data, returning a dictionary mapping document ID to label
def read_label_data(file_name):
    label_data = {}
    with open(file_name, ’r’) as file:
        lineNum = 1
        for line in file:
            label_data[lineNum] = int(line.strip())
            lineNum += 1
            # docId = line number
    file.close()
    return label_data

# Function to read word data, returning a dictionary mapping word ID to word
def read_word_data(file_name):
    word_data = {}
    with open(file_name, ’r’) as file:
        lineNum = 1
        for line in file:
            word_data[int(lineNum)] = line.strip()
            lineNum += 1
            # wordId = line number
    # Define the Number of features
    global NUM_FEATURES
    NUM_FEATURES = lineNum
    file.close()
    return word_data

# Function to read train or test data
# document_data[n] = arrray of word_id that n+1'th doc have.
def read_document_data(file_name):
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    document_data = {i: [] for i in range(1, NUM_SAMPLE + 1)} # 1 to 1500
    with open(file_name, 'r') as file:
        for line in file:
            doc_id, word_id = line.strip().split()
            document_data[int(doc_id)].append(int(word_id))
    file.close()
    return document_data

##### COMPUTING #####

# functions that does log_2(x/x+y).
# the # of bits to encode x.
def entropy(x, y):
    #print(f"x = {x}, y = {y}")

    # corner case, for our purpose log_2(0) = 0.
    if x == 0:
        return 0

    val = x / (x + y)
    #print(f"val = {val}")
    #print(f"log2(val) = {log2(val)}")
    return - log2(val)

def information_content(dataset, doc_subreddit_dict):
    # calculate the information content of the dataset
    # dataset needs to be a dictionary

    atheism_count = 0
    books_count = 0

    doc_ids = [key for key, _ in dataset.items()]

    # Loop over all elements and calculate # belongs to atheism or books respectively
    for doc_id in doc_ids:
        if doc_subreddit_dict[doc_id] == ATHEISM_ID:
            atheism_count += 1
        elif doc_subreddit_dict[doc_id] == BOOKS_ID:
            books_count += 1

    etp1 = entropy(atheism_count, books_count)
    etp2 = entropy(books_count, atheism_count)

    # Corner case
    if (atheism_count + books_count) == 0:
        return 0

    return ((atheism_count) / (atheism_count + books_count) * etp1) + ((books_count) / (atheism_count + books_count) * etp2)

def information_gain(num1, num2):
    etp1 = entropy(num1, num2)
    etp2 = entropy(num2, num1)

    # Corner case
    if (num1 + num2) == 0:
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        return 0

    return ((num1) / (num1 + num2) * etp1) + ((num2) / (num1 + num2) * etp2)

# Function to calculate the delta information gain for a given split
def delta_information_gain(elements, doc_subreddit_dict, word_to_split, method, debug=False):

    atheism_count_E = 0
    books_count_E = 0

    doc_ids = [key for key, _ in elements.items()]

    # Loop over all elements and calculate # belongs to atheism or books respectively
    for doc_id in doc_ids:
        if doc_subreddit_dict[doc_id] == ATHEISM_ID:
            atheism_count_E += 1
        elif doc_subreddit_dict[doc_id] == BOOKS_ID:
            books_count_E += 1

    # TODO: not sure if for IE we going to use different methods.
    IE = information_gain(atheism_count_E, books_count_E)
    if debug:
        print(f"IE = {IE}, atheism_count_E = {atheism_count_E}, books_count_E = {books_count_E}")
    # We now proceed to split the elements by the word_to_split.

    # E1
    has_word_to_split = [key for key, value in elements.items()
                         if word_to_split in value]
    # E2
    not_have_word_to_split = [key for key, value in elements.items()
                              if word_to_split not in value]

    #if debug:
    #    print(f"len has_word_to_split = {len(has_word_to_split)}, len not_have_word_to_split = {len(not_have_word_to_split)}")

    # note the values in the arrays are doc_id has or has not the word to split.
    atheism_count_E1 = 0
    books_count_E1 = 0

    # Loop over all elements and calculate # belongs to atheism or books respectively
    for doc_id in has_word_to_split:
        if doc_subreddit_dict[doc_id] == ATHEISM_ID:
            atheism_count_E1 += 1
        elif doc_subreddit_dict[doc_id] == BOOKS_ID:
            books_count_E1 += 1

    # TODO: not sure if for IE we going to use different methods.
    IE1 = information_gain(atheism_count_E1, books_count_E1)
    if debug:
        print(f"IE1 = {IE1}, atheism_count_E1 = {atheism_count_E1}, books_count_E1 = {books_count_E1}")

    atheism_count_E2 = 0
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    books_count_E2 = 0

    # Loop over all elements and calculate # belongs to atheism or books respectively
    for doc_id in not_have_word_to_split:
        if doc_subreddit_dict[doc_id] == ATHEISM_ID:
            atheism_count_E2 += 1
        elif doc_subreddit_dict[doc_id] == BOOKS_ID:
            books_count_E2 += 1

    # TODO: not sure if for IE we going to use different methods.
    IE2 = information_gain(atheism_count_E2, books_count_E2)
    if debug:
        print(f"IE2 = {IE2}, atheism_count_E2 = {atheism_count_E2}, books_count_E2 = {books_count_E2}")

