# Model Development Documentation

## Citi Branded Cards Champion Loss Projection (Net Credit Loss) Model

**October 2017**

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# MODEL INFO SHEET

| | | | |
|---|---|---|---|
| Model Name and Model ID: | **GRMI ID** | **iMRM Model ID** | **Model Name** |
| | 43US076 | 134624 | Branded Cards CCAR Account Level PD Model (Champion) 2018 |
| | 43US077 | 134725 | Branded Cards CCAR Account Level EADF Model (Champion) 2018 |
| | 43US078 | 134576 | Branded Cards CCAR Vintage Recovery Rate Model (Champion) 2018 |
| | 43US079 | 134726 | Branded Cards CCAR NCL Model (Champion) 2018 |

| | |
|---|---|
| Adjusted ENR: | $66.91 BN (As of Q2'17) |
| Portfolio Classification | Citi Bank Cards |
| Model Usage | CCAR Loss forecasting and other uses mentioned in Section 1.2. |
| Model Type | Custom |
| Model Risk Level (MRL) | High |
| Model Sponsor | Neeraj Singh |
| Model Developer | Kunal Palit, Xin Li, Ankita Sinha, Tarannum Girdhar, Ujjwal Kumar, Saurav Gandhi, Bhavnish Walia, Rajat Tripathi, Anuj Jain, Ibtesam Siddiqui, Shubhranshu Atray, Deepak Madugula |
| Line(s) of Business | Cards |
| PMACS ID and Description: | 391001 (AA Business), 391002 (Citi Business), 391003 (ATT Business), 1611167 (AA Relationship), 8897876 (Cash), 8897936 (Rewards), 1611501 (Other Co-Brands), 1611500 (Other PPM) |
| Business Unit(s) | NA Branded Cards |
| Product(s) | Consumer Cards, Small business Cards |
| Date Model Placed in Use | 1st Jan 2018 |

| | |
|---|---|
| Model Processing System | UNIX, SAS |
| Benchmark Model | NA |
| Estimation Technique | Compound Model (Components Models PD and EADF are developed using logistic regression and ordinary least square regression respectively; Recovery model was developed using Linear Regression) |
| Date Documentation Submitted | 10/18/2017 |

## VERSION INFORMATION

[Describe whether the model is new or an update/revision to an existing model. If a previous version exists, please describe when it was developed and the last time it was documented.]

| Version | Date | Author | Description of Change |
|---|---|---|---|
| 1.0 | 09/30/2017 | Xin Li, Ankita Sinha, Tarannum Girdhar, Ujjwal Kumar, Saurav Gandhi | Draft |
| 2.0 | 10/14/2017 | Tarannum Girdhar | Updated Section 7 |
| 3.0 | 10/15/2017 | Ankita Sinha | Updated Section 5 |
| 4.0 | 10/18/2017 | Bhavnish Walia | Updated Section 8 |

## PORTFOLIO INFO SHEET

***Example:***

| | Portfolio Description | Product | Portfolio ID | Open Accounts | ENR (US $M) | Annual NCL (US $M) | NCL % | Date (as of) |
|---|---|---|---|---|---|---|---|---|
| 1 | Citi Branded Cards NAM (Excluding Costco) | 1 | | 24.68 MM | $66.91 Billion | $1,438 MM | 2.14% | Q2-2017 |

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# Executive Summary

## 1. Overview and Background

Branded Cards is an unsecured consumer lending branch of the Citi Global Consumer Bank that offers credit cards to customers across the US. There are five major product groups that make up the Consumer portfolio: AAdvantage, Cash, Rewards, Other Co-Brands and Other-PPM. Branded Cards also consists of a business cards portfolio, however the percentage of business cards is relatively small (<5% of the portfolio). Branded Cards also acquired Costco portfolio in June'16. Due to lack of through the cycle sufficient data for Costco, a separate approach has been taken for Costco portfolio. The Champion NCL model for CCAR 2018 is developed for Branded Cards excluding the Costco portfolio. In this document, Branded Cards might be referred to as "the portfolio excluding the Costco accounts" at some places.

The Comprehensive Capital Analysis and Review (CCAR) is an annual and Dodd-Frank Act Stress Test (DFAST) submission is a semi-annual exercise respectively undertaken by the Federal Reserve to ensure that Bank Holding Companies (BHCs) have robust, forward-looking capital planning processes that account for their idiosyncratic risks. Additionally, it is to ensure that BHCs have sufficient capital to continue operations throughout times of economic and financial stress. As sophisticated credit risk models are the foundation for this exercise, it becomes paramount to evaluate and mitigate risks associated with the development and implementation of these models.

Loss forecasting is one of the most critical steps from planning, regulatory and monitoring perspective and hence requires a high degree of precision. As a standard industry wide practice, consistent with Basel definitions, Loss forecasting in account level Champion models has been considered as three components: the risk of default or Probability of Default (PD); the Exposure at Default (EAD), representing average account balance at default; and Loss Given Default (LGD), representing the percentage of the exposure at default that is eventually lost. Probability of Default (PD) indicates the incidence of defaults. Exposure at Default (EAD) provides an estimate of the severity of loss. Expected gross credit loss is the product of PD and EAD and is given by:

**Equation 1**

$$\textit{Gross Expected loss (GCL)} = PD * EAD$$

The business is using the following different modeling approaches to estimate GCL for the CCAR 2018 submission:

- **Account Level Discrete time Survival Models**: Champion PD & EAD models (Next generation of CCAR 2017 Champion models)
- **Account level State transition models**: Challenger model

- **Roll Rate model:** Baseline model for estimating GCL.

Gross Credit Loss of a credit cards portfolio is an important through the cycle metric as it is most sensitive to macro-economic shocks. In order to understand the impact of credit attributes and macro-economic factors on the overall portfolio, GCL needs to be adequately estimated and monitored. Recoveries typically follow Gross Credit Losses and are often reflective of the magnitude of GCL in the previous quarters. Net Credit Loss (NCL) is derived using the component models (PD, EAD and Recovery). The relationship between Net Credit Loss (NCL) and GCL can be defined by:

***NCL = GCL – Net Recoveries – Adjustment & Refunds***

Net recoveries can be decomposed into three components: Cash Collected recoveries, Agency fees and Asset Sales. Below are the approaches used by the Business for modeling cash collected recovery rate.

- **Vintage Based Recovery Rate Model**: Champion Model (Similar methodology to CCAR 2017 challenger approach)
- **Recovery Rate ERM**: Challenger Model (CCAR 2017 Champion Recovery model)

Vintage based recovery rate models are more granular and sophisticated than portfolio based approaches. Accordingly, vintage based Recovery rate model would be used as the Champion model for CCAR 2018 submission. The details would be covered in subsequent sections.

This MDDT describes the Champion Account Level PD & EADF Models and Vintage based Recovery models. The model is designed for CCAR stress testing thereby generating loss forecasts under baseline and stressed economic scenarios.

## 2. Model History

For 2016 CCAR submission, Branded Cards modeled PD and EAD using LookAhead model as the champion model and Account Level Models as Challenger Model. LookAhead model is a segment level model based on age period cohort methodology. Given the success of challenger account level models in CCAR 2016 submission, Branded Cards decided to develop 2nd generation of PD and EAD models at an account level using Discrete Time Survival Approach, as the Champion Model for CCAR 2017 Submission.

For CCAR 2018, Branded Cards is continuing to develop next generation of PD and EAD models for CCAR 2018. LookAhead PD and EAD models for CCAR 2017 submission will continue to serve as the challenger models for CCAR 2018 models. Additionally, the business is developing an alternative account level State transition based approach as a challenger approach for CCAR 2018.

For Recoveries, the business is developing the next generation of Vintage based Recovery rate model (CCAR 2017 challenger approach) as the champion approach. This approach is more sophisticated as

compared to Recovery ERM model (Champion model for CCAR 2017 submission), which would be used as the challenger approach for CCAR 2018 submission.

For CCAR 2018, Branded Cards continued with some of the important enhancements/changes that were made in previous submissions:

**Localized macroeconomic variables in modeling**: As a follow-up of December 2013 submission, Branded Cards received a feedback from the FRB in form of "Matter Requiring Immediate Attention (MRIA)" to ensure that the segmentation approach is sufficiently granular. Specifically, FRB has asked to explore unemployment and other macroeconomic variables at a local or state level. The comments received in the FRB letter are given below verbatim:

*"…ensure that the segmentation approaches used for loss estimation are sufficiently granular to capture exposures that react differently to risk drivers under stressed conditions…"*

*"…The firm's loss estimation methodology for U.S. Cards uses the national unemployment rate rather than localized unemployment rates because the model only has two geographic segments. The unemployment rate is a material risk driver for consumer lending, and, to the extent that the local unemployment rate is higher than the national rate in regions where Citi has a higher concentration of mortgages and/or credit cards, the impact of this risk may not be fully captured in the loss estimates. Citi is required to re-evaluate its variable selection process to ensure key portfolio and macroeconomic risk drivers are identified and all macroeconomic variables that directly impact these risk drivers are incorporated, including state or local-level unemployment rates."*

In response to the above feedback, the regional segmentation granularity was included in the models which resulted into 8 segments in the 2015 and 2016 CCAR submission for the champion models from 2 segments in 2014 submission. In CCAR 2017 and CCAR 2018 submission, the champion models have used state level Unemployment Rate and HPI. The macroeconomic forecasts for these states / regional segments are provided by Global Country Risk team.

## 3. Description of Model Methodology

The Branded Cards account-level PD and EAD models are based on discrete time survival approach. The Discrete Time approach was selected since it was found to be favorable in terms of granularity, model transparency, reporting flexibility and macroeconomic sensitivity. Exposure at Default factor (EADF) approach has been used to model EAD. EADF is defined as $ charge-off amount divided by $ line amount. Justification of the modeling approach selection is provided in Section 4.1. The final forms of equation for PD, EADF (EAD) are as given below:

**Equation 2**

$$PD/EADF = f(Behavioral\ Variables, Macroeconomic\ Variables, time\ since\ observation)$$

In order to ensure robustness of the framework across alternate macroeconomic scenarios, the models for each of the components (PD and EAD) were developed on through-the-cycle data starting q4 2006 till the last available data point i.e. performance till June'16. Last 1 year (4Q) of data (July'16 to June'17) was kept as holdout (Out Of Time) sample for validation.

The PD model is developed using logistic regression technique on a 10% random sample of customers. The EAD model was developed using linear regression technique on the default accounts within the sample used for the development of the PD model. The final model specifications can be found in Section 6.6.

The PD and EAD estimates Gross Credit Loss (for all accounts on books at the time of snapshot) for each of the thirteen quarters in the forecast period as a function of on-us (internal) variables, off-us (bureau) variables, macro-economic factors, portfolio indicators and seasonality. The on-us and off-us variables are fixed at snapshot, whereas the macro-economic factors are allowed to vary over time. Recovery is then calculated at a portfolio level as per formula mentioned above using forecasted GCL to arrive at the final Net Credit Loss. GCL is computed using the PD and EADF models in conjunction with a line amount assumption, as shown in the formula below:

$$\$GCL = PD * \$Line\ Amount * EADF$$

NCL is derived using the component models (PD, EAD and Recovery). The relationship between Net Credit Loss (NCL) and GCL can be defined by:

***NCL = GCL – Net Recoveries – Adjustment & Refunds***

For CCAR 2018 submission, Account Level Models were developed as the next generation of CCAR 2017 champion models. In the current submission, we have enhanced these models in terms of data availability, increasing the candidate pool of variables for modelling, improvement in variable selection process etc. Regular meetings were held with Senior Management in Branded Cards and

Independent Risk (IRMO) and the feedback received was incorporated in the modelling process. The below table lists out the key enhancements made to Account Level Modelling process for this submission:

Table 1-1 Enhancements in Account Level Modeling Process

| Modeling Process | Description |
|---|---|
| **Data** | • **FICO'08 purchased and used in Model development** (In CCAR 2017 aligned FICO had been used)<br>• **More than ~800 Million through-the-cycle records** used for model development compared to ~500 Million used in CCAR 2017 |
| **Design** | • Incremental Variable interactions with Product, FICO, MOB and Delinquency explored<br>• Use of MOB in the forecast horizon |
| **Modeling Framework** | • **EADF approach used instead of EAD approach that was used in CCAR 2017**<br>• Inclusion of age (MOB) of the account in the model |
| **Governance** | • **Regular review with Independent Risk** |
| **Feedback** | • **Time invariant coefficient of macroeconomics tested in the model** |

## 4. Model Performance

Model performance is assessed through back testing and sensitivity analysis.

The models underwent rigorous testing, including out of time validation, in order to ensure accurate prediction over a wide range of macro-economic conditions. A brief summary of results is given below. More details will be provided in the subsequent sections.

Table 1-2: Branded Cards Validation Summary

| | Back Test | Error (%) |
|---|---|---|
| PD | Recent 9Q Back test[1] | 3.9% |
| | Stress 9Q Back test[2] | -0.3% |

[1] Recent 9Q backtest refers to Q2'14 – Q2'16 performance vintages
[2] Stress 9Q backtest refers to Q1'08 – Q1'10 performance vintages

| | | |
|---|---|---|
| | Most Recent 9Q Back test[3] | -3.9% |
| EAD | Recent 9Q Back test | 1.2% |
| | Stress 9Q Back test | -0.5% |
| | Most Recent 9Q Back test | -0.1% |
| GCL | Recent 9Q Back test | 8.0% |
| | Stress 9Q Back test | -2.6% |
| | Most Recent 9Q Back test | 2.2% |
| Recovery | Recent 9Q Back test | -1.2% |
| | Stress 9Q Back test | -3.1% |
| | Most Recent 9Q Back test | -1.0% |
| NCL | Recent 9Q Back test | 7.7% |
| | Stress 9Q Back test | -0.8% |
| | Most Recent 9Q Back test | 3.4% |

It can be observed from the table above that the PD, EAD, Recovery and GCL and NCL obtained through combination of PD, EAD and recovery rate models are well within the 10% threshold for back test errors.

The nine quarter forecast for Branded Cards is presented below for both the MCST base[4] and stress scenarios. A comparison of GCL and NCL is also done with the Challenger Model.

**Table 1-3: Branded Cards 9 Quarters Loss Forecast and Comparison (GCL)**

| $GCL* | Champion | Account Level Models 2017 |
|---|---|---|
| MCST Base ( As of Jun'17) | 5,718 | 5,595 |
| MCST Adverse ( As of Jun'17) | 7,524 | 7,487 |
| MCST Severely Adverse ( As of Jun'17) | 8,916 | 8,993 |

* The GCL presented below is for ECM account only

**Table 1-4: Branded Cards 9 Quarters Loss Forecast and Comparison (NCL)**

| $NCL** | Champion | Account Level Models 2017 |
|---|---|---|
| MCST Base ( As of Jun'17) | 4,843 | 4,623 |
| MCST Adverse ( As of Jun'17) | 6,938 | 6,927 |
| MCST Severely Adverse ( As of Jun'17) | 8,406 | 8,453 |

* *The NCL includes new origination as well.

[3] Most Recent 9Q backtest refers to Q3'15 – Q3'17 performance vintages
[4] MCST scenarios refers to CCAR 2018 submission (Forecast from Q3'17 – Q3'19)

In terms of key drivers of losses, Unemployment Rate and HPI have been identified as the major drivers of Branded Cards losses. The plots for these are provided below. These two macroeconomic variables are also observed to be severely stressed in the Supervisory Severely Adverse scenario and hence it makes intuitive sense that losses in this scenario would be appropriately stressed.

**Figure 1: Unemployment Rate (UR) Comparison**

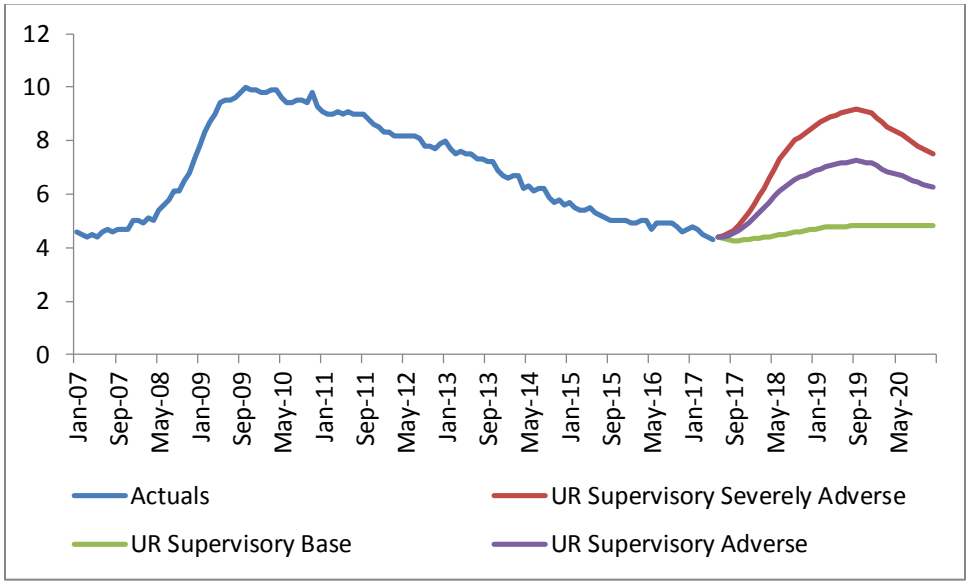
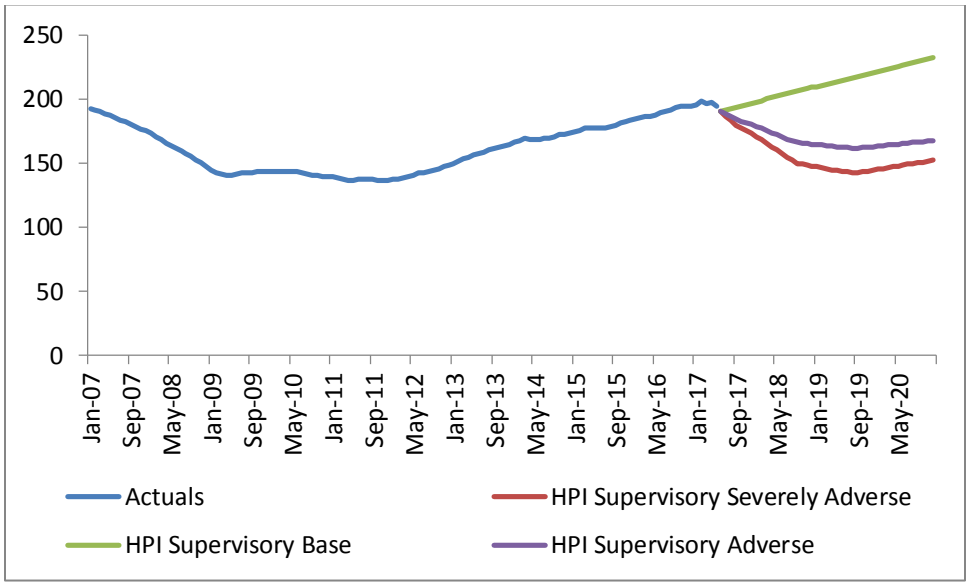



**Figure 2: House Price Index (HPI) Comparison**



# I. Functional Soundness Considerations

[*Part I: Functional Soundness Considerations* to be filled out by model sponsor]

## 1 Model Scope, Purpose and Use

### 1.1 Model Risk Level

[Please provide model risk level assessment.]

***Example (may not apply for all business lines, format layout as necessary.)***

Table 5: Model Risk Level

| | |
|---|---|
| PMACS Portfolio ID(s) | 391001 (AA Business), 391002 (Citi Business), 391003 (ATT Business), 1611167 (AA Relationship), 1611502 ( Cash), 1611502 ( Rewards), 1611501 ( Other Co-Brands), 1611500 ( Other PPM) |
| Legacy Model ID | 43US076,43US077,43US078,43US079 |
| PMACS Data As of Date | 10/16/2017 |
| **1a) Model Exposure**<br>PMACS ENR ($ US b) [0, 9] | 9 (5+ b) |
| 2a) Model Type Use [0,3] | 3 (Loss Forecasting: CCAR or Other Regulatory purposes) |
| 2b) Model Methodology (Complexity) [0,1] | 1 (Complexity (CCAR stress models, Complex Scoring)) |
| 2c) Model Performance [0,2] | 0 (No performance issues ) |
| 2d) Reputational Risk [0,2] | 2 (Significant reputational risk) |
| 2e) Portfolio Classification [0,3] | 3 |
| 2f) Penalty Function [-3,3] | 0 (No Penalty) |
| **Model Specific Risk (MSR)**<br>2a+2b+2c+2d+2e+2f | 3+1+0+2+3+0 = 9 |
| Portfolio Model Risk Index (MRI)<br>1a x MSR | 9 x 9 = 81 |
| **Portfolio Model Risk Level (MRL)**<br>HI, MED, LO, IMM | HI |

| | |
|---|---|
| **3a ) Adjusted ENR ($ US b)*** [0,9] | NA |
| Adjusted Model Risk Index (MRI)<br>3a x MSR | NA |
| **Adjusted Model Risk Level (MRL)**<br>HI, MED, LO, IMM | NA |
| **Model Risk Level** | HI |
| Overlay Reasons | NA |
| Adjustment Rationale & Supporting documentation | NA |

** Loss forecasting models, including those used for CCAR, consider Portfolio Exposure, no Adjusted Exposure (ENR) is required.*

## 1.2 Model Objectives and Business Purpose

The primary objective of the development of these models is to forecast Branded cards portfolio losses over a pre-determined future horizon. The aim is to develop statistically robust suite of models to obtain granular and stable loss forecasts for various purposes. The loss forecasts would be primarily used for CCAR 2018 submission.

The Comprehensive Capital Analysis and Review (CCAR) is an annual and Dodd-Frank Act Stress Test (DFAST) submission is a semi-annual exercise by the Federal Reserve. These exercises are meant to ensure that Bank Holding Companies (BHC's) have robust, forward-looking capital planning processes that account for their idiosyncratic risks and ensure sufficient capital to continue operations throughout times of economic and financial stress. Forecasting losses under varying levels of economic stress or alternate macro-economic scenarios is a key and integral part of this exercise.

The Net Credit Loss Model has been developed as a champion model for annual Comprehensive Capital Analysis and Review (CCAR) and semi-annual Dodd-Frank Act Stress Test (DFAST). The process is designed to forecast losses under various economic conditions. Accordingly, these models forecast nine quarters losses as a part of the Y14-A CCAR/DFAST submission.

Per Y14-A CCAR / DFAST requirements, this model will be used in:

1. **Annual Submission** which analyzes and forecasts losses based on data through the fourth quarter of the given year.

2. **Mid-Year Submission** which is based on data through the first two quarter of the given year.

Additionally, for CCAR 2018, these models will be leveraged to generate forecasts corresponding to a total of six scenarios, which will be in turn used by the business. (Note: Provider of forecast is in brackets):

1. MCST Baseline Scenario (GCRM)
2. MCST Adverse Scenario (GCRM)
3. MCST Severely Adverse Scenario (GCRM)
4. Supervisory Baseline Scenario (FRB/GCRM)
5. Supervisory Adverse Scenario (FRB/GCRM)
6. Supervisory Severe Adverse Scenario (FRB/GCRM)

The PD Model is used in conjunction with EAD and recovery models to generate gross and net credit losses for the portfolio at an overall level and gross loss at an account level for each of the Fed and BHC Scenarios. The NCL generated as a result of this model will be directly used in CCAR 14-A semi-annually. Some of the outcomes can be potentially used as a proxy for the Costco portfolio.

The loss forecast may also be used in conjunction with other business loss forecasting models for strategic planning. The stress captured by the models at a portfolio and sub-portfolio level may be used to enhance the Risk Appetite Framework of the business. The loss forecast from model is also used for LLR for CCAR purpose.

Given that the quarterly NCL prediction are used for various scenarios and the back test errors are well under MRM threshold, the business is comfortable in its usage. Branded Cards takes into consideration the outcome from the CCAR process in various ways:

**Business Planning (CCAR Baseline)** - The semi-annual strategic planning exercise requires a loss forecast over the entire planning horizon. The results from the Account Level, LookAhead and Roll Rate models serve as a triangulation point for the Baseline forecast.

**New Portfolio Acquisition** - BHC and Supervisory CCAR scenarios are used to stress test new portfolios as part of due diligence

## 1.3 Business Scope of the Model

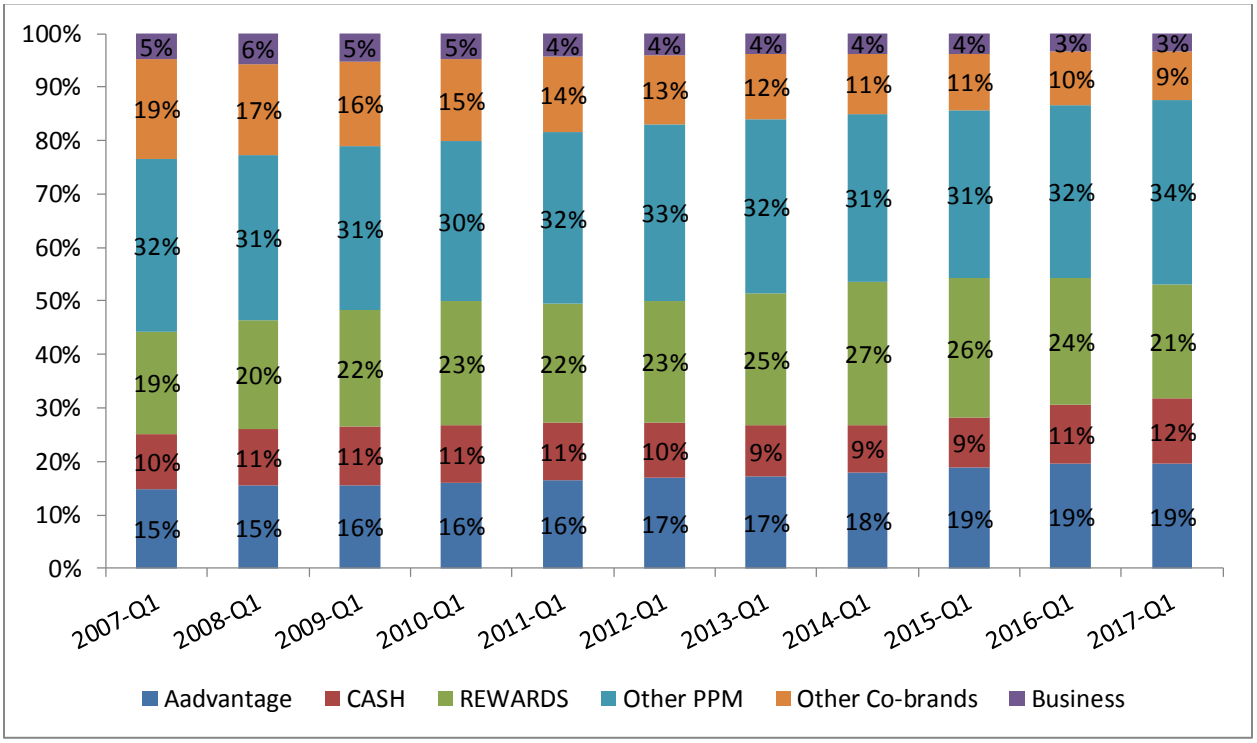
- Which business line does the model describe?

This model is developed for Citi North America Branded Cards portfolio.

- Portfolio description

Branded Cards is an unsecured consumer lending branch of the Citi Global Consumer Bank that provided credit cards to customers across the US. There are five major product groups that make up the Consumer portfolio: AAdvantage, Cash, Rewards, Other Co-Brands and Other-PPM. Branded Cards also consists of business portfolio, however the percentage of business cards is relatively small (<5% of the portfolio). As shown in chart below, the distribution of ENR within each of these products has remained stable over the past among these 5 products and business cards.

**Figure 3 : Distribution of ENR by products[5]**

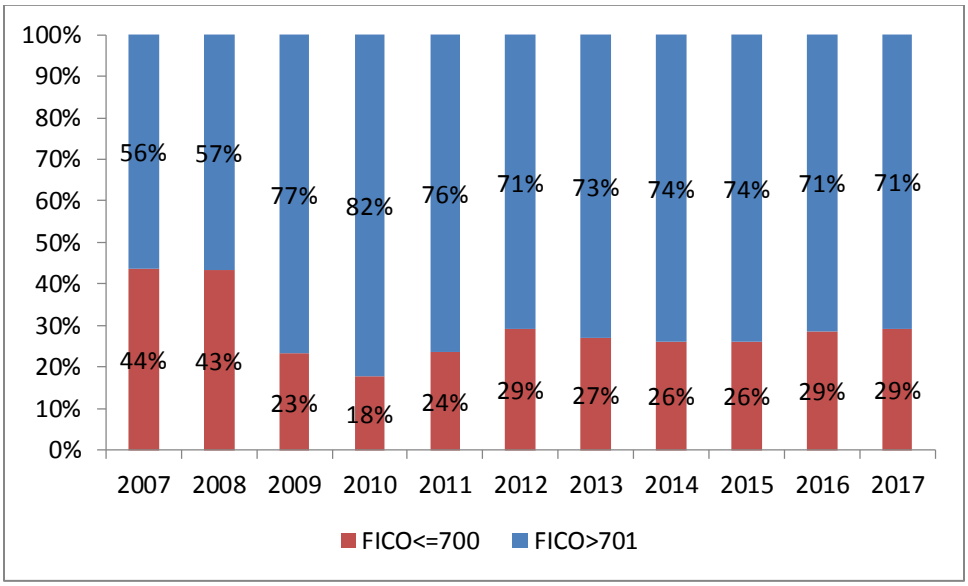



In addition to the five products and business portfolio mentioned above, Branded Cards also acquired Costco portfolio in June'16. However, due to insufficient performance data on Costco portfolio, Costco model has been developed separately.

[5] 100% ENR in Q2'17 = $66.91BN

Branded Cards promotes a strong risk culture with best in class processes and frameworks. Underwriting strategy is designed to target low risk customers who are more likely to show stable performance throughout an economic cycle. New originations are of high quality with stable performance. In recent years, Branded cards have acquired ~71% of new accounts in FICO score greater than 700.

Figure 4 : Distribution of number of acquisitions by FICO




Strong underwriting policies coupled with strict customer credit risk management policies has resulted in improved health of portfolio since the recession in 2008. Improved credit discipline is reflected in 11% reduction in credit line in June 2016 compared to June 2008. Credit actions mentioned above have resulted in lowering of average credit limit, balances and open to buy in the portfolio.

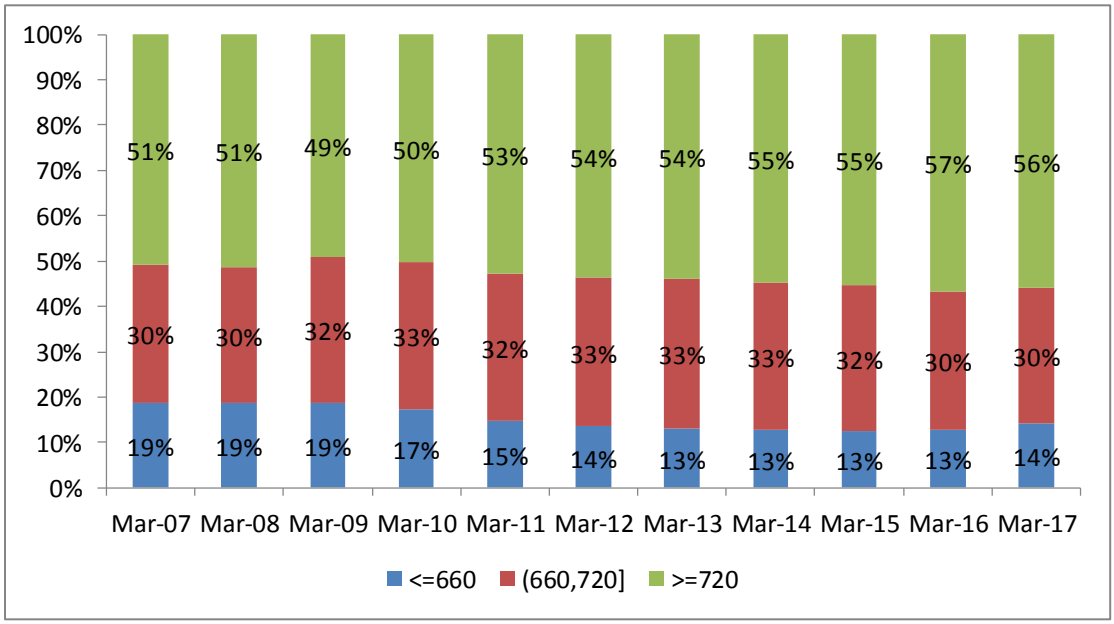
Table 1-6 : Credit metrics in 2008 V/S 2016

| Metric | Jun-08 | Jun-16 | % Change |
|---|---|---|---|
| Open to Buy | $333B | $219B | -34% |
| Average Line | 12,842 | 11,416 | -11% |
| Average Balance | 2,622 | 2,543 | -3% |

Distribution ENR has improved by FICO. From the

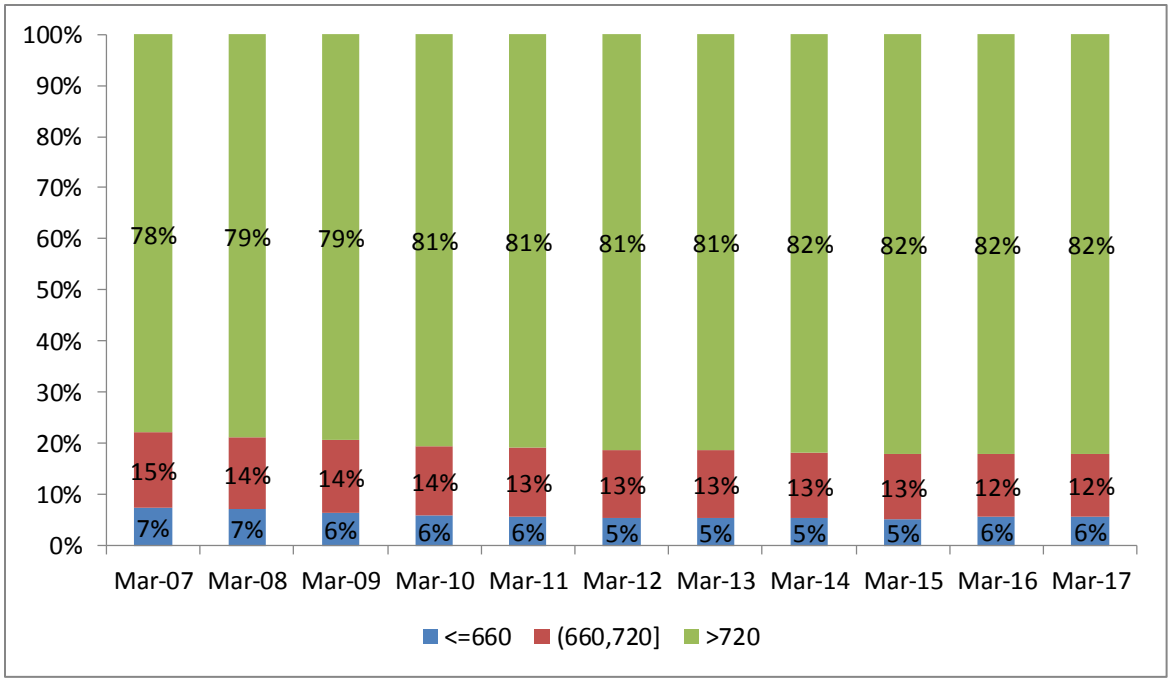
Figure 5 , it can be observed that there is a steady increase in the proportion of ENR (~56% in FICO > 720) for higher FICO bands. This is mainly due to the acquisition policies set forth by the Branded Cards team where a much smaller number of accounts are being approved in the lower FICO Bands (less than 700) as shown above. This increased average FICO of both the overall ECM portfolio and of the new originations has led to less risky portfolio.

**Figure 5 : ENR by refreshed FICO**



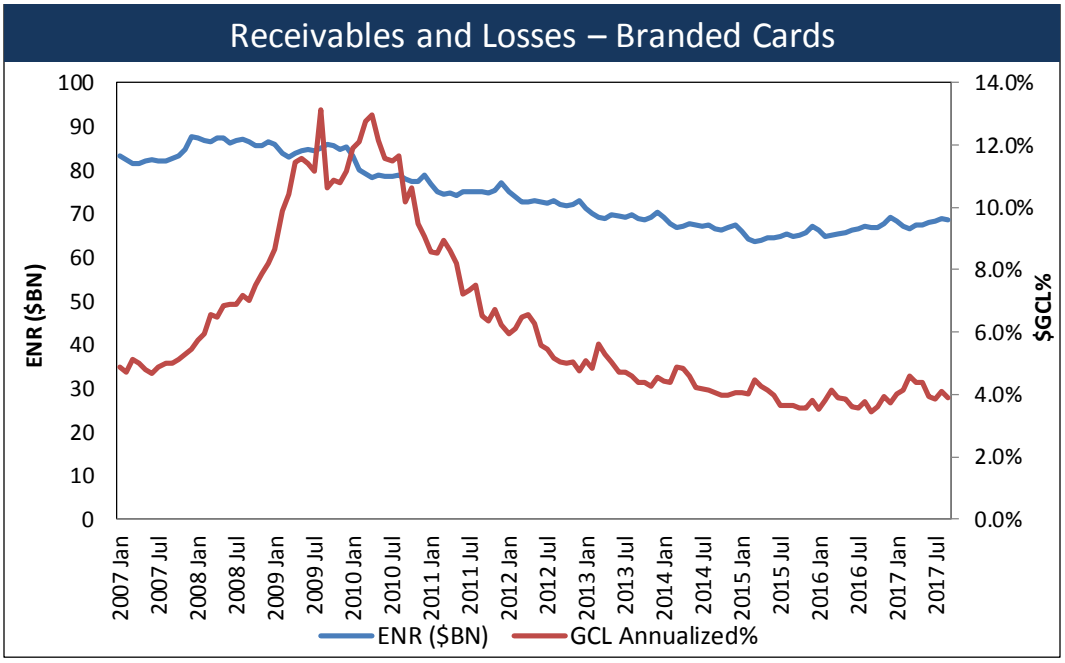

Exposure by FICO has seen an improvement. From Figure 6, it can be observed that there is a steady increase in exposure of High FICO accounts (~82% in FICO > 720)

**Figure 6: Exposure by FICO**




Lower GCL rates and improved recovery rates are evidence of the turnaround in portfolio health. Figure 7 illustrates the trend of Receivables and Losses for Branded Cards. It can be observed that the loss rates of the portfolio are trending downwards and are even lower than the pre-recession levels.

