

```

vector<vector<int>>> grassmanCalcul(vector<vector<int>>> S, int k){
    vector<vector<int>>> SI;
    int n = S.size();
    vector<pairs> row1;
    vector<pairs> row2;
    vector<string> combinaisons = comb(n, k);

    for(int h=0; h<combinaisons.size(); h++){
        vector<pairs> result;
        for (int a = 0; a<S[0].size(); a++){
            result.push_back(makePairs(S[stringToInt(combinaisons[h], 0)][a], std::to_string(a)));
        }

        for(int i = 1; i<combinaisons[h].size(); i++){
            row1 = result;
            result.clear();
            row2.clear();

            for (int a = 0; a<S[0].size(); a++){
                row2.push_back(makePairs(S[stringToInt(combinaisons[h], i)][a], std::to_string(a)));
            }

            result = multipli2Vects(row1, row2);

            vector<string> combTemp = comb(maxIndex(result)+1, result[0].second.size());

            int size = result.size();
            for(int j = 0; j<size; j++){
                if(checkExactConcordance(result[j].second, combTemp) == -1){
                    int combinaisonNumber = checkIndexConcordance(result[j].second, combTemp);
                    if(combinaisonNumber != -1){
                        int sign = parity(result[j].second, combTemp[combinaisonNumber]);
                        displayPairVector(result);
                        result = useSym(result, j, sign*result[j].first, combTemp[combinaisonNumber]);
                        size--;
                    }
                }
            }
            else{
                j++;
            }
        }
        vector<int> vectorTemp;
        vectorTemp.clear();
        for(int k = 0; k<result.size(); k++){
            vectorTemp.push_back(result[k].first);
        }
        SI.push_back(vectorTemp);
    }
    return SI;
}

```

```

1 GrassmanCalcul(matrix S, int K){
2     matrix S*;
3     int n = size of S;
4     Vector<pair> row1, row2;
5     Vector<string> combinaisons = comb(n, k);
6
7
8     For h From 0 To size of combinaisons, h<-h+1{
9         Vector<pair> result;
10
11         result <- the row of S corresponding to the first element of combinaisons[h];
12
13
14
15     For i From 0 To K, i<-i+1{
16
17         row1 <- result;
18         clear result and row2;
19
20
21
22
23         row2 <- the row of S corresponding to the ith of combinaisons[h];
24
25
26         result = row1*row2;
27
28         Vector<string> comTemp = comb (max index of result +1, size of second of result elements);
29
30
31         For all elements j of result{
32             if result[j].second != any elements of comTemp{
33
34                 if result[j].second correspond to the combinaisonNumberth element of comTemp but with a element ≠
35                 order{
36                     int sign = parityOfPermutation(result[j].second, comTemp(combinaisonNumber));
37                     add sign*result[j].first to the element of result who has the same combination but in the
38                     right order;
39                     delete the line j of result;
40
41                 }
42             }
43             else{
44                 j <- j+1;
45             }
46         }
47         else{
48             j <- j+1;
49         }
50     }
51 }
52
53 Add the first column of result to S*;
54
55 Return S*;
56
57
58
59
60
61
62

```

Vector<pair> :
Int first, string second

```

5, 0
1, 1
4, 2

```

$5 \cdot e_0 + 1 \cdot e_1 + 4 \cdot e_2$

row1: $\begin{pmatrix} 5 & 0 \\ 1 & 1 \\ 4 & 2 \end{pmatrix}$ * row2: $\begin{pmatrix} -2 & 0 \\ 3 & 1 \\ 1 & 2 \end{pmatrix}$

```

15, 01
5, 02
-2, 10
1, 12
-8, 20
12, 21

```

Max index of result:2

Size of second of result : 2
because combination of
2 vectors

before sym:
15, 01
5, 02
-2, 10
1, 12
-8, 20
12, 21
replacing -2 with index 10 by adding 2 to index 01
after sym:
17, 01
5, 02
1, 12
-8, 20
12, 21

Calcul of the parity of the permutation by counting cycles
If even \Rightarrow parity $\leftarrow 1$
If odd \Rightarrow parity $\leftarrow -1$

end of loop	end of loop	end of loop
17, 01	10, 01	-4, 01
13, 02	35, 02	-14, 02
-11, 12	-1, 12	19, 12

```

17 13 -11
10 35 -1
-4 -14 19

```

Complexity in function of n , k and $s =$ number of elements of $\text{comb}(n, k)$:
We have 3 main nested loops of lengths s , k and s again.

Consequently, the complexity of this algorithm is $T(n, k, s) \in O(s^2 k)$

Choice of the matrix :

Generation of a random generation of a non-invertible $n \times n$ matrix

Generation of a $n \times (n-1)$ matrix, the last column is the sum of all other columns:

Choice of n and k :

n k s s^2 s^2/k (with s=number of combination(n, k))

4	2	6	36	18
5	2	10	100	50
5	3	10	100	33.3333
6	2	15	225	112.5
6	3	20	400	133.333
6	4	15	225	56.25
7	2	21	441	220.5
7	3	35	1225	408.333
7	4	35	1225	306.25
7	5	21	441	88.2
8	2	28	784	392
8	3	56	3136	1045.33
8	4	70	4900	1225
8	5	56	3136	627.2
8	6	28	784	130.667
9	2	36	1296	648
9	3	84	7056	2352
9	4	126	15876	3969
9	5	126	15876	3175.2
9	6	84	7056	1176
9	7	36	1296	185.143
10	2	45	2025	1012.5
10	3	120	14400	4800
10	4	210	44100	11025
10	5	252	63504	12700.8
10	6	210	44100	7350
10	7	120	14400	2057.14
10	8	45	2025	253.125
11	2	55	3025	1512.5
11	3	165	27225	9075
11	4	330	108900	27225
11	5	462	213444	42688.8
11	6	462	213444	35574
11	7	330	108900	15557.1
11	8	165	27225	3403.12
11	9	55	3025	336.111
12	2	66	4356	2178
12	3	220	48400	16133.3
12	4	495	245025	61256.2
12	5	792	627264	125453
12	6	924	853776	142296
12	7	792	627264	89609.1
12	8	495	245025	30628.1
12	9	220	48400	5377.78
12	10	66	4356	435.6
13	2	78	6084	3042
13	3	286	81796	27265.3
13	4	715	511225	127806

Choose