    # Finally the calculation
    if method == 1:
        # Method 1: Average information gain across the leaves
        return IE - ((IE1 / 2) + (IE2 / 2))
    elif method == 2:
        # Method 2: The one discussed in Class

        sum_E1 = atheism_count_E1 + books_count_E1
        sum_E2 = atheism_count_E2 + books_count_E2

        # Corner case
        if sum_E1 == 0 and sum_E2 == 0:
            return IE

        return IE - ((sum_E1 / (sum_E1 + sum_E2)) * IE1) - ((sum_E2 / (sum_E1 + sum_E2)) * IE2)

##### Decision Tree #####

class Node:
    def __init__(self, dataset, point_estimate, feature_to_split=None, info_gain=0, splitted_feature=[]):
        self.dataset = dataset  # The subset 'E' of the dataset at this node
        self.feature_to_split = feature_to_split  # 'X_prime': The feature to split on at this node
        self.point_estimate = point_estimate # Point estimation of current node.
        self.splitted_feature = splitted_feature[:] # array of already splitted feature
        self.info_gain = info_gain  # 'delta_I': Information gain of the split
        self.left = None      # Left child (with feature)
        self.right = None     # Right child (without feature)

    # This is needed for the PriorityQueue to compare Nodes based on information gain
    def __lt__(self, other):
         # since priority queue is a min_heap, so we define lt as gt
        return self.info_gain > other.info_gain

def calculate_point_estimate(dataset, label_dict):
        # Calculate the dominant label (subreddit) in the dataset
        label_counts = {ATHEISM_ID: 0, BOOKS_ID: 0}
        for doc_id in dataset:
            label = label_dict[doc_id]
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            label_counts[label] += 1
        # The point estimate is the label with the highest count
        point_estimate = max(label_counts, key=label_counts.get)
        return point_estimate

def find_best_to_split(dataset, label_dict, method, words=None, splitted_feature=[]):
    # Find the best feature to split next
    # by try to compute the delta_information_gain on all features
    # appeared in the dataset but not the ones in splitted_feature, and return a tuple containing
    # the feature to split that will give the biggest delta_information_gain
    # and the delta_information_gain.
    best_feature = None
    best_info_gain = 0  # Start with 0 to ensure any gain is better, but not no split

    # Iterate through each feature in the dataset to find the best one to split on
    for feature in range(1, NUM_FEATURES + 1):

        # skips the feature we already split on
        if feature in splitted_feature:
            continue

        # Compute the information gain for splitting on the current feature
        current_info_gain = delta_information_gain(dataset, label_dict, feature, method)

        # If the information gain of the current feature is better than the best one so far, update the best feature and gain
        if current_info_gain > best_info_gain:
            #print(f"better word = {words[feature]}, info gain = {current_info_gain} ")
            #print(f"delta I = {delta_information_gain(dataset, label_dict, feature, method, True)}")
            #print("------------")
            best_feature = feature
            best_info_gain = current_info_gain

    #print(f"delta I = {delta_information_gain(dataset, label_dict, best_feature, method, True)}")
    return best_feature, best_info_gain

def split_dataset(dataset, feature_to_split):
    # Datasets to hold the split
    dataset_with_feature = {}
    dataset_without_feature = {}
    #print(f"Splitting on {feature_to_split}")
    # Iterate over each entry in the dataset
    for doc_id, word_ids in dataset.items():
        # Check if the feature to split on is in the document's word IDs
        if feature_to_split in word_ids:
            # Add this document to the dataset with the feature
            #print(f"with_feature: doc_id = {doc_id}, features = {word_ids}")
            dataset_with_feature[doc_id] = word_ids[:] # [:] to make a shallow copy

            #print(f"remove feature: {feature_to_split}")
            # remove the spliting feature
            #dataset_with_feature[doc_id].remove(feature_to_split)
        else:
            # Add this document to the dataset without the feature
            #print(f"without_feature: doc_id = {doc_id}, features = {word_ids}")
            dataset_without_feature[doc_id] = word_ids
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    #print(f"L size: {str(len(dataset_with_feature))}, R size:  {str(len(dataset_without_feature))}")
    return dataset_with_feature, dataset_without_feature

# Function to build the decision tree
def build_decision_tree(train_data, train_labels, method, subreddit_dict, max_nodes=100, words=None):
    MAX_NODES = max_nodes
    pq = PriorityQueue()
    root = Node(dataset=train_data, point_estimate=calculate_point_estimate(train_data, subreddit_dict))

    best_feature, best_info_gain = find_best_to_split(root.dataset, train_labels, method, words=words)

    # Update root node with split info
    root.feature_to_split = best_feature
    root.info_gain = best_info_gain

    pq.put(root)

    while not pq.empty() and max_nodes > 0:

        current_node = pq.get()

        # We only counts the number of internal nodes with max_nodes
        max_nodes -= 1
        #print(f"\ndoing the {MAX_NODES - max_nodes}'th node")
        #print(f"info gained = {info_gained}, split word = {words[current_node.feature_to_split]}")

        # Split the dataset based on the best feature
        current_node.splitted_feature.append(current_node.feature_to_split) # add splitted feature