**Figure 7 : Branded Cards -Receivables and Loss Rates (Source: Essbase)**



Further, as shown below in

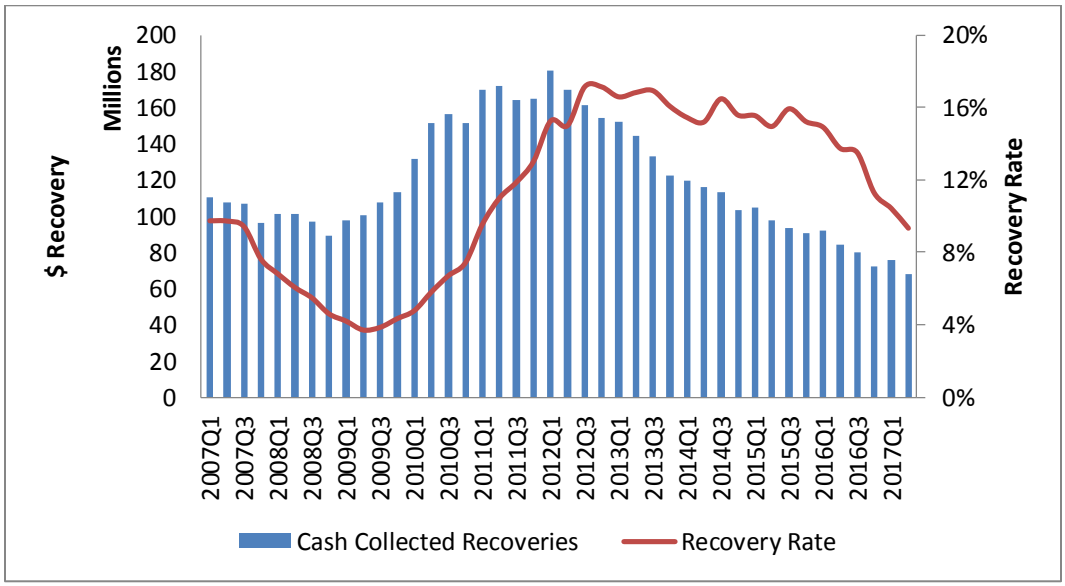
Figure 8, Recovery rate has improved post-recession. However, $Recoveries have decreased in the recent years despite improvement in economy. This can be attributed to the decrease in charge-off volume that has been seen recently.

Recovery rate in the below chart has been defined as the following:

**Equation 3**

$$Recovery\ Rate\ for\ Curr\ Quarter = \frac{Net\ Cash\ Collected\ (\$)\ for\ Current\ Quarter}{GCL\ (\$)\ (Previous\ Quarter\ )}$$

**Figure 8 Trend of Cash Collected Recovery**




## 1.4 Modeling Data

- Why does the model sponsor believe the data was sufficient for the intended purpose?

The model sponsor strongly believes that the data used for the development of these models is sufficient for the purpose. The models were built on through the cycle portfolio data starting Q4 2006 till Q2 2016 containing more than 800M records. The period in consideration covers both stressed and benign economic environments and is therefore adequate and sufficient to build robust models which would hold true for any economic cycle. The data used for modelling includes data from historical stress period i.e. Q1 2008-Q1 2010 (Period of Recession) and data from recent period. As can be observed from the Gating Principle 1-3 attachment in Section 4.2, the portfolio is sufficiently robust and mature for building CCAR models.

## 1.5 Model Assumptions and Framework

- Describe model specification and list model assumptions.

Branded cards have developed account level discrete time survival based PD model and an Ordinary Least Square regression based EADF. The PD and the EADF models are combined and used to calculate quarterly Gross Credit Losses (GCL) for the next thirteen quarters. Further, vintage based recovery models estimate the net recoveries on the portfolio, which when subtracted from GCL provides the Net Credit Losses. The various drivers of these models include and are not limited to Credit attributes like FICO, Credit Balances, Utilization, Delinquency history, Product indicators, Macro-economic factors, Seasonality indicators and Dynamic age of account. The detailed specifications of each of the contributing models can be found in section 6.2.2.

The suite of models includes several model assumptions. These assumptions correspond to the discrete time survival framework, logistic regression based models and ordinary least square regression based models. Additionally, there are some specific assumptions corresponding to the vintage based recovery models which have also been documented. The detailed assumptions corresponding to each of the categories have been documented in section 6.3.

Additionally, it is also assumed that there are no major changes in the regulatory environment going forward.

- Are the assumptions reasonable, taking into consideration the modeling objective, approach, and available data?

All assumptions considered are reasonable, given the modeling objective, approach and available data. Each of the model assumptions has been adequately tested (wherever applicable) and the modelers have ensured that the models pass those tests. Best in class industry literature have been referenced in case any framework specific assumption has been adopted. Additionally any other assumption that has been considered has been empirically tested and documented. Given the large size of the Branded Cards data, the success of the models in the back tests is evidence that the assumptions are reasonable. Further, the assumptions are required to be monitored continuously for usage and future development requirements.

- Have the model assumptions been adequately tested and justified?

The Modeling Team has tested the assumptions, where feasible. The model stability, back tests and loss forecasts are well within expectations. The detailed results for assumption testing are given in Section 8.4.5.

- Impact of assumptions on the model output

The Back Tests for model outputs viz, PD and EADF are well within the prescribed error rates of +/- 10%. Accordingly, there is no evidence of an adverse impact of these assumptions on the model output. The detailed results for assumption testing are given in Section 8.4.5.

## 1.6 Model Performance

- Are model-fit performance results acceptable and do they hold well across samples (hold-out, out-of-time)?

The model performance results are acceptable based on their back testing results as well as model fit statistics. The results hold well across different samples. As seen in the below table the models perform well across the validations that are required by MRM. The most recent validation includes 4 quarters of most recent development data (Q3-2015 to Q2-2016) and 5 quarters of OOT (Q3-2016 to Q3-2017)

Table 1-7 Branded Cards Back Test Summary

| | Validation Summary | Champion Model |
|---|---|---|
| GCL | Recent 9Q Back test[6] | 8.0% |
| | Stress 9Q Back test[7] | -2.6% |
| | Most Recent 9Q Back test[8] | 2.2% |
| NCL | Recent 9Q Back test | 7.7% |
| | Stress 9Q Back test | -0.8% |
| | Most Recent 9Q Back test | 3.4% |

Further details on the back testing results and sensitivity analysis can be seen in Section 8.

- Are back-testing results reasonable (i.e., within the acceptable threshold)?

The model performs well across stress, recent and holdout time periods with quarterly error rates for both GCL and NCL well within MRM threshold of +/- 25% and CERR (Cumulative Error Rate) of 9Q within +/-10%.

Please refer to Section 8.3 for detailed results.

[6] Recent 9Q backtest refers to Q2'14 – Q2'16 performance vintages
[7] Stress 9Q backtest refers to Q1'08 – Q1'10 performance vintages
[8] Most Recent 9Q backtest refers to Q3'15 – Q3'17 performance vintages

- Are the results of sensitivity analysis and stress testing reasonable and acceptable?

Yes, the models give a GCL sensitivity of ~176% in supervisory severely adverse scenario which is very reasonable and comparable to the Challenger Model and previous submissions for same scenarios. Please refer to section 8.3 and section 8.4 for details on model back testing and sensitivity results.

## 1.7 Model Outputs

- What are the specific model outputs and over what time period do they apply?

The models provide account level quarterly PD and EADF estimates for next thirteen quarters. The account level PD model forecasts and EADF model forecasts are multiplied with credit limit to project write-off amounts for the required projection periods (i.e., nine quarters for the CCAR/DFAST GCL forecast plus four additional quarters for the loan loss provision forecast). An additional model output is the recovery rate which is obtained from the vintage level recovery model. NCL is derived using the component models (PD, EAD and Recovery). The relationship between Net Credit Loss (NCL) and GCL can be defined by:

***NCL = GCL – Net Recoveries – Adjustment & Refunds***

- How do the outputs support the business use?

The NCL forecast based on PD, EAD and Recovery is submitted to the FRB for the purposes of CCAR and DFAST stress test reporting and other business purposes (e.g. annual business plan). Additionally the thirteen quarter PD and GCL forecasts can be used for strategic planning, LLR and Risk Appetite estimations.

- Were multiple dependent variables and their transformations tested? Is the choice of the transformed dependent variable appropriate?

For the PD Account level model, the dependent variable is binary i.e. either an account has defaulted or is current. Logistic technique is used to estimate PD at an account level. This is an industry wide practice and business is comfortable in using the binary dependent variable.

For EADF Model, EAD and EADF were both explored as the dependent variable. In CCAR 2017 Submission Branded Cards modeled EAD however as a feedback from IRMO in CCAR 2018 the Branded cards has modeled EADF as it is stationary and relatively more intuitive. The detailed reasons can be found in the attached document.

For Recovery Model, two recovery models with different dependent variable were developed. The dependent variable for the Vintage Recovery Model (Champion Model 2018) has Vintage recovery

rate as the dependent variable. For the portfolio recovery model, the dependent variable is the portfolio recovery rate defined as the recovery payment in a quarter divided by the GCL of the previous quarter.

## 1.8 Business Practices and Complexities

The business has observed significant changes in portfolio composition and roll-forward rates post 2012. The above is a result of several policy changes that had taken place around that vintage, significant being change in forbearance policies. Accordingly the modelers have devised way to capture this effect in the models. The champion model has used regime dummies and the challenger model has used different apportioning probability weights for transitions in the pre and post 2012 period. The details of the method adopted in the champion model have been documented in section 5.2

The business expects there to be no major changes in the way that it operates in the future. Therefore it is not expected for any major complexities to occur in the model development process and it is expected to provide accurate results for a future forecast for the portfolio.

## 1.9 Business Assumptions

Following are the key business assumptions –

- New originations are assumed to have similar characteristics as recent originations.
- No major change in regulatory environment such as changes in Bankruptcy losses etc.

# 2 Functional Soundness Assessment

## 2.1 Contribution of Key Stakeholders

.

- Who were the stakeholders and how were they involved?

The list of key personnel involved at multiple stages of model development and finalization for Branded Cards is presented below:

**Table 2-1 Key Stakeholders**

| Responsibility Description | PD Model | EADF Model | Vintage Recovery Model | Functional Responsibility |
|---|---|---|---|---|
| Performance Data Pull (Maker) | Larry Hanus | | | Analyst |
| Performance Data Pull (Checker) | Mark La Fleur | | | Sr Risk Manager |
| Data Preparation at account level | Ibtesam Siddiqui, Anuj Jain, Deepak Madugula | Tarannum Girdhar, Anuj Jain | Ujjwal Kumar, Deepak | Consultant |
| Future originations forecast | FP&A | | | |
| Model Development | Ankita Sinha, Ujjwal Kumar, Rajat Tripathi, Ibtesam Siddiqui, Deepak Madugula | Tarannum Girdhar, Anuj Jain, Shubhranshu Atray | Bhavnish Walia, Ujjwal Kumar, Deepak | Analyst /Consultant |
| Model Re-performer | Ibtesam Siddiqui | Anuj Jain | Deepak | Consultants |
| Model Development Supervision | Kunal Palit | | | Sr. Risk Manager |
| Model and Process Documentation | Bhavnish Walia, Tarannum Girdhar, Ankita Sinha, Ujjwal Kumar. Aniruddha Chatterjee | | | Consultant |
| Y14-A CCAR / DFAST Coordinator (Model User) | Neeraj Singh | | | Group Manager |
| **Upon completion of model development process, the model results and complete documentation package is shared with MVG, FP&A Team, Y14-A CCAR / DFAST Coordinator and Senior Management** | | | | |
| In-business Risk Team Model Sponsor | Neeraj Singh | | | CRO |
| In-business Risk Team Model Approver | Neeraj Singh | | | Group Manager |
| FP&A team approver and reviewer | FP&A | | | CFO |
| Independent Review and Challenge | William Hunt, Zhou Liu | | | Independent Risk |

Table 2-1 MRM Team for Reviewing Complete Model Package

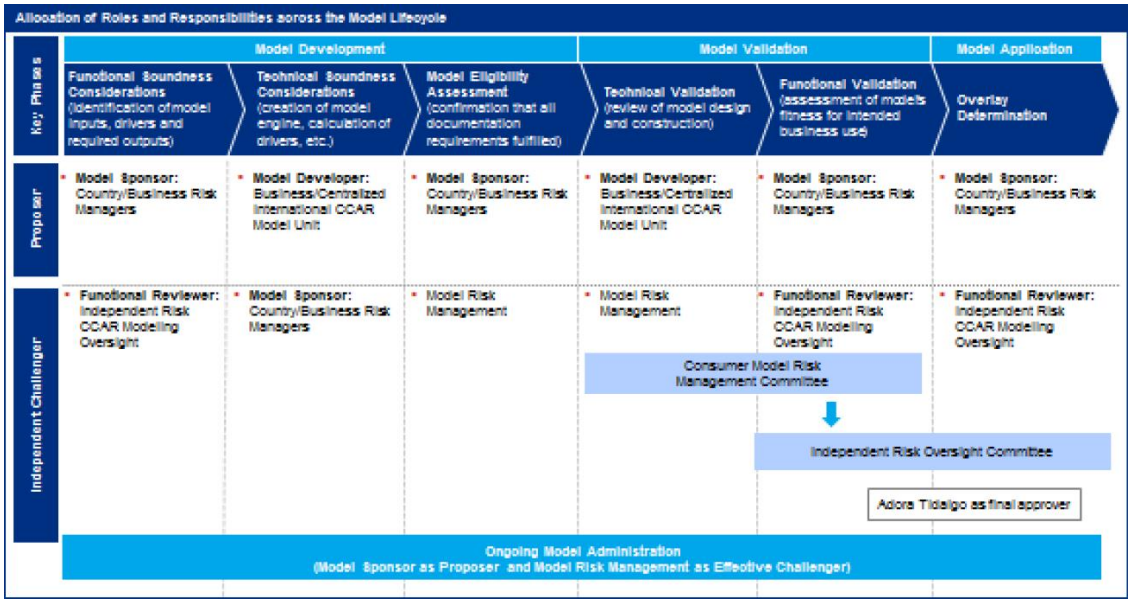
| MRM Team for Reviewing model results and complete documentation package | | |
|---|---|---|
| Modeling Validation Group model validation and reviewer | Yuri Yermakov, Lalit Kumar Agarwalla | MRM |
| Model Review and Management Committee model validation and reviewer | Yuri Yermakov, Lalit Kumar Agarwalla | MRM |

Table 2-2 Escalation Matrix

| Escalation matrix | | |
|---|---|---|
| Model Developers | Tarannum Girdhar, Ujjwal Kumar, Ankita Sinha, Xin Li, Bhavnish Walia | Analyst, Consultant |
| Escalation-1 (Supervisors) | Kunal Palit | Sr. Risk Manager |
| Escalation-2 | Neeraj Singh | Group Manager |
| Escalation-3 (MRM) | Yuri Yermakov, Lalit Kumar Agarwalla | MRM |

Model sponsor defines functional objective and provides business experience/insight to support modelers. Through intensified communications with business, modelers aim to provide effective proposals based on technical analysis and build valid models satisfying functional objective and required performance.

Figure 9: Citibank's Model Risk Governance Framework




As shown in Figure 9, Citi model risk governance framework is further strengthened this year with functional responsibilities allocated to different roles throughout key process of the model lifecycle:

**Model Sponsor** – Act as a main proposer to develop hypothesis and functional requirements for models based on business insights/experience. Ensure model documentation is complete for validation. Engage other resources for model development and documentation. Assess model limitation and model fit from business perspective. Determine business overlays (if any). Use model for capital planning or other business decisions. Propose changes to model if necessary during the ongoing model usage. Make decision on alternative model choices.

**Model Developer** – Develop and present key model choices consistent with functional requirements. Build model in line with key model decisions. Implement tests required for validation.

**Independent Risk** – Challenge proposal on key model functional requirements and application constraints. Give go ahead for model implementation. Challenge model fitness for intended business use. Challenge choice of overlays to remediate model limitations.

**Model Risk Management** – Conduct preliminary assessment of model. Provide feedback and guidelines for documentation update. Reviews validation test results and passes/fails models based on predefined measures. Performs comprehensive and produces validation reports (incl. model limitations) in line with Validation Standards. Monitor and assess ongoing performance of model vs. standards.

The champion NCL models have been developed by the in-business Branded Cards Risk team. The account level PD & EAD models is the next generation of the CCAR 2017 models with enhancements as

discussed in section 7 of this MDDT. Recovery model is an extension of challenger recovery rate model submitted by Branded Cards in CCAR 2017.

The models are created with extensive consultations with in house Senior Modelers. In the initial phase, a detailed plan on modeling approach was shared with Independent risk and Modeling Oversight (IRMO), regional modeling head and CRO.

During the Model Development phase, there have been weekly meetings with the Regional Modeling Head. The purpose of these meetings was to give an update on how the team was progressing and receive feedback on the model development process. Any feedback received on the ex- Costco or Costco models was then incorporated in the development.

Meetings were held with Independent risk in order to receive feedback on the modeling approach. A detailed plan on modeling was shared in the beginning and regular meetings were then held to update on the progress of the model and receive feedback to ensure functional soundness of the model.

In addition, there have meetings with the CRO during the model development phase as well. These meeting started with the development team presenting the approaches that the business was working on. During the middle stages of development the team presented the initial findings to the CRO to receive feedback. Once the development stage was complete, meetings were organized to receive feedback.

- What is the key evidence of stakeholders' contribution?

**Model Development Stage**

The modeling team developed a comprehensive model development framework in consultation with in-house senior modelers and external consultants. The framework was proposed to and discussed in details with the key stakeholders. Upon approval, modeling team proceeded to draft requirements required to build the model.

This includes weekly meetings giving the supervisors updates on the progress that has taken place and obtain feedback. The evidence of regular meetings with the supervisors is attached below:

Weekly CCAR Update.msg

During the modeling process, several meetings were held with the RMH to discuss the project status. Many key decisions about how to proceed came out of these discussions. The content of these discussions are summarized below.

| Date | Discussion items and Recommendations |
|---|---|
| 2/9/2017 | During this meeting, the purchase of FICO V8 for vintages prior to March 2011 was discussed. |
| 3/30/2017 | For purchasing the FICO V8, data preparation was performed and status updated |
| 4/6/2017 | Historical back test of the models was discussed in detail and opportunity areas were identified and discussed. |
| 5/5/2017 | The impact of EAD vs EADF was analyzed. The results of the last year's model were discussed with one year data added. At the same time data from Costco was received from the previous issuer. |
| 5/11/2017 | The responses of the questions from FED on the prior models were discussed. The data integrity checks were initiated on the Costco data that was received. |
| 5/18/2017 | Potential drivers for segmentations were discussed, but given that the model was performing well across all dimensions, it was determined that segmentation would not be necessary. |
| 5/26/2017 | The plan for CCAR 2018 was finalized. Enhancements for the champion 2017 model were decided. |
| 6/1/2017 | The initial results of the State Transition Models were discussed and the results were promising. |
| 6/8/2017 | The impact of using performance MOB was evaluated. |
| 6/15/2017 | The impact of having fixed coefficients across all quarters for macro variables was evaluated. Analysis on replacing Fico v2 with Fico v8 on pre-March-2011 data was performed. |
| 6/29/2017 | The initial results for the EAD and the Costco models were evaluated. Models were being extended to 13 quarters from 9 quarters. |
| 7/6/2017 | Analysis to be shared with IRMO was discussed with RMH and the feedback incorporated. Various Costco modeling approaches were being attempted and the results of these attempts were discussed. RMH recommended building separate models for the Costco portfolio |
| 7/20/2017 | Straight transition models were discussed and the segmented EADF approach was finalized for the state transition approach. |
| 8/10/2017 | Based on the results of EADF and EAD, the RMH recommended that EADF be the final approach. |
| 8/17/2017 | Introduction of APR in the model was discussed, and RMH recommended the inclusion of the variable in the model given the improvements noticed. |

Ongoing meetings between model developer and model sponsor were established to ensure that the model meets the required standards in conceptual / functional soundness. There were regular meetings with the CRO and model sponsor to get feedback on the progress made and seek the challenge as a part of the model development process. For instance, it is observed significant changes

in roll-forward rates post Q1 2012. Key changes in policies include but are not limited to significant tightening of the eligibility criteria of the short-term forbearance program (UPP), implementation of more conservative forbearance policies like direct charge-off post any breach in forbearance terms etc. The directional impact of these stricter forbearance policies would be to increase losses in the initial quarters and reduce losses in the later quarters of the forecast horizon. Such business intuition and risk challenged model specifications provided by CRO and model sponsor. The evidence of a key meeting with the CRO and model sponsor attached below:

CCAR Update to Model Sponsor - 10 11.msg

CCAR Update to Model Sponsor - 9 19.msg

Throughout the development process, regular meetings were held with IRMO to discuss the status of model development. One feedback received from the IRMO was to fix the macro variable coefficients. Based on feedback from IRMO Implemented a constant macro variable coefficient across performance quarters especially Q3-Q9. Another input was to explore more macroeconomic variables. The team also finalized on the transformation of EADF models based on the IRMO feedback. The minutes of the meetings with the Independent Risk team (IRMO) are attached below:

20170706_IRMO_Update.docx

20171010_IRMO_Update.docx

12070907_IRMO_Update

**Model Validation Stage**

Model developer implemented the tests required for validation and produced model validation results. Model risk management and independent risk challenge the validation results based on Validation Standards. Model risk management will gave pass/fail assessment to model based on predefined measurement standards. Model sponsor assessed model limitations and model fit from business perspective.

**Model Application & Administration Stage**

Model sponsor will determine which overlay should be used in order to remediate model limitations post the validation.

For an ongoing basis, model sponsor will use the model for capital planning and other business uses. If situations change, model sponsor may propose changes where necessary. Model risk management will also assist monitor model performance.

Quarterly model monitoring will be implemented in accordance with Model Risk Management Policy and Model Monitoring Protocol. Model production teams will execute model runs and generate error

reports. Sponsors and Developers will review the results and evaluate the cause of any error rates that breach Model Risk Management thresholds. Action plans will be developed to remediate performance issues and address critical limitations identified during technical or functional validation. The Independent Risk Model Oversight team and the Independent Risk Oversight Committee will review and challenge action plans and prioritization of those plans.

It is of utmost importance that the models in scope for loss calculation and regulatory approval be tracked and monitored on a regular basis. The primary goal is to ensure that the constituent models and the overall loss framework are adequate and performing equally well on an ongoing basis i.e. the models are representative and appropriate for out of time population. Models created to forecast losses, especially losses expected to occur over the next several years, inherently possess a high degree of uncertainty. There are two components that drive forecasts of this nature are projections for the economy and the model itself.

The most important theme and objective of Loss Forecasting model monitoring is the tracking of actual vs. predicted loss forecasts. Hence every quarter based on updated actuals of internal and macroeconomic variables models will be back tested over the most recent 9 quarter and 13 quarter period.

Based on the model performance, the following actions may be taken:

- Re-evaluation of the model & data related assumptions and limitations
- Monitoring of subsequent quarterly performances
- Recalibration or if necessary Redevelopment

Additional details on the scope, process and frequency of testing, and the performance metrics that will be included in ongoing testing. Additional metrics will be added to monitoring on an as needed basis. The standard Model Monitoring Protocol templates are as per the Global Consumer CCAR Standards.

## 2.2 Review and Challenge Process as Part of Model Development

Throughout the development process, regular meetings were held to discuss the status and execution of the models. These interactions involved model developers, model owners, and business leaders, and ensured effective challenge both at the business line level and at the executive governance level. Model developers and owners met throughout the model development process to evaluate the model methodology and results. The business met throughout the process to promote informal business engagement and constant discussion about loss forecasting progress. As models were completed and run, they were presented to business leaders who were able to review model methodology and results and challenge presented information.

The challenges put forth by the IRMO team were documented. The attached document provides the detailed interaction summary.

IRMO Meeting Questions

The below table summarizes the key interactions as a part of review and challenge process.

**Table2-1: Review and Challenge Notes**

| Review and Challenge Meeting | IRMO Question | Business Response | IRMO Expectations | Question Type (Clarifying, Guidance, Functional Evaluation) | Question Category (input, technique, output) |
|---|---|---|---|---|---|
| 7/6/2017 | Is segmentation by client addressed effectively? If there are separate models for each Retail Services client, then will additional segmentation be examined? | Yes, there is additional backtest by product and FICO. | | Functional Validation | technique - variables |
| 7/6/2017 | Did you intend to investigate different baselines hazard functions by delinquency bucket for the Retail Services CCAR 2018 Champion Account Level PD and EAD models? If delinquent accounts were tagged separately from accounts that are cured would it be easier to examine differences for the account level survival models? | Yes. Segmentation by delinquency did not add value. | Share analysis of delinquent vs. non-delinquent accounts in MDDT. | Functional Validation | technique - logic |
| 7/6/2017 | Is the approach for evaluating segmentation of delinquent accounts going to be specifically described in the MDDT? | Yes | Include with a description of the evaluation of delinquency segmentation in the MDDT. | Functional Validation | technique - logic |

| Review and Challenge Meeting | IRMO Question | Business Response | IRMO Expectations | Question Type (Clarifying, Guidance, Functional Evaluation) | Question Category (input, technique, output) |
|---|---|---|---|---|---|
| 7/6/2017 | How will the accuracy of the Retail Services CCAR 2018 Champion Account Level PD and EAD Models over a full 13Q window be evaluated? | Two approaches will be used - 1) flatline 9Q forecasts and see if backtest results are reasonable; 2) Re-calibrate the model equation for all 13Q. | Document testing for both approaches in the MDDT | Functional Validation | technique - logic |
| 7/6/2017 | Will the business purpose identified in the MDDT extend beyond 13Q? | No, CECL will be addressed at a later date. | | Functional Validation | technique - logic |
| 7/6/2017 | Have both teams looked at the new MRM backtesting requirements? | No. | Review requirements | Clarifying | technique - logic |
| 7/6/2017 | Does MOB add any benefit or have any relevance to PD? What evidence will be included to show how the MOB as the baseline model curve was included in the analysis? | Yes. Intercepts will automatically capture the attribute of MOB. It adds value in terms of interpretation of the model. | Include analysis in MDDT | Functional Validation | technique - variables |
| 7/6/2017 | Can you identify the mechanism that is driving decisions for including and excluding variables? | This will be included in the analysis. | Describe the process for deciding which variables to include/exclude | Functional Validation | technique - variables |
| 7/6/2017 | Clearly demonstrate the relationship between PD and EADF vs the selectedmodel variables | Will be added in the MDDT | Demonstrate the relationship in the MDDT | Functional Validation | technique - variables |
| 7/6/2017 | Does the model capture either policy driven events or interest rate shocks? | Interest rate is evaluated in variable selection. There are issues in estimating sensitivity as interest rates have been flat. | Identify policy changes that could be impacting sensitivity | Functional Validation | input - data |
| 7/6/2017 | What is planned regarding voluntary account closures? Accounts closed with and without balances should be separately analyzed | There is no known significant impact. Business will do additional research. | Provide additional analysis on the treatment of closed accounts | Functional Validation | input - assumptions |
| 7/6/2017 | What is the definition of a closed account? How is "active" defined? | In this instance, it is no longer an active line where active is defined based on a snapshot | Provide parameters of how closed accounts are identified and consider | Functional Validation | input - assumptions |

| Review and Challenge Meeting | IRMO Question | Business Response | IRMO Expectations | Question Type (Clarifying, Guidance, Functional Evaluation) | Question Category (input, technique, output) |
|---|---|---|---|---|---|
| | | | refinement of active definition beyond snapshot | | |
| 7/6/2017 | Why would the impact of macrovariables change with quarters since snapshot? | The explanatory power of economic variables should increase relative to that of bureau and internal variables further into the forecasting horizon. | Analyze macrovariable movements and the impact of changing relationships | Functional Validation | technique - variables |
| 7/6/2017 | Was there any feedback from MRM on discrete time versus continuous time? | No | | Clarifying | technique - logic |
| 7/6/2017 | Will the Branded Cards AMRs reflect use for longer time horizons? | Model usage will be described. | Provide analysis on model use for the Roll Rate model | Functional Validation | output - suitability for objective |
| 7/6/2017 | Does the credit bureau provide Branded Cards with data for the FICOv8 conversion? Is there anything that translates the old into new scores? | We purchased FICOv8 for the development sample. | | Clarifying | input - data |
| 7/6/2017 | Could FICO retroscore FICOv8 if you could collect and process account level data back to 2002? | No, account level data is not available before 2007. | | Clarifying | input - data |
| 7/6/2017 | Why is there a hump after age 60 in the maturation curve? | The hump may be explained by other variables such as quality and economy. | Explain the hump in the MDDT | Functional Validation | input - data |
| 7/6/2017 | Why is monthly data not used? | From a data and degrees of freedom perspective, this is better. | | Functional Validation | input - data |
| 7/6/2017 | Are state level macrofactors for HPI being used (slide 6)? What analysis could be performed to show that state level is granular enough and there is no need to go to county level? | Regional forecasts below state level are not used but there are various dimensions that satisfy CCAR requirements | Explore additional granularity | Functional Validation | technique - variables |

| Review and Challenge Meeting | IRMO Question | Business Response | IRMO Expectations | Question Type (Clarifying, Guidance, Functional Evaluation) | Question Category (input, technique, output) |
|---|---|---|---|---|---|
| 7/6/2017 | Did anyone from MRM give feedback around looking maturation curves for parametric? | No | | Guidance | technique - logic |
| 7/6/2017 | How is backtesting performed for the whole portfolio if FICOv8 is not available for all accounts? | Fico v8 is purchased for the stress periods for all accounts. | | Clarifying | input - data |
| 7/6/2017 | Will other interest rate variables and combinations of interest rate variables be considered? For instance, how is quantitative easing of bond buying evaluated to understand the impact of this on card performance? | The business will try to do this at 1 and 2 year intervals. If they are using the appropriate transformation a positive correlation is expected. | Business will document how this will be incorporated. | Functional Validation | technique - variables |
| 7/6/2017 | Can it be shown whether event dummies are considered and if not included in the final model, why? | The MDDT will detail the logic. | Business needs to include an explanation for these spikes and why or why they were not considered. | Functional Validation | technique - variables |
| 9/7/2017 | What product segmentation is anticipated for recovery this year? (Slide 8) | There will most likely be two products: AA and non-AA. The portfolio is quite homogenous. Most of the variables are AA versus non AA and consumer versus business. For instance, when we looked at the loans by product, the AA had the highest recovery we observed. | Segmentation needs to be justified in the MDDT. | Functional Validation | technique - logic |
| 9/7/2017 | How do you control for a higher credit limit client? | A higher FICO typically indicates a higher creditlimit. A higher FICO meant that the recovery dollar was higher. Customers who have higher credit limits have higher Recovery dollars but low Recovery Rates. (Slide 9). | | Clarifying | input - assumptions |
| 9/7/2017 | What other variable can explain or capture the characteristics between AA and Non AA? | AA are mostly transactors and therefore have a higher recovery rates. | | Clarifying | technique - variables |

| Review and Challenge Meeting | IRMO Question | Business Response | IRMO Expectations | Question Type (Clarifying, Guidance, Functional Evaluation) | Question Category (input, technique, output) |
|---|---|---|---|---|---|
| 9/7/2017 | Can you use variables that are fixed at origination such as FICO? (Slide 6) | FICO was not available for all accounts originated earlier than 2007 since those accounts might not have had the acquisition data. | | Clarifying | technique - variables |
| 9/7/2017 | Are these loans subject to an outside agency? | Inventory any that would have happened before December 2017. We will try incremental variables. The model without these variables we can still use the inventory. | Provide an analysis on incremental variables. | Functional Validation | input - assumptions |
| 9/7/2017 | Are these only modeling the cash recoveries? | Yes | | Clarifying | input - data |
| 9/7/2017 | Before netting out fees, what is the recovery rate of the accounts from the collection firms (asset sales)? | It is volatile and unclear. | | Clarifying | input - data |
| 9/7/2017 | Do you include the recoveries in backtests? | Yes | | Clarifying | input - data |
| 9/7/2017 | What occurs during the blip in QSC1 and QSC2? (Slide 7) | The first quarter is the quarter of Charge-off. That' why the recovery dollar is lower than the second quarter. | | Clarifying | input - data |
| 9/7/2017 | Are recoveries still sensitive to the economy, for instance, 6 quarters after the Charge-off? | The log raw function by default assign different sensitivities to high and low recovery rates. | | Functional Validation | technique - variables |
| 9/7/2017 | Did you consider additional testing on macrovariables outside of the stress ranges to see how much the moves? For instance, did you observe effects of higher home prices or changes in the unemployment rate? What sort of degree of non-linearity would you expect? What happens if you decide to come up with different scenarios especially in nonlinear forms? | We will design additional scenarios to enhance sensitivity analysis approaches. | Perform additional testing on macrovariables outside of the stress ranges and provide analysis on additional scenarios to observe effects of other economic shifts. | Functional Validation | output - reasonablen ess of output |
| 9/7/2017 | Did you test level? | Not significant. There are very low correlations. | | Clarifying | technique - variables |

| Review and Challenge Meeting | IRMO Question | Business Response | IRMO Expectations | Question Type (Clarifying, Guidance, Functional Evaluation) | Question Category (input, technique, output) |
|---|---|---|---|---|---|
| 9/7/2017 | Any other macroeconomic variable tested? | We tried GDP. | | Functional Validation | technique - variables |
| 9/7/2017 | Did you observe the concept of the conscientious borrower based on the APR or the total amount of purchases on the account? | Once it charges off, there is no concept of APR. We looked at this behavior and whether the account holder had a spending pattern. For instance, if a person had an issue with spending, then their payment becomes too much for them to pay and they likely give up altogether. | Provide analysis on the comparison between Approach 1 and 2. | Clarifying | input - assumptions |
| 9/7/2017 | Is the recent period the same for the 2 models? (Slide 13) | Yes. From a bactest perspective it is the same. Some are out of time. | | Clarifying | input - data |
| 9/7/2017 | When did credit abuse occur? | Most of it happened in June 2016. The impact happened Q1 2017. It does not change the estimates. We have already fixed the policies. That activity is captured as third party fraud and taken care of by operational risk. | | Clarifying | input - assumptions |
| 9/7/2017 | In the portfolio delinquency transitions 2+ months going to CO is increasing. Did you observe any pattern on recoveries as a result? | Business replied that they do not observe any pattern on the recovery rate as a result of the increase in the 2+ to CO rates. This is because the Recoveries is more impact by the total GCL. Also the total GCL has been decreasing due to the significant decrease in Current to Bucket 1 since the great recession. The evidence of the low error rates in both stress and recent time period of the recovery model inspite to not including any regime change or similar drivers further strengthens the argument of no change in trend of recovery rate because of the increased 2+ to CO rates | | Functional Validation | technique - logic |
| 10/10/2017 | Did you use anything that was similar to a debt service burden? Economic intuition suggests that this could be a relevant variable. | For CCAR, we did not use a debt service burden. So far, we have not received that variable from the Fed or GCRM. We will experiment this variable in the next round of model development. | | Clarifying | Technique-variables |

| Review and Challenge Meeting | IRMO Question | Business Response | IRMO Expectations | Question Type (Clarifying, Guidance, Functional Evaluation) | Question Category (input, technique, output) |
|---|---|---|---|---|---|
| 10/10/2017 | What is the recovery model used together with the Look Ahead Challenger model? | The recovery model was the same as last time. | | Clarifying | input - data |
| 10/10/2017 | How do you solve the problem that since 2012 the model persistently underestimates the first 2 quarters? (IRMO Touchpoint Slide 7) | Regime change indicators have been incorporated to solve the problem. | | Clarifying | Technique-mode logit |
| 10/10/2017 | Why does the data set stop at July 2016? (IRMO Touchpoint Slide 7 and 25) | We received the data in the middle of the development cycle. More recent data will be used as out of time testing which will be reported in the MDDT. | | Clarifying | input - data |
| 10/10/2017 | Did you change the way you predict 13Q? | We developed model in a 9-quarter since snapshot structure but recalibrated the model in 1 13-quarter since snapshot structure. | | Functional validation | technique - model logic |
| 10/10/2017 | What proportion of full recoveries are payment recoveries? (IRMO Touchpoint Slide 22) | Please refer to slide 22. | | Clarifying | input - data |
| 10/10/2017 | Is it useful to show a benchmarking analysis? | This will be in the MDDT. | | Clarifying | output – reasonablen ess of output |
| 10/10/2017 | What are the risk drivers of PD and EAD? | Refer to Slide 10. | | clarifying | Technique-variables |
| 10/10/2017 | What does negative recovery mean? | Agency fee higher than recovery amount. | | Clarifying | input - data |
| 10/10/2017 | How do you account for the closure in the forecasting period? | There is no closure assumption. We will document why and the impact in the MDDT. | | Functional Validation | Input-assumption |
| 10/10/2017 | What is the percentage of closed accounts? (Slide 17) | 1% quarterly | | Clarifying | input - data |
| 10/10/2017 | Do closed accounts remain in the dataset after they are closed with future performance back-filled? | Yes. | IRMO suspects this will lead to biased estimation but given the tiny percentage, it is not a big problem. | Functional Validation | input - data |

| Review and Challenge Meeting | IRMO Question | Business Response | IRMO Expectations | Question Type (Clarifying, Guidance, Functional Evaluation) | Question Category (input, technique, output) |
|---|---|---|---|---|---|
| 10/10/2017 | What is the historical percentage of revolvers? | It has been around 55% revolvers for Branded Cards. | | Clarifying | input - data |
| 10/10/2017 | Will the Account Level model for Costco be submitted? | Yes | | Functional Validation | input - data |

Based on the discussions with the IRMO below are some of the enhancements made in the models and the documentation

- Models have been back tested by Current / Delinquent and this information is added to MDDT.
- Model development details in the MDDT include the selection of specific delinquency indicators amongst potential alternatives
- 13Q back tests have been included in the MDDT
- MDDT includes the latest MRM back testing templates.
- The model development details in the MDDT include the selection of specific Month on book indicators and interactions amongst potential alternatives
- Model development and variables selection is documented in details in the MDDT
- Analysis of EAD vs EADF approach is included in the MDDT
- Policy change indicator for forbearance also included and documented in the MDDT
- No significant impacts observed in back testing due to voluntary closure. The rationale of not having separate assumptions for attrition and stable voluntary attrition evidence is documented in the MDDT
- Based on feedback from IRMO implemented a constant macro variable coefficient across performance quarters especially Q3-Q9
- Inclusion of interest variables will be explored further in later submissions
- Business is not currently forecasting APR. Need to be discussed further
- Discussed with IRMO and shared the analysis on the transformation of recovery rate

There was no limitation on PD and EAD models. For recovery the potential omission of variables was called out. Both businesses have now developed vintage level recovery models with a bigger set of predictors

# 3 Restrictions on Model Usage

The restrictions on model usage can be summarized as follows:

1) The models have been developed for loss forecasting and for CCAR stress testing purpose. The models could also be used for non-CCAR purpose subject to the model usage approval from eligible model risk management authorities.