        #print(f"splitted feature list is  {current_node.splitted_feature}")
        left_dataset, right_dataset = split_dataset(current_node.dataset, current_node.feature_to_split)

        best_feature_L, best_info_gain_L = find_best_to_split(left_dataset, train_labels, method, words=words, splitted_feature=current_node.splitted_feature)
        best_feature_R, best_info_gain_R = find_best_to_split(right_dataset, train_labels, method, words=words, splitted_feature=current_node.splitted_feature)


        left_node = Node(dataset=left_dataset, point_estimate=calculate_point_estimate(left_dataset, subreddit_dict), info_gain=best_info_gain_L, feature_to_split=best_feature_L, splitted_feature=current_node.splitted_feature)
        right_node = Node(dataset=right_dataset, point_estimate=calculate_point_estimate(right_dataset, subreddit_dict), info_gain=best_info_gain_R, feature_to_split=best_feature_R, splitted_feature=current_node.splitted_feature)

        # Update current node with split info
        current_node.left = left_node
        current_node.right = right_node

        # Add child nodes to the priority queue
        if best_feature_L is not None:
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            pq.put(left_node)
            #print(f"Adde L feature = {words[best_feature_L]}, info gain = {best_info_gain_L}")
        if best_feature_R is not None:
            pq.put(right_node)
            #print(f"Added R feature = {words[best_feature_R]}, info gain = {best_info_gain_R}")

    return root

def print_tree(node, depth=0, feature_names=None):
    # Base case: if the node is a leaf, it will not have a child
    if node.left is None or node.right is None:
        print("-" * depth + "Leaf, estimate: " + LABEL_STR[node.point_estimate])
        return

    # Recursive case: print the current node's split information
    if feature_names and node.feature_to_split in feature_names:
        feature_name = feature_names[node.feature_to_split]
    else:
        feature_name = str(node.feature_to_split)

    print("-" * depth + f"Node: Split Feature = {feature_name}, Info Gain = {node.info_gain:.10f}")

    # Recursively print the left subtree
    print("-" * depth + "L (w/ feature):")
    print_tree(node.left, depth + 1, feature_names)

    # Recursively print the right subtree
    print("-" * depth + "R (wo/ feature):")
    print_tree(node.right, depth + 1, feature_names)

##### TESTING ######

# Use decision tree to predict the label for a single document
def predict_label(node, document_word_array):
    # If we have reached a leaf node, return its point estimate
    if node.left is None and node.right is None:
        return node.point_estimate
    # If the document contains the word_id at the current node, go left
    elif node.feature_to_split in document_word_array:
        return predict_label(node.left, document_word_array)
    # If the document does not contain the word_id, go right
    else:
        return predict_label(node.right, document_word_array)

# Function to calculate the accuracy of the decision tree
def calculate_accuracy(tree, data, labels):
    correct_predictions = 0
    # Iterate over all documents in the test data
    for doc_id, document_word_array in data.items():
        # Use the tree to predict the label for the current document
        predicted_label = predict_label(tree, document_word_array)
        # If the predicted label matches the actual label, increment the correct predictions count
        if predicted_label == labels[doc_id]:
            correct_predictions += 1
    # Calculate the percentage of correctly classified samples
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    accuracy = (correct_predictions / len(data)) * 100
    return accuracy

##### CONCURRENCY #####

# Function to build a tree and calculate accuracies for a given number of nodes
def compute_accuracies_for_nodes(max_nodes):
    words = read_word_data('./words.txt')
    train_data = read_document_data('./trainData.txt')
    train_labels = read_label_data('./trainLabel.txt')
    test_data = read_document_data('./testData.txt')
    test_labels = read_label_data('./testLabel.txt')

    # Build trees using both methods
    tree1 = build_decision_tree(train_data, train_labels, method=1, subreddit_dict=train_labels, words=words, max_nodes=max_nodes)
    tree2 = build_decision_tree(train_data, train_labels, method=2, subreddit_dict=train_labels, words=words, max_nodes=max_nodes)

    # Calculate accuracies for both trees
    accuracy_train_tree1 = calculate_accuracy(tree1, train_data, train_labels)
    accuracy_test_tree1 = calculate_accuracy(tree1, test_data, test_labels)
    accuracy_train_tree2 = calculate_accuracy(tree2, train_data, train_labels)
    accuracy_test_tree2 = calculate_accuracy(tree2, test_data, test_labels)

    # Return a tuple of accuracies
    print(f"Done max_node = {str(max_nodes)}.")
    return (max_nodes, accuracy_train_tree1, accuracy_test_tree1, accuracy_train_tree2, accuracy_test_tree2)

##### MAIN ######

if __name__ == '__main__':

    # Reading data from files
    words = read_word_data('./words.txt')
    train_data = read_document_data('./trainData.txt')
    train_labels = read_label_data('./trainLabel.txt')
    test_data = read_document_data('./testData.txt')
    test_labels = read_label_data('./testLabel.txt')

    ##### b) #####

    print("--- building tree 1 ---\n")
    tree1 = build_decision_tree(train_data, train_labels, method=1, subreddit_dict=train_labels, words=words, max_nodes=10)
    print("\n--- method 1 tree ---\n")
    print_tree(tree1, feature_names=words)

    print("\n--- building tree 2 ---\n")
    tree2 = build_decision_tree(train_data, train_labels, method=2, subreddit_dict=train_labels, words=words, max_nodes=10)
    print("\n--- method 2 tree ---\n")
    print_tree(tree2, feature_names=words)