2) The model has been built on the US Branded Cards portfolio and should be used for any other portfolio only with caution and post due diligence.

3) The model cannot be utilized for Adverse Actions such as denial of credit line increase etc.

# 4 Justification of Modeling Approach

## 4.1 Description of Applicable Modeling Approach(es) (MAs)

Stress testing is still a relatively new and evolving modeling activity for many financial institutions. First generation CCAR and DFAST models were often portfolio level time series models. However, over a period of time, the requirement to capture granular portfolio behavior led to modelers adopting more sophisticated modeling approaches at portfolio segment as well as at account-level.

Under the expected loss approach, losses are estimated as a function of three components: Probability of Default (PD); Loss Given Default (LGD) / Recovery Rate Models (RRM); and Exposure at Default (EAD). This approach has been selected as the preferred approach for the estimation of losses within Citi's Retail portfolios and is aligned with regulatory guidance and supported by academic research[9]. Model development leveraging the preferred approach can be achieved using a variety of techniques, which are summarized in the Table 4.1-1 below. These techniques are considered to be robust and are well supported by academic research and the Federal Reserve Bank's (FRB) SR 11-7 requirements for modeling approaches.

A more detailed description of the relevant techniques for PD, EAD and Recovery follows, along with a discussion of the advantages and disadvantages of each.

### 4.1.1 Probability of Default Modeling

Among the many variations that are used successfully for termination event modeling, the following is the primary classification for various approaches: state transition models versus survival models; and loan-level versus aggregate-level models.

Table 4.1-1 : Probability of Default Modeling

| | State Transition Approach | Survival Approach |
|---|---|---|
| **Aggregate-level (portfolio or segment level)** | - Roll rate model | - Dual-time dynamics (LookAhead segment level approach) |
| **Account-level** | - Loan level state transition model | - Cox proportional hazard model<br>- Discrete time survival model |

[9] Refer to the Federal Reserve Bank's "Capital Planning at Large Bank Holding Companies: Supervisory Expectations and Range of Current Practice"

**Roll rate models** measure the rate at which groups of accounts transition from one payment status to another over a specific time period (e.g., a month, quarter, year). These transition rates form a matrix, with each element of the matrix representing the transition from a source status to a destination status. A roll rate model may include transition rates for every possible transition or a selection of transitions (e.g., current-to-30dpd, 30dpd-to-60dpd, 60dpd-to-default). The simplest roll rate models assume constant transition rates over the forecast time horizon. Alternatively, it could also employ rolling historical average transition rates. In either case, the transition matrix is applied to the portfolio at the start of the forecast period to generate future distributions of accounts across payment status buckets. Roll rate models may be developed at different levels of aggregation based on population characteristics such as product, FICO score, etc., for which transition propensities differ observably.

Roll rate models can be extended to take into account macroeconomic scenarios. This is most typically accomplished by estimating one or more transition rates for each segment (e.g., current-to-30dpd for a credit card segment) as a function of macroeconomic variables, based on statistical regressions. As an aggregate modeling approach, cross-sectional variation across accounts within a segment may not be fully captured. Although this approach can be made more granular by increasing the number of separate modeling segments, it should be noted that a large number of separately estimated models could potentially result in non-intuitive patterns of estimated sensitivities across related segments and transitions. As a general rule, aggregate level roll rate models, especially those not directly accepting macroeconomic scenario inputs, are considered a lagging practice by the Federal Reserve.10

**State transition** models are granular analogs of roll rate models that use a variety of account-level input factors, in addition to macroeconomic factors, to parameterize the roll rates via regression analyses, typically in the form of discrete choice multinomial equations conditional on a starting payment status. Similar to the roll rate models, the account-level transition matrix is iterated forward through the forecast time horizon to generate the future probability distributions of individual accounts across payment statuses. The addition of account-level risk factors contributes to the model's ability to capture portfolio compositional change. As delinquency profile, FICO distribution, loan age distribution, etc., change over time with underwriting tightening or loosening, runoff, and new bookings, account-level parameters help capture these changes, while aggregate-level models cannot.

Dual-time Dynamics models (popularized through the LookAhead software product) are in the class of age, period, cohort (APC) models because they operate by attributing losses to loan age, calendar time (period), and vintage (cohort) dimensions. The framework can be operationalized at the segment level through separate dual time dynamics (i.e., loan age and calendar time)

[10] *Capital Planning at Large Bank Holding Companies: Supervisory Expectations and Range of Current Practice*, August 2013, Board of Governors of the Federal Reserve System.

decompositions performed separately for risk based (e.g., FICO, product, etc.) portfolio subpopulations. The calendar time component resulting from the dual time decomposition is referred to as the exogenous component, which is further decomposed into special event effects, seasonality effects, and an exogenous residual. The exogenous residual may be modeled as a function of macroeconomic variables by means of equations known as Economic Response Models (ERMs). ERMs are typically estimated using time series techniques or ordinary least squares regressions.

The dual time dynamics approach is often referred to as an "originations" based model because origination cohorts are followed across their loan age loss development curve (maturation curve). From a loss forecasting standpoint, the loan age orientation of the framework does not naturally allow use of updated predictive account attributes such as current FICO, observed delinquency history, or current delinquency status. The model segmentation scheme must also be based on static account attributes, making the framework conceptually less risk sensitive—in the sense that it will be less able to adapt to changing account-level portfolio risk characteristics than the state transition framework discussed above and the account-level survival model approaches discussed below. Within a segment, the dual time dynamics decomposition relies on information related only to loss evolution decomposed into loan age and calendar time effects, controlling for origination vintage. The account level Cox proportional hazard model and discrete time survival modeling approaches discussed below are able to exploit the data both in cross section and over time.

**Cox proportional hazard models** were developed specifically to combine survival modeling with the other explanatory factors, so that this type of model can serve stress testing purposes. The Cox proportional hazard model and discrete time survival model frameworks are natural account-level choices to model losses in support of CCAR and DFAST stress testing exercises. Reasons include the ability to include time varying macroeconomic and other risk factors, jointly estimated with additional static risk factors, to directly exploit cross sectional factor variation in estimation, and to directly model the timing of loss. In this sense, these frameworks approach the conceptual extent of risk sensitivity inherent in the loan level state transition approach. Since both are survival model frameworks, they also accommodate censored observations. Censored observations include performance records that end for unknown reasons, performance records that reach the end of the observation period without a termination event, or termination events that are not observed because a competing event occurred instead, such as a charge-off event that cannot occur after an account is closed (or a closure event that cannot occur after an account is charged-off).

The traditional Cox proportional hazard model relies on a key assumption that there is no interaction between the explanatory variables and analysis time. In other words, the effects of the coefficients of the risk drivers do not change with time. For the most basic Cox proportional

hazard model setting where there are no time-varying explanatory variables, this is typically referred to as the "proportional hazard assumption", because the ratio of the hazard rates for any two units is not a function of time in the original Cox proportional hazard model. When there are time-varying explanatory variables, the hazard ratio will not be a constant. The model can be extended to handle time-varying explanatory variables, but is more complicated to implement. In this case, the discrete time hazard model allows for incorporating these interactions more easily

Both models belong to the family of generalized linear models. In the Cox proportional hazard model, the log hazard rates (relative to the baseline rates) are assumed to be a linear function of explanatory variables. In the discrete time hazard model, the log-odds of discrete time hazard events are assumed to be a linear function of explanatory variables.

Another feature of the Cox proportional hazard model is that the effects of explanatory variables are estimated without specifying a particular functional form for the baseline hazard function. This is a substantial advantage if the goal is just to understand the sensitivity of the hazard rate to changes in explanatory variables. The Cox proportional hazard model, as implemented in SAS procedure PHREG, uses a semi-parametric approach to model the baseline hazard function; whereas the discrete time hazard model estimates the baseline hazard explicitly through the use of number of months/quarters, or time, as an explanatory variable in the estimated logistic regression equation. Using the semi-parametric baseline hazard in model estimation and forecasting has a few disadvantages, including a lack of transparency and flexibility, as well as a potential for over-fitting. In contrast, discrete time hazard models allow the baseline hazard rate to be estimated parametrically, which is often desirable to maintain or enforce smoothness and parametric prior beliefs on the appropriate shape of baseline hazard curve as a function of time.

One more advantage of the survival model framework is that these can also be applied in a single termination event setting. In that case, all termination events are grouped together as one type and modeled together. The competing risk approach is used when it is believed the drivers of termination differ by subtype (charge-off vs. closure). A further alternative is to recognize one type of termination event only. In this case closure would be considered a "good" and not distinguished from accounts that experience no termination event at all.

For Branded Cards Champion model, the discrete time survival modeling approach using logistic regression is used to predict quarterly probability of default at account level leveraging on-us historical performance, current account delinquency status, off-us credit bureau, FICO score and macro-economic variables (UR, HPI), thus forecasting PD as a function of macro-economic factors and behavioral factors combined in a single equation.

Further, this PD model is expected to forecast probability of default for the next nine quarters at quarterly / monthly level. Different factors are responsible for explaining probability of default of an account for different time horizons in the given nine quarters. Example - Delinquency plays a

big role in explaining PD of an account for the next two quarters. In order to capture this effect into the model for accurate predictions at quarterly / monthly level, single PD equations capable of estimating next 9 quarters are estimated which are effective in explaining the probability of default accurately for different time horizons.

Moreover, the discrete time survival framework framework meets all of our key guiding principles:

- **Granularity**: Model is granular and takes into account level information including FICO andstate level macro-economic information
- **Accuracy**: Model performs well in back test not just in one or two snapshots but thorughout the data used for model development. Moreover, an out-of-sample is kept to ensure the model is robust and stable even in the data not used for modeling
- **Sensitivity**: Model is sensitive to macro-economic stress as seen in prior stress periods
- **Business Intuition**: Model variables make sense from business perspective

It is also noted that the methods for forecasting losses at the loan level have been receiving increasing attention in the available literature.[11]

For the PD model, all the accounts, which can potentially give losses, are considered for modeling purpose. All the events, which can result into a positive dollar loss (contractual charge-off, bankruptcy, pre charge-off settlements) are considered as the bad events. Given the relative smaller size of voluntary closures, the business has decided to model only one type of termination event (bad) for the champion approach. In this case, closure would be considered a "good" and not distinguished from accounts that experience no termination event at all. The model methodology is described in detail in Section 6.

### 4.1.2 Exposure at Default

Model selection for estimating Exposure at Default involves two key decisions: the particular transformof the EAD dependent variable and the methodology for estimating the EAD.

In this model, Exposure at Default estimation is accomplished through modeling Exposure at Default Fraction (EADF). For line of credit (credit card) products, EADF is defined as (charge-off balance/ credit line at observation/forecast date). EADF is the exposure at default as a percentage of the credit line at the time of capital calculation, i.e. the expected utilization at charge off and is defined as:

[11] The following reference examines the types of approaches used for termination models: Engelmann and Rauhmeier (eds) (2011): *The Basel II Risk Parameters – Estimation, Validation, and Stress Testing*, a compilation of detailed expositions covering scoring and rating methods, modeling LGD, termination model validation methods, and validation for Internal Rating Systems.

Equation 4.11.1

$$EADF = \frac{Outstanding\ Balance\ (T = T)}{Total\ Credit\ Limit\ (T = 0)}$$

For Credit cards, one alternative to modeling the EADF would be to project the absolute charge-off amount directly. Modeling EAD in the form of a ratio rather than a dollar figure is considered as a standard industry practice, as EAD in the form of a ratio is typically found to be statistically stationary over a period of time. In addition, EADF ratios are more intuitive and the estimates are easy to interpret by the business.

The modeling approach for both the EAD and EADF model would be similar, except for the dependent variable transformation. The EAD (or EADF) can be modeled through an OLS regression or using decision trees. OLS regression is naturally amenable to the inclusion of macroeconomic explanatory variables while a decision tree approach is not. In a decision tree based approach, explanatory variables are split into discrete buckets whereas macroeconomic factors are required to be naturally entered in a stress testing model in its original continuous form. The decision tree approach is not able to incorporate important predictive information brought in by the macroeconomic variables with respect to account propensities to charge-off.

Additionally, the OLS framework can accommodate a larger number of predictive relationships.

In summary, given the accuracy and granularity benefit for OLS regression, the business decided to go ahead with OLS regression technique.

For CCAR 2017, Branded Cards modeled directly the charge-off amount or EAD. However, for CCAR 2018, Branded Cards is using the EADF approach due to following reasons:

For revolving exposures, EADF is defined as the charge-off amount as a fraction of the line amount as of the snapshot date, as indicated by t=0:

$$EADF(t) = \frac{Charge\text{-}off\ amount(t)}{Line\ Amount(0)} \qquad (6.2.2.1)$$

Also EADF is not capped at 1.0 as customers can go above existing credit limit. Increases in the credit line following the snapshot date and associated balance draw behavior can also result in charge-off amounts greater than line amount at observation. EADF should, however be bounded below by zero to avoid nonsensical negative charge-off amounts or balances.

One of the major decisions for estimating Exposure at Default involves the selection of the functional form of the EAD dependent variable. The two options considered are

1. EAD: Modelling the absolute charge-off amount directly

2. EADF: Modeling of an Exposure at Default Fraction (EADF). EADF is the exposure at default as a percentage of the credit line at the time of capital calculation, i.e. the expected utilization at charge off and is defined as:

Equation 4.11.2

$$\text{EADF} = \frac{\text{Outstanding Balance (T = T)}}{\text{Total Credit Limit (T = 0)}}$$

As credit Limit as of snapshot is used, there is no need to forecast the Credit Line in the current approach.

To model each of the above (EAD or EADF), OLS regression was selected due to the reasons stated in Section 4.

For CCAR 2017, Branded Cards modeled directly the charge-off amount or EAD. After the CCAR 2017; IRMO (Independent Review Modelling Oversight) provided the feedback to evaluate modelling EADF instead of EAD (to check for stationarity, stress etc.). EAD in the form of a ratio is typically found to be more stationary. In addition, EAD ratios are be more intuitive from business judgement point of view.

For CCAR 2018, Branded Cards employed the EADF approach based on the feedback from IRMO due to following reasons:

1. Stationarity: While modelling exposure using Linear Regression technique, the independent and dependent variable should be stationary. Branded cards tested the stationarity for both EAD and EADF using the through the cycle data. The results demonstrate that EADF being a ratio comes out to be stationary while EAD being dollar amount is non-stationary

***Stationarity Results – ADF Test****

| Stationarity Test | p value | |
|---|---|---|
| EAD | 0.62 | **Non Stationary** |
| EADF | 0.11 | **Stationary** |

2. Boundedness :
Intercepts and Indicators make it difficult to bound EAD; however such variables make more intuitive sense with EADF .For Eg. A product indicator with coefficient of 5000 will increase the EAD for the all accounts with that product by 5000 dollars, even if the credit limit of an account may be significantly less than 5000.

****Note** : The data used for ADF Test calculation is from Jan'07-Jun'15*

Branded cards analyzed the Predicted EAD and EADF for toy models built during this decision. Using EADF Approach, Only ~0.02% accounts were found while using Predicted EADF where the Predicted EAD exceeded the Limit by 3 times. The same number for EAD Model was ~3%. In the sample cases, the same thing is observed, with an account having Limit of $2000 dollars, the EAD Model predicts the EAD to be $6424 and the EADF model predicts the EAD to be $3877 (much closer to the actual $2462).

*Sample Cases*

| Cl_tid | Line | Actual EAD | Predicted EAD - EAD Model | Predicted EAD - EADF Model |
|---|---|---|---|---|
| 327966805 | $1,500 | $3,618 | $5,239 | $3,666 |
| 166369 | $2,000 | $2,462 | $6,424 | $3,877 |
| 324493701 | $1,000 | $994 | $3,035 | $1,045 |
| 324532333 | $1,000 | $1,224 | $3,086 | $1,289 |

3. Intuitive Rationale:

The relationships of some of the variables are not very clear with EAD as dependent variables because EAD is a function of risk and credit limit. For Example; FICO; an account with higher FICO should default with less amount as he is less risky; however when he defaults; he will have higher Limit and hence higher EAD. Hence, it becomes difficult to decide the direction of the relationship while modelling EAD.

The effect of Limit is detached from EADF and hence the direction of the relationship is easy to decide. For Example; Higher FICO will default with lower EADF with high likelihood.

Hence, modelling EADF was considered as the final approach for predicting Exposure at default for CCAR2018 Models.

### 4.1.3 Recovery Rate models

NCL is derived using the component models (PD, EAD and Recovery). The relationship between Net Credit Loss (NCL) and GCL can be defined by:

***NCL = GCL – Net Recoveries – Adjustment & Refunds***

Net recoveries can be decomposed into three components: Cash Collected recoveries, Agency fees and Asset Sales. Hence, the net recoveries are calculated by the below formula:

$$Net\ Recoveries = Cash\ Collected\ Recoveries - Agency\ fees + Asset\ Sales$$

Cash collected recoveries form the majority of net recoveries (~90-95%). Hence, Branded Cards has modeled Cash collected Recovery Rate.

Asset Sales are driven by business policy decisions such as the amount of assets that will be sold in a given month as well as the pricing of those assets. Hence they do not have a high correlation with macroeconomic factors and would not be modeled. Instead a Judgmental Model approach has been adopted for the same.

The Agency Fees as a percentage of dollar recoveries does not have a correlation with macroeconomic conditions and are by and large stable. Moreover, Agency fees contribute to less than 2-3% of the total NCL and hence, small variations would not result in significant changes in the NCL forecast given minor changes in agency fees. Hence they are not modeled with macroeconomic variables but are incorporated as a judgmental approach.

Adjustments and refunds are included in final NCL Calculation. However they are a contra neutral item to the P&L. Post charge-offs, the unearned interest and fees (or adjustments and refunds) are netted out of GCL dollars to determine the Net Credit Losses (NCL). The changes in trends for unearned (% of GCL) are more a result of the internal pricing and fee related policy changes. They are not impacted by the changes in economic environments.

Below are the approaches used by the Business for modeling cash collected recovery rate:

- **Vintage Based Recovery Rate Model**: Champion Model (Similar methodology to CCAR 2017 challenger approach)
- **Recovery Rate ERM**: Challenger Model (CCAR 2017 Champion Recovery model)

**Vintage based Recovery Rate Model** has been developed using the life cycle curves (time since charge-off) and macroeconomic attributes using discrete time linear regression technique. The life cycle component can be estimated through period dummies or as and exponential function of empirical curves. In addition to the lifecycle curve, this model contains national level macroeconomic attributes in the form of transformations incorporating lags and windows. The final model equation consists of both the life cycle variables as well as the macroeconomic attributes along with seasonality factors. All the model factors are captured in a single step of linear regression.

**Economic Response Models (ERM)** in recovery models provide a mathematical relationship between the dependent variable (recovery rate), and independent macroeconomic time-series. Multiple permutations of macroeconomic attributes, transformations, lags and windows are analyzed to determine the most predictive model. Models were ultimately selected based upon the theoretical underpinnings of the variable, intuitive correlation with the dependent variables and statistical measures.

The vintage based recovery rate model has been selected as the champion approach because it is more granular than the Recovery Rate ERM challenger model as it is built at an additional level of charge-off vintage and gives better prediction accuracy.

### 4.1.4 Preferred choices for CCAR 2018 champion models

In conclusion, Branded Cards has decided to leverage Discrete Time Survival based approach to estimate PD for the champion model.. Logistic Regression is the modeling technique adopted. The business is modeling EADF using Ordinary Least Square (OLS) regression. For cash recoveries, the business is using Vintage based Recovery rate modeling approach.

LookAhead PD and EADF models, which are based on Dual time dynamics approach, constitute the first set of challenger models. These models are same as those submitted in the CCAR 2017 cycle.

Additionally, the business is developing an account level state transition matrix based model as a second challenger model to be submitted for CCAR 2018.

For Recovery models, time series recovery rate models (CCAR 2017 champion approach) would be used as a challenger model for CCAR 2018 submission.

## 4.2 Evolution of Modeling Approaches Attempted

The purpose of this section is to provide an overview of the alternatives considered in arriving at the chosen MA.

### 4.2.1 For the preferred Modeling Approach identified in Section 4.1, please describe the challenges faced, if any. (If the preferred MA is not your chosen MA, at least one of the following must be addressed in support of the chosen MA.)

[Model Sponsor must adhere to the Gating Principles guidance and the Gating Principles excels. In case Coping strategy has been employed to address the lack of portfolio data and/or stress period, Model Sponsor must clearly provide the rationale for Coping in terms of risk behavior of customers for e.g. CLTV etc) , regulatory guidance etc. Model Sponsor must demonstrate the appropriateness of Coping and provide the GP excels for both home and target country as required by the Guidance. For detailed information of Gating Principles please refer to the Gating Principle Guidance and Excels attached below]

Gating Principle Guidance Jun2017.dc

The Gating Principle 1 and 2 is attached below:

Gating principle1_2.xlsx

The Gating Principle 3, 4, 5 are attached below for all the models:

| PD | EAD | EADF | Recovery Rate |
|---|---|---|---|
| Gating Principal 4-5_PD.xlsx | Gating Principal 4-5 EAD.xlsx | Gating Principal 4-5 EADF.xlsx | Gating Principal 4-5_Recovery.xlsx |

**Inputs:**

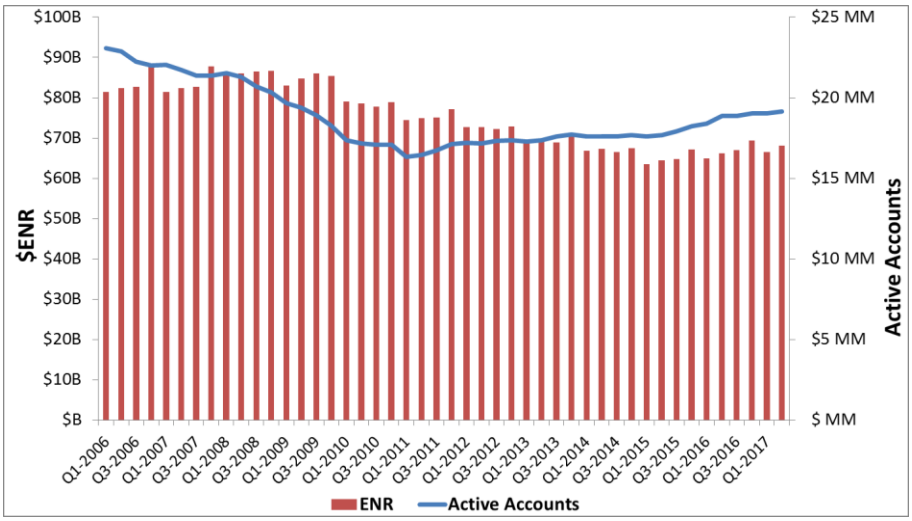
**Gating Principle 1: Current Portfolio Profile**

- Is the reference portfolio in a mature state (e.g., has a long and stable history of active accounts, pay-off, and default accounts) or have a history covering at least one business cycle, or one interest rate cycle.
- Does the portfolio cover at least 10 years of data
- If the portfolio was not observed to be in mature state in terms of number of defaults/ sufficient amount of balance or revenue, was coping strategy employed.

*[*Required: Please provide response here]*

Yes, the portfolio was sufficiently mature at the beginning of the observation period as observed from the ENR, active accounts.

Figure 10 Branded cards Portfolio Size (Existing Accounts and ENR)




The portfolio has more than 10 years of history. In fact, the data used in the modeling starts 2006, which indicates the portfolio maturity. Since the portfolio in terms of number of defaults, ENR etc is in a mature state, no coping strategy was needed.

**Gating Principle 2: Future Portfolio Profile**

- Please describe whether or not the "go-forward" portfolio set is similar to the development portfolio. Please consider a comprehensive set of potential business changes that could impact the MA including changes in portfolio characteristics (e.g., average limit, exposure, remaining maturity, interest rate, LTV, FICO, delinquency, CD maturity, time-on-file), product mix, product features (e.g., terms and conditions), underwriting criteria, regulation, business / collection / risk management strategy, marketing campaigns, macroeconomic / industry / political conditions, etc.

*[*Required: Please provide response here]*

The business assumes that the portfolio is stable and mature. The material change to the portfolio is the acquisition of the Costco portfolio, which would be modeled separately due to lack of data availability. The branded cards portfolio (excluding Costco) is expected to be very similar to the development dataset.

**Gating Principle 3: Portfolio Segmentation**

- To show that the segmentation approach has appropriate granularity (i.e., segments have sufficient data to develop robust and testable estimates that capture different underlying portfolio

characteristics) given the modeling objective of forecasting losses under normal and stressed macroeconomic environments, please provide:

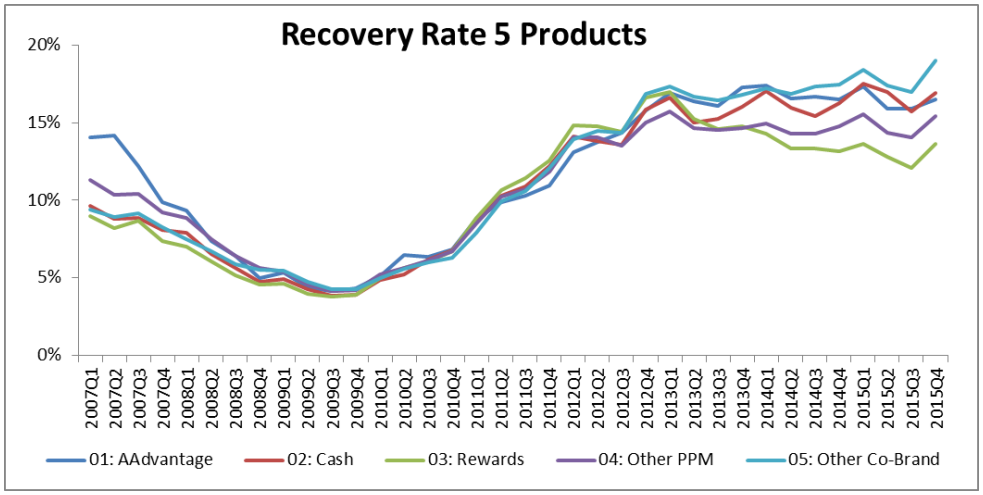
- Details on segmentation scheme explored and how the chosen segmentation scheme meets the modeling objective.
- Business rationale for the segmentation, which could be either the business requirements driving a non-statistical segmentation or the business intuition for the segmentation variables of a statistical segmentation
- Evidence of what leads you to believe that developing a model based on this segmentation approach is feasible (e.g., number of defaults per segment)
- For statistical segmentations, discussion of the trade-offs compared to alternative segmentations such as an MA with appropriate explanatory variables but no segmentation
- For statistical segmentation, if available, analysis to justify differences between segments (e.g., central tendency and / or dispersion of distributions, risk factors, sensitivities to common risk factors)

*[*Required: Please provide response here. Even if there is no segmentation, please provide the rationale for no segmentation.]*

- In the current submission, the Account Level PD and EAD models are developed at overall Branded Cards portfolio level. For CCAR 2017, Consumer cards and Business cards were planned to be modeled separately initially. However, the pre-final models of consumer and business cards were very similar and hence these were combined to get one portfolio level model for Branded Cards.
-
- To take into account Portfolio specific risk dynamics, portfolio indicator variables are used in the models. Additionally, FICO Score and delinquency based indicator variables are used to take into account specific risk characteristics of a customer. These variables have led to an improvement in model performance and all the errors are well within the specified MRM guidelines.
-
- The account level models were evaluated for accuracy for various granular back-testing e.g. portfolio, FICO and Delinquency status. For CCAR 2017 models, all the errors were well within the MRM thresholds. However, for CCAR 2018 submission, the models have been back-tested for even further granular dimensions. Necessary interactions of the various variables have been considered and hence, the models perform very accurately by various granular dimensions. Hence, the models have been developed at the portfolio level
-
- For CCAR 2017, the Branded Cards team is not going ahead with any segmentation and is modeling the entire Consumer portfolio as one. For CCAR 2018, the Branded Cards team is modeling the entire Consumer and business portfolio as one. Business Cards were not modelled separately as they are a small percentage of overall losses. However, for CCAR 2016 Submission NA Branded Cards had successfully developed models on 5 product portfolios as segments which were approved by MRM.

The recovery rate charts for the 5 products are as below. As seen from Figure 11, the 5 recovery rates are almost overlapping in stress.

Figure 11 Recovery Rate - 5 Products




The final models submitted for the 5 products are as below:

Table 4-2 CCAR 2016 Recovery Rate Models

| Segment | Var 1 | Lag1 | Window 1 | Trans1 | Coeff 1 | Var2 | Lag2 | Window 2 | Trans2 | Coeff2 |
|---|---|---|---|---|---|---|---|---|---|---|
| AA | UR | 9 | 12 | Avg Diff | -0.24 | HP | 0 | 12 | Avg Diff | 0.03 |
| Other CoBrand | UR | 9 | 12 | Avg Diff | -0.27 | HP | 0 | 12 | Avg Diff | 0.03 |
| Cash | UR | 9 | 9 | Avg Diff | -0.22 | HP | 0 | 12 | Avg Diff | 0.03 |
| Rewards | UR | 9 | 9 | Avg Diff | -0.20 | HP | 0 | 12 | Avg Diff | 0.03 |
| Other-PPM | UR | 6 | 12 | Avg Diff | -0.24 | HP | 0 | 12 | Avg Diff | 0.02 |

As seen from the models above, the independent variable transformations are very similar for all 5 products. Also, the variable coefficients were similar, indicating that the 5 segments are not very different.

The recovery rate for the 5 products also failed the ANOVA test recommended in the MDDT. The ANOVA test determines if the segments are significantly different.

Table 4-3 ANOVA Test Results for Product Segmentation

| Start Time | End Time | Product Segmentation | F-Stat | Result |
|---|---|---|---|---|
| 2007Q1 | 2015Q4 | AA, Co-Brands, Cash, Other-PPM, Rewards | 0.5 | Fail |

Because of the above mentioned reasons, it was decided to drop the product segmentation for CCAR 2017. The same has been continued for CCAR 2018 submission.

Gating Principle 4: Stress Period Identification (Applicable for Stress Models only)

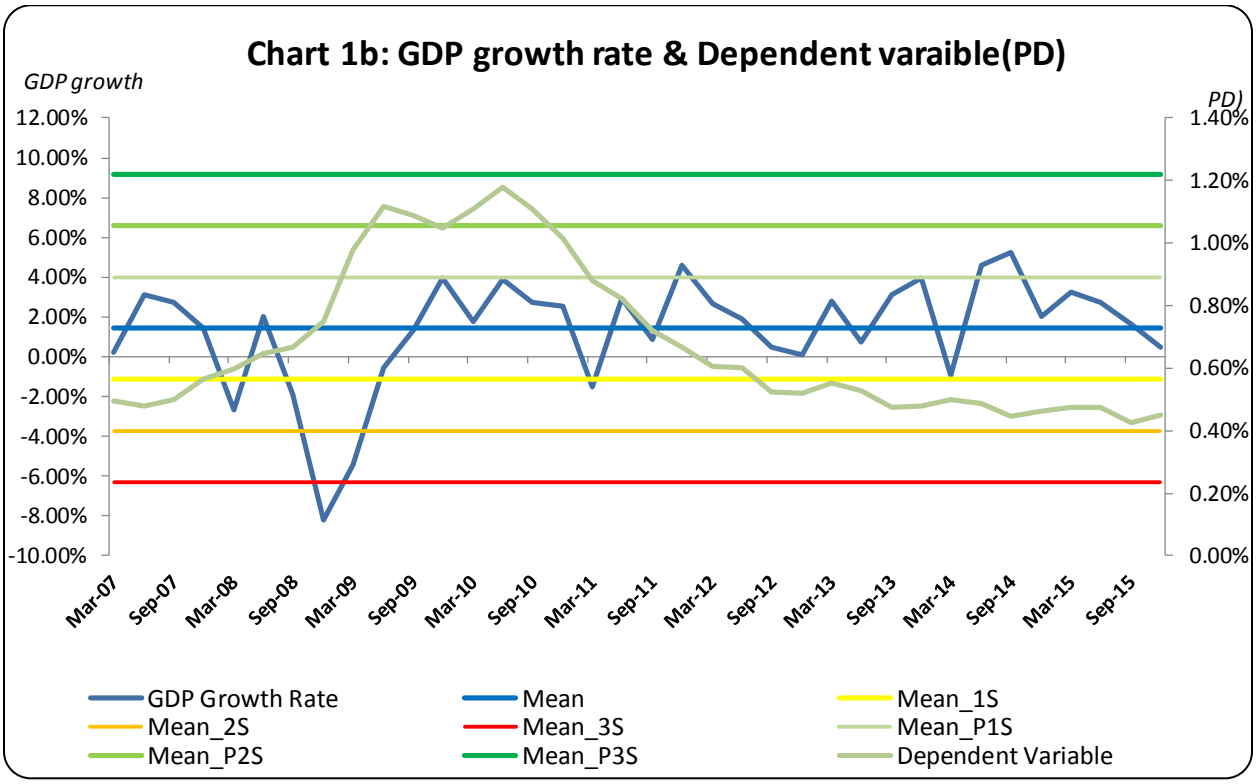
- Please provide evidence of whether or not the modeling data set has a macroeconomic stress period. Any one of the three options below are acceptable:
- If there were at least two quarters of negative GDP growth, please indicate which time period experienced this stress (no additional evidence is necessary)
- If there were at least two consecutive quarters of GDP growth rate that fall below one standard deviation of the historical mean (10 years), please indicate which time period experienced this stress (no additional evidence is necessary)
- If the stress period is defined differently (e.g., using other macroeconomic variables from the FRB approved list), please indicate what variables are being used to define the stress period and which time period experienced stress. In addition, please provide evidence that the that this was a sufficiently stressed period (e.g., behavior of the dependent variable over the stress period)

*[*Required: Please provide response here]*

The US economy experienced the Great Recession that lasted from Dec '07 to Jun '09 that began with the bursting of an eight trillion dollar housing bubble. In 2008 and 2009 the U.S. labor market lost 8.4 million jobs, or 6.1% of overall payroll employment. This was the most dramatic employment contraction of any recession since the Great Depression. From the chart below it can be seen that a severe contraction was observed in GDP between Q1 '08 to Q2 '09. In Q4 '08, the GDP growth rate was very close to three standard deviations away from the mean (shown in the red line), which illustrates the enormity of the Great Recession. Between Sep'08 and Mar'09 the GDP contraction was more than one standard deviation.

Although the economy started growing from Q4 '09 onwards, the growth was extremely feeble and losses in the Branded Cards business continued to rise and reached a peak in around Q1 '10. Taking the macroeconomic environment and portfolio performance into consideration, Branded Cards decided to use Q1 '08 to Q1 '10 as the Stress Period.

Figure 12: GDP Q-o-Q SAAR Historical




The 9Q stress period for modeling was selected as **Q1 2008 – Q1 2010** (a 9-quarter stress period) in alignment with previous CCAR submissions.

Gating Principle 5: Influence of Macro-Economic Variables on dependent variables or portfolio losses/balances

- Please provide evidence of correlation between target/dependent variable or portfolio losses/balances and key macro-economic variables
  - Did the target/dependent variable or portfolio losses/balances showed correlation with standard set of macro-economic variables mentioned in the guidance.
  - If dependent variable or portfolio losses does not show correlation with any standard set of macro-economic variables, a set of regional or industry specific macro-economic variables must be explored.
  - Please refer to the Gating Principles guidance for correlation thresholds and provide the rationale in case of no and weak correlation.

*[*Required: Please provide response here]*

Account level PD and EAD models have been built using both behavioral and macroeconomic attributes.

As can be seen from chart below, the macroeconomic variables such as Unemployment Rate (at a state level), Housing Price Index (at a state level) show intuitive relationships with the portfolio PD Rates and Exposure at default (EAD).

**Figure 13 : Relationship of PD (Dependent Variable) with HPI**

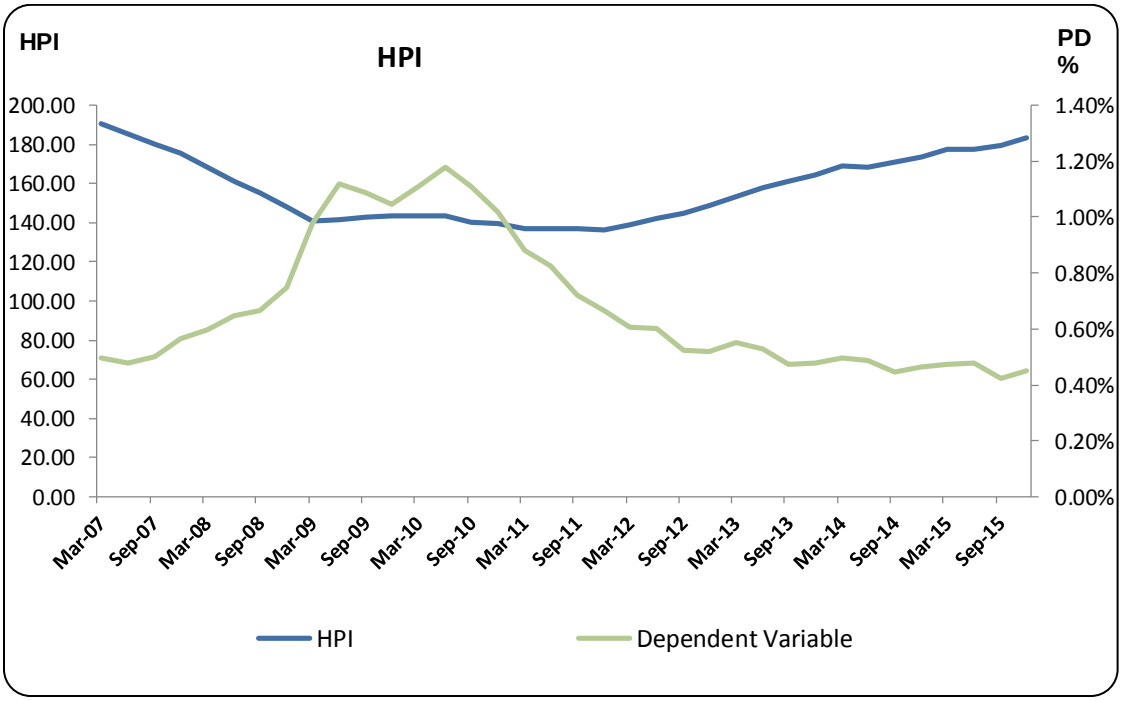


Figure 14 : Relationship of PD with Unemployment Rate

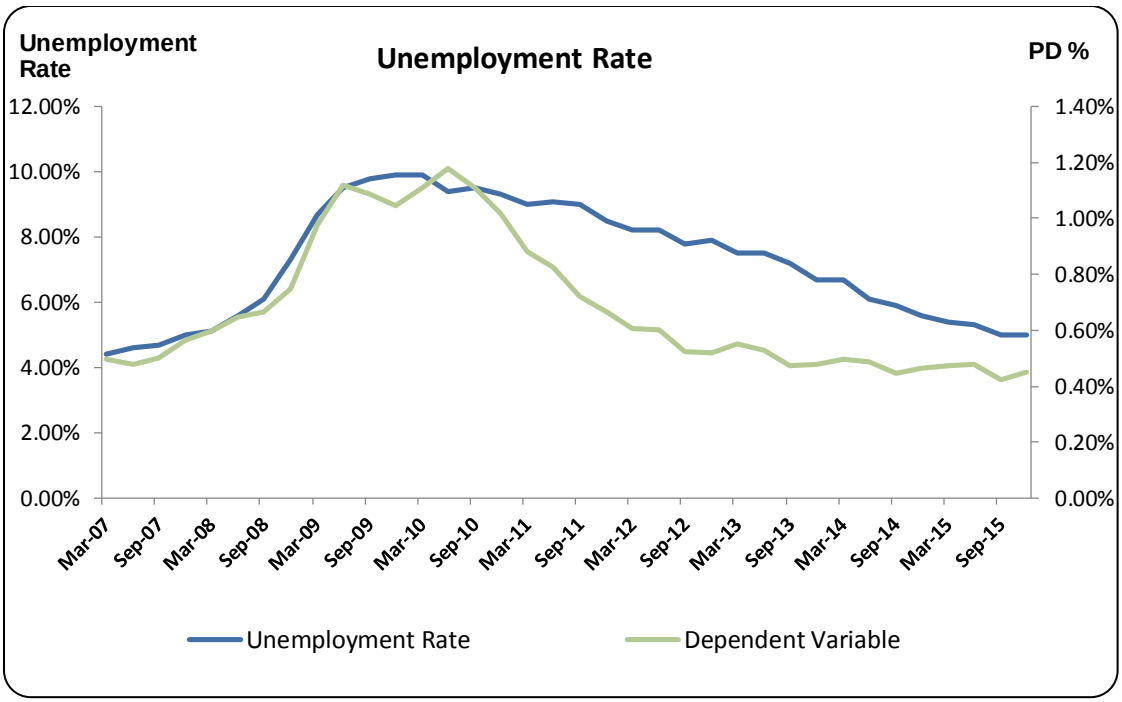



Figure 15 - Branded Cards - EAD and Unemployment Rate

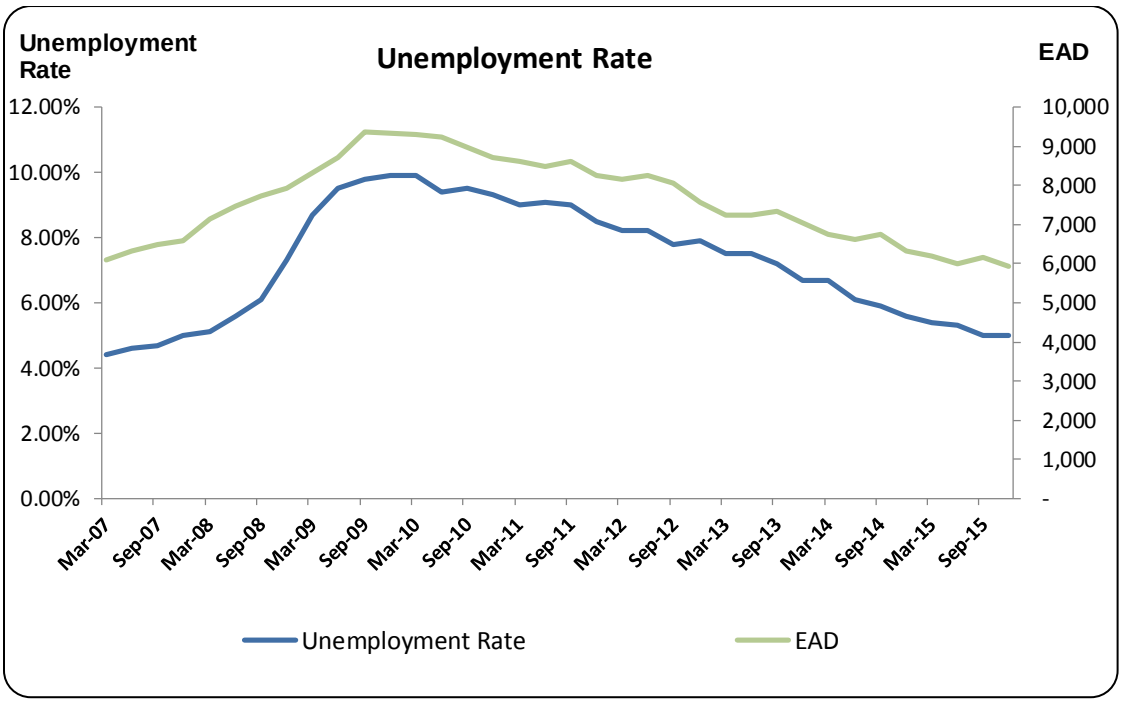
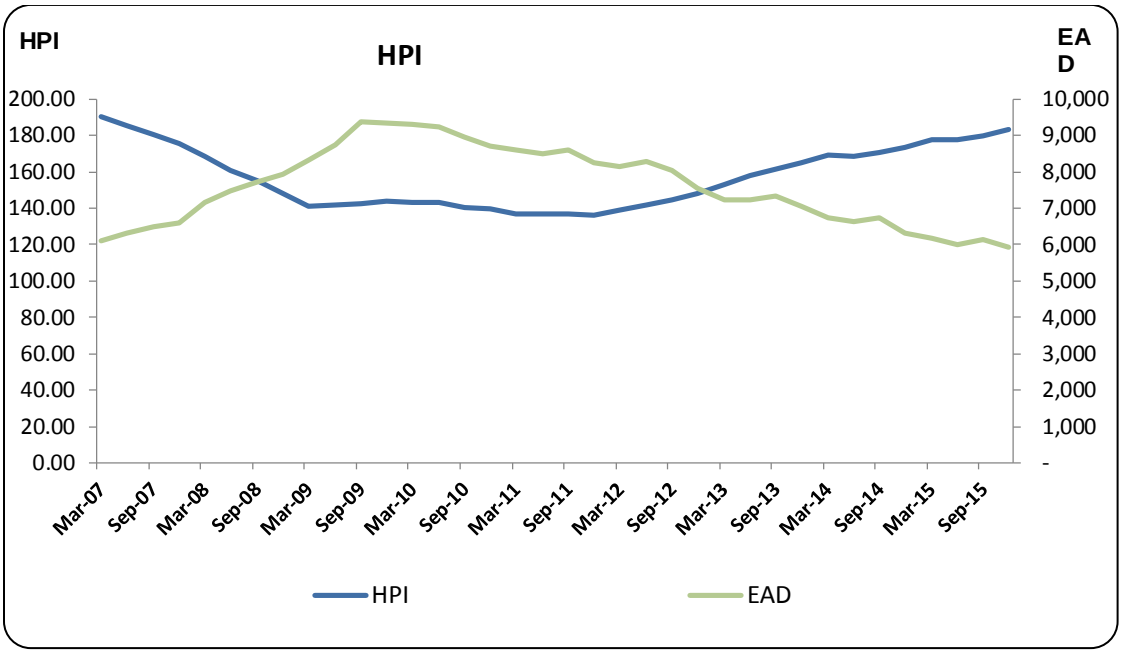


Figure 16 - Branded Cards - EAD and House Price Index




In order to capture risk characteristic at an account level, various risk drivers both internal and external which can explain dependent variable or PD and EAD have been explored. A robust category of risk drivers as shown in table below has been considered in model development:

Branded Cards business utilized the macroeconomic variables Unemployment Rate and Housing Price Index as they show intuitive relationships with the Recovery Rates. The variables have been present in previous development efforts also.

Figure 17 Branded Cards Recovery Rate And Unemployment Rate

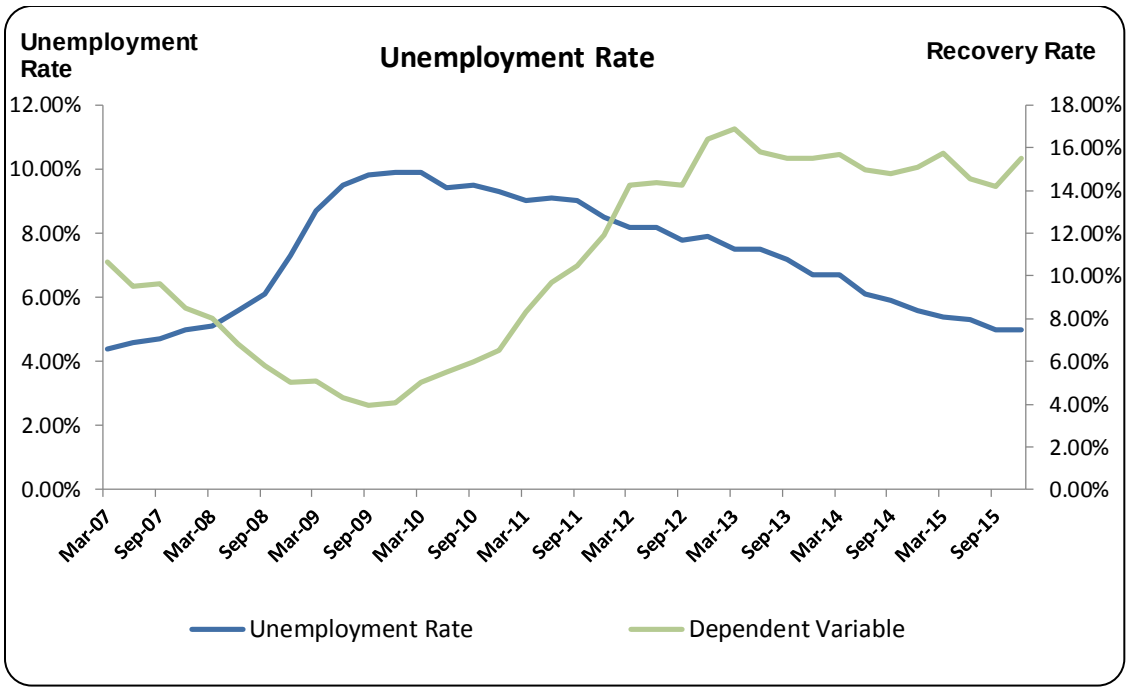
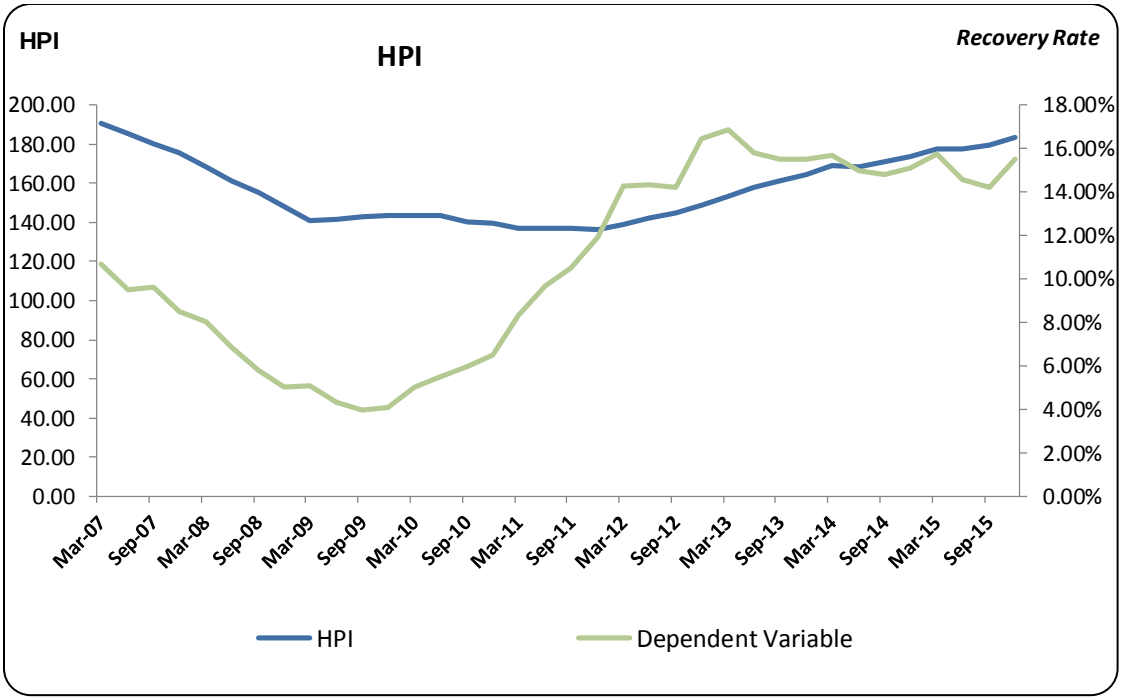



Figure 18 Branded Cards Recovery Rate And Housing Price Index



- Appropriateness of Coping:

  Since coping was not required, this is not applicable.

- Please articulate which variables influence the dependent variable (these could include macroeconomic variables, behavioral variables, etc.), and justify using any of the three options below:
  - Regulatory guidance and industry literature
  - Qualitative description of business justification
  - Statistical evidence, if available from current or prior development efforts

| PD | EAD | Recovery |
|---|---|---|
| Final_model_vars_PD.xlsx | Final_model_vars_EAD.xlsx | Final_model_vars_recovery.xlsx |

- If there were other input challenges, please describe

  There are no other input challenges.

**Technique:**

- If there were any challenges with technique in current or prior development efforts, please describe (e.g., variables were not predictive or had a counterintuitive relationship with model output)

  There are no challenges with the technique in current or prior development efforts.

**Output:**

- If there were any output challenges in current or prior development efforts, please describe (e.g., statistical tests exceeded prescribed error thresholds, or outputs under stress events do not match business hypotheses)

  There are no output challenges in current or prior development efforts.

**Other Challenges:**

- If there were any other challenges, please describe

  There are no other challenges.

### 4.2.2 Describe the progression of MAs attempted, and highlight why an approach was rejected. Then describe how you chose to refine the MA. Highlight the challenges faced, (e.g., a direct model of the reference portfolio could not be estimated because the number of necessary observations were insufficient) and provide evidence of appropriate mitigating actions to overcome the challenges before proceeding to another alternative Modeling Approach.

#### 4.2.2.1 Alternative Modeling Approach #1

The choice of the modeling approach and considerations for not selecting other approaches has been discussed in details in section 4.1. 0nce the relevant modeling approach was agreed upon, the modelers did not face any data or portfolio specific challenges. The approach that was agreed upon was leveraged to develop the model.

#### 4.2.2.2 Alternative Modeling Approach #2

The choice of the modeling approach and considerations for not selecting other approaches has been discussed in details in section 4.1. 0nce the relevant modeling approach was agreed upon, the modelers did not face any data or portfolio specific challenges. The approach that was agreed upon was leveraged to develop the mod

# II. Technical Soundness Considerations

[*Part II: Technical Soundness Considerations* to be filled out by model developer]

## 5 Model Data

[*Part II: Technical Soundness Considerations* to be filled out by model developer]

NCL Model is a compound model and not a standalone model with a separate data construct. It is derived using the component models (PD, EAD and Recovery). The following section talks about modeling data for PD, EAD and Recovery models.

### 5.1 Model Input

#### 5.1.1 Data Sources and Controls

The data required for champion models can be divided into 3 categories: Account level historical attributes, macro-economic series and future origination assumptions. Detailed description on how the data was accessed, processed and stored is as follows:

**Historical Data/ Account Level attributes** - The data is extracted from the System of Records (SOR) which comprises of Apollo and IBS. Apollo is the application sourcing system where new applications are processed. Once an account is processed in Apollo and approved a record is established in the IBS system which is the account management system.

Teradata Server is used to extract information from Enterprise Data warehouse (EDW) which contains the extracts from SOR.

**Macro-Economic Series** – For various macro-economic series and scenarios, historical data is provided by the Global Country Risk Management. Forecasted data through March 2021 is also provided by Citi Treasury for baseline and stressed BHC scenarios. Forecasts for supervisory baseline and stressed scenarios are provided by Federal Reserve at a national level. These supervisory forecasts are taken by the GCRM team to forecast state level variables that are not provided by the Federal Reserve Board.

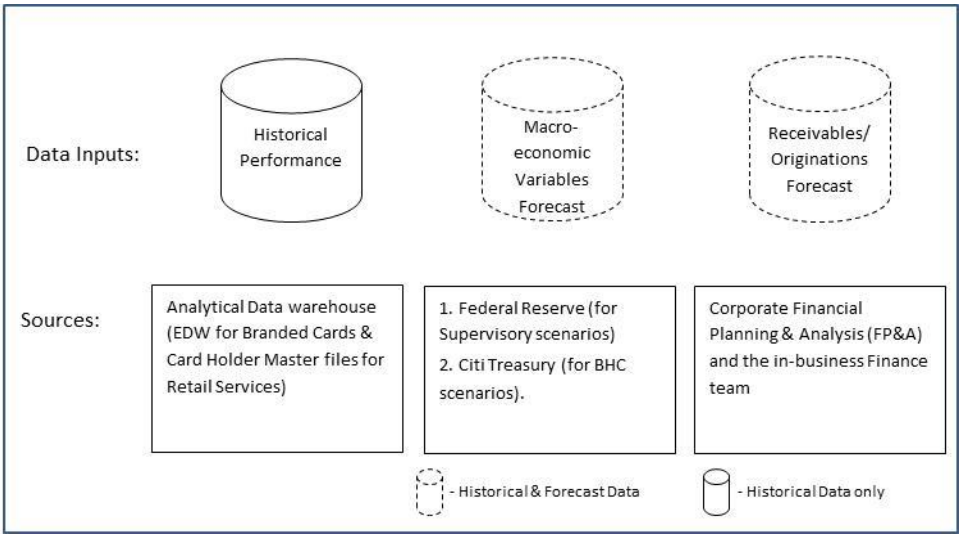
**Future originations** – Forecasts for new account plans through Q1-2021 is provided by respective in-business Financial Planning and Analysis (FP&A) teams for various economic scenarios. Following files contain new origination forecast provided for CCAR 2017 submission. Similar file will be sourced for CCAR 2018 submission from FP&A team.

Table 5-1 : Originations Forecast from FP&A

| MCST | FRB |
| --- | --- |
| FW MCST RiskFinance Weekly M | FRB New Originations.msg |

The various sources/teams involved for providing the data is given in Figure 19.

Figure 19: Data sources for Account level modeling




- What are the performance, segmentation and account data sources for the model?

The source data for the account level attributes (including performance and segmentation) originates from two different systems of record (SOR):

- **Apollo** - Citi Cards' system for processing applications for credit cards; provides account characteristics at acquisition

- **IBS** - Citi Cards' system for processing records for Existing Cardholder Management (ECM); provides account-level monthly credit card data on account statuses, balances, credit lines, and transactions

EDW (Enterprise Data Warehouse) is a Teradata server containing database tables of extracts from these SOR. The data to EDW is uploaded using automated scripts by Global BI team.

The data was created for model development sourcing Enterprise Data Warehouse (EDW). The warehouse contains information on card member's internal behavior and external data from credit bureaus and is extensively used by teams for business insights and model development across Citibank. Assessment of Card Master File (CMF) and Enterprise Data Warehouse (EDW) data sources was carried out to investigate the pros and cons of each data source for daily scoring. Due to the various advantages of EDW over CMF like ease of data access, availability of data for 10+ years, availability of time series variables for historical information, presence of derived variables and bureau data, EDW was chosen as the primary data source for model development.

The EDW databases used by CCAR models are:

- **P_BCD_V_I_CONSUMER** - Acquisition characteristics data (from Apollo) and account-level monthly credit card data on account statuses and balances (from IBS)

Detailed data attributes pulled for PD and EAD models, sub-tables in P_BCD_V_I_CONSUMER along with definitions of each variable can be found in following attached file:

5.1.1 Variable_Consolidated

Details of final model attributes for PD, EADF and Recovery Models along with the definition can be found in the attachment below:

5.1.1_Final_model_variables.xlsx

- What are the macroeconomic data sources for the model including stress scenarios?

The historical macro-economic data was provided by Global Country Risk Management (GCRM) team. The forecasts for the BHC baseline and stress scenarios were provided by GCRM and the forecasts for the Supervisory baseline, adverse and severe adverse scenarios were provided by the Federal Reserve Board at a national level. However, the final forecast generated by the business for the supervisory scenarios comes from the GCRM team as there are regional/ state level macroeconomic variables that do not come from the Federal Reserve but rather need to be forecasted by the BHC.

Following economic scenarios were considered for baseline and stressed forecasting:

1. Supervisory Baseline Scenario
2. Supervisory Adverse Scenario
3. Supervisory Severely Adverse Scenario
4. BHC / Mid Cycle Stress Test (MCST) Baseline Scenario
5. BHC / Mid Cycle Stress Test (MCST) Adverse Scenario
6. BHC / Mid Cycle Stress Test (MCST) Severely Adverse Scenario

- What process checks and controls are in place to ensure data integrity and that the data is properly sourced and reliable?

There are two major sources of data used in Account level models – behavioral and macroeconomic. There are comprehensive process checks and controls in place to ensure data integrity and reliability.

a. Behavioral data

For all analytical purposes, EDW is used to extract the data for all business and regulatory models. For all prior CCAR submissions, EDW has been the source for the account level data. Every month the EDW load team (IT) sets triggers when they have completed loading the EDW. The process is executed automatically each month utilizing Maestro scheduling software. These triggers initiate many of our queries that extract data from the EDW database, process this data, and create some of the data sources needed for all of our regulatory reporting, not just CCAR. Once these queries have successfully completed, it triggers our CCAR process to begin execution. In a Typical month, the EDW is loaded on the 2nd day of the month and our CCAR process usually runs on the same day. Of course there are exceptions throughout the year, but typically the CCAR SAS processes runs by the 2nd- 3rd day of each month.

There is a standard error checking route that is part of the automated execution that checks each SAS log for errors, warnings, and uninitialized variables, numeric converted to characters, characters converted to numeric, division by zero and missing values. If any of these checks are positive the production process will amend and a warning mail is circulated within the group to look into it. The execution after investigating into it (ensuring EDW is loaded etc.) is again implemented using Maestro and rechecked for the above possible abnormalities. This ensures that none of the above abnormalities are present in the data.

Each step of the process is performed independently by two different analysts (performer/re-performer). The results are then compared to reduce the likelihood of human error. Following are the key steps to ensure accuracy of data and model outputs.

1) Consistency between data requested and received

*Please find below the detailed mail on data requirement along with variables required to be pulled from EDW are sent to the data pull team. Post receiving the data it is ensured all variables sent are received and the number of records seem logically justifiable. It is also ensured that the sampled data matches with the Independent Reconciliation system*

FW Data Request for FICO.msg

2) As the first step in the modeling, the modelers investigate the submitted data using various data views to confirm data reasonability and share the output of the analysis. Data distribution of each attribute is looked into details:

a. In case of missing variable – appropriate justification is noted
b. In case higher percentage of missing in any attribute – the same is dropped after investigation
c. Distribution is intuitive and makes business sense. Example – Close balance amount should vary between 0 and $75K for Percentile 1 – Percentile 99

The details of the checks performed are given in Section 5.1.2

3) Data is consistent across various models in terms of key metrics such as #defaults, gross credit loss. Refer to email below for reconciliation process carried out :

RE Maker Checker Process on Data and D

4) Portfolio data pulled from EDW for Branded Cards is reconciled against known-good system (GL/PGPearl/NCLFcst) by two different analysts. Additionally, we also performed statistical quality check on the performance data used in the model. Please refer to section 5.1.2 for data reconciliation with independent system.

5) The work-stations on the floor are single-user stations for use only by the authorized user. This ensures that there is no external tempering at any step of the modeling process.

<u>Macroeconomic data</u>

The macroeconomic data is sourced from the macroeconomic templates provided by the GCRM team. It is a manual process however the Branded Cards team completes a validation to make sure

that the data is aligned with the previously held historical data. In addition the Branded Cards team performs checks with the Retail Services team so that the historical and forecasted data matches up as the businesses use the same macroeconomic variables in modeling. Please refer to attached email confirming maker checker process.

RE CCAR MCST Forecast Scenarios - I

RE CCAR Forecast Scenario File from GC

The maker checker email confirming the reconciliation internally is attached below:

RE Updates Historical Macro-economic Files

- Is the macroeconomic data source and data itself approved for use in Citi?

Yes. Macroeconomic data is provided by GCRM and is acceptable for use in Citi models and sufficient forecast period is available to model.

### 5.1.2 Data Reconciliation

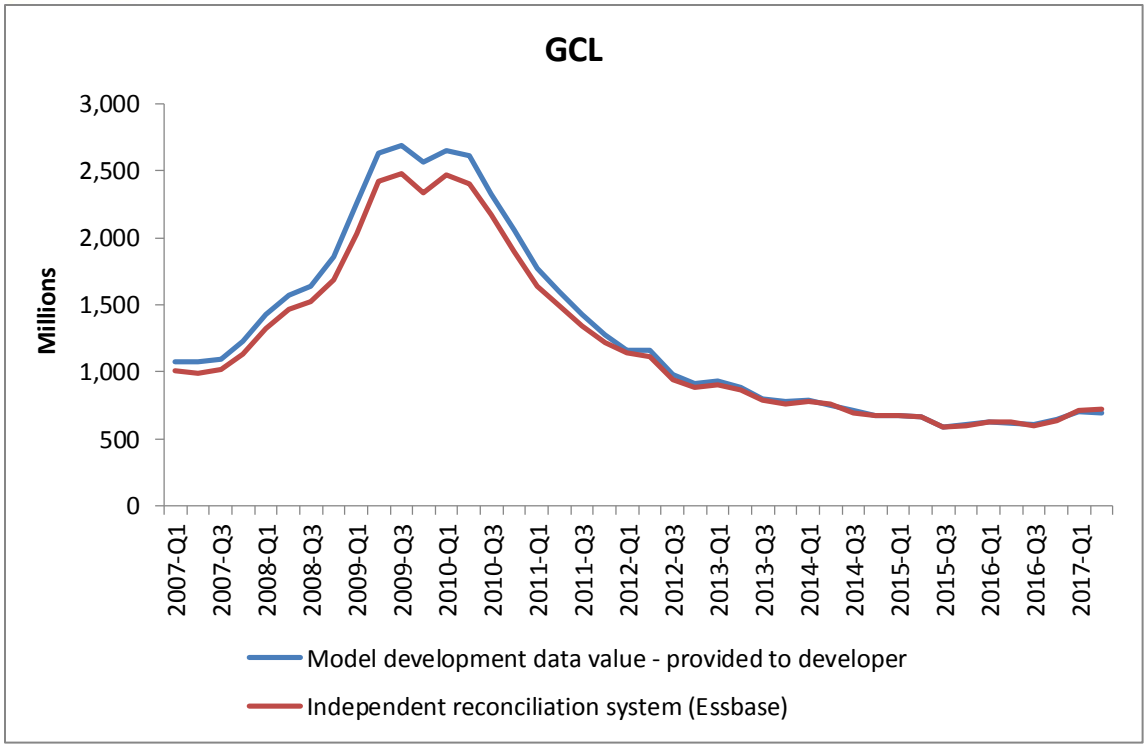
- Is the data source regularly reconciled to another independent data source for all key model variables?

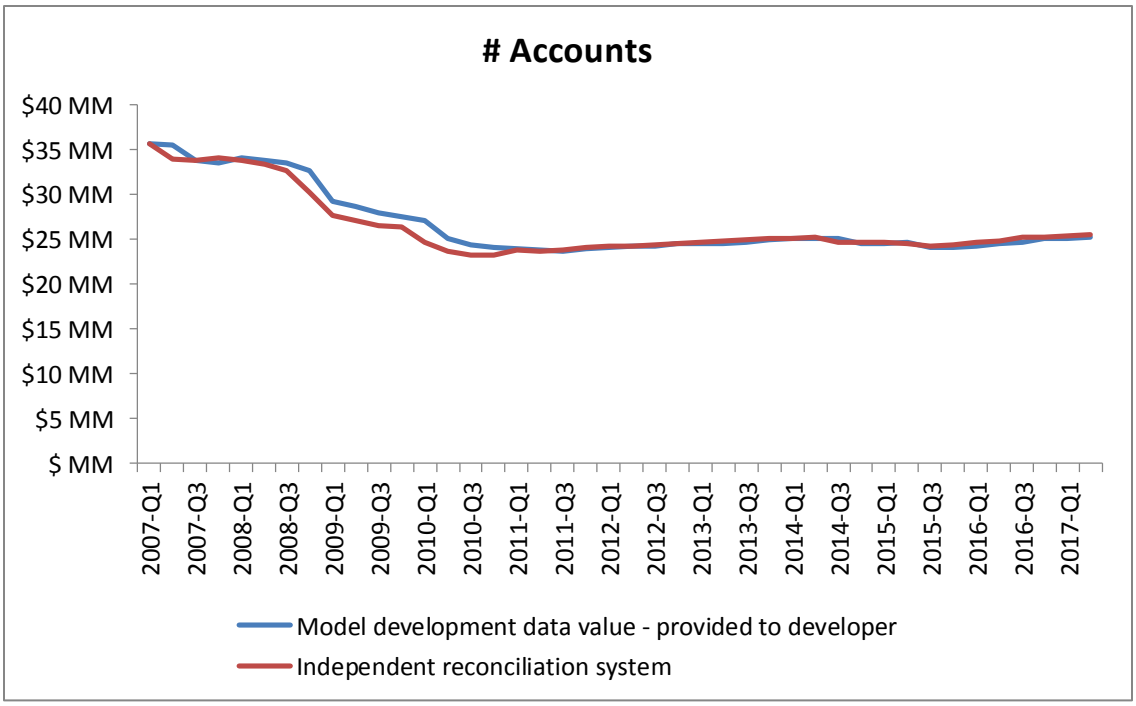
The data reconciliation is carried out with the independent reconciliation system. The reconciliation template is attached below.

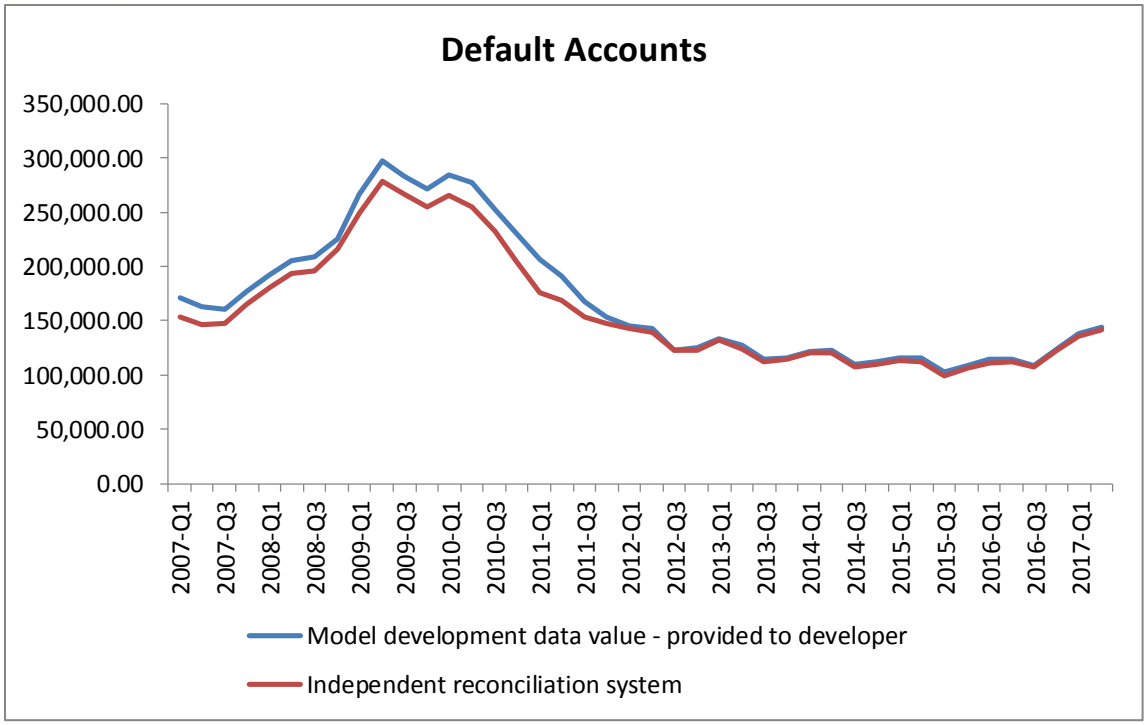
5.1.2 Data_Reconciliation.xls

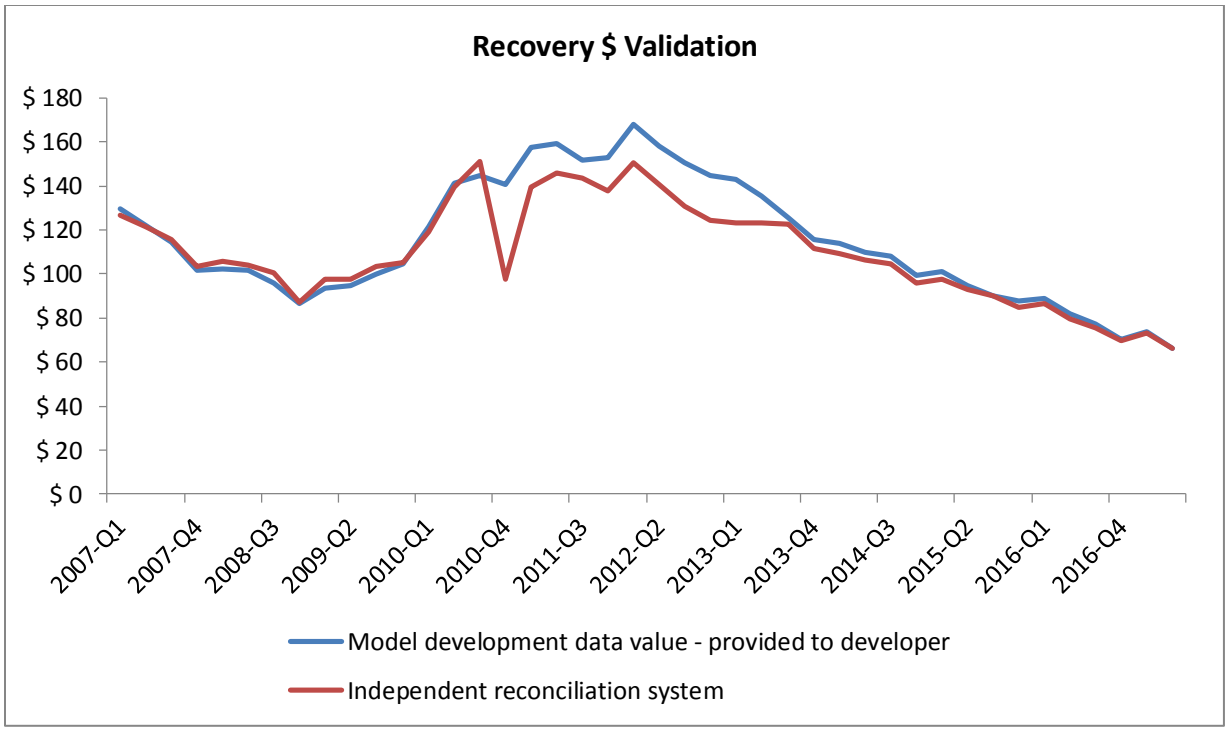
As shown in charts below, the data used in the Y14-A CCAR / DFAST process is validated against the data present in General Ledger. As observed from the sheet, the numbers for Y14-A CCAR / DFAST data compare well with the records in the General Ledger. The wider difference number of accounts, charge off accounts and gross credit loss in the period prior to 2011 is due to inclusion of certain portfolios not considered in General Ledger (Essbase). The same has been corrected post 2011 where Business observed a significantly high match rate.