    # Validate the decision trees tree1 and tree2
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    accuracy_tree1 = calculate_accuracy(tree1, test_data, test_labels)
    accuracy_tree2 = calculate_accuracy(tree2, test_data, test_labels)

    # Print the accuracies
    print(f"\nAccuracy of tree1 (Method 1): {accuracy_tree1:.5f}%\n")
    print(f"\nAccuracy of tree2 (Method 2): {accuracy_tree2:.5f}%\n")

    ##### C) #####

    # Number of processes to use
    num_processes = 100 # For school server, laptop will explode with this

    # Create a pool of processes
    with Pool(processes=num_processes) as pool:
        # Map the compute_accuracies_for_nodes function to each number of max_nodes
        results = pool.map(compute_accuracies_for_nodes, range(1, 101))

    # Now we will process the results to plot them
    # Initialize lists to hold accuracies for plotting
    nodes = []
    accuracy_train_method1 = []
    accuracy_test_method1 = []
    accuracy_train_method2 = []
    accuracy_test_method2 = []

    # Populate the lists with data
    i = 1
    for result in results:
        print(f"{str(i)} node, train_m1 = {result[1]:.5f}, test_m1 = {result[2]:.5f}, train_m2 = {result[3]:.5f}, test_m2 = {result[4]:.5f}")
        nodes.append(result[0])
        accuracy_train_method1.append(result[1])
        accuracy_test_method1.append(result[2])
        accuracy_train_method2.append(result[3])
        accuracy_test_method2.append(result[4])
        i += 1

    # Plotting the results
    plt.figure(figsize=(14, 7))
    # Plot for method 1
    plt.subplot(1, 2, 1)
    plt.scatter(nodes, accuracy_train_method1, label='Training Accuracy (Method 1)')
    plt.scatter(nodes, accuracy_test_method1, label='Testing Accuracy (Method 1)')
    plt.xlabel('Number of Nodes')
    plt.ylabel('Accuracy (%)')
    plt.title('Training and Testing Accuracies (Method 1)')
    plt.legend()
    # Plot for method 2
    plt.subplot(1, 2, 2)
    plt.scatter(nodes, accuracy_train_method2, label='Training Accuracy (Method 2)')
    plt.scatter(nodes, accuracy_test_method2, label='Testing Accuracy (Method 2)')
    plt.xlabel('Number of Nodes')
    plt.ylabel('Accuracy (%)')
    plt.title('Training and Testing Accuracies (Method 2)')
    plt.legend()
    plt.savefig('method_accuracies.png')
```
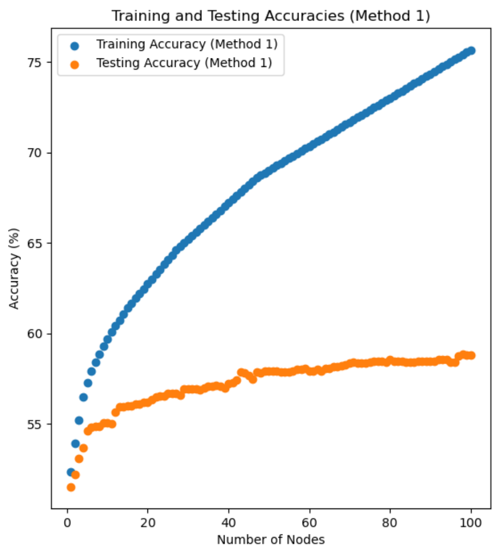
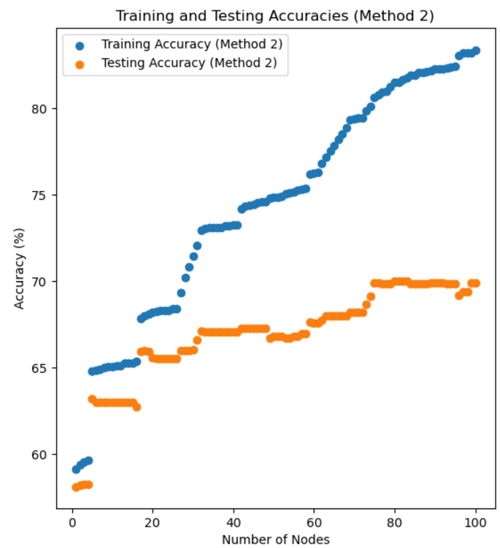

b)

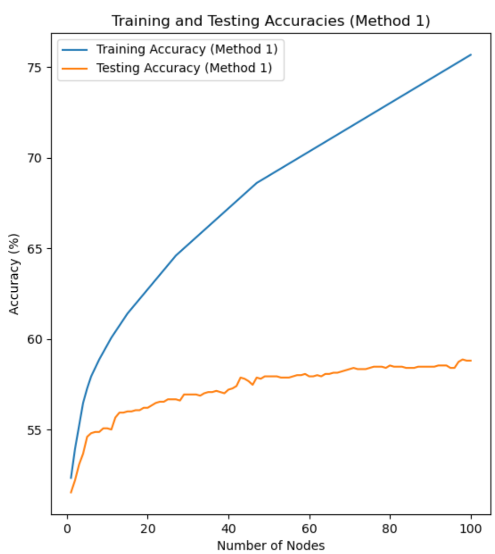
```
--- method 1 tree ---