**Figure 20 : Reconciliation between development data and general ledger**


GCL
Millions
3,000
2,500
2,000
1,500
1,000
500
0
2007-Q1
2007-Q3
2008-Q1
2008-Q3
2009-Q1
2009-Q3
2010-Q1
2010-Q3
2011-Q1
2011-Q3
2012-Q1
2012-Q3
2013-Q1
2013-Q3
2014-Q1
2014-Q3
2015-Q1
2015-Q3
2016-Q1
2016-Q3
2017-Q1
Model development data value - provided to developer
Independent reconciliation system (Essbase)


# Accounts
$40 MM
$35 MM
$30 MM
$25 MM
$20 MM
$15 MM
$10 MM
$5 MM
$ MM
2007-Q1
2007-Q3
2008-Q1
2008-Q3
2009-Q1
2009-Q3
2010-Q1
2010-Q3
2011-Q1
2011-Q3
2012-Q1
2012-Q3
2013-Q1
2013-Q3
2014-Q1
2014-Q3
2015-Q1
2015-Q3
2016-Q1
2016-Q3
2017-Q1
Model development data value - provided to developer
Independent reconciliation system


Default Accounts
350,000.00
300,000.00
250,000.00
200,000.00
150,000.00
100,000.00
50,000.00
0.00
2007-Q1
2007-Q3
2008-Q1
2008-Q3
2009-Q1
2009-Q3
2010-Q1
2010-Q3
2011-Q1
2011-Q3
2012-Q1
2012-Q3
2013-Q1
2013-Q3
2014-Q1
2014-Q3
2015-Q1
2015-Q3
2016-Q1
2016-Q3
2017-Q1
Model development data value - provided to developer
Independent reconciliation system


Recovery $ Validation
$ 180
$ 160
$ 140
$ 120
$ 100
$ 80
$ 60
$ 40
$ 20
$ 0
2007-Q1
2007-Q4
2008-Q3
2009-Q2
2010-Q1
2010-Q4
2011-Q3
2012-Q2
2013-Q1
2013-Q4
2014-Q3
2015-Q2
2016-Q1
2016-Q4
Model development data value - provided to developer
Independent reconciliation system

It can be observed that Essbase and EDW are very close except for the period from Q3'2010 to Q2'2013. It was discovered that recoveries in Essbase were understated by $6.5-$7MM per month, which was adjusted in both contractual losses and adjustments and refunds. This was because adjustments were made in recoveries to counter for refunds made; hence the impact to NCL was flat. Beginning Q3'2013, the understatement of recoveries has been addressed. The modeling data is retrieved from EDW which is unaffected by this understatement.

However, some differences will always occur in the reconciliation between the EDW Data and Essbase because of the timing in which the data is set in these sources and slight difference in the definitions. The key difference in timing is that EDW is based on the billing cycle data and Essbase is based on period end data which can therefore results in differences between the two sources.

- If the data source is not regularly reconciled, how was the data used for model development reconciled?

CCAR Data Source has been regularly reconciled. Once data sample is pulled, a thorough reconciliation process is carried out. Key metrics such as default rate, number of accounts, gross credit loss amount and receivables are compared against known system such as general ledger. Reconciliation is also performed among various models to ensure data reconciles well across CCAR models. A maker checker process is performed to eliminate any error arising due to human error. Further, distribution analysis of all model variables is performed to ensure distribution is well within intuitive range and does not have any observed anomalies.

During model execution as well a similar maker checker process of reconciliation is followed

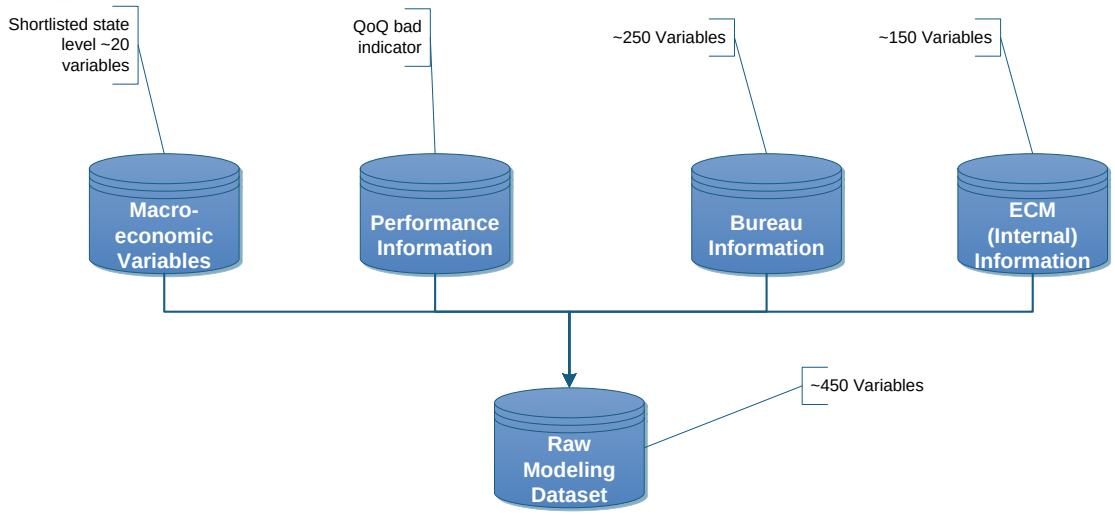
- Were there any reconciliation differences?

There are some differences in the reconciliation between the CCAR Data and Essbase as mentioned above. However the main concerns number of accounts, charge offs and GCL in the period prior to 2011 is due to inclusion of certain portfolios not considered in Essbase. This difference has been documented and well observed during past CCAR submissions. Therefore, Business is comfortable with the data and considers it sufficiently suitable for modeling.

### 5.1.3 Data Characteristics, Fields and Definitions

As mentioned above, account level characteristics data is extracted from EDW and macro-economic data is obtained from Citi GCRM team. Following illustration provides high level data assimilation process:

**Figure 21 Data Assimilation Process**




The process of collation of data has been mentioned in section 5.1.1 above. Details on structure of data for processing in the model development can be found in section 6.3 of the MDDTs.

Detailed data attributes pulled, sub-tables in P_BCD_V_I_CONSUMER along with definitions of each variable for PD, EAD and Recovery model can be found in following attached file:

5.1.1 Variable_Consolidated

- Please provide descriptive statistics for the underlying data

  Following files provides univariate distribution analysis of all variables considered in the PD and EAD.

  Sec5_descriptive_statistics_EAD_model.xls

  sec5_descriptive_statistics_PD_model.xlsx

- If a vendor model is used, describe any model-specific data requirements.

  There is no vendor model used.

### 5.1.4 Data Sampling Methodology and Results

**<u>Account level PD & EAD model data structure</u>**

For the development of the Account Level PD and EAD models a Through the Cycle (TTC) data structure is used. Branded Cards have modeled through the cycle PD and EAD (Dec'06 – March'16) of portfolio using behavioral and macroeconomic variables.

The Account level PD model is a Through-the-Cycle (TTC) model built on data prepared by stacking 10% samples of 38 different quarterly vintages ranging from Dec'06 to March'16. The open accounts and the closed accounts with remaining closing balance which have not charged off before or in the observation vintage are kept. Random 10% samples are chosen from each vintage and stacked together to obtain through the cycle random sample of the portfolio. This sampling leads to ~100 MM number of unique accounts in the sample chosen.

The data structure of account level PD and EAD model is the following:

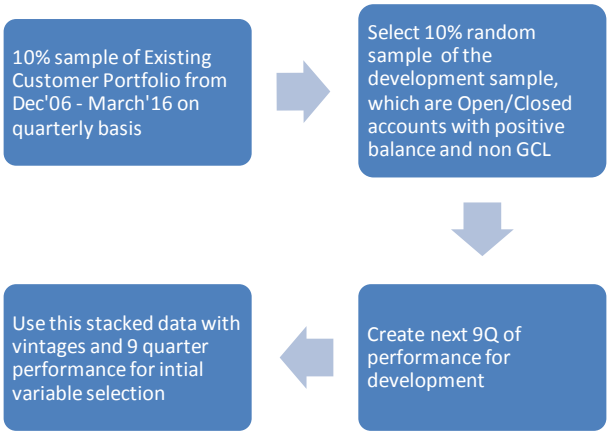
**Table 5-2 : Data structure for PD model**

| | | Performance | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **Qtr1** | **Qtr2** | **Qtr3** | **Qtr4** | **Qtr5** | **Qtr6** | **Qtr7** | **Qtr8** | **Qtr9** |
| **Observation** | **Dec-06** | Mar-07 | Jun-07 | Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 |
| | **Mar-07** | Jun-07 | Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 |
| | **Jun-07** | Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 |
| | **Sep-07** | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 |
| | **Dec-07** | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 |
| | **Mar-08** | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 |
| | **Jun-08** | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 |
| | **Sep-08** | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 |
| | **Dec-08** | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 |
| | **Mar-09** | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 |
| | **Jun-09** | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 |
| | **Sep-09** | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 |
| | **Dec-09** | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 |
| | **Mar-10** | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 |
| | **Jun-10** | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 |
| | **Sep-10** | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 |
| | **Dec-10** | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 |
| | **Mar-11** | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 |
| | **Jun-11** | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 |
| | **Sep-11** | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 |
| | **Dec-11** | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 |
| | **Mar-12** | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 |
| | **Jun-12** | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 |
| | **Sep-12** | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **Dec-12** | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 |
| | **Mar-13** | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 |
| | **Jun-13** | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 |
| | **Sep-13** | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 |
| | **Dec-13** | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 |
| | **Mar-14** | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 |
| | **Jun-14** | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 |
| | **Sep-14** | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 |
| | **Dec-14** | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 |
| | **Mar-15** | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 |
| | **Jun-15** | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | |
| | **Sep-15** | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | |
| | **Dec-15** | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | |
| | **Mar-16** | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | |
| | **Jun-16** | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | | |
| | **Sep-16** | Dec-16 | Mar-17 | Jun-17 | | | | | | |
| | **Dec-16** | Mar-17 | Jun-17 | | | | | | | |
| | **Mar-17** | Jun-17 | | | | | | | | |
| | Development sample | | | | | | | | | |
| | Hold out Sample | | | | | | | | | |

For the initial variable selection purpose, further 10% of the development sample has been taken, which is 1% of the through the cycle data. Final model has been built on complete 10% development sample.

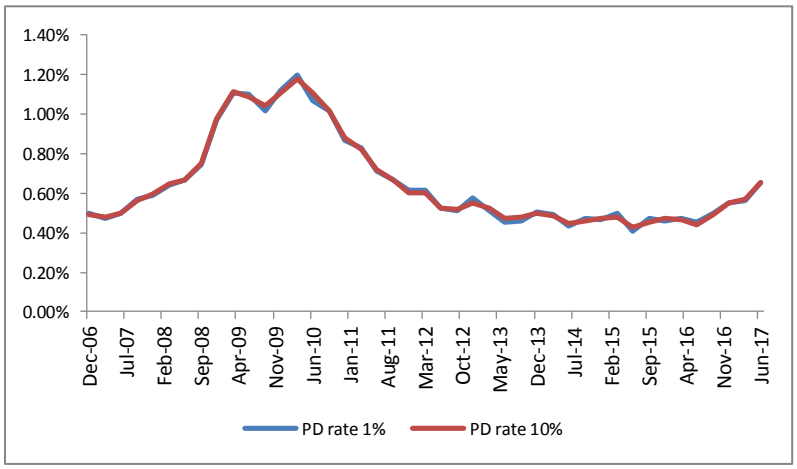
**Figure 22 : Data sampling flowchart**




Both development sample and 10% sample of the development sample reconciles well in terms of performance variables such as number of charge offs. Following chart represents #defaults in first

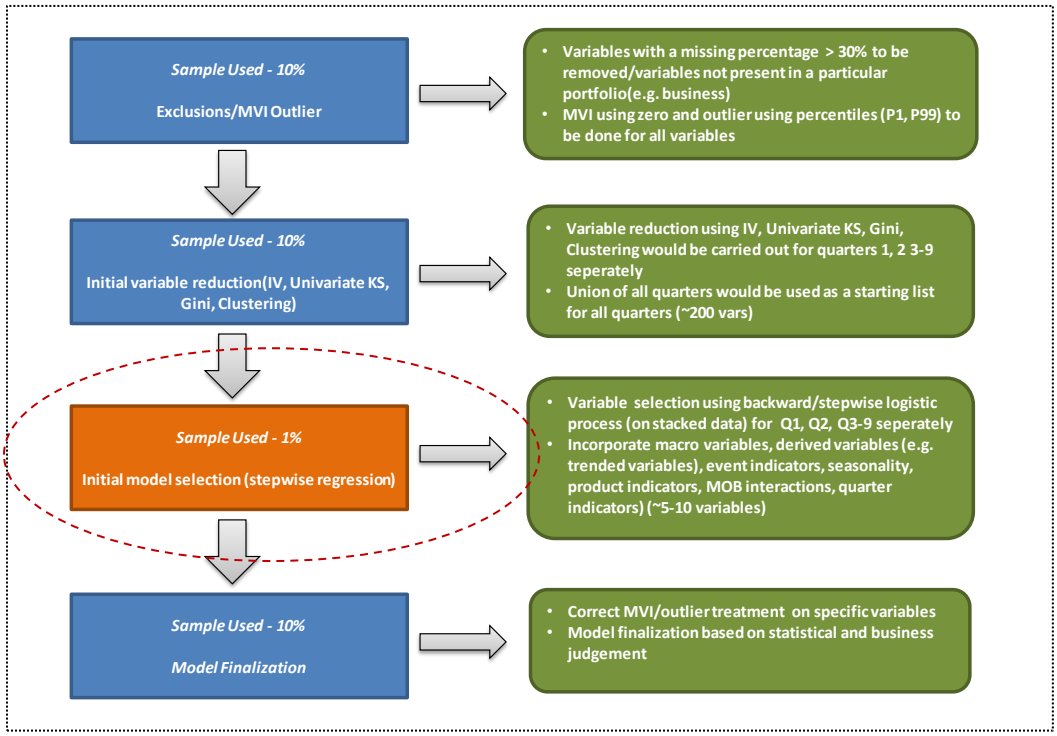
quarter performance in the two samples considered: (*Please note #defaults 1% sample has been multiplied by 10 to make the two samples comparable)*

**Figure 23 : Reconciliation of 1% sample and 10% sample for 9Q cumulative PD rate[12]**




The following illustration depicts the step by step model development process carried out for the PD model on the 1% and 10% samples:

**Figure 28: PD model development steps on 1% vs 10% sample**




[12] 9Q cumulative PD rate over next 9Q. Example : For Dec'06 the two points are PD rate over next 9Q (Q1'07 – Q1'09)

*Only one step in the model development process is carried out on a 1% sample to avoid complexities and save time and server space.*

*The variable selection process and model development is carried out for 9 quarters. The 9th quarter variables are used to recalibrate the 10th to 13th quarter model. No separate variable selection/reduction are carried out for the 10th to 13th quarter.*

**EADF Model**

EADF model has been developed on default population from the PD model. For CCAR 2017, EAD model was developed on 100% sample as opposed to 10% sample this time. However, it was analyzed that this does not lead to lower model accuracy.

The major reason for this is that Branded cards purchased FICO Version 8 for pre-alignment vintages for this sample for CCAR2018. In addition, analysis was done to see that this sample is representative of the population and this does not lead to lower model accuracy. Please see below the detailed analysis.

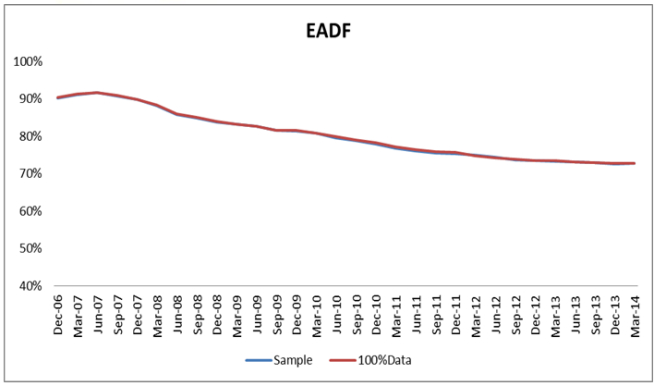
Table 5-3 : Comparison of 10% and 100% data

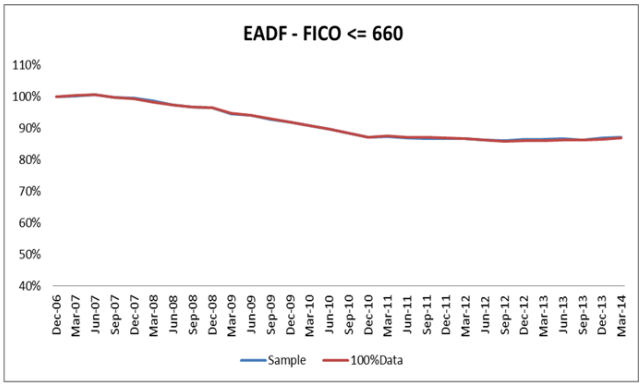
| Key Dimensions | 10% PD Data | 100% Data |
|---|---|---|
| Sample Size | Sufficiently Representative | Full Sample |
| Availability of FICO V8 | Yes | No |
| Model Accuracy | Comparable | |

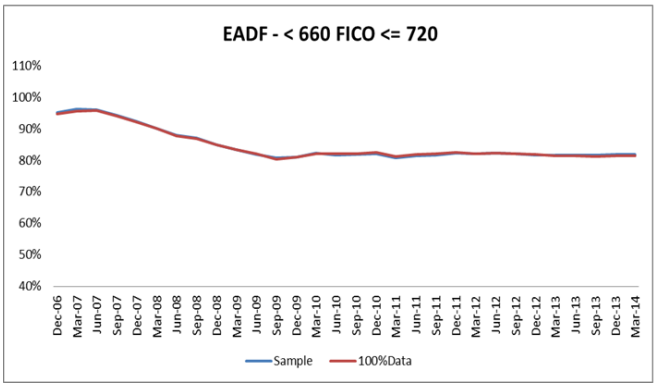
1. **Sample EADF data is representative of the 100% EADF Data**

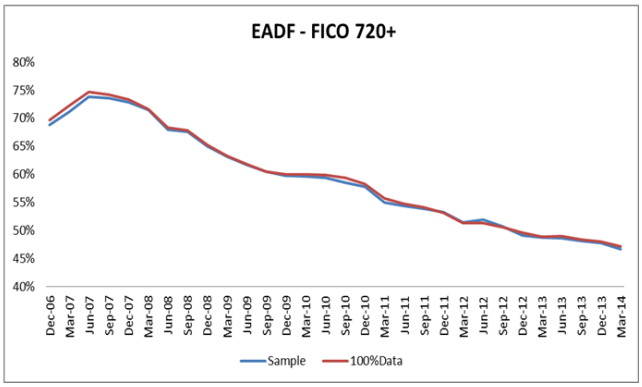
On comparing the EADF through the cycle, at an overall and FICO bin level, it is seen that the sample is representative of the complete default population

Figure 28: Comparison of 10% and 100% data across FICO Bins




## 2. Accuracy at 10% Sample is comparable to 100% data

A quick model for EADF was developed in order to do the sampling analysis. Back testing was done on 100% data and error rates were analyzed. At an overall level, results from both Models – developed on 10% vs. 100% look fairly similar.

**Table 5-4 : Comparison of errors rates on 10% and 100% data**

| | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Dec-07 | 0% | -4% | -6% | -7% | -6% | -3% | 0% | 4% | 0% | -2% |
| Mar-08 | -1% | -3% | -5% | -5% | -2% | 0% | 3% | 1% | -1% | -1% |
| Jun-08 | 0% | -3% | -4% | -1% | 1% | 3% | 1% | 0% | 2% | 0% |
| Sep-08 | -1% | -3% | 0% | 0% | 2% | 1% | 0% | 3% | 1% | 0% |
| Dec-08 | -1% | 0% | 1% | 2% | 1% | 1% | 4% | 3% | -2% | 1% |
| Mar-09 | 1% | 0% | 1% | 0% | 1% | 4% | 3% | -2% | 2% | 1% |
| Jun-09 | 0% | 0% | 0% | 1% | 4% | 4% | 0% | 3% | 4% | 1% |
| Sep-09 | -3% | 0% | 2% | 5% | 5% | 2% | 5% | 5% | 5% | 2% |
| Dec-09 | -2% | 1% | 4% | 5% | 2% | 5% | 5% | 5% | 5% | 3% |
| Mar-10 | -2% | 3% | 3% | 0% | 3% | 4% | 3% | 4% | 4% | 2% |
| Jun-10 | 0% | 2% | 1% | 2% | 3% | 3% | 3% | 4% | 1% | 2% |
| Sep-10 | -1% | 1% | 3% | 3% | 3% | 3% | 4% | 2% | 3% | 2% |
| Dec-10 | 0% | 3% | 3% | 3% | 3% | 4% | 2% | 3% | 6% | 3% |
| Mar-11 | -1% | 2% | 2% | 2% | 3% | 1% | 2% | 5% | 3% | 2% |
| Jun-11 | -1% | 1% | 2% | 3% | 2% | 2% | 5% | 3% | 2% | 2% |

| | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Dec-07 | 1% | -2% | -5% | -6% | -5% | -2% | 1% | 5% | 1% | -1% |
| Mar-08 | 0% | -2% | -4% | -4% | -1% | 1% | 4% | 2% | 0% | 0% |
| Jun-08 | 1% | -2% | -3% | 0% | 1% | 4% | 2% | 1% | 3% | 1% |
| Sep-08 | 0% | -1% | 1% | 2% | 3% | 2% | 1% | 3% | 2% | 1% |
| Dec-08 | 0% | 2% | 2% | 3% | 2% | 2% | 5% | 3% | -1% | 2% |
| Mar-09 | 2% | 2% | 2% | 1% | 2% | 5% | 4% | -1% | 3% | 2% |
| Jun-09 | 0% | 1% | 1% | 2% | 5% | 5% | 1% | 4% | 5% | 3% |
| Sep-09 | -2% | 1% | 3% | 6% | 6% | 2% | 5% | 6% | 6% | 3% |
| Dec-09 | -1% | 3% | 5% | 6% | 3% | 5% | 6% | 5% | 6% | 4% |
| Mar-10 | -1% | 4% | 4% | 2% | 4% | 5% | 4% | 4% | 5% | 3% |
| Jun-10 | 1% | 4% | 2% | 4% | 4% | 4% | 4% | 5% | 2% | 3% |
| Sep-10 | 0% | 3% | 4% | 4% | 4% | 4% | 5% | 2% | 4% | 3% |
| Dec-10 | 1% | 4% | 4% | 4% | 4% | 5% | 3% | 4% | 7% | 4% |
| Mar-11 | 0% | 3% | 3% | 3% | 4% | 2% | 3% | 6% | 4% | 3% |
| Jun-11 | 0% | 2% | 3% | 4% | 3% | 3% | 5% | 3% | 2% | 3% |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Sep-11 | -1% | 1% | 3% | 2% | 3% | 4% | 3% | 2% | 1% | 2% |
| Dec-11 | 0% | 2% | 2% | 3% | 4% | 3% | 2% | 1% | 0% | 2% |
| Mar-12 | -1% | 1% | 2% | 3% | 2% | 1% | 0% | -1% | -1% | 1% |
| Jun-12 | 0% | 0% | 2% | 2% | 1% | 1% | -1% | -1% | -3% | 0% |
| Sep-12 | 1% | 0% | 2% | 2% | 1% | -1% | -1% | -2% | -2% | 0% |
| Dec-12 | 2% | 1% | 2% | 2% | 0% | 0% | -2% | -1% | -1% | 1% |
| Mar-13 | 2% | 1% | 1% | 0% | 0% | -2% | -2% | -1% | 0% | 0% |
| Jun-13 | 1% | 0% | 0% | 0% | -1% | -1% | -1% | 0% | | 0% |
| Sep-13 | 1% | 0% | 1% | 0% | 0% | 0% | 1% | | | 0% |
| Dec-13 | 3% | 0% | 1% | 1% | 1% | 1% | | | | 1% |
| Mar-14 | 2% | 0% | 0% | 0% | 1% | | | | | 1% |
| Jun-14 | 1% | 1% | 1% | 1% | | | | | | 1% |
| Sep-14 | 1% | 1% | 2% | | | | | | | 1% |
| Dec-14 | 3% | 1% | | | | | | | | 2% |
| Mar-15 | 2% | | | | | | | | | 2% |

**EADF Model developed on 100%, Validated on 100%**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Sep-11 | 0% | 2% | 4% | 3% | 4% | 5% | 3% | 2% | 2% | 3% |
| Dec-11 | 1% | 3% | 3% | 4% | 5% | 4% | 3% | 2% | 0% | 3% |
| Mar-12 | 0% | 2% | 3% | 4% | 3% | 2% | 1% | -1% | 0% | 2% |
| Jun-12 | 1% | 2% | 4% | 3% | 2% | 1% | -1% | 0% | -2% | 1% |
| Sep-12 | 1% | 2% | 3% | 3% | 2% | 0% | 0% | -2% | -1% | 1% |
| Dec-12 | 3% | 2% | 3% | 3% | 1% | 1% | -1% | -1% | 0% | 2% |
| Mar-13 | 3% | 2% | 2% | 1% | 1% | -1% | -1% | -1% | 1% | 1% |
| Jun-13 | 2% | 1% | 1% | 1% | 0% | 0% | 0% | 1% | | 1% |
| Sep-13 | 2% | 1% | 2% | 1% | 1% | 0% | 1% | | | 1% |
| Dec-13 | 4% | 2% | 2% | 1% | 1% | 2% | | | | 2% |
| Mar-14 | 3% | 2% | 1% | 1% | 2% | | | | | 2% |
| Jun-14 | 2% | 2% | 2% | 2% | | | | | | 2% |
| Sep-14 | 2% | 2% | 3% | | | | | | | 2% |
| Dec-14 | 4% | 3% | | | | | | | | 3% |
| Mar-15 | 4% | | | | | | | | | 4% |

**EADF Model developed on 10%, Validated on 100%**

## Recovery model

The complete consumer portfolio recovery payments from January 2007 through June 2016 are used in the purpose of modeling Recovery rate. Therefore no sampling has been conducted. Recovery payments from July 2016 through June 2017 have been used for holdout validation of the recovery model. Payment information is taken for accounts charged-off after December 2004.

Recovery payment performance is tracked for 32 quarters post charge-off (including the charge-off quarter) for each charged-off account. The close balance amount at the time of charge-off for an account is the GCL for the account. A shorter version of recovery model data structure is given below.

**Table 5-5 : Data structure for Recovery model**

| | Performance Quarter | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **CO Qtr** | **Qtr1** | **Qtr2** | **Qtr3** | **Qtr4** | **Qtr5** | **Qtr6** | **…** | **Qtr32** |
| **Mar-05** | | | | | | | … | Dec-12 |
| **Jun-05** | | | | | | | … | Mar-13 |
| **Sep-05** | | | | | | | … | Jun-13 |
| **Dec-05** | | | | | | Mar-07 | … | Sep-13 |
| **Mar-06** | | | | | Mar-07 | Jun-07 | … | Dec-13 |
| **Jun-06** | | | | Mar-07 | Jun-07 | Sep-07 | … | Mar-14 |
| **Sep-06** | | | Mar-07 | Jun-07 | Sep-07 | Dec-07 | … | Jun-14 |
| **Dec-06** | | Mar-07 | Jun-07 | Sep-07 | Dec-07 | Mar-08 | … | Sep-14 |
| **Mar-07** | Mar-07 | Jun-07 | Sep-07 | Dec-07 | Mar-08 | Jun-08 | … | Dec-14 |
| **Jun-07** | Jun-07 | Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | … | Mar-15 |
| **Sep-07** | Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | … | Jun-15 |
| **Dec-07** | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | … | Sep-15 |
| **Mar-08** | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | … | Dec-15 |
| **Jun-08** | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | … | Mar-16 |
| **Sep-08** | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | … | Jun-16 |
| **Dec-08** | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | … | Sep-16 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Mar-09** | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | … | Dec-16 |
| **Jun-09** | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | … | Mar-17 |
| **Sep-09** | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | … | Jun-17 |
| **Dec-09** | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | … | |
| **Mar-14** | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | … | |
| **Jun-14** | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | … | |
| **Sep-14** | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | … | |
| **Dec-14** | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | … | |
| **Mar-15** | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | | |
| **Jun-15** | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | | |
| **Sep-15** | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | | |
| **Dec-15** | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | | |
| **Mar-16** | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | |
| **Jun-16** | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | |
| **Sep-16** | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | |
| **Dec-16** | Dec-16 | Mar-17 | Jun-17 | | | | | |
| **Mar-17** | Mar-17 | Jun-17 | | | | | | |
| **Jun-17** | Jun-17 | | | | | | | |
| | Development sample | | | | | | | |
| | Hold out Sample | | | | | | | |

Detailed data structure of the recovery model is given in below attachment.

Recovery_Model_Data_Structure.xlsx

### 5.1.5 Data cleaning and preparation

- What were the data cleaning steps?

The data is treated and analyzed for any special values, outliers or missing values before being used for modeling purposes. Prior to modelling the behavioral variables, the variables were treated so as to impute the missing values, reduce the redundant variables and remove the outliers from the data. Flooring and capping of the selected variables were also performed.

During the process of initial variable selection, missing values were treated with mean or zero depending on business judgment and variables with outliers were floored at 1$^{st}$ percentile and capped at 99$^{th}$ percentile.

However, further detailed analysis was carried out on the final list of variables and all treatment was based on multiple factors such as plot of log odds, business judgment to ensure appropriate MVI/Outlier treatment of final variables.

For example in the PD model the variables have been treated in the following manner:

Table 5-6 : Data treatment for PD Model

| Variable | Minimum | p_99 | p_99_9 | Maximum | Miss % | Floor | Cap | Missing value imputation |
|---|---|---|---|---|---|---|---|---|
| BILL_ACTV_IND | 0 | 1 | 1 | 1 | 0.0% | 0 | 1 | 1 |
| bucket_cd1 | 0 | 2 | 6 | 6 | 0.0% | 0 | 6 | 0 |
| CRM_E1_NIBS_IBS_DELQ_CD | -1 | 2 | 6 | 6 | 18.7% | -1 | 6 | 0 |
| DPD60_NCC_BAL_AMT | 0 | 705 | 199999 | 999999 | 3.5% | 0 | 700 | 0 |
| DPD90_NCC_NDIS_AMT | 0 | 24739 | 525730 | 999999 | 3.5% | 0 | 25000 | 0 |
| FICO_8 | 0 | 874 | 99999 | 99999 | 1.9% | 250 | 900 | 0 |
| INQ_6MON_CT | 0 | 5 | 9 | 86 | 3.5% | 0 | 10 | 0 |
| MON_ON_BOOK_CT | 1 | 379 | 489 | 1286 | 0.0% | 1 | 500 | 0 |
| PURCH_APR | 0 | 30.08 | 32.31 | 99.99 | 0.0% | 0 | 33 | 0 |
| SALES_ACTV_IND | 0 | 1 | 1 | 1 | 0.0% | 0 | 1 | 1 |
| TOTL_RVLV_BAL_AMT | 0 | 82581 | 99999 | 99999 | 3.5% | 0 | 99999 | 0 |
| UTIL_RATE | -999.99 | 104.68 | 138.56 | 999.99 | 0.0% | 0 | 300 | 0 |

The decision on the final variable treatment has been done by analyzing variable trends and through multiple internal discussions to ensure the treatment makes business sense.

***Flooring***

Most of the variables have been floored at their minimum value, since the minimum value and the first percentile value are relatively similar. However utilization has been floored at zero which is higher than its minimum value. This is done to remain conservative.

***Capping***

Variables have been capped in the following manner:

a. If the 99$^{th}$ percentile value is not significantly different from the 99.9$^{th}$ percentile value then the variable has been capped at the 99.9$^{th}$ percentile (e.g. Annual percentage rate)
b. If the 99$^{th}$ percentile value is significantly lower than the 99.9$^{th}$ percentile, then the variable has been capped at the 99$^{th}$ percentile (e.g. bureau balance)

***Missing value imputation***

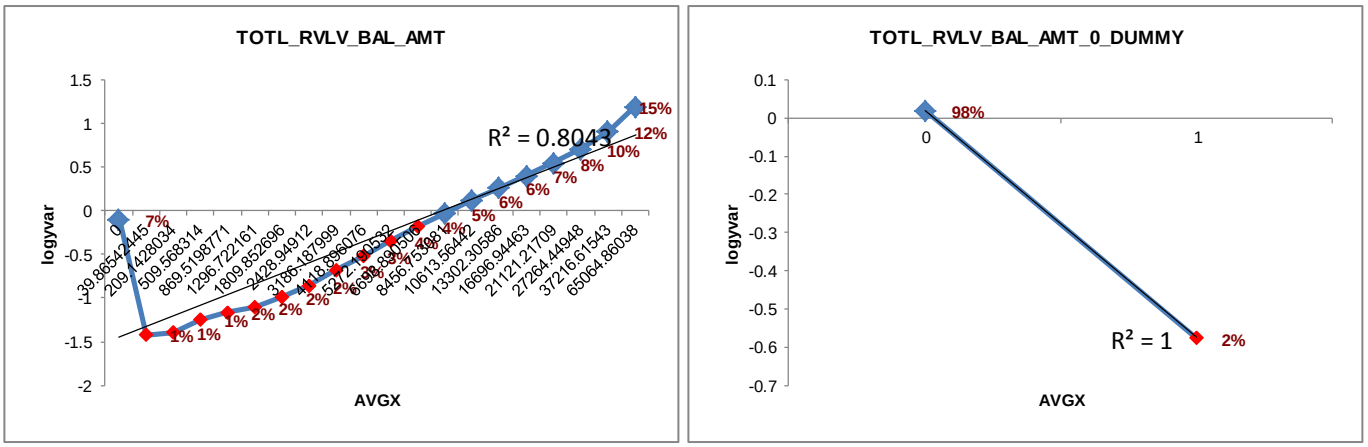
In general it has been observed from historical trends that internal variables do not have missing values and bureau variables have a constant bureau no hit or missing percentage. These bureaus no

hit accounts are constant across all variables. Missing values have been treated by imputing them by zero and an additional missing dummy indicator has been introduced

***Variable Trends***

A dummy for the zero value of the variable ***revolving balance amount*** has been used since the bivariate plot for this variable depicts break in monotonicity at the zero value since the bad rate for the zero value is different

**Figure 29 : Bivariate plot for revolving balance amount variable**




A similar treatment has been done for all variables in the EADF model. However the missing value has been imputed with the mean EADF.

Attached are the documents for more details on the Variable treatment for all models:

5.1.5_MVI_Outlier_PD.xlsx

5.1.5_MVI_Outlier_EAD.xlsx

The macro-economic variables did not have any missing values or outliers, so this treatment does not apply to macroeconomic variables.

- Were any data proxies used?

No data proxies have been used.

- Was any data excluded for data quality or business reasons? What was the rationale for the exclusion?

The PD model is developed on all the accounts, which can give potential future losses. ***The base population includes all the open accounts and closed accounts with positive balance.*** EAD and Recovery models are developed on only the defaulted population.

The Following set of Exclusions were made (for observation as well as performance) in champion models (PD/EAD/Recovery):

**Table 5-7 : Exclusions**

| Exclusions applied | Observation | *Performance |
|---|---|---|
| Retail services Portfolio IDs | ● | |
| Sold PID's | ● | |
| Accounts default at observation | ● | |
| Accounts in settlement at observation | ● | |
| Accounts with EAD <=0 | ● | ● |
| Costco Accounts | ● | |

*Please note that performance exclusions are not valid for forecasting datasets

The Branded Cards team purchased FICO version 8 this year for vintages prior to March 2011.To obtain FICO_v8 for accounts for these vintages, Branded cards had sent the data for these vintages to the bureau (Experian). This data consisted of 10% Consumer and Business cohorts as well as the 100% Dec'07 Cohort (to be used for back testing). This data was sent to bureau with exclusions already applied to make the purchase of FICO cost efficient to the business. The exclusions for these datasets were accounted for before it was sent to the bureau.

For vintages post March 2012 data was created using 10pct samples of Consumer and Business cohorts and its exclusions are accounted for separately.

The exclusions applied to the data are as follows:

1. Portfolio ID's not a part of the Branded Cards portfolio any longer.
2. Costco Accounts
3. Accounts already in charge off or settlement state at observation
4. Accounts with zero or negative EAD at performance

The waterfall table for the exclusions made in the PD dataset is given below:

Table 5-8 : Exclusions

| Exclusion Waterfall | Exclusion | Percentage |
|---|---|---|
| **Data sent to Bureau** | Accounts which are closed with a balance <=0 | 39.50% |
| | Portfolio ID's not a part of Branded Cards | 4.50% |
| | Costco Accounts | 0.00% |
| **Data not sent to Bureau** | Accounts which are closed with a balance <=0 | 22.80% |
| | Retail Services portfolio ID's | 0.70% |
| | Costco Accounts | 3.10% |
| **Exclusions made on combined data** | Costco business accounts | 0.30% |
| | Settlement/Charge off at Snapshot quarter | 0.00% |
| | Negative or zero EAD at default | 0.01% |

The EADF model is developed on the PD data using only charge off accounts. The final data creation of the EADF model is done as follows:

Table 5-9 : Exclusions

| Waterfall Table | Total Records | Removed |
|---|---|---|
| **PD Data** | 112,328,857 | |
| **Extraction of 13 quarter defaults** | 7,003,381 | 105,325,476 |
| **Credit Limit < = 0 exclusion** | 7,003,366 | 14 |
| **Model Development Sample** | 6,479,589 | - |

- What data discrepancies were observed and what actions were taken to mitigate them? What was the impact of data discrepancies, cleaning, data exclusions and remediation?

No major data discrepancies were observed in the data.

### 5.1.6 Data transformation

The dependent variable transformations are as given below:

For the PD Account level model, the dependent variable is binary i.e. either an account has defaulted or is current. Logistic technique is used to estimate PD at an account level. This is an industry wide practice and business is comfortable in using the binary dependent variable.

For EADF Model, EAD and EADF were both explored as the dependent variable. In CCAR 2017 Submission Branded Cards modeled EAD however as a feedback from IRMO in CCAR 2018 the Branded cards has modeled EADF as it is stationary and relatively more intuitive.

For Recovery Model, two recovery models with different dependent variable were developed. The dependent variable for the Vintage Recovery Model (Champion Model 2018) has Vintage recovery rate as the dependent variable. For the portfolio recovery model, the dependent variable is the portfolio recovery rate defined as the recovery payment in a quarter divided by the GCL of the previous quarter.

Also independent variables or covariates and macroeconomic variables have been transformed to enhance the predictive power of these attributes.

**Behavioral Variables:**

Some additional transformations are created to leverage on predictive power of significant variables .For some variables; a transformation of the variable shows a greater correlation with the dependent variable, is able to capture historical information as opposed to point in time information and makes more business sense. For example – Payment behavior in last 6 months is a stronger predictor of an account charge off than snapshot payment information. The excel sheet attached lists the created and the transformed variables used for PD modeling.

Derived Variables.xlsx

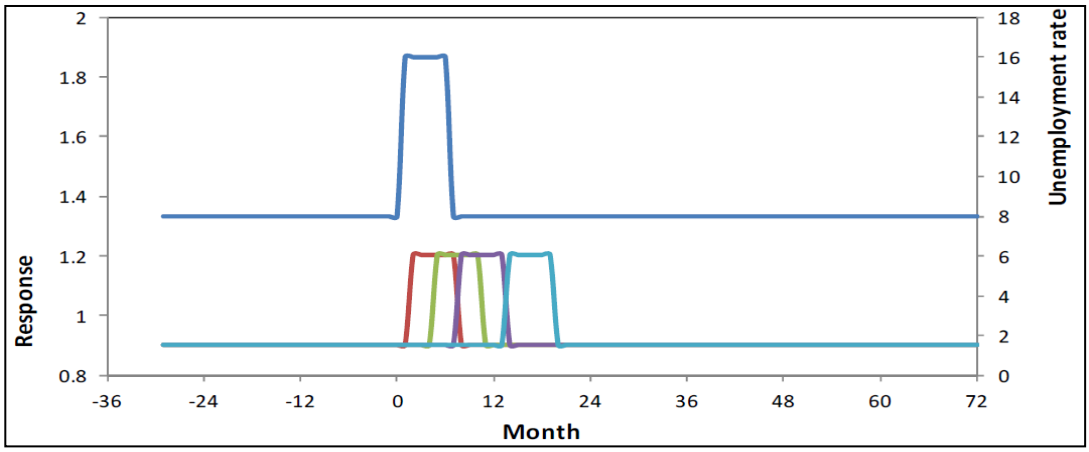
**Macro-Economic Variables:**

In time-series data, it is often necessary to transform one or more variables in order to find meaningful relationships over time. Transformations can also be used to normalize data ranges and facilitate more direct comparison between time-series. Transformations applied to macro-economic variables determine the shape of the response of their dependent variables.

**Windows and Lags**

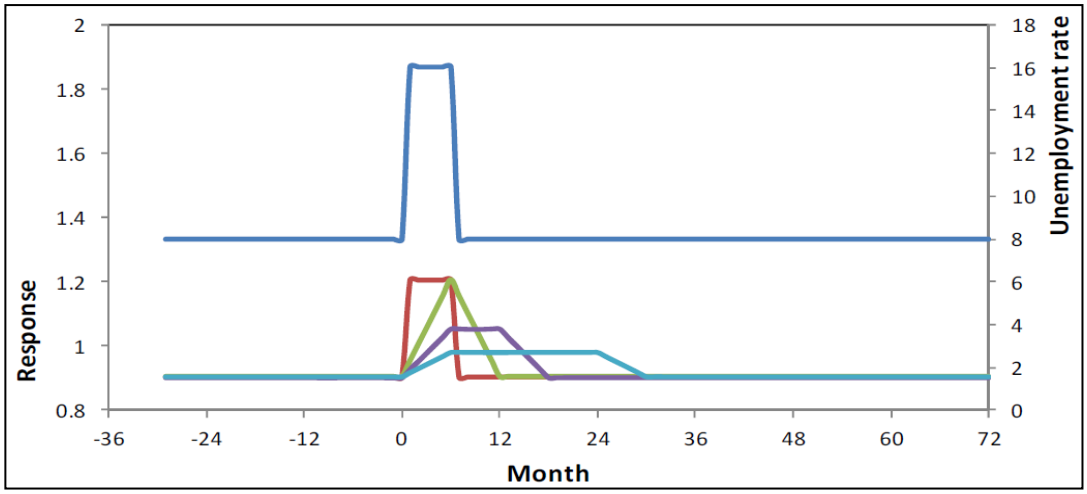
The timing of the response can be further adjusted by judicious selection of windows and lags.

Figure 24: Effect of Lag on a Square pulse




**Lags:** If the lag is greater than zero, the response is delayed by the magnitude of lag. For example, when the lag is zero, the response is immediate. However, if the lag is set to 12 months, the response doesn't begin until 12 months after the initial increase in the independent macro-economic variable in consideration. Changing the lag affects the delay of the response, but does not affect the shape of the response. Above figure explains the effect of increasing the lag (window here is held constant).

Figure 25 Effect of changing Window on a square pulse




*Windows affect the spread in the delay of the response of the Log-Average transform of the dependent variable to a square pulse in the independent variable (unemployment rate), but not the starting point of the response*

**Window**: The window defines the number of months over which an average or difference has been considered while transforming the variable. Increasing the window spreads out the response of the dependent variable there by flattening the curve. 4.1-2 demonstrates the effect of changing the window. The response is immediate as lag is kept equal to zero. The duration of the response,

however, is changing based on the value of the window. For example, the green curve with a window of 6 months shows a response spread over 6 months after the end of the pulse (event). The magnitude of the response is not important and can be adjusted by changing values of the slope and intercept.
The lags are evaluated for last four quarters and windows for a period of four quarters before lag.

**Transformations**

Four different transformations (LogAverage, LogDiff, Raw Average, Raw Difference) are evaluated for Unemployment Rate and three different transformations (Log Difference, Raw Difference, Ratio) are evaluated for House Price Index. In 2016 CCAR Champion and Challenger models only Log Transformations of macro-economic variables were evaluated. For the CCAR 2017 and 2018 submission, following the guidelines provided by Independent Risk Modelling Oversight (IRMO) team, raw transformations of macro-economic variables were also evaluated. Further, ratio transformation of HPI has been considered for raw transformation in order to ensure that quantity is comparable across different regions. Example – Raw difference of 20 points may not mean same for NY and CA, however when standardized by ratio, these quantities can be comparable. Various lags and windows (as explained below) of the transformations are explored. Following are the details for the same.

For variable X at time t, with window W (W > 0) and lag L (L >= 0):

If ***W = 0*** then

Equation 3

$$\{LogAverage(X)\}_{t,0,L} \equiv \{LogDiff(X)\}_{t,0,L} \equiv Ln[X_{t-L}]$$

$$\{RawAverage(X)\}_{t,0,L} \equiv \{RawDiff(X)\}_{t,0,L} \equiv [X_{t-L}]$$

Else if ***W > 0*** then

$$\{LogAverage(X)\}_{t,W,L} \equiv Ln\left[\prod_{s=0}^{W-1} X_{t-L-s}\right]\Big/W$$

$$\{RawAverage(X)\}_{t,W,L} \equiv \left[\sum_{s=0}^{W-1} X_{t-L-s}\right]\Big/W$$

$$\{LogDiff(X)\}_{t,W,L} \equiv Ln\left[\frac{X_{t-L}}{X_{t-L-W}}\right]$$

$$\{RawDiff(X)\}_{t,W,L} \equiv X_{t-L} - X_{t-L-W}$$

$$\{Raw\ ratio^{13}/\ (X)\}_{t,W,L} \equiv \frac{X_{t-L}}{X_{t-L-W}} - 1$$

The LogAverage and Raw Average transformation of HPI are not evaluated as we have seen historically that these series are not stationary.

In all Branded Cards PD models, the transformation for Unemployment Rate of Raw difference and ratio of HPI came out to be predictive.

- What is the rationale for performance, segmentation and account data transformation?

The rationale for using macro-economic data transformation is that in time-series data, various transformations have differential impact on the output: for example, average unemployment rates typically impact with a lag of 3-6 months while difference of unemployment rate takes a longer time to have a material impact.

The rationale for transforming behavioral attributes is to capture historical averages and trend information as opposed to capturing snapshot information. These historical averages and trends enhance the predictive power of attributes.

- Does the approach to macroeconomic variable selection or transformation significantly differ from approaches in similar countries in the region?

No, the approach does not differ across NA cards. The approach taken by the Branded Cards team for transformation and selection of macroeconomic variables is the same approach used by the Retail Services team.

[13] Ratio refers to rate of change of the variable throughout the document

- Are data transformations based on business intuition?

  Yes, lags of macroeconomic component are restricted to 12 months and this is in line with business judgment because it typically takes up to 7 months for an account to charge off from current bucket and it might take some time for the impact to be percolated to the population in general. Windows are restricted to 12 because anything beyond 12 months is a longer duration to be considered.

- Do data transformations satisfy statistical requirements?

  Yes, the transformations pass all the stationarity requirements. The below table gives final transformations utilized in the PD, EAD and Recovery model and their stationarity test results:

**Table 5-10 Stationarity Result of Macroeconomic Variables Used in Model**

| | Variable | | | | |
|---|---|---|---|---|---|
| LABEL | var | lag(in months) | window(in months) | Transformation | Pass/Fail |
| UR_L3_w4_t3 | UR | 9 | 12 | Difference | PASS |
| UR_l2_w4_t3 | UR | 6 | 12 | Difference | PASS |
| UR_l2_w2_t3 | UR | 6 | 6 | Difference | PASS |
| UR_l0_w4_t2 | UR | 0 | 12 | Average | PASS |
| UR_l0_w3_t3 | UR | 0 | 9 | Difference | PASS |
| UR_l0_w2_t2 | UR | 0 | 6 | Average | PASS |
| HPI_l3_w4_t4 | HPI | 9 | 12 | Ratio | PASS |
| HPI_l3_w4_t4 | HPI | 9 | 12 | Ratio | PASS |
| HPI_l1_w4_t4 | HPI | 3 | 12 | Ratio | PASS |
| HPI_l1_w3_t4 | HPI | 3 | 9 | Ratio | PASS |
| HPI_l1_w2_t4 | HPI | 3 | 6 | Ratio | PASS |
| HPI_l0_w4_t4 | HPI | 0 | 12 | Ratio | PASS |

All other variables considered have also been tested for stationarity. The details on stationarity can be found in Section 8.1.4.

### 5.1.7 Data sufficiency

- What data quality checks have been done?

  Data quality checks address the following areas:

  1. **Business data quality checks** - quick overview of data based on charts to visually detect any outliers or anomalies
  2. **Technical data quality checks** - detailed data analysis including:
  - Primary keys and duplicates check

- Statistical checks -- univariate distribution (mean, median, minimum, maximum, number of observations, and where applicable frequencies) or segmentation, dependent, final independent variables
- Missing or zero values check, date/time conformity, value format conformity
- Account level data checks (checks for default values, duplicates, record mapping -- joins, completeness of fields, data and business logic integrity within and across records and frequency checks)

Please refer to attached templates for the checks conducted on the Account level models.

| | Account Level PD Model | Account Level EADF Model |
|---|---|---|
| **Business** | 5.1.7_Biz_Data_Quality_PD.xlsx | 5.1.7_Biz_Data_Quality_EAD.xlsx |
| **Technical** | 5.1.7_Tech_Data_Qualtity_PD.xlsx | 5.1.7_Tech_Data_Quality_EAD.xlsx |

What statistical tests were run to verify whether the available data quality is sufficient to develop the model? What were the results of those tests?

The Branded cards portfolio data is stable and large portfolio so the data is sufficient for model development. In terms of data quality tests are attached in the immediately preceding section.

- Were there any significant events that have affected data quality in the past?

The major event that occurred was the changing of bankruptcy laws that went into effect from December 2005. Inclusion of data for 2005 might cause a major outlier in the modeling dataset. Therefore the business starts using modeling data from Dec'06 onwards. Further, Business has considered exogenous events or one-time events as described in section 5.2 below.

- Is the data coverage sufficient?

The model sponsor strongly believes that the data used for the development of these models is sufficient for the purpose. The models were built on through the cycle portfolio

data starting Q4 2006 till Q2 2016 containing more than 500M data points. The period in consideration covers both stressed and benign economic environments and is therefore adequate and sufficient to build robust models which would hold true for any economic cycle. The data used for modelling spans from December 2006 to June 2016. It includes data from historical stress period i.e. 2008-2009 (Period of Recession) and data from recent period as well. Both the time periods are equally necessary for predicting account level PD, EAD as model should preform accurately during stress as well as base conditions.

### 5.1.8 Vendor-model specific inputs

The model is not a Vendor model.

### 5.1.9 Code and Data Guidance

[Model Sponsors must adhere to the coding practices provided in the Code and Data Guidance. The information regarding the flow of execution of codes and datasets must be provided in the attached template. All the materials must be shared as per the data sharing process. All the embedded templates should be mandatorily provided.]

- Please explain if model is SAS based or excel based. Please provide details on what software was used in development?

  The models are based on SAS.

- Attach the final version of development code

| PD GCL | EAD | Recovery |
|---|---|---|
| 5.1.9_Model_Code_Info_PD_GCL.xlsx | 5.1.9_Model_Code_Info_EAD.xlsx | 5.1.9_Model_Code_Info_recovery.xlsx |
| Model_Dev_PD.sas | Model_Dev_EAD.sas | Model_Dev_Recovery.sas |

- Provide the location and access to the data used for model development.

  The data for PD and EAD and Recovery Models can be found at the path specified below:

***Model Development Data***

*PD:*

/ccr/ccar_mvg/1_businessdata/US_Unsecured/ccar2018/Branded_Cards_NCL_Model/Account_level_NCL_model/01.PD/01.Data_Preparation/Data/pct10_model_data.sas7bdat

*EAD:*

/ccr/ccar_mvg/1_businessdata/US_Unsecured/ccar2018/Branded_Cards_NCL_Model/Account_level_NCL_model/02.EAD/EADF_Datasets/Modelling_Datasets/ead_final_dev_data.sas7bdat

*Recovery:*

/ccr/ccar_mvg/1_businessdata/US_Unsecured/ccar2018/Branded_Cards_NCL_Model/Recovery/OUTPUT/model_final.sas7bdat

## 5.2 Data Assumptions and Limitations

- What are the key performance, segmentation and account data assumptions?

The data assumptions considered in the model are as follows:

- The forecast for the volume of future originations at portfolio level is provided by the FP&A team. Business assumptions are made regarding the new booking (originations) that will take place. These originations will have an impact on the newest part of the portfolio for a multi-year forecast. The attached mail is received from the FP&A team with the volume of future originations forecast

- The percentage of voluntary closures is assumed to remain stable over time.

- What are the key macroeconomic data assumptions?

The key macroeconomic data assumptions are as follows:

- Impact of Unemployment Rate and HPI will happen with a lag of 0 to 12 months and with window (duration of impact) between 3 and 12 months with increments of 3 months for both lags and windows
- The forecast regarding future economic scenarios for fall submission are received from Federal Reserve for each submission at a national level. State level information is then provided by GCRM team. Business assumes that the granular level information generated by GCRM team is accurate and fit for forecasting purposes.

- The assumptions (forecasts) regarding future economic scenarios Mid-cycle submission are received from internal GCRM team. Business assumes that the granular level information generated by GCRM team is accurate and fit for forecasting purposes.

- What are the key limitations of the data?

  The following were identified as limitations of the data:

  - The data only covers one recessionary phase. Data prior to 2006 is not used by the model. Therefore the modeled relationship between target variables and macro-economic variables based on the single cycle is assumed to be representative of economic cycles, in general. This is a relatively common limitation of industry stress-testing models.
  - Post the last recession, the forbearance polices have become relatively strict in nature, which leads to higher roll-forward rates post the recession. There is no stress time period with the current forbearance policy. Hence there have been regime dummies used in the PD model to account for the impact of the changing forbearance policy over time. The details of this are explained later.

- Were any exogenous events/shocks evident in the historical data? How were they accounted for?

  Exogenous Events are incorporated into the estimation process so that they do not tamper with the natural seasonal patterns and cycles. Discussion with management determines events which are clearly singular and idiosyncratic, hence appropriate for inclusion as an event. Events in account level PD model have been included in the form of period dummies. Following table illustrates events that have been included in the PD model:

**Table 5-11 : Events in PD model**

| Description | Date |
|---|---|
| Change-in-terms & APR increase | Aug'09 |
| Change in policy for forbearance charge-off from 180 DPD to 120 DPD | Sep-Oct'10 |

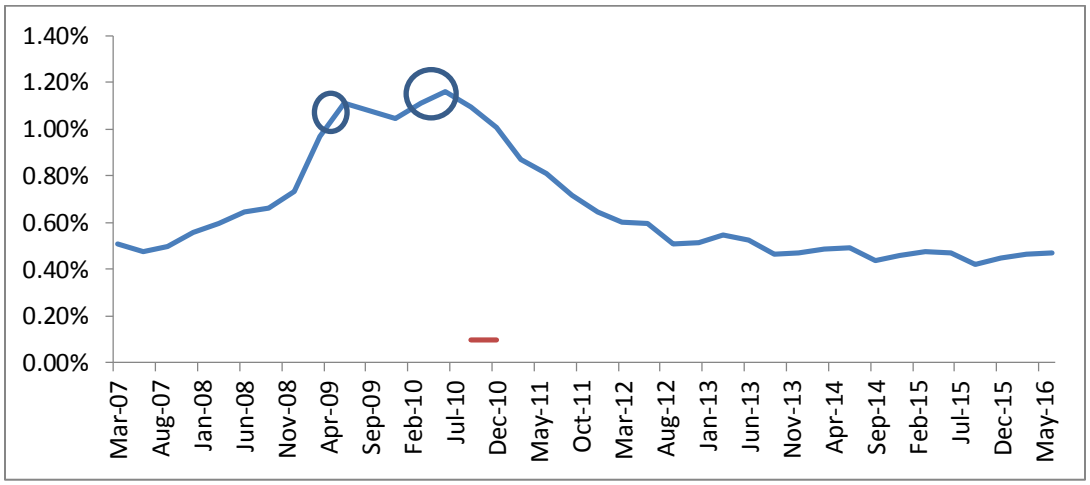
Events mentioned have been included in the exogenous component of LookAhead models in previous CCAR submissions. These events were found to be significant in the Account level models for previous year submission and this year 2018 CCAR submission and removing those causes these particular vintages to have high error rates.

Although these events have been added during model development, these dummies do not impact forecast.

Attached is the email exchange with business for confirmation of these events:

RE Q3-2012 - Risk Events for CCAR.msg

Figure 26 : Events in Branded Cards




There are no events used in the EADF model.

- What factors, if any, mitigate the limitations of the data?

The following steps have been taken to ensure the limitations of the data have been satisfactorily mitigated

- The models cover a financial cycle of 10 years and covers great recession vintage from Q1'08 – Q1'10 and hence ensures sufficient data coverage for model development. Further, given the stress observed in branded cards during last great recession coupled with account level information for ~100MM observations with performance over 9 quarters ensures sufficient data coverage to build any stress model. The same is corroborated by sensitivity seen in forecast for any macroeconomic scenario.

- For CCAR 2018, the business has also applied a "***regime shift***" dummy to account for the policy changes in the forbearance programs. The business has observed significant changes in roll-forward rates post Q1 2012. This is attributed to several policy changes and change in portfolio composition. Key changes in policies include but are not limited to significant tightening of the eligibility criteria of the short-term forbearance program (UPP),

implementation of more conservative forbearance policies like direct charge-off post any breach in forbearance terms etc. For CCAR 2018, the business has applied a ***"regime shift"*** dummy to account for the above observed phenomenon. The directional impact of these stricter forbearance policies would be to increase losses in the initial quarters and reduce losses in the later quarters of the forecast horizon.

In order to effectively capture the regime change in the model, it is important to differentiate the interaction of the regime dummy for current and delinquent population. As discussed above, the key policy change has been forbearance related and the business understands that delinquent accounts will be more affected than the current. It is also critical to examine the effect by quarter since snapshot. For the initial quarters from snapshot (Quarter 1 and 2), it is important to consider regime dummies with all accounts. Delinquent accounts would be definitely impacted and some of the current accounts might also be under forbearance program and can potentially witness accelerated charge-off. Accordingly interaction with portfolio roll-rates would be an important consideration. For slightly distant quarters from snapshot (Quarter 3, 4 and 5), it makes sense to consider regime dummy interactions with only delinquent accounts as all current accounts which are susceptible to undergo accelerated charge-off, would have already been charged-off in the first couple of quarters and would have not survived till quarter three. For very distant quarter from snapshot (Quarter 9), it is unlikely that delinquent accounts at snapshot will still be under forbearance programs (as they might have cured or charged-off), while the current accounts remaining can be considered as good accounts and part of a relatively cleaner portfolio that is expected to have an adverse contribution to loss. Accordingly, it is prudent to consider regime dummy interaction with current population

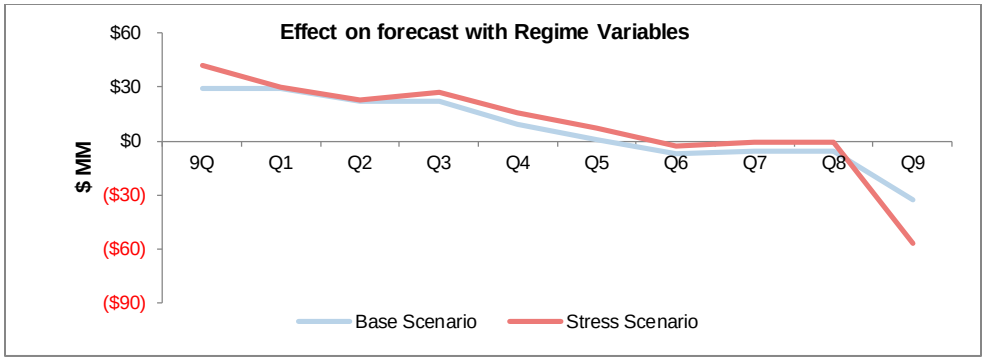
There is no significant change in the sensitivity of the models due to the application of the Regime Dummy. This can be observed from the Table 5-12 and Figure 31 below:

**Table 5-12 : Impact of Regime dummies on forecast**

| | With Regime Dummies | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | 9Q |
| STRESS (Dec'07) | $642 | $590 | $662 | $663 | $616 | $621 | $673 | $671 | $580 | **$5,716** |
| RECENT (Mar'14) | $643 | $601 | $808 | $924 | $1,010 | $1,158 | $1,316 | $1,335 | $1,123 | **$8,920** |
| Sensitivity | 0% | 2% | 22% | 39% | 64% | 87% | 96% | 99% | 94% | **56%** |

| | Without Regime Dummies | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | 9Q |
| STRESS (Dec'07) | $613 | $568 | $640 | $654 | $615 | $628 | $679 | $677 | $613 | **$5,687** |
| RECENT (Mar'14) | $613 | $578 | $781 | $908 | $1,003 | $1,161 | $1,317 | $1,336 | $1,180 | **$8,878** |
| Sensitivity | 0% | 2% | 22% | 39% | 63% | 85% | 94% | 97% | 92% | **56%** |

Figure 30: Impact of Regime dummies on forecast




The back-tests improve given the more accurate prediction across quarters with the impact of more conservative forbearance policies taken into consideration.

Table 5-13 : Impact of Regime dummies on back-test

| | Recent 9 Q back test of GCL (Q2'14- Q2'16) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | 9Q |
| Without Regime | -3% | -10% | 2% | 5% | 12% | 11% | 4% | 12% | 18% | **5%** |
| With Regime | 2% | -6% | 6% | 8% | 14% | 10% | 3% | 11% | 12% | **6%** |

# 6 Model Methodology and Specification

## 6.1 Model Methodology Overview

### 6.1.1 Model Objective

Under the expected loss approach, losses are estimated as a function of three components: Probability of Default (PD); Loss Given Default (LGD) / Recovery Rate Models (RRM); and Exposure at Default (EAD). This approach has been selected as the preferred approach for the estimation of losses within Citi's Branded Cards portfolio and is aligned with regulatory guidance and supported by academic research[14]. Expected gross losses (GCL) is computed using the PD and EADF models in conjunction with a line amount assumption, as shown in the formula below:

**Equation 4**

$$\$GCL = PD * \$Line\ Amount * EADF$$

Net Credit Loss (NCL) is further calculated by netting off the Gross Credit Loss for recoveries and unearned fees and finance charges.

**Equation 5**

***NCL = GCL – Net Recoveries – Adjustment & Refunds***

The Account Level PD and EAD Models, along with the recovery model will be used to forecast thirteen quarters' losses for different scenarios as a part of the Y14-A CCAR/DFAST submission. The model will be used by Branded Cards for the Comprehensive Capital Analysis and Review (CCAR) and Dodd-Frank Act Stress Test (DFAST) stress testing. The process is designed to forecast losses under various macro-economic conditions. The NCL forecast is produced at a quarterly level over a forecast horizon of 13 quarters, covering the mandated 9-quarter CCAR/DFAST forecast horizon and an additional year that is used in downstream CCAR/DFAST loan loss provision forecasting.

Citi's experience with these models, during the previous Y14-A CCAR/DFAST submission demonstrated the ability of these models to provide robust forecast of losses under a variety of macro-economic conditions including stressed losses under the regulatory scenarios. The account level models for PD and EAD are a significant enhancement over LookAhead models used for prior submissions.

[14] Refer to the Federal Reserve Bank's "Capital Planning at Large Bank Holding Companies: Supervisory Expectations and Range of Current Practice

### 6.1.2 Design, Theory, and Logic of the Model

NCL is derived using the component models (PD, EAD and Recovery).

- **Probability of Default (PD)** measures the likelihood of incidence of loss; or the likelihood that a borrower will be unable to meet its debt obligations on the account and go into default at time T
- **Exposure at Default (EAD)** measures the balance owed on the account at time of default
- **Recovery** measures the amount that the bank is able to recover back from charged-off loan

Net Credit Loss (NCL) is calculated by netting off the Gross Credit Loss for **Recoveries** and unearned fees and finance charges.

Net recoveries can be decomposed into three components: Cash Collected recoveries, Agency fees and Asset Sales.

- **Cash Collected** refers to cash payments made on charged-off accounts in inventory to internal or external collection agencies. The amount of Cash Collected is driven by customer behavior and is further stimulated by the Bank's collections and recoveries practices.

- **Agency fees** are the fees collected by a recovery agency for the recoveries they collect. They are a proportion of the recovery dollars collected by the agency.

- **Asset Sales** are the sum of proceeds from the sale of the charged-off accounts. Asset Sales are driven by management decisions based on supply and demand of debt markets which fluctuate throughout the economic cycle.

This section describes PD, EAD and Recovery in detail below:

#### 6.1.2.1 *Probability of Default Methodology, Design, and Logic*

**Modeling Approach: Discrete Time Account Level Survival Model**

The discrete time survival model methodology is a commonly used approach in credit risk modeling. The discrete time survival model structure predicts account termination given survival to date. The drivers of PD model include both account-level attributes and macroeconomic factors.

An important feature of a survival model is the "baseline hazard" function. The approach adopted for the current model development effort relies on a **quarter end portfolio snapshot** based data structure where account performance is measured quarterly following a given portfolio snapshot date. Multiple portfolio snapshots (through the cycle quarter end snapshots– Dec'06 to Mar'16)

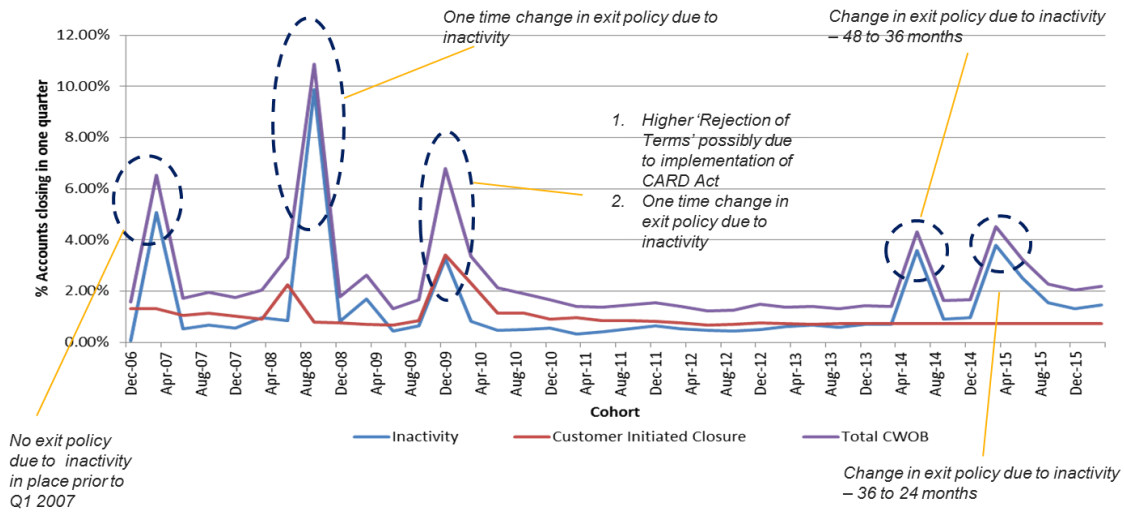
and associated account level performance histories are stacked into a panel data set for model estimation. Using this approach Quarter Since Snapshot(QSS) is defined relative to the forecast start date (snapshot date) and Quarter=t corresponds to period t of the forecast horizon for all accounts. In contrast, loan age during the forecast period will vary across accounts based on loan age at the snapshot date and time elapsed. Specifically, loan age at time t during the forecast period equals loan age at snapshot date plus 3*QSS (t). Within this framework, QSS (t) plays the role as a pure function of time since observation and acts as the model baseline variable. Because loan age is not the same function of time for all accounts, it is not considered a formal baseline variable, but rather as another candidate explanatory variable for use in model specification.

The portfolio snapshot based structure of the framework naturally allows leveraging the most recent account level loan attributes, including delinquency status, balance, historical payment patterns to date, loan age, etc., available as of the forecast start date. For each quarter following the snapshot date, 2 performance outcomes are recognized: good or bad. Macroeconomic data series and account level risk factors are attached to the panel data framework and directly included in the modeling. Once an account has been recognized as a bad, there will not be any further records for that. For example, if an accounts defaults in 5$^{th}$ quarter as observed from the observation date, it would be tagged as a good ('0') for the first 4 quarters and it would be tagged as a bad for 5$^{th}$ quarter ('1') and it would not be present for 6$^{th}$ to 9$^{th}$ quarter. On the other hand, if an account does not charge-off in the performance window, it would be tagged as 'good' for all the 9 quarters. This modeling approach meets the CCAR objective that macroeconomic scenarios are direct model inputs used to generate scenario specific model outputs.

From a competing risk survival approach standpoint, Branded Cards is considering just one termination event (defined as 'bad'). The closure (or attrition) cases, defined as accounts which close either by voluntary customer requests or are closed by bank due to prolonged inactivity, are treated as good accounts. For example, if an account attrites in 5$^{th}$ quarter as observed from the observation date, it would be tagged as a good ('0') for all the 9 quarters. Treatment of voluntarily attriting accounts as good accounts can be considered as one of the possible treatments to simplify the model framework, especially where the attrition rates have remained relatively stable over a period of time. The following example illustrates the rationale for such an approach. Consider a portfolio with 100 accounts out of which 5 default and 10 attrite by the next quarter. In our current approach, the PD rate will be (5 / 100) = 0.05. In another method where we exclude the attriting population from the base and then apply an attrition probability, the calculated PD will be (5/90) X (1-(10/100)) = 0.05. The second method is applicable in cases where there is a relatively stable attrition rate over a long period of time (considered as 10% here). We can clearly observe that in both the cases the PD rates are coming to be the same. Method 1 was chosen as it is easy to interpret. Additionally, historical attrition rates, especially customer initiated closure rates have been relatively stable over a period of time as can be observed in the figure below.

Peaks in inactivity based closures can be observed, however these closures do not have any significant contribution to losses.

**Figure 27 : Historical trend for attrition accounts**




As can be seen in the figure above, customer initiated closure has remained stable over a through the cycle period, apart from a minor spike in Sep'09 , which could be explained by the introduction of card act and corresponding change in terms and conditions. A Sep'09 event dummy is present in the model to capture this specific effect.

Additionally, the model has been back tested extensively and the results have been excellent and well within MRM established thresholds. This is further evidence to the feasibility of the approach considered above. Also continuing with the tradition of introducing changes through the challenger route, attrition as a separate state in a competing risk setup has been considered in the account level state transition matrix based challenger model.

The framework was developed to enable the model to simultaneously capture a) impact of macro-economic factors and b) impact of behavioral (on-us and bureau) risk drivers on the probability of default. These have further been isolated from variations in probability of default due to product characteristics and seasonality. The modeling construct can be summarized using the equation below.

**Equation 6**

$$PD_t = f(I_0, B_0, P_0, M_t, Q_t, QSS_t, AGE_t)$$ Where,

$\mathbf{I_0}$ = internal attributes of the account at observation (t=0)
$BI_t$ = Time varying co-variate for internal attribute
$\mathbf{B_0}$ = credit-bureau attributes for the account at observation (t=0)

$BB_t$ = Time varying co-variate for credit bureau
**$P_0$** = merchant / product indicator for the account
$BP_t$= Time varying covariate for product indicator
**$M_t$** = macro-economic indicators at performance time t
$BM_t$= Time varying co-variate for macro variable
**$Q_t$** = seasonality indicators representing calendar quarter of performance
**$QSS_t$** = time indicators representing time difference between the observation performance period and the performance period
**$AGE_t$** = Month on books at the performance time (= AGE (t=0) + 3* QSS)

The function f represents the logit link function given by

**Equation 7**

$$Logit(PD_t) = Ln\left\{\frac{PD_t}{1 - PD_t}\right\}$$
$$= QSS_t + BI_t * I_0 + BB_t * B_0 + BP_t * P_0 + BM * M_t + BAGE * AGE_t + BQ_t * Q_t$$

The formulation above allows for time-varying covariates for ($BI_t$, $BB_t$, $BP_t$, $BM_t$), which has been incorporated to develop robust PD prediction as contribution of covariates change over time. As an example, current delinquency of an account will have higher impact in short term horizon (next 2 quarters) and will have lower impact at a long term horizon ($8^{th}$ – $9^{th}$ quarter). In order to simplify the forecasting methodology, Branded Cards have adopted this approach which incorporates single equation of PD to forecast over different time horizons by incorporating time varying coefficients. Time varying coefficients has been discussed in detail in subsequent paragraphs below.