Node: Split Feature = christian, Info Gain = 0.5002058812
L (w/ feature):
-Leaf, estimate: Atheism
R (wo/ feature):
-Node: Split Feature = atheism, Info Gain = 0.5001930379
-L (w/ feature):
--Leaf, estimate: Atheism
-R (wo/ feature):
--Node: Split Feature = christians, Info Gain = 0.4998761311
--L (w/ feature):
---Leaf, estimate: Atheism
--R (wo/ feature):
---Node: Split Feature = beliefs, Info Gain = 0.4995539340
---L (w/ feature):
----Leaf, estimate: Atheism
---R (wo/ feature):
----Node: Split Feature = atheists, Info Gain = 0.4987661687
----L (w/ feature):
-----Leaf, estimate: Atheism
----R (wo/ feature):
-----Node: Split Feature = brain, Info Gain = 0.4982474614
-----L (w/ feature):
------Leaf, estimate: Atheism
-----R (wo/ feature):
------Node: Split Feature = aa, Info Gain = 0.4976745555
------L (w/ feature):
-------Leaf, estimate: Atheism
------R (wo/ feature):
-------Node: Split Feature = murder, Info Gain = 0.4973448755
-------L (w/ feature):
--------Leaf, estimate: Atheism
-------R (wo/ feature):
--------Node: Split Feature = proof, Info Gain = 0.4969295610
--------L (w/ feature):
---------Leaf, estimate: Atheism
--------R (wo/ feature):
---------Node: Split Feature = logic, Info Gain = 0.4966008344
---------L (w/ feature):
----------Leaf, estimate: Atheism
---------R (wo/ feature):
----------Leaf, estimate: Books
```

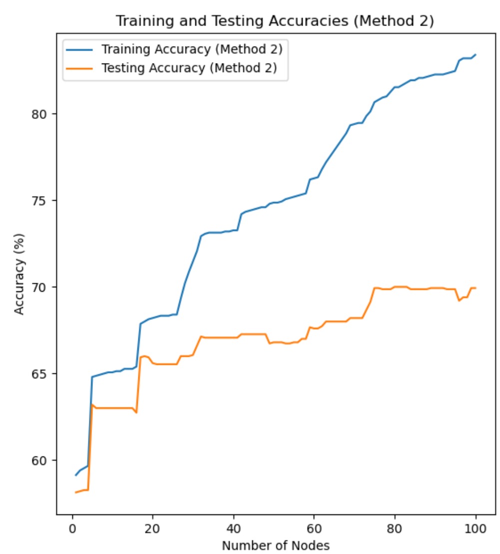
```
  --- method 2 tree ---