Seasonality is an exception to this. Seasonality, by definition, does not vary year over year. It merely captures regular quarterly variations over the course of the year not expected to be impacted by other risk characteristics. Therefore the interaction of seasonality with performance window has not been allowed in the model.

Further, it is important to note the interaction effects captured in the models in addition to those with time. Interactions with FICO and products have been considered wherever appropriate. These interactions are relevant as some of the factors are significant only in conjugation with specific risk profiles and product groups. Interaction terms of such kind capture the aforesaid effect.

Allowing the coefficients to vary over time by using interaction with time dummies also impacts the variable selection process. This will be described in detail in section 6.5 on variable selection. Briefly, the logic followed is to create indicators for default over varying performance windows

and select variables based on their predictive power over these alternate performance windows, along with the cumulative 9Q error from the resulting model.

This section presents the general theory and mathematics of survival analysis used to model account termination likelihood:

### *a) Survival analysis fundamentals*

Survival analysis methods are characterized by the following modeling features: (1) the dependent variable or response is the waiting time (duration) until the occurrence of a well-defined event and consequently event timing is central to the analysis, (2) observations can be subject to censoring, meaning that for some units the event of interest is not observed prior to the end of the observation period, and (3) predictors may be included, which have an effect on the waiting time.[15] General examples of waiting time events that can be modeled using survival analysis include age at first marriage, the time to death following a medical treatment, or time to termination of a banking exposure.

Given the terminology of survival analysis, referring to the event of interest as 'death' and to the waiting time as 'survival' time, we denote $T$ as a non-negative random variable representing the waiting time until the occurrence of an event. For generality, $T$ is a continuous random variable with Probability Density Function (PDF), $f(t) = Pr(T=t)$, and Cumulative Distribution Function (CDF), $F(t) = Pr\{T < t\}$, giving the probability that the event has occurred by time $t$.

It will often be convenient to work with the complement of the CDF, the ***survival*** function

$$S(t) = \Pr\{T \geq t\} = 1 - F(t) = \int_t^{\infty} f(x)dx, \qquad (6.2.1.1)$$

giving the probability of being alive until $t$, or more generally, the probability that the event of interest has *not occurred by t*.

An alternative characterization of the distribution of $T$ is the ***hazard*** function, or instantaneous rate of occurrence of the event

$$\lambda(t) = \lim_{dt \to 0} \frac{\Pr\{t \leq T < t+dt \mid T \geq t\}}{dt}. \qquad (6.2.1.2)$$

The numerator of this expression is the conditional probability that the event will occur in the interval $[t, t + dt)$ given that it has not occurred before, and the denominator is the length of the

[15] See Paul Allison, 1982, Discrete-Time Methods for the Analysis of Event Histories, Sociological Methodology, Vol. 13 pp. 61-98; German Rodriguez, Survival Models, http://data.princeton.edu/wws509/notes/c7.pdf, September, 2010; David Collett, Modelling Survival Data in Medical Research (2003).

time interval. Dividing one by the other we obtain a rate of event occurrence per unit of time. Taking the limit as *dt* approaches 0, we obtain an instantaneous rate of occurrence.

The conditional probability in the numerator may be written as the ratio of the joint probability that *T* is in the interval *[t, t + dt)* and *T >= t* (which is the same as the probability that *t* is in the interval), to the probability of the condition *T >= t*. The former may be written as *f(t)dt* for small *dt*, while the latter is *S(t)* by definition. Dividing by *dt* and passing to the limit gives the useful result

$$\lambda(t) = \frac{f(t)}{S(t)}, \tag{6.2.1.3}$$

which is a definition of the hazard function. Put differently, the rate of occurrence of the event at *t* equals the density of events at *t*, divided by the probability of surviving to *t,* without experiencing the event. Recall that *-f(t)* is the derivative of *S(t)*, which allows rewriting the above equation such that

$$\lambda(t) = f(t)\frac{dlog(S(t))}{d(S(t))} = f(t)\frac{dlog(S(t))}{-f(t)dt} = -\frac{d}{dt}logS(t). \tag{6.2.1.4}$$

Integrating from *0* to *t* and solving the above (differential) equation subject to the boundary condition *S(0) = 1* (since the event is sure not to have occurred by duration 0), we obtain a formula for the probability of survival to *t* in terms of the hazard function at all times until *t*:

$$S(t) = \exp\left\{-\int_0^t \lambda(x)dx\right\}. \tag{6.2.1.5}$$

A distinguishing feature of survival analysis is its ability to address censoring. The most typical case is right censoring, where for some units the event of interest has not occurred before the observation period ends. For right-censored observations, all that is known is that the waiting time exceeds the observation time. Right censoring occurs when:

a) The observation window is shorter than the underlying duration (or lifetime). Given that the model's objective is to project charge-offs for thirteen quarters, a decision was made to track account performance for a maximum of thirteen quarters since observation. Therefore, performance is right-censored for accounts with greater than 9 quarters of available (surviving) performance.
b) The end of the observation period (calendar month) is reached before thirteen quarters of performance are observed. This occurs, for example, when a development data snapshot date is less than thirteen quarters prior to the last historical observation date and the account is surviving at the end of the observation period .
c) Account performance ceases to be recorded for an unknown reason before the observation period ends.

d) In a competing risk survival model context right censoring occurs for one termination event of interest if another termination event occurs first. This means that account closure is a right censoring event for the charge-off event, and vice versa.

A basic assumption of the analysis is that all right censoring is *non-informative*, which means that the distribution of censoring times is independent of event times.

Specifically for d), Branded Cards is taking just one termination event (defined as 'bad'). For the closure (or voluntary attrition) cases, they are treated as good accounts.

Assume the population consists of *n* units with lifetimes governed by a survival function *S(t)* with associated density *f(t)* and hazard *λ(t)*. For a given snapshot date and a unit *i,* either an event is observed or else right censoring occurs. Time $t_i$ represents the earlier of these two possibilities. If $t_i$ represents an event, the unit contributes to the likelihood function as follows:

$$L_i = f(t_i) = S(t_i)\,\lambda(t_i). \tag{6.2.1.6}$$

Alternatively, if $t_i$ represents censoring, the unit's contribution to the likelihood function is:

$$L_i = S(t_i)\,. \tag{6.2.1.7}$$

Consider the non-censored and censored observations collectively and let $\delta_i$ be an observed event indicator, which takes the value of 1 if an event occurs for unit *i,* and 0 otherwise. The likelihood function of the observed population would be

$$L = \prod_{i=1}^{n} L_i = \prod_{i=1}^{n} \lambda(t_i)^{\delta_i} S(t_i). \tag{6.2.1.8}$$

***b. Modeling in discrete time***

Survival modeling fundamentals have been introduced in a continuous time setting above. Measurement of events actually occurs in discrete time in many instances. For the application of survival analysis to loan termination modeling, termination status is most typically measured at the end of each quarter. Therefore, using a discrete time survival model for loan termination modeling is a natural choice, given that the model output too is at discrete time intervals.

In discrete time, assume T is a non-negative integer value random variable taking the values $t_1$, $t_2$, … ($t_1 < t_2 < \ldots$ ) representing the waiting time until the occurrence of an event of interest. The probabilities are

$$f(t_j) = f_j = Pr\{T = t_j\}. \tag{6.2.1.9}$$

The associated survival function is

$$S(t_j) = S_j = Pr\{T \geq t_j\} = \sum_{k=j}^{\infty} f_k, \qquad (6.2.1.10)$$

and the hazard at time tj is the conditional probability of an event occurring during the specific discrete time interval (month), given that the unit survived to the beginning of the interval

$$\lambda(t_j) = \lambda_j = Pr\{T = t_j \mid T \geq t_j\} = \frac{f_j}{S_j}. \qquad (6.2.1.11)$$

Note that, in discrete time, the hazard function has an intuitive interpretation, e.g., a conditional probability. A further result of interest in discrete time is that the survival function at time $t_j$ can be written in terms of the hazard at all prior times $t_1, \ldots, t_{j-1}$ as

$$S_j = (1 - \lambda_1)(1 - \lambda_2) \cdots (1 - \lambda_{j-1}). \qquad (6.2.1.12)$$

This result makes it clear that in order to survive to time $t_j$, the unit must first survive to $t_1$, then it must survive to $t_2$ given that it survived to $t_1$, and so on, finally surviving to $t_j$ given survival to $t_{j-1}$. This result is analogous to the result linking the survival function in continuous time to the integrated, or cumulative hazard, at all previous times.

For the population in discrete time, the likelihood function of the observed data would be the following, according to Equation (6.2.1.8):

$$L = \prod_{i=1}^{n} Pr\{T = t_i\}^{\delta_i} \Pr\{T > t_i\}^{1-\delta_i}. \qquad (6.2.1.13)$$

### *c. Discrete time survival model and the logistic link*

Cox (1972) proposed a family of continuous time models to describe the hazard function.[16] The simplest model proposed is the proportional hazard model, where the hazard at time $t$ for a unit with covariates $x_i$ is assumed to be

$$\lambda_i(t|x_i) = \lambda_0(t)\, exp\{\boldsymbol{x_i}'\boldsymbol{\beta}\}, \qquad (6.2.1.14)$$

Where $\lambda_0(t)$ is a baseline hazard function that describes the risk for a unit with $\boldsymbol{x_i} = 0$. The baseline hazard at each $t$ serves as a reference cell or pivot, and $exp\{\boldsymbol{x_i}'\boldsymbol{\beta}\}$ is the relative risk associated with the set of characteristics $x_i$. The vector $\boldsymbol{\beta}$ represents the effects of the explanatory variables on the hazard rate $\lambda_0(t)$. This model assumes that the increase or decrease in hazard is proportional at all durations $t$. In other words, the ratio of the hazard rates for any two units is not a function of time in the original Cox proportional hazard model.[17]

[16] See Cox, 1972, "Regression models and life-tables" Journal of the Royal Statistical Society, 34: 187-202.

[17] The proportional hazard model with time-invariant covariates can be easily extended to allow for time-varying covariates. The generalized proportional hazard model with covariates that change over time becomes $\lambda_i(t|x_i(t)) = \lambda_0(t)\, exp\{x_i(t)'\beta\}$. Note that the separation of duration and covariate effects is not so

An extension of the continuous time proportional hazard model to discrete time involves working with the conditional odds of an event at each time $t_j$, given survival up to that point.[18] Specifically, in discrete time, the model takes the following form:

$$\frac{\lambda(t_j|x_i)dt}{1-\lambda(t_j|x_i)dt} = \frac{\lambda_0(t_j)dt}{1-\lambda_0(t_j)dt}\ exp\{\boldsymbol{x}_i{}'\boldsymbol{\beta}\}, \tag{6.2.1.15}$$

where $\lambda(t_j|x_i)$ is the hazard at time $t_j$ for a unit with covariate values $x_i$, $\lambda_0(t_j)$ is the baseline hazard at time $t_j$, $dt$ is an increment of time, and $exp\{x_i{}'\beta\}$ is the relative risk associated with covariate values $x_i$. In the limit as $dt$ approaches to zero, discrete equation 6.2.1.15 becomes the continuous equation 6.2.1.14.

Setting $dt$ to 1 and taking logs on both sides of equation 6.2.1.15, we obtain a model on the logit of the discrete time hazard of an event at $t_j$ given survival up to that time[19]

$$\log\left(\frac{\lambda(t_j|x_i)}{1-\lambda(t_j|x_i)}\right) = logit\ \lambda(t_j|x_i) = \alpha_j + \boldsymbol{x_i'\beta} \tag{6.2.1.16}$$

Note that the model essentially treats time as a discrete factor by introducing one parameter $\alpha_j$ for each possible time of event $t_j$. We leave the parameter unspecified, while noting it could be a set of constants (dummies) or other functional form. Interpretation of the parameters $\boldsymbol{\beta}$ associated with the other covariates follows the same logic as in any logistic regression. The left side of Equation (6.2.1.16) is a logit transformation of discrete time hazard.

Based on Equation (6.2.1.13), each probability contributing to the likelihood function can be expressed as a function of the hazard rate:

$$Pr\{T = t_j\} = \lambda_j \prod_{s=1}^{j-1}(1-\lambda_s) \tag{6.2.1.17}$$

$$Pr\{T > t_j\} = \prod_{s=1}^{j}(1-\lambda_s) \tag{6.2.1.18}$$

Substituting these into Equation (6.2.1.13) and taking the logarithm, we obtain

$$log\ L = \sum_{i=1}^{n} \delta_i log\{\lambda_{i,t_{j,i}}/(1-\lambda_{i,t_{j,i}})\} + \sum_{i=1}^{n}\sum_{s=1}^{t_{j,i}} log\{1-\lambda_{i,s}\} \tag{6.2.1.19}$$

The above equation can be re-written by introducing the event history indicator $y_{i,tj}$. We define a dummy variable $y_{i,tj}$ equal to 1 if unit $i$ experiences an event at time $t_j$, otherwise zero. We then have

clear now and the calculation of the survival function is more complicated. Nonetheless, one might still prefer a time-varying covariate as a more meaningful predictor of time varying risk than time alone.

[18] See, e.g. Cox, 1972, "Regression models and life-tables" Journal of the Royal Statistical Society, 34: 187-202.

[19] Natural logarithms are used throughout the document. Note that $\alpha_j = log(\lambda_0(t_j)/(1-\lambda_0(t_j))$.

$$\sum_{s=1}^{t_{j,i}} y_{i,t_{j,i}} log\left\{\frac{\lambda_{i,s}}{1-\lambda_{i,s}}\right\} = \begin{cases} log\left\{\frac{\lambda_{i,t_{j,i}}}{1-\lambda_{i,t_{j,i}}}\right\}, & when\ \delta_i = 1 \\ 0, & when\ \delta_i = 0 \end{cases}$$

$$= \delta_i log\{\lambda_{i,t_{j,i}}/(1-\lambda_{i,t_{j,i}})\} \qquad (6.2.1.20)$$

Substituting (6.2.1.20) into (6.2.1.19), the log likelihood equation can be written as

$$log\ L = \sum_{i=1}^{n}\sum_{s=1}^{t_{j,i}} y_{i,t_{j,i}} log\{\lambda_{i,s}/(1-\lambda_{i,s})\} + \sum_{i=1}^{n}\sum_{s=1}^{t_{j,i}} log\{1-\lambda_{i,s}\} \qquad (6.2.1.21)$$

Equation (6.2.1.21) is recognized to be the exact log-likelihood function for the binomial logistic regression model.[20] This identity implies that a proportional discrete time hazard model can be estimated using a logistic regression, provided model development data is suitably constructed as a panel data set based on loan event histories. We will elaborate on the specific details of panel data set construction in following section.

The transformation in Equation (6.2.1.16) also makes the logistic regression coefficient estimates interpretable as in any linear regression, where the magnitude of $\boldsymbol{\beta}$ represents the increase in the logit of discrete time hazard for one unit change of $x_i$.

### *d. Competing risk survival models*

The goal of survival analysis is to assess the risk that a unit with certain characteristics incurs an event of interest, or terminates, at time *t*. As shown below, the methodology generalizes to the case of multiple competing termination risks. In medical research, patients are exposed concurrently to different types of disease and causes of death. In loan termination modeling, termination can result from account closure or account charge-off. The competing risk framework models a unit at risk for more than one type of termination event at each time *t*, provided no termination event has been experienced to date.

Consistent with previous notation, we use *T* to represent survival time. When a termination event occurs, it could be one of *m* distinct event types, indexed by *k* in *{1, 2, … , m}*, with *K* representing a random variable measuring the waiting time to an event of type *k*. Note that the earliest event to occur, or the minimum event duration among the competing risks is the one realized. The cause-specific hazard rate becomes:

$$\lambda_k(t) = \lim_{dt\to 0} \frac{\Pr\{t \le T < t+dt, K=k \mid T \ge t\}}{dt} \qquad (6.2.1.22)$$

[20] Cox, 1970, The analysis of binary data. Nerlov and Press 1973, Univariate and multivariate log-linear and logistic models.

The intuition is that one can calculate the conditional probability that a unit terminates in the interval *[t, t+dt)* from a cause of type *k*, given that the unit survived all risks until *t*. Because a termination event can be due to one and only one of the m causes, by the law of total probability, we have

$$\lambda(t) = \sum_{k=1}^{m} \lambda_k(t) \tag{6.2.1.23}$$

As before, we set $\delta_{\mathrm{i}} = 1$ if an account experiences an event and otherwise set it to zero. The likelihood function according to Equation (6.2.1.13) can be shown to be

$$L = \prod_{i=1}^{n} \left\{ \left[ \frac{\lambda_{t_{j,i}k}}{1-\sum_{k=1}^{m} \lambda_k(t_{j,i}|x_i)} \right]^{\delta_i} \prod_{s=1}^{t_{j,i}} [1 - \sum_{k=1}^{m} \lambda_k(s|x_i)] \right\} \tag{6.2.1.24}$$

Consider a generalization of the binary logistic discrete time hazard model (6.2.1.16), to the multinomial logistic competing risk hazard model, specifically

$$\lambda_{t_j k} = \frac{e^{\alpha_{kt_j} + \mathrm{x}'\beta_k}}{1+\sum_{r=1}^{m} e^{\alpha_{rt_j} + \mathrm{x}'\beta_r}} \tag{6.2.1.25}$$

This reduces to Equation (6.2.1.16) when *m*=1.

Substituting (6.2.1.25) into (6.2.1.24) and taking the logarithm, we can obtain the log likelihood for a multinomial logistic model, in which all individuals are treated as separate, independent observations.[21] That is, just as the single-event discrete time hazard model can be estimated with a binomial logistic model, the discrete time competing risk hazard model can be estimated by using a multinomial logistic model.

However, as mentioned above, from a competing risk survival approach standpoint, Branded Cards is taking just one termination event (defined as 'bad'). For the closure (or voluntary attrition) cases, they are treated as good accounts.

### *f. Baseline hazard*

This section elaborates on the baseline hazard function introduced in section 6.1.1. As previously mentioned, an important choice in specifying a survival model is the orientation of the baseline hazard function.

Given the modeling objective of projecting portfolio charge-offs from a single calendar start date, months since snapshot measures time to termination event and acts as the baseline hazard by representing a function of time that is the same for every account. Time varying loan

[21] See Hanushek and Jackson, 1977, Statistical methods for social scientists.

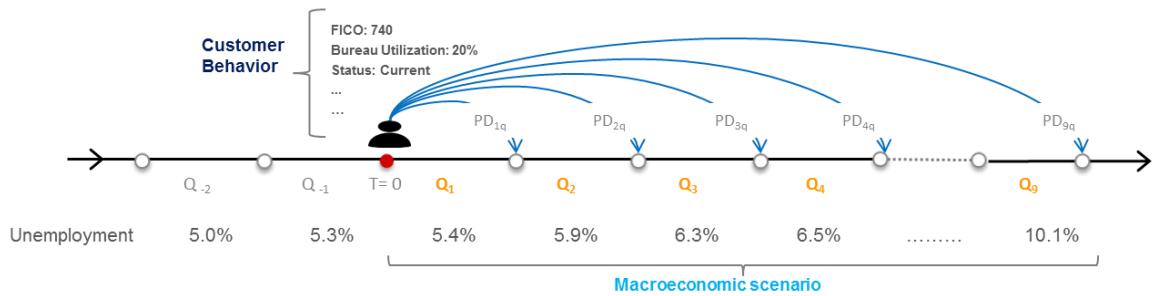
age following the snapshot date is also a function of time, but varies across loans through time based on different loan age at the portfolio snapshot date. Time varying loan age is therefore not considered to be a survival model baseline variable within the portfolio snapshot based panel data framework used in model development.

### *g. Time varying coefficients.*

For illustrative purposes, the Model design for PD Model design can be seen in Figure

**Figure 28 : Model design of PD and EAD Model**

***1. The objective is to predict PD at an account level in each of the next 9 quarters under a given economic scenario***




***2. Developed a single equation capturing dynamic relationship between customer behavior and macroeconomic factors to predict PD***

$$PD = f\,(\text{historical behavior of the customer},\ \text{macroeconomic scenario in future},\ \text{time since observation})$$

*Affects relative importance of customer behavior . Impact of macroeconomic factors is the same*

$$PD = \text{Logit}\left(\sum \alpha_i * q_i + \sum X_i * (\beta_{1i}\, q_1 + \beta_{2i}\, q_2 \ldots \beta_{9i}\, q_9) + \sum Y_i * (\delta_i)\right)$$

*Impact of $X_i$ in 2nd qtr.* *Impact of Yi in all qtr.*

***3. Similar to PD, EADF is also modeled using a single equation. However EADF uses linear regression whereas PD is modeled using logistic regression***

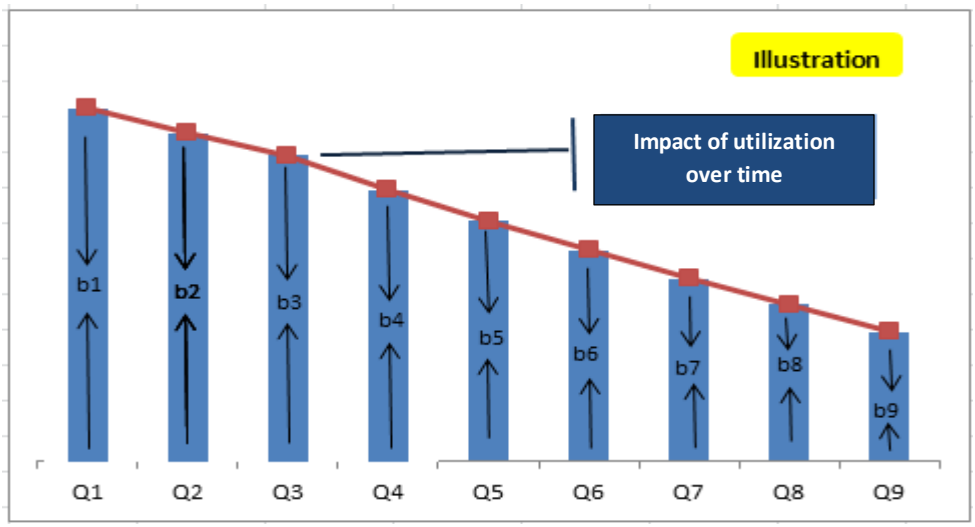
*The above approach is utilized in Costco and Ex- Costco Models*

The Y14-A Comprehensive Capital Analysis and Review (CCAR) stress test and Dodd-Frank Act Stress Test (DFAST) requires the PD to be forecasted over 9 quarter horizon with loss forecast at quarterly level over the given horizon. Given the PD model at an account level, this will translate to building separate models (9 different for each quarter) for each quarter over next 9Q since different behavioral variables will impact differently over time horizons. As an example, the current utilization of an account will have higher impact in near term horizon and lower impact at a long term horizon (24th – 27th month). In order to simplify the forecasting methodology, Branded Cards have adopted the methodology of time varying interaction effects. It

incorporates single equation of PD to forecast over different time horizons by incorporating Interaction effects.

Interaction effects adjust the contribution of the variable tailored to different quarters being forecasted. The graphical representation of this technique has been further explained below:

**Figure 29 : Graphical representation of time varying interaction technique - Contribution of internal variables (balance) in predicting PD**




Since the contribution of the independent variables in predicting the PD depends on the window of forecast, the time varying interaction incorporates this by including the variables for different time horizons in a single model equation. Theoretically, the equation for PD will contain different coefficients for each quarter it'll then use the necessary coefficient while making all others equal to zero.

After incorporating the behavioral attributes and macro-economic sensitivity, a single equation for predicting PD for next 9 quarters is obtained.

Finally, this is a single model which has multiple independent variables at account level

1. Behavioral variables with quarterly interaction effects, varying for each account

2. Macro-economic variables varying at state Level (all accounts in the same state have same values)

This methodology gives the benefit of having a single equation that gives more control to measure the relationship (coefficients) of variables across different quarters capturing the continual decay effect. Additionally, it also provides computational and maintenance advantages of 9 different models vis-à-vis one single model.

### *h. Panel data set construction*

As explained above, the approach adopted for the current model development effort relies on a **quarter end portfolio snapshot** based data structure where account performance is measured quarterly following a given portfolio snapshot date. Multiple portfolio snapshots (through the cycle quarter end snapshots– Dec'06 to Mar'16) and associated account level performance histories are stacked into a panel data set for model estimation. Using this approach Quarter Since Snapshot(QSS) is defined relative to the forecast start date (snapshot date) and Quarter=t corresponds to period t of the forecast horizon for all accounts. In contrast, loan age during the forecast period will vary across accounts based on loan age at the snapshot date and time elapsed. Specifically, loan age at time t during the forecast period equals loan age at snapshot date plus 3*QSS(t). The structure of panel data used for Account level PD and EAD Modeling is illustrated below

| | | Performance | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **Qtr1** | **Qtr2** | **Qtr3** | **Qtr4** | **Qtr5** | **Qtr6** | **Qtr7** | **Qtr8** | **Qtr9** |
| **Observation** | **Dec-06** | Mar-07 | Jun-07 | Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 |
| | **Mar-07** | Jun-07 | Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 |
| | **Jun-07** | Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 |
| | **Sep-07** | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 |
| | **Dec-07** | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 |
| | **Mar-08** | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 |
| | **Jun-08** | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 |
| | **Sep-08** | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 |
| | **Dec-08** | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 |
| | **Mar-09** | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 |
| | **Jun-09** | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 |
| | **Sep-09** | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 |
| | **Dec-09** | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 |
| | **Mar-10** | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 |
| | **Jun-10** | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 |
| | **Sep-10** | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 |
| | **Dec-10** | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 |
| | **Mar-11** | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 |
| | **Jun-11** | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 |
| | **Sep-11** | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 |
| | **Dec-11** | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 |
| | **Mar-12** | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **Jun-12** | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 |
| | **Sep-12** | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 |
| | **Dec-12** | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 |
| | **Mar-13** | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 |
| | **Jun-13** | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 |
| | **Sep-13** | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 |
| | **Dec-13** | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 |
| | **Mar-14** | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 |
| | **Jun-14** | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 |
| | **Sep-14** | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 |
| | **Dec-14** | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 |
| | **Mar-15** | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 |
| | **Jun-15** | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | |
| | **Sep-15** | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | |
| | **Dec-15** | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | |
| | **Mar-16** | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | |
| | **Jun-16** | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | | |
| | **Sep-16** | Dec-16 | Mar-17 | Jun-17 | | | | | | |
| | **Dec-16** | Mar-17 | Jun-17 | | | | | | | |
| | **Mar-17** | Jun-17 | | | | | | | | |
| | Development sample | | | | | | | | | |
| | Hold out Sample | | | | | | | | | |

The portfolio snapshot based structure of the framework naturally allows leveraging the most recent account level loan attributes, including delinquency status, balance, historical payment patterns to date, loan age, etc., available as of the forecast start date. Each account would have a maximum of 9 rows, each representing 1 quarter. For each quarter following the snapshot date, 2 performance outcomes are recognized: good or bad. Macroeconomic data series and account level risk factors are attached to the panel data framework and directly included in the modeling. Once an account has been recognized as a bad, there will not be any further records for that. For example, if an accounts defaults in 5$^{th}$ quarter as observed from the observation date, it would be tagged as a good ('0') for the first 4 quarters and it would be tagged as a bad for 5$^{th}$ quarter ('1') and it would not be present for 6$^{th}$ to 9$^{th}$ quarter. On the other hand, if an account does not charge-off in the performance window, it would be tagged as 'good' for all the 9 quarters. This modeling approach meets the CCAR objective that macroeconomic scenarios are direct model inputs used to generate scenario specific model outputs.

As described earlier, from a competing risk survival approach standpoint, Branded Cards is considering just one termination event (defined as 'bad'). For the closure (or attrition) cases, they are treated as good accounts. For example, if account attrites in 5$^{th}$ quarter as observed from the

observation date, it would be tagged as a good ('0') for all the 9 quarters. Treatment of voluntary attrited accounts as good accounts is one of possible ways in survival analysis.

When constructing the panel data set, potentially all Ki(s) performance observations are included for account i and starting observation month s. However, for datasets containing a large number of accounts over a long observation period, the number of rows in the panel data set can become very large. Therefore, certain sampling strategies, such as capping the length of an event history sequence at, say 39 months, or deciding on a rule, such as one randomly selected calendar month per quarter as potential s dates, may be required to reduce the modeling data set to a more manageable size. Data sampling is discussed in Section 5.1.4.

#### *i. Converting conditional charge-off and closure outcomes to unconditional outcomes*

In the survival framework, for an active loan as of the forecast date, a sequence of unconditional PD's can be calculated over the 9-quarter forecast horizon. It is derived from the conditional charge-off model outputs. Specifically, the survival probability at time $t$ is the probability of not being terminated by a charge-off event at time $t$. Mathematically, the survival probability can be expressed as

$$S_t = S_{t-1} \times [1 - Pr\{Default\ at\ t|\ x, alive\ at\ \mathrm{t}-1\})] \qquad (6.2.1.32)$$

Combining the survival probability and conditional probability of charge-off and closure, unconditional probabilities are defined as follows

$$(\text{Unconditional Probability Charge} - \text{off})_{\mathbf{t}} = S_{t-1} \times Pr\{Default\ at\ t|\ x, alive\ at\ \mathrm{t}-1\} \qquad (6.2.1.33)$$

### 6.1.2.2 *Exposure at Default Methodology, Design, and Logic*

Exposure at Default is defined as a dollar charge-off amount in the event of default. While the PD model generates the incidence of default, the EAD model estimates the exposure with which an account will default. Among many ways of estimating EAD, the most popular is through the estimation of Exposure at Default Factor, or EADF, which when multiplied by line amount, results in an estimate of the $ charge-off amount or the EAD.

The EAD Model framework was developed to enable the model to simultaneously capture a) impact of macro-economic factors and b) impact of behavioral (on-us and bureau) risk drivers on

the Exposure at default. The estimation of EADF was done leveraging OLS regression technique. The modeling construct can be summarized using the equation below.

**Equation 8**

$$EAD_t = f(QSS_t, I_0, B_0, P_0, M_t)$$

Where,

t = performance quarter (1 to 9)
**$I_0$** = Internal (on-us) attributes of the account at observation (t=0)
**$B_0$** = credit-bureau attributes for the account at observation (t=0)
**$P_0$** = merchant / product indicator for the account
**$M_t$** = macroeconomic indicators at performance time t
**$QSS_t$** = time dummy representing time difference between the snapshot/observation and performance
**f()** = linear function

The formulation above allows for time-varying covariates for the independent variables. The value of these covariates is allowed to vary during the performance period; and these can therefore capture information up to the time for which the prediction is being generated.

The model explicitly takes into account the fact that the predictive power of a variable can change over time. For example, the current balance of an account will have higher impact in near term horizon and lower impact at a long term horizon (9$^{th}$ quarter). This change in predictive power over time is captured by allowing the coefficient of each of the variables to change over time.

***Target Variable Definitions***

For revolving exposures, EADF is defined as the charge-off amount as a fraction of the line amount as of the snapshot date, as indicated by t=0:

$$EADF(t) = \frac{Charge\text{-}off\ amount(t)}{Line\ Amount(0)} \qquad (6.2.2.1)$$

Also EADF is not capped at 1.0 as customers can go above existing credit limit. Increases in the credit line following the snapshot date and associated balance draw behavior can also result in charge-off amounts greater than line amount at observation. EADF should, however be bounded below by zero to avoid nonsensical negative charge-off amounts or balances.

One of the major decisions for estimating Exposure at Default involves the selection of the functional form of the EAD dependent variable. The two options considered are

1. EAD: Modelling the absolute charge-off amount directly
2. EADF: Modeling of an Exposure at Default Fraction (EADF). EADF is the exposure at default as a percentage of the credit line at the time of capital calculation, i.e. the expected utilization at charge off and is defined as:

$$\text{EADF} = \frac{\text{Outstanding Balance (T = T)}}{\text{Total Credit Limit (T = 0)}}$$

As credit Limit as of snapshot is used, there is no need to forecast the Credit Line in the current approach.

To model each of the above (EAD or EADF), OLS regression was selected due to the reasons stated in Section 4.

For CCAR 2017, Branded Cards modeled directly the charge-off amount or EAD. After the CCAR 2017; IRMO (Independent Review Modelling Oversight) provided the feedback to evaluate modelling EADF instead of EAD (to check for stationarity, stress etc.). EAD in the form of a ratio is typically found to be more stationary. In addition, EAD ratios are be more intuitive from business judgement point of view.

For CCAR 2018, Branded Cards employed the EADF approach based on the feedback from IRMO due to following reasons:

1. Stationarity: While modelling exposure using Linear Regression technique, the independent and dependent variable should be stationary. Branded cards tested the stationarity for both EAD and EADF using the through the cycle data. The results demonstrate that EADF being a ratio comes out to be stationary while EAD being dollar amount is non-stationary.

**Table 6-1 Stationarity ADF[22] Test**

| Stationarity Test | p value | |
|---|---|---|
| EAD | 0.62 | **Non Stationary** |
| EADF | 0.11 | **Stationary** |

2. Boundedness :
   Intercepts and Indicators make it difficult to bound EAD; however such variables make more intuitive sense with EADF. For example a product indicator with coefficient of 5000 will increase the EAD for the all accounts with that product by 5000 dollars, even if the credit limit of an account may be significantly less than 5000.

[22] The data used for ADF Test calculation is from Jan'07-Jun'15

Branded cards analyzed the Predicted EAD and EADF for toy models built during this decision. Using EADF Approach, Only ~0.02% accounts were found while using Predicted EADF where the Predicted EAD exceeded the Limit by 3 times. The same number for EAD Model was ~3%. In the sample cases, the same thing is observed, with an account having Limit of $2000 dollars, the EAD Model predicts the EAD to be $6424 and the EADF model predicts the EAD to be $3877 (much closer to the actual $2462).

Table 6-2 Sample Cases

| Account ID | Line | Actual EAD | Predicted EAD - EAD Model | Predicted EAD - EADF Model |
|---|---|---|---|---|
| 327966805 | $1,500 | $3,618 | $5,239 | $3,666 |
| 166369 | $2,000 | $2,462 | $6,424 | $3,877 |
| 324493701 | $1,000 | $994 | $3,035 | $1,045 |
| 324532333 | $1,000 | $1,224 | $3,086 | $1,289 |

3. The relationships of some of the variables are not very clear with EAD as dependent variables because EAD is a function of risk and credit limit. For Example; FICO; an account with higher FICO should default with less amount as he is less risky; however when he defaults; he will have higher Limit and hence higher EAD. Hence, it becomes difficult to decide the direction of the relationship while modelling EAD.

The effect of Limit is detached from EADF and hence the direction of the relationship is easy to decide. For Example; Higher FICO will default with lower EADF with high likelihood.

Hence, modelling EADF was considered as the final approach for predicting Exposure at default for CCAR2018 Models.
The EADF model is developed on the same data as PD model, however, only the accounts that experienced a charge-off event with corresponding charge-off amounts, were included in the model development.

Moving to model estimation, the Account Level EAD model is estimated using linear regression based on Ordinary Least Squares (OLS) technique .Linear regression is one of the most fundamental quantitative approaches used to model EAD (EADF). It estimates a linear relationship between EADF and independent variables (i.e., on-us & off-us behavioral variables and macroeconomic factors in this case), using a linear function, given below:

Equation 9

$$EAD_t = \alpha_t QSS_t + \beta_t^I I_0 + \beta_t^B B_0 + \beta_t^P P_0 + \beta_0^M M_t$$

As has been mentioned earlier in this the coefficients (β) are allowed to vary over time. This is achieved by allowing interaction between the input variables and indicators for ($T_t$) for time between the observation and performance period.

A detailed variable selection process has been employed to narrow down to the final set of variables in the model. This includes a number of univariate and bivariate techniques along with business intuition to iteratively select the most relevant and predictive variables among the more than 250+ variables .Allowing the coefficients to vary over time by using interaction with time dummies also impacts the variable selection process. This will be described in detail in section 6.5 on variable selection.

**GCL Forecast Generation**

Since economic trends in the model are forecasted using macro-economic variables, multiple future scenarios can be considered for stress scenario forecasting. Citi Treasury's forecast for macro-economic variables under different future economic scenarios enables us to forecast losses for stressed economic scenarios like Deep Downturn or a Global Trade Shock. By replacing the baseline forecasts with these stressed scenarios forecasts in the model provides relationship of portfolio losses with economy under stressed scenarios. However, seasonality, and vintage quality (as measured through on-us and off-us attributes) remains the same. Only macro-economic attributes and future vintage origination volumes are stressed. In order to stress test the Branded Cards' portfolio losses under different economic conditions, a set of forecasts was constructed using following scenarios:

1. Supervisory Baseline Scenario
2. Supervisory Adverse Scenario
3. Supervisory Severely Adverse Scenario
4. BHC / Mid Cycle Stress Test (MCST) Baseline Scenario
5. BHC / Mid Cycle Stress Test (MCST) Adverse Scenario
6. BHC / Mid Cycle Stress Test (MCST) Severely Adverse Scenario

For each stress scenario, forecasts were created, following the assumptions described in this and previous section using the economic series and the originations scenarios.