Node: Split Feature = book, Info Gain = 0.0770187045
L (w/ feature):
-Node: Split Feature = bible, Info Gain = 0.1153579525
-L (w/ feature):
--Leaf, estimate: Atheism
-R (wo/ feature):
--Node: Split Feature = call, Info Gain = 0.0694586299
--L (w/ feature):
---Leaf, estimate: Atheism
--R (wo/ feature):
---Node: Split Feature = sent, Info Gain = 0.0842240762
---L (w/ feature):
----Leaf, estimate: Atheism
---R (wo/ feature):
----Node: Split Feature = controlling, Info Gain = 0.0587581571
----L (w/ feature):
-----Leaf, estimate: Atheism
----R (wo/ feature):
-----Leaf, estimate: Books
R (wo/ feature):
-Node: Split Feature = books, Info Gain = 0.0599261848
-L (w/ feature):
--Node: Split Feature = sure, Info Gain = 0.0976949605
--L (w/ feature):
---Node: Split Feature = soon, Info Gain = 0.9182958341
---L (w/ feature):
----Leaf, estimate: Books
---R (wo/ feature):
----Leaf, estimate: Atheism
--R (wo/ feature):
---Node: Split Feature = spirit, Info Gain = 0.0969446061
---L (w/ feature):
----Leaf, estimate: Atheism
---R (wo/ feature):
----Leaf, estimate: Books
-R (wo/ feature):
--Node: Split Feature = religion, Info Gain = 0.0356734070
--L (w/ feature):
---Leaf, estimate: Atheism
--R (wo/ feature):
---Leaf, estimate: Atheism
```

c) Traing & Testing Accuracies:



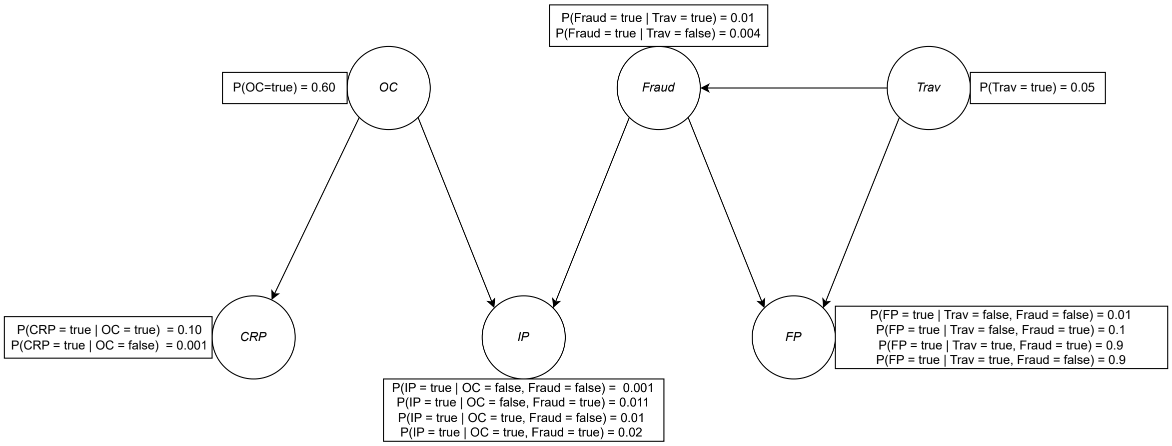






Problem 2

a) Bayes Network:



b) The prior probability that the current transaction is a fraud is:

$P(Fraud = true) = \sum_{Trav} P(Fraud = true|Trav)P(Trav)$

$= \sum_{Trav} f_1(Trav)f_0(Trav)$

where $f_0(Trav) = P(Trav)$

$Trav$	$f_0(Trav)$
t	0.05
f	0.95

where $f_1(Trav) = P(Fraud = true|Trav)$

$Trav$	$P(Fraud = true)$
t	0.01
f	0.004

Thus we get $P(Fraud = true) \propto \sum_{Trav} f_1(Trav)f_0(Trav)$

$= 0.05 \times 0.01 + 0.95 \times 0.004 = 0.0043$

$f_0(Trav) = P(Trav) =$

$Trav$	$f_0(Trav)$
t	0.05
f	0.95

$f_1(Fraud, Trav) = P(Fraud|Trav) =$

$Fraud$	$Trav$	$f_1(Fraud|Trav)$
t	t	0.01
t	f	0.004
f	t	$1 - 0.01 = 0.99$
f	f	$1 - 0.004 = 0.996$

$f_2(OC) = P(OC) =$

OC	$f_2(OC)$
t	0.6
f	0.4

$f_3(Fraud, Trav) = P(FP = true|Fraud, Trav) =$

$Fraud$	$Trav$	$f_3(Fraud, Trav)$
t	t	0.9
t	f	0.1
f	t	0.9
f	f	0.01

$f_4(OC) = P(CRP = true|OC) =$

OC	$f_4(OC)$
t	0.1
f	0.001

$f_5(Fraud, OC) = P(IP = false|Fraud, OC) =$

$Fraud$	OC	$f_5(Fraud, OC)$
t	t	$1 - 0.02 = 0.98$
t	f	$1 - 0.011 = 0.989$
f	t	$1 - 0.01 = 0.99$
f	f	$1 - 0.001 = 0.999$

So that $P(Fraud|FP = true, IP = false, CRP = true)$

$\propto \sum_{OC} \sum_{Trav} f_0(Trav)f_1(Fraud, Trav)f_2(OC)f_3(Fraud, Trav)f_4(OC)f_5(Fraud, OC)$

$= \sum_{OC} f_2(OC)f_4(OC)f_5(Fraud, OC) \sum_{Trav} f_0(Trav)f_1(Fraud, Trav)f_3(Fraud, Trav)$

$= \sum_{OC} f_2(OC)f_4(OC)f_5(Fraud, OC)f_6(Fraud)$

Where $f_6(Fraud) = \sum_{Trav} f_0(Trav)f_1(Fraud, Trav)f_3(Fraud, Trav)$

=

$Fraud$	$f_6(Fraud)$
t	$0.05 \times 0.01 \times 0.9 + 0.95 \times 0.004 \times 0.1 = 0.00083$
f	$0.05 \times 0.99 \times 0.9 + 0.95 \times 0.996 \times 0.01 = 0.054012$

Then, we get $\sum_{OC} f_2(OC)f_4(OC)f_5(Fraud, OC)f_6(Fraud) = f_7(Fraud)f_6(fraud)$

Where $f_7(Fraud) = \sum_{OC} f_2(OC)f_4(OC)f_5(Fraud, OC)$

=

$Fraud$	$f_7(Fraud)$
t	$0.6 \times 0.1 \times 0.98 + 0.4 \times 0.001 \times 0.989 = 0.0591956$
f	$0.6 \times 0.1 \times 0.99 + 0.4 \times 0.001 \times 0.999 = 0.059799599999999994$

Thus, we get $f_8(Fraud) = f_7(Fraud)f_6(fraud)$

=

$Fraud$	$f_8(Fraud)$
t	$0.00083 \times 0.0591956 = 0.000049132348$
f	$0.054012 \times 0.059799599999999994 = 0.0032298959951999997$

Thus, we get $P(Fraud = true|FP = true, IP = false, CRP = true)$ as
$\frac{0.000049132348}{0.000049132348+0.0032298959951999997} = 0.014983813147541082 \approx 0.01498$

c) $f_0(Fraud) = P(Fraud|Trav = true) =$

$Fraud$	$f_0(Fraud)$
t	0.01
f	$(1 - 0.01) = 0.99$

$f_1(OC) = P(OC) =$

OC	$f_1(OC)$
t	0.6
f	0.4

$f_2(Fraud) = P(FP = true|Fraud, Trav = true) =$

$Fraud$	$f_2(Fraud)$
t	0.9
f	0.9

$f_3(OC) = P(CRP = true|OC) =$

OC	$f_3(OC)$
t	0.1
f	0.001

$f_5(Fraud, OC) = P(IP = false|Fraud, OC) =$

$Fraud$	OC	$f_5(Fraud, OC)$
t	t	$1 - 0.02 = 0.98$
t	f	$1 - 0.011 = 0.989$
f	t	$1 - 0.01 = 0.99$
f	f	$1 - 0.001 = 0.999$

So, $P(Fraud|FP = true, IP = false, CRP = true, Trav = true)$
$= \sum_{OC} f_0(Fraud)f_1(OC)f_2(Fraud)f_3(OC)f_5(Fraud, OC)$
$= f_0(Fraud)f_2(Fraud)\sum_{OC} f_1(OC)f_3(OC)f_5(Fraud, OC)$
$= f_0(Fraud)f_2(Fraud)f_6(Fraud)$

Where $f_6(Fraud) = \sum_{OC} f_1(OC)f_3(OC)f_5(Fraud, OC)$

$=$

$Fraud$	$f_6(Fraud)$
t	$0.6 \times 0.1 \times 0.98 + 0.4 \times 0.001 \times 0.989 = 0.0591956$
f	$0.6 \times 0.1 \times 0.99 + 0.4 \times 0.001 \times 0.999 = 0.0597996$

Thus, we get $f_7(Fraud) = f_0(Fraud)f_2(Fraud)f_6(Fraud)$

$=$

$Fraud$	$f_7(Fraud)$
t	$0.01 \times 0.9 \times 0.0591956 = 0.0005327604$
f	$0.99 \times 0.9 \times 0.0597996 = 0.0532814436$

Thus, we get $P(Fraud = true|FP = true, IP = false, CRP = true, Trav = true)$
$= \frac{0.0005327604}{0.0005327604+0.0532814436} = 0.00989999591929298 \approx 0.0099$

d) We will need to calculate $P(Fraud|IP = true)$.

$f_0(Trav) = P(Trav) =$

$Trav$	$f_0(Trav)$
t	0.05
f	0.95

$f_1(Fraud, Trav) = P(Fraud|Trav) =$

$Fraud$	$Trav$	$f_1(Fraud|Trav)$
t	t	0.01
t	f	0.004
f	t	$1 - 0.01 = 0.99$
f	f	$1 - 0.004 = 0.996$

$f_2(OC) = P(OC) =$

OC	$f_2(OC)$
t	0.6
f	0.4

$f_3(FP, Fraud, Trav) = P(FP|Fraud, Trav) =$

FP	$Fraud$	$Trav$	$f_3(Fraud, Trav)$
t	t	t	0.9
t	t	f	0.1
t	f	t	0.9
t	f	f	0.01
f	t	t	$1 - 0.9 = 0.1$
f	t	f	$1 - 0.1 = 0.9$
f	f	t	$1 - 0.9 = 0.1$
f	f	f	$1 - 0.01 = 0.99$

$f_4(CRP, OC) = P(CRP|OC) =$

CRP	OC	$f_4(CRP, OC))$
t	t	0.10
t	f	0.001
f	t	$1 - 0.10 = 0.9$
f	f	$1 - 0.001 = 0.999$

$f_5(Fraud, OC) = P(IP = true|Fraud, OC) =$

$Fraud$	OC	$f_5(Fraud, OC)$
t	t	0.02
t	f	0.011
f	t	0.01
f	f	0.001

Thus we get $P(Fraud|IP = true)$
$\propto \sum_{CRP} \sum_{OC} f_2(OC) f_4(CRP, OC) f_5(Fraud, OC) \sum_{FP} \sum_{Trav} f_0(Trav) f_1(Fraud, Trav) f_3(FP, Fraud, Trav)$
$= \sum_{CRP} \sum_{OC} f_2(OC) f_4(CRP, OC) f_5(Fraud, OC) \sum_{FP} f_6(Fraud, FP)$

Where $f_6(Fraud, FP) = \sum_{Trav} f_0(Trav) f_1(Fraud, Trav) f_3(FP, Fraud, Trav)$

$=$

$Fraud$	FP	$f_6(Fraud, FP)$
t	t	$0.05 \times 0.01 \times 0.9 + 0.95 \times 0.004 \times 0.1 = 0.00083$
t	f	$0.05 \times 0.01 \times 0.1 + 0.95 \times 0.004 \times 0.9 = 0.00347$
f	t	$0.05 \times 0.99 \times 0.9 + 0.95 \times 0.996 \times 0.01 = 0.054012$
f	f	$0.05 \times 0.99 \times 0.1 + 0.95 \times 0.996 \times 0.99 = 0.941688$

So, $\sum_{CRP} \sum_{OC} f_2(OC) f_4(CRP, OC) f_5(Fraud, OC) \sum_{FP} f_6(Fraud, FP)$
$= \sum_{CRP} \sum_{OC} f_2(OC) f_4(CRP, OC) f_5(Fraud, OC) f_7(Fraud)$

Where $f_7(Fraud) = \sum_{FP} f_6(Fraud, FP)$

$Fraud$	$f_7(Fraud)$
t	$0.00083 + 0.00347 = 0.0043$
f	$0.054012 + 0.941688 = 0.9957$

So, $\sum_{CRP} \sum_{OC} f_2(OC) f_4(CRP, OC) f_5(Fraud, OC) f_7(Fraud) = \sum_{CRP} f_8(Fraud, CRP)$

Where $f_8(Fraud, CRP) = \sum_{OC} f_2(OC) f_4(CRP, OC) f_5(Fraud, OC) f_7(Fraud)$

=

$Fraud$	CRP	$f_8(Fraud, CRP)$
t	t	$0.6 \times 0.1 \times 0.02 \times 0.0043 + 0.4 \times 0.001 \times 0.011 \times 0.0043 = 0.00000517892$
t	f	$0.6 \times 0.9 \times 0.02 \times 0.0043 + 0.4 \times 0.999 \times 0.011 \times 0.0043 = 0.00006534108$
f	t	$0.6 \times 0.1 \times 0.01 \times 0.9957 + 0.4 \times 0.001 \times 0.001 \times 0.9957 = 0.00059781828$
f	f	$0.6 \times 0.9 \times 0.01 \times 0.9957 + 0.4 \times 0.999 \times 0.001 \times 0.9957 = 0.00577466172$

So, $f_9(Fraud) = \sum_{CRP} f_8(Fraud, CRP)$

=

$Fraud$	$f_9(Fraud)$
t	$0.00000517892 + 0.00006534108 = 0.00007052$
f	$0.00059781828 + 0.00577466172 = 0.00637248$

So, we get $P(Fraud = true|IP = true) = \frac{0.00007052}{0.00007052+0.00637248} = 0.010945211857830203 \approx 0.01095.$

Now we proceed with $P(Fraud = true|IP = true, CRP = true)$:

$f_0(Trav) = P(Trav) =$

$Trav$	$f_0(Trav)$
t	0.05
f	0.95

$f_1(Fraud, Trav) = P(Fraud|Trav) =$

$Fraud$	$Trav$	$f_1(Fraud|Trav)$
t	t	0.01
t	f	0.004
f	t	$1 - 0.01 = 0.99$
f	f	$1 - 0.004 = 0.996$

$f_2(OC) = P(OC) =$

OC	$f_2(OC)$
t	0.6
f	0.4

$f_3(FP, Fraud, Trav) = P(FP|Fraud, Trav) =$

FP	$Fraud$	$Trav$	$f_3(Fraud, Trav)$
t	t	t	0.9
t	t	f	0.1
t	f	t	0.9
t	f	f	0.01
f	t	t	$1 - 0.9 = 0.1$
f	t	f	$1 - 0.1 = 0.9$
f	f	t	$1 - 0.9 = 0.1$
f	f	f	$1 - 0.01 = 0.99$

$f_4(OC) = P(CRP = true|OC) =$

OC	$f_4(OC))$
t	0.10
f	0.001

$f_5(Fraud, OC) = P(IP = true|Fraud, OC) =$

$Fraud$	OC	$f_5(Fraud, OC)$
t	t	0.02
t	f	0.011
f	t	0.01
f	f	0.001

Thus we get $P(Fraud|IP = true, CRP = true)$
$\propto \sum_{OC} f_2(OC)f_4(OC)f_5(Fraud, OC) \sum_{FP} \sum_{Trav} f_0(Trav)f_1(Fraud, Trav)f_3(FP, Fraud, Trav)$
$= \sum_{OC} f_2(OC)f_4(OC)f_5(Fraud, OC) \sum_{FP} f_6(Fraud, FP)$

Where $f_6(Fraud, FP) = \sum_{Trav} f_0(Trav)f_1(Fraud, Trav)f_3(FP, Fraud, Trav)$

=

$Fraud$	FP	$f_6(Fraud, FP)$
t	t	$0.05 \times 0.01 \times 0.9 + 0.95 \times 0.004 \times 0.1 = 0.00083$
t	f	$0.05 \times 0.01 \times 0.1 + 0.95 \times 0.004 \times 0.9 = 0.00347$
f	t	$0.05 \times 0.99 \times 0.9 + 0.95 \times 0.996 \times 0.01 = 0.054012$
f	f	$0.05 \times 0.99 \times 0.1 + 0.95 \times 0.996 \times 0.99 = 0.941688$

So, $\sum_{OC} f_2(OC)f_4(OC)f_5(Fraud, OC) \sum_{FP} f_6(Fraud, FP)$
$= \sum_{OC} f_2(OC)f_4(OC)f_5(Fraud, OC)f_7(Fraud)$

Where $f_7(Fraud) = \sum_{FP} f_6(Fraud, FP)$

$Fraud$	$f_7(Fraud)$
t	$0.00083 + 0.00347 = 0.0043$
f	$0.054012 + 0.941688 = 0.9957$

So, $\sum_{OC} f_2(OC)f_4(OC)f_5(Fraud, OC)f_7(Fraud) = f_8(Fraud)$

=

$Fraud$	$f_8(Fraud)$
t	$0.6 \times 0.1 \times 0.02 \times 0.0043 + 0.4 \times 0.001 \times 0.011 \times 0.0043 = 0.00000517892$
f	$0.6 \times 0.1 \times 0.01 \times 0.9957 + 0.4 \times 0.001 \times 0.001 \times 0.9957 = 0.00059781828$

So, we get $P(Fraud = true|IP = true, CRP = true) = \frac{0.00000517892}{0.00000517892+0.00059781828} = 0.008588630262296408$
≈ 0.00859.

So by having $CRP = true$ prior to my internet purchase, the chance of the transaction being rejected as a possible fraud is reduced by $0.010945211857830203 - 0.008588630262296408 = 0.00235658 \approx 0.00236$.