To obtain forecast for the new originations, accounts with MOB 1 to 6 are scored with the models. Average forecasted GCL is then weighted according to origination assumptions received from FP&A.

#### 6.1.2.3 *Recovery rate Model Methodology, Design, and Logic*

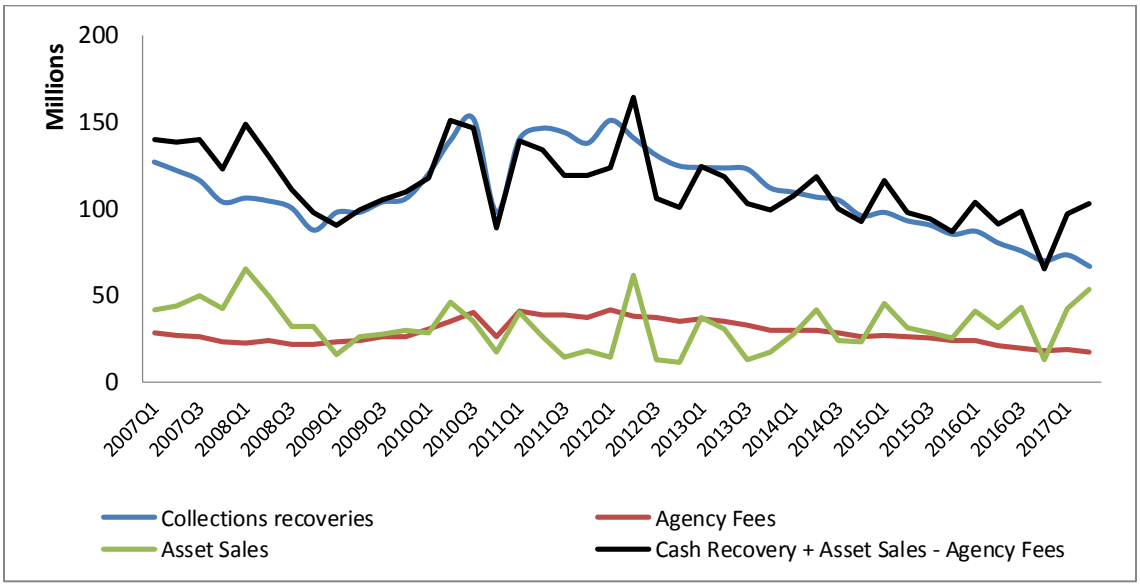
Net Recoveries can be decomposed into three components: Cash Collected recoveries, Agency Fees and Asset Sales.

1. **Cash Collected** refers to cash payments made on charged-off accounts in inventory to internal or external collection agencies. The amount of Cash Collected is driven by customer behavior and is further stimulated by the Bank's collections and recoveries practices.

2. **Agency Fees** are the fees collected by a recovery agency for the recoveries they collect. They are a proportion of the recovery dollars collected by the agency

3. **Asset Sales** are the sum of proceeds from the sale of the charged-off accounts. Asset Sales are driven by management decisions based on supply and demand of debt markets which fluctuate throughout the economic cycle.

Figure 30 Recovery Components




Hence, the net recoveries are calculated by the below formula:

Equation 10

$$Recoveries = Cash\,Collected\,Recoveries - Agency\,Fees + Asset\,Sales$$

For the CCAR 2018 submission, Branded Cards has modeled Cash collected Recovery Rate since it is driven by the macroeconomic factors.

Asset sales as a percentage of cash collected recoveries have been observed to be 33% in the recent 9 quarters of model development sample (April 2014-June 2016). Agency Fees as a

percentage of cash collected recoveries have been observed to be 27% in the recent 9 quarters of model development sample (April 2014-June 2016).

Agency fee is observed to be a constant percentage of the cash collected recoveries whereas Asset sales are driven by policy decisions. In recent months, business has observed that there has been a change in Asset sales policy as a result of which asset sales has increased. However, the model will not be able to capture this policy change. If the asset sales increase persists in future the business might consider applying an overlay.

Detailed modeling approaches for all 3 components of recoveries are explained below:

**Cash Collected Recoveries**

Modeling Recovery Rate using time- series forecasting technique is an industry wide practice. Additionally, for modeling Recovery Rate, it is observed that there is high correlation of the dependent variable with the macro-economic variables such as Unemployment Rate, House Price Index and GDP. One of the reasons that cash collected recovery dollars are not modeled as they are a function of both the losses and recovery rates.

Figure 31 illustrates the distribution of cash collected recovery in different years from different charge-off vintage years. The cash collected recovery rate for a given time period(t) coming from a particular charge-off vintage(T) is defined as the cash collected amount in that time period(t) divided by the initial charge-off balance for the corresponding charge-off vintage (T). This implies that Cash Collected in the current time period (t) consists of components corresponding to all of the historical charge-off vintages which are within the statute of limitations. Therefore, Cash Collected in a future time period depends on the current charge-off inventory in addition to any future charge-offs that would have occurred till that calendar time period.

Figure 31 Distribution of Recovery dollars in a given year coming from different Charge-off years

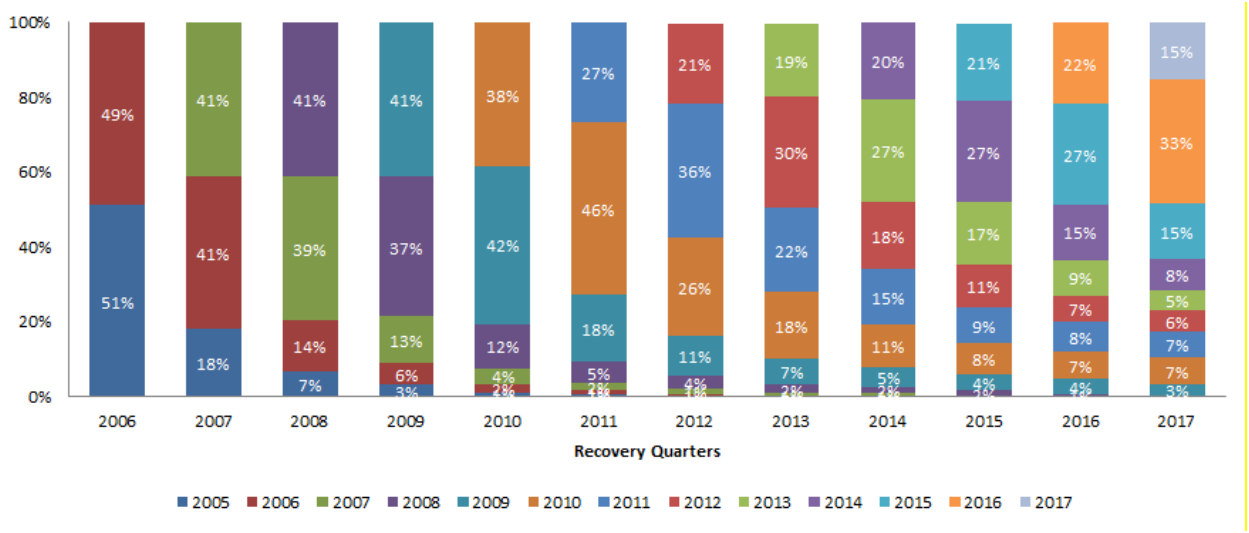
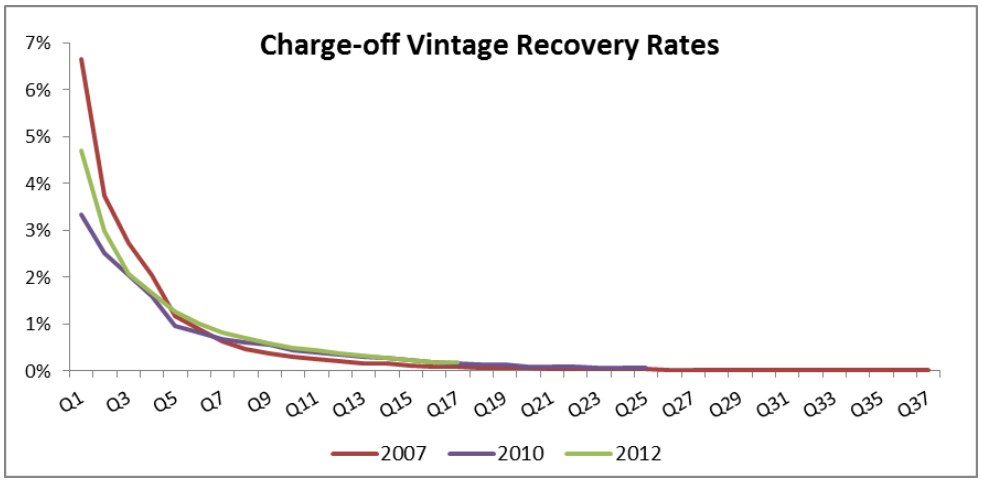



Figure 32 illustrates the Cash Collected recovery rate for a sample of charge-off vintage years. The Cash Collected recovery rate for a time period(t) coming from a particular charge-off vintage(T) is defined as the cash collected amount in that month divided by the initial charge-off balance for the corresponding charge-off vintage (T). The curve is steep in the 1st year after charge-off which gradually decreases in the 2nd and 3rd years and becomes stable after the 4th year. It can be seen from the graph that accounts charged-off during the pre-recessionary period (2007 vintage) have a higher recovery rate in the initial quarters, while recessionary period (2010 vintages) have lower initial recovery rates.

Figure 32 Charge-off Vintage Level Recovery Rates




OLS regression modeling technique is employed for the Recovery Rate model using a combination of recovery rate life cycle effect for (using time since charge off) and the effect of

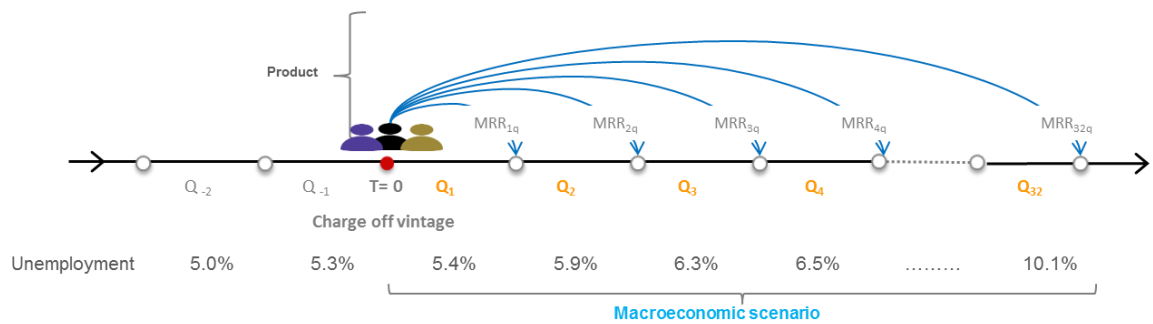
macroeconomics (using appropriate macroeconomic variable transformations). This approach is more granular compared to the Recovery Rate Challenger model, as the model is built using data at the level of charge off vintage.

The recovery rate life cycle component is estimated through Time Since charge-off dummies. In addition to the life cycle component, this model contains national macroeconomic variables. Macro-economic variables are transformed by incorporating lags and windows of the transformations.

The final model equation consists of both the life cycle impact variables as well as the macroeconomic impact variables, capturing both in a single step of linear regression.

**Figure 33 Recovery rate Model Methodology**

***1. The objective is to predict Recovery Rate at a charge off vintage level for 8[1] years under a given economic scenario***




***2. Developed a single equation capturing dynamic relationship between months since charge and macroeconomic factors to predict MRR***

$$MRR^2 = f\,(\text{Product},\ \text{macroeconomic scenario in future},\ \text{time since charge off})$$

*Affects relative importance of customer behavior. Impact of macroeconomic factors is the same*

$$\text{Log MRR} = \left(\sum \alpha_i * q_i + \sum \text{Product}_i * \beta_i + \sum Y_i * \delta_i\right)$$

*Impact of Yi in all qtr.*

[1] *Recovery beyond 8 years (32 quarters) is assumed to be 0*

[2] *MRR= Marginal Recovery Rate ( Recovery in the quarter/ total charge off $)*

The target variable is given by the below formula:

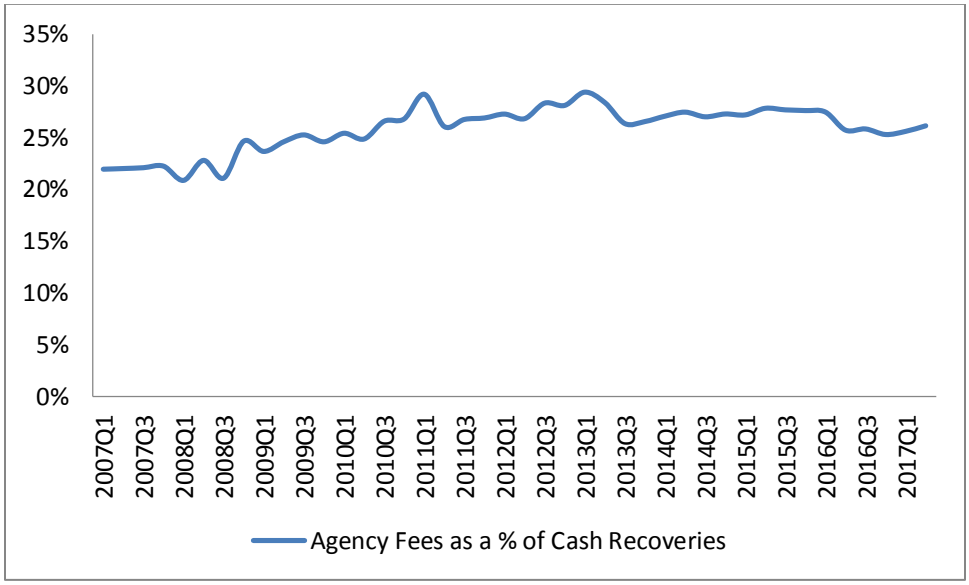
$$\begin{matrix} Recovery\ Rate\ (at\ time\ t = Payment) \\ (for\ a\ given\ charge\ off\ vintage = T) \end{matrix} = \frac{Recovery\ Dollar\ (at\ time\ t = Payment\ \ from\ Charge\ off\ = T)}{GCL\ (at\ charge\ off\ = T)}$$

**Agency Fees**

The Agency Fees as a percentage of dollar recoveries do not have a correlation with macroeconomic conditions and are by and large stable as shown in Figure 34. Hence they are not modeled with macroeconomic variables but are incorporated as a judgmental approach.

The recent 9 quarter in the development trend shows that agency fees as a percentage of recoveries is consistently in the range of 27-28%. However, to get conservative estimates, the Agency Fees as a percentage of Recoveries is taken as 30%, which is higher than observed in any of the quarters in our observation period.

Figure 34 Agency Fees as a % of Recoveries



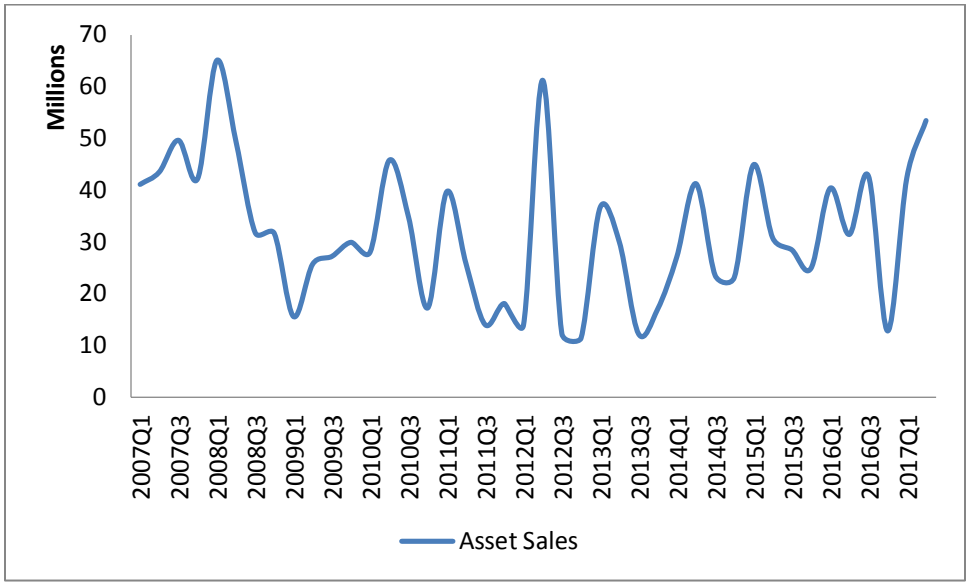

Please find attached the data on agency fees.

Agency_Fees.xlsx

## Asset Sales

Asset sales recoveries are driven by management decisions based on the dynamics of supply and demand in the debt markets, and it will fluctuate through the economic cycle. As a result, the model development team decided not to model asset sales and instead used a judgmental approach for the same.

Figure 35 Asset Sales




Below is the methodology for forecasting asset sales for both the baseline and stressed scenarios.

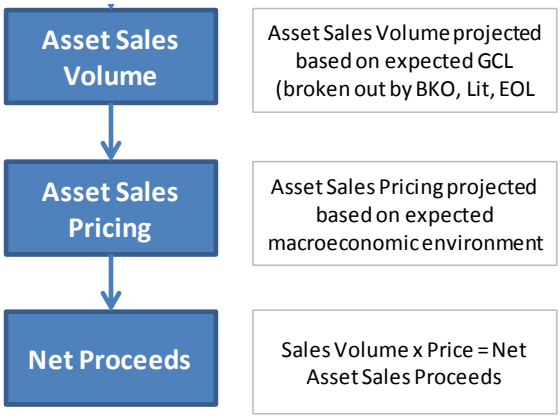
### Base Scenario

The process to forecast Asset Sales involves a series of handoffs with Risk working in conjunction with partners in Loss Forecasting / Modeling team, Collections, Operations, and BP&A Finance to derive the net proceeds expected from Asset Sales.

Table 6-3 Parties Responsible for Calculation of Base Asset Sales

| Roles and Responsibilities | |
|---|---|
| Collections | Projects Asset Sales Volume (End of Life segment) |
| BP&A Finance | Projects Asset Sales Volume (Bankruptcy and Litigation segments) |
| Operations | Projects Asset Sales Pricing |

Figure 36 Process Flow of Baseline Asset Sales Calculation




**Stress Scenario**

Once the base asset sale forecast is received, the stress asset sales are forecasted using a combination of stress ratio of forecasted cash collected recoveries and pricing.

### A *Stressed Volume Change*

Although quarterly asset sales are volatile, it has been observed that their 9 quarter relationship with cash collected recoveries is stable. Hence, to calculate the change in asset sales volume due to stress, the quarterly base asset sales are multiplied by the stress in cash recoveries.

Table 6-4 Historical Ratio of Asset Sales with Cash Collected Recoveries

| Time period | $\frac{\text{Asset sales}}{\text{Cash Collected Recoveries}}$ |
|---|---|
| 2008Q1-2010Q1 | 33.0% |
| 2014Q2-2016Q2 | 33.2% |

### B *Stressed Price Change*

To calculate the change in price due to stress, asset sales pricing for both stress and base scenarios provided by the Operations team are used. The average portfolio price in both scenarios is calculated by using a weighted average of different sale types. Pricing change is calculated using the below formula:

Equation 11

$$Stressed\ Asset\ Sales = Base\ Asset\ Sales \times \underbrace{\left\{\frac{Stressed\ Quarterly\ Recovery\ Forecast}{Base\ Quarterly\ Recoveries\ Forecast}\right\}}_{A} \times \underbrace{\left\{\frac{Portfolio\ Average\ StressPrice}{Portfolio\ Average\ Base\ Price}\right\}}_{B}$$

Attached is the Asset Sales Computation Methodology for CCAR 2017 Submission

Asset_Sales.xlsx

## NCL Calculation

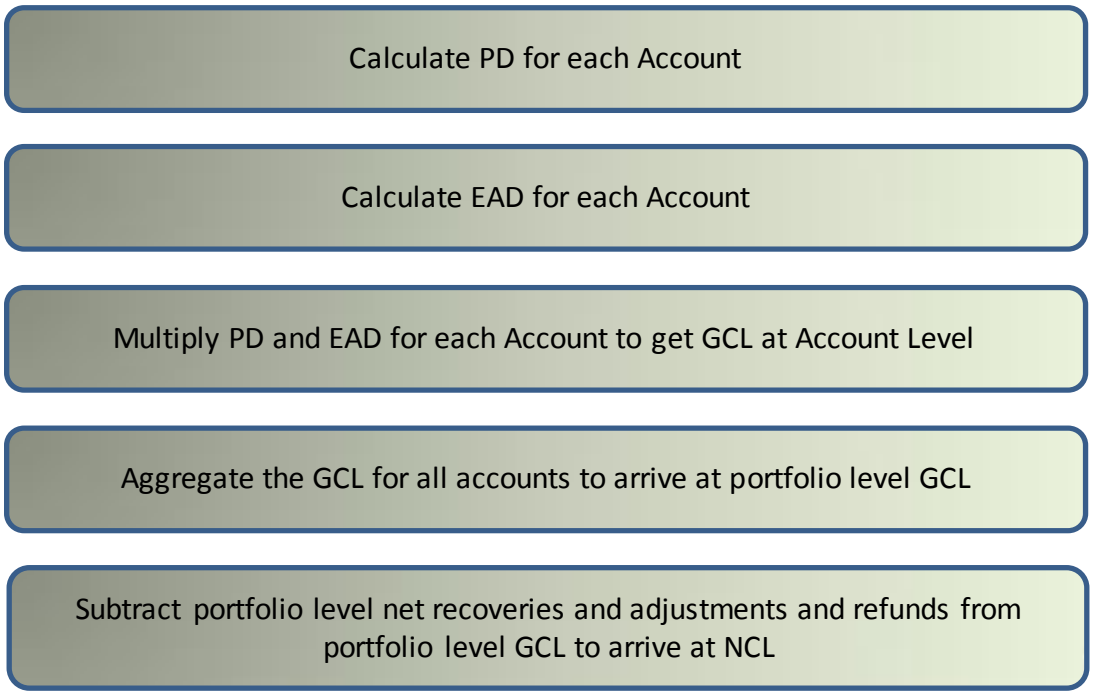
As mentioned earlier, NCL is derived by subtracting off recoveries from GCL.

Equation 12

$$NCL = GCL - Net\ Recoveries - Adjustment\ \&\ Refunds$$

Putting the pieces described above in detail, the flowchart for calculating NCL can be summarized as below

Figure 37 Overall $ NCL Estimation Process



The components used in calculation of NCL as follows:

## 1. GCL

**Equation 13**

$$GCL = PD*EAD$$

Predicted PD and EAD are derived out of the component models.

## 2. Net Recoveries

**Equation 14**

$$Net\ Recoveries = Cash\ Collected\ Recoveries - Asset\ Sales + Agency\ fees$$

## 3. Adjustment & Refunds

These are included in final NCL Calculation. However they are a contra neutral item to the P&L Post charge-offs, the unearned interest and fees (aka adjustments and refunds) are netted out of GCL dollars to determine the Net Credit Losses (NCL). The changes in trends for unearned (% of GCL) are more a result of the internal pricing and fee related policy changes. They are not impacted by the changes in economic environments.

## 4. Overlay in prior versions

Additionally, since 2010, Branded Cards has implemented more conservative forbearance policies, including elimination of pre-delinquent forbearance programs and tightening eligibility for the short-term forbearance program (UPP). The business collection policy team performed an analysis to estimate the impact of forbearance policy changes at a portfolio level NCL. Based on that analysis a small conservative adjustment will be applied for this.
In CCAR 2017 submission, the business applied the forbearance overlay for forecasting. This component is used only in the stress scenarios. For details please refer to the attachment below.

NA Cards - Forbearance Overlay

However, as mentioned above, since the business is using the regime shift indicators in the models, the business will evaluate to remove this overlay for CCAR 2018 submission. Details on the regime shift indicator is available in section 5.2.

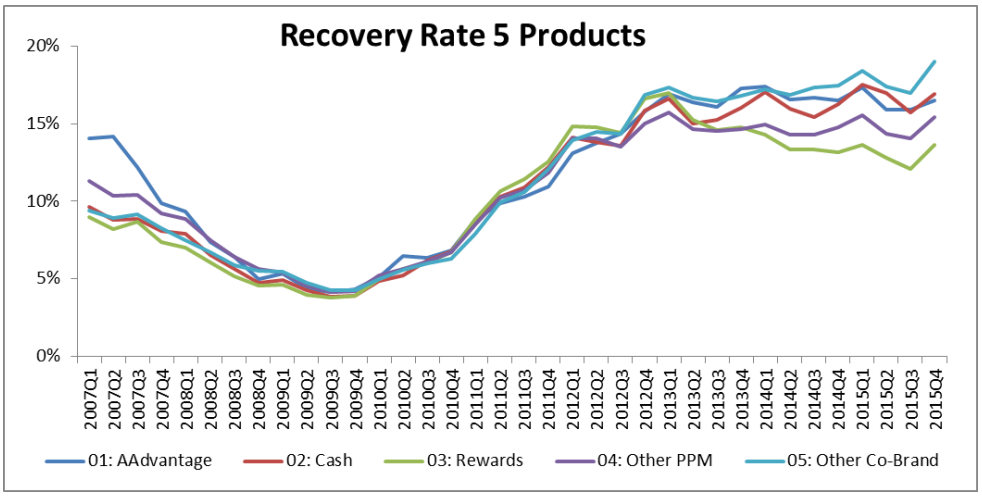
### 6.1.3 Model Segmentation

In the current submission, the Account Level PD and EAD models are developed at overall Branded Cards portfolio level. For CCAR 2017, initially, the Consumer cards and Business cards were planned to be modeled separately, however the pre-final models of consumer and business cards were very similar and hence these portfolios were combined to get one portfolio level model for Branded Cards. Additionally, the business cards portfolio is materially much smaller compared to that of consumer portfolio.

The account level models were evaluated for accuracy by back testing for several granular sub-segments viz. Portfolio, FICO and Delinquency status. For CCAR 2017 models, all the Back Test errors were well within the MRM thresholds. Models have been also back tested for each of the product segments and strong back test results further reinforce the fact that portfolio segmentation was not necessary. The detailed granular level back test has been discussed in Section 8.3.2.

Also, for CCAR 2017 and CCAR 2018, Branded Cards have developed recovery models on the entire portfolio. For CCAR 2016 Submission NA Branded Cards had successfully developed recovery models on 5 product portfolios as segments which were approved by MRM. The recent recovery rate charts for the 5 products are as below. As seen from Figure 11, the 5 recovery rates are almost overlapping in stress.

Figure 38 Recovery Rate - 5 Products




The final models submitted for the 5 products for CCAR 2016 submission are as below:

Table 6-5 CCAR 2016 Recovery Rate Models

| Segment | Var1 | Lag1 | Window1 | Trans1 | Coeff1 | Var2 | Lag2 | Window2 | Trans2 | Coeff2 |
|---|---|---|---|---|---|---|---|---|---|---|
| AA | UR | 9 | 12 | Avg Diff | -0.24 | HP | 0 | 12 | Avg Diff | 0.03 |
| Other CoBrand | UR | 9 | 12 | Avg Diff | -0.27 | HP | 0 | 12 | Avg Diff | 0.03 |
| Cash | UR | 9 | 9 | Avg Diff | -0.22 | HP | 0 | 12 | Avg Diff | 0.03 |
| Rewards | UR | 9 | 9 | Avg Diff | -0.20 | HP | 0 | 12 | Avg Diff | 0.03 |
| Other-PPM | UR | 6 | 12 | Avg Diff | -0.24 | HP | 0 | 12 | Avg Diff | 0.02 |

As seen from the table above, the independent variable transformations are very similar for all 5 products. Also, the variable coefficients are very similar, indicating that the 5 segments are not very different.

The recovery rate for the 5 products also failed the ANOVA test recommended in the MDDT. The ANOVA test determines if the segments are significantly different.

Table 6-6 ANOVA Test Results for Product Segmentation

| Start Time | End Time | Product Segmentation | F-Stat | Result |
|---|---|---|---|---|
| 2007Q1 | 2015Q4 | AA, Co-Brands, Cash, Other-PPM, Rewards | 0.5 | Fail |

Because of the above mentioned reasons, it was decided to drop the product segmentation for CCAR 2017. The same has been continued for CCAR 2018 submission. However, in the CCAR 2018 submission, cash recoveries from business cards have also been modeled along with consumer cards. Interaction of business data with maturation effect dummies (quarter since charge-off dummies) and HPI incorporates segmentation effect in the model. Business cards were not separately modeled due to lack of sufficient data points, hence combined with consumer cards and interaction variables were used to improve model prediction power.

- Was segmentation explored and if no segmentation is possible appropriate rationale/documentation should be provided?

Possibilities of segmentation were explored for both PD and EADF models, however based on the homogeneity of product groups, such segments were not producing a significant differentiation in model results. The back tests on the PD model, built on the overall portfolio, indicated highly accurate performance, well within the prescribed error rate threshold. The models were also back tested for logical sub-populations based on product, FICO; Month on book and delinquency. The back test performance was well within MRM threshold. This was further evidence to the fact that no portfolio segmentation was required. The detailed granular back test is discussed in given below:

PD Back Test

Table 6.1.7: PD Back Test by Product for Stress, Recent and Most Recent

| Period | AA | Cobrand | CASH | REWARDS | PPM | BUSINESS | Grand Total |
|---|---|---|---|---|---|---|---|
| Stress (13Q) | 0% | -9% | 9% | -3% | -1% | -4% | -1% |
| Recent (13Q) | 2% | 10% | 5% | 2% | -1% | 6% | 2% |
| Most Recent (9Q) | -5% | -2% | -1% | 4% | -6% | -1% | -4% |

Table 6.1.8: PD Back Test by FICO for Stress, Recent and Most Recent

| Period | <= 660 | 661-720 | 721+ | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | -2% | -6% | 10% | -1% |
| Recent (13Q) | 2% | 10% | -8% | 2% |
| Most Recent (9Q) | -3% | 3% | -16% | -4% |

Table 6.1.9: PD Back Test by Delinquency Status for Stress, Recent and Most Recent

| Period | Current | Delinquent | Grand Total |
|---|---|---|---|
| Stress (13Q) | -2% | 2% | -1% |
| Recent (13Q) | 3% | -1% | 2% |
| Most Recent (9Q) | -4% | -4% | -4% |

Table 6.1.10: PD Back Test by Month on Book Count for Stress, Recent and Most Recent

| Period | 1-24 | 25-60 | 61 + | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | -5% | 2% | 0% | -1% |
| Recent (13Q) | 0% | 2% | 4% | 2% |
| Most Recent (9Q) | -7% | -6% | 0% | -4% |

EAD Back test

Table 6.1.11: EAD Back Test by Product for Stress, Recent and Most Recent

| Period | AA | Cobrand | CASH | REWARDS | PPM | BUSINESS | Grand Total |
|---|---|---|---|---|---|---|---|
| Stress (13Q) | -2% | 0% | -2% | -1% | -3% | 3% | -1% |
| Recent (13Q) | 5% | 4% | 4% | 1% | 1% | 1% | 2% |
| Most Recent (9Q) | 1% | 2% | 1% | -1% | 0% | -1% | 0% |

Table 6.1.12: EAD Back Test by FICO for Stress, Recent and Most Recent

| Period | <= 660 | 661-720 | 721+ | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | 0% | -3% | -2% | -1% |
| Recent (13Q) | 1% | 0% | 7% | 2% |
| Most Recent (9Q) | 0% | -2% | 4% | 0% |

Table 6.1.13: EAD Back Test by Delinquency Status for Stress, Recent and Most Recent

| Period | Current | Delinquent | Grand Total |
|---|---|---|---|
| Stress (13Q) | -2% | 1% | -1% |
| Recent (13Q) | 2% | 3% | 2% |
| Most Recent (9Q) | -1% | 2% | 0% |

Table 6.1.14: EAD Back Test by Month on Book Count for Stress, Recent and Most Recent

| Period | 1-24 | 25-60 | 61 + | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | -5% | -3% | 0% | -1% |
| Recent (13Q) | -2% | 1% | 3% | 2% |
| Most Recent (9Q) | -5% | 0% | 2% | 0% |

GCL Back Test

Table 6.1.15: GCL Back Test by Product for Stress, Recent and Most Recent

| Period | AA | Cobrand | CASH | REWARDS | PPM | BUSINESS | Grand Total |
|---|---|---|---|---|---|---|---|
| Stress (13Q) | 4% | -3% | -2% | -8% | -9% | -7% | -6% |
| Recent (13Q) | 14% | 9% | 10% | 7% | 5% | 10% | 8% |
| Most Recent (9Q) | 6% | 5% | 6% | 3% | -1% | 3% | 2% |

Table 6.1.16: GCL Back Test by FICO for Stress, Recent and Most Recent

| Period | <= 660 | 661-720 | 721+ | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | -4% | -12% | -2% | -6% |
| Recent (13Q) | 6% | 12% | 6% | 8% |
| Most Recent (9Q) | 1% | 6% | -2% | 2% |

Table 6.1.17: GCL Back Test by Delinquency Status for Stress, Recent and Most Recent

| Period | Current | Delinquent | Grand Total |
|---|---|---|---|
| Stress (13Q) | -7% | 0% | -6% |
| Recent (13Q) | 11% | -2% | 8% |
| Most Recent (9Q) | 5% | -4% | 2% |

Table 6.1.18: GCL Back Test by Month on Book Count for Stress, Recent and Most Recent

| Period | 1-24 | 25-60 | 61 + | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | -8% | -8% | -5% | -6% |
| Recent (13Q) | 7% | 5% | 9% | 8% |
| Most Recent (9Q) | -2% | -4% | 6% | 2% |

- What tests were performed to confirm the segmentation scheme meets the modeling objective and/or generates better forecasting results?

  The back tests performed on the stress and recent sample were well within the expected thresholds. Given the accurate back test performance and the homogeneity of other possible portfolio segments, it was concluded that model objectives were satisfied even without considering any segmentation schema. The back tests carried out on sub-populations were adequate and within prescribed thresholds.

- What were the results of the segmentation tests?

  The back tests carried out on sub-populations were adequate and within prescribed thresholds. It was observed that all desirable attributes of segmentation were captured by the model developed on the overall portfolio. FICO, Product etc. based dummies and interaction terms with variables were adequate to capture sub-portfolio specific nuances. This was evident from the sub-population specific back tests which were within prescribed thresholds. Accordingly, no portfolio segmentation was deemed to be required for meeting model objectives

- How were the test results interpreted to refine the segmentation?

  As the models were not built on any segment, refining of segmentation was not required.

- What was the final segmentation scheme used?

  No segmentation has been used for Champion NCL models. The model was built on the overall portfolio.

- If segmentation is based in Business judgment, appropriate rationale must be provided to support the segmentation scheme?

  The decision to not have portfolio segmentation for the account level models was based on strong sub-population Back Test performance. No business judgment was involved.

- What is the evidence of segmentation optimality? How do the optimization criteria relate to the overall aim of the model?

  The champion model for CCAR 2018 submission has not considered any segmentation and has yet performed very well on overall portfolio as well as sub-population specific Back Tests. Additionally, products under the branded cards show consistent homogeneity and there weren't any evidence of strong segmentation requirements. FICO and Utilization are more appropriate as explanatory variables. The only other dimension that could have been leveraged was delinquency status and the same has been used along with FICO as indicators wherever applicable. Interaction terms for these variables have also been leveraged in model development

- Is the segmentation technique applied robust over time (i.e. is it invariant to the starting point)?

  No segmentation has been performed on the portfolio; accordingly robustness over time is accounted for.

- Are the segments internally homogeneous, and do they differ significantly between each other? What measure is used to determine homogeneity / heterogeneity of segments?

  As the portfolio is considered as a whole, internal homogeneity is maintained.

### 6.1.4 Model Variable Selection

Variable selection is a crucial step in model building. The objective of variable selection is to evaluate a large number of potential predictors in order to select the most predictive, stable and intuitive set of variables from among these.

Overall the variable selection process involves both business intuition and empirical analysis to ensure the key risk drivers are included and the final selected variables are meaningful from a business perspective and can logically and sufficiently explain movements in the dependent variable. The empirical analysis includes two major steps – bivariate analysis and multivariate analysis. The bivariate analysis was first performed to examine the relationship of the target variable with individual explanatory variable in order to ensure the strong and intuitive predictors are selected as the candidate variables to be further evaluated. The transformation is

applied only when necessary and appropriate. The multivariate analysis further examine the modeled relationship has a sufficient goodness of fit and each variables is still statistically significant and business intuitive given other variables presents. As a supplement to multivariate analysis, various model specifications are tested and compared with the final model.

The final model specification includes macroeconomic variables, behavioral variables, time from forecast date and time interactions with behavior variables. The variable selection process for each variable type is described below.

#### 6.1.4.1 *PD Model variable selection*

Based on existing internal research, academic papers, and industry practice, branded cards have generated long lists of factors that potentially explain different aspects of credit card loan performance. The variable selection process draws on the insights from this rich body of research and, more importantly, on the quantitative and qualitative properties of the explanatory variables.

Branded Cards has found that a good balance between business intuitions, economic reasoning, and statistical power provides a robust guide to effective variable selection.

The criteria for explanatory variable selection on a high level can be summarized as:

**Business Intuition / Economic Theory:** Variables should make good business sense and there should be an economic rationale for their use in credit cards loan modeling.

First, variables were excluded from the modeling candidate set based on an evaluation of their business meanings. Variables were dropped if they:

A) Have zero/constant value throughout
B) Have missing rate greater than 90%
C) Are character variables such as account id, region code, etc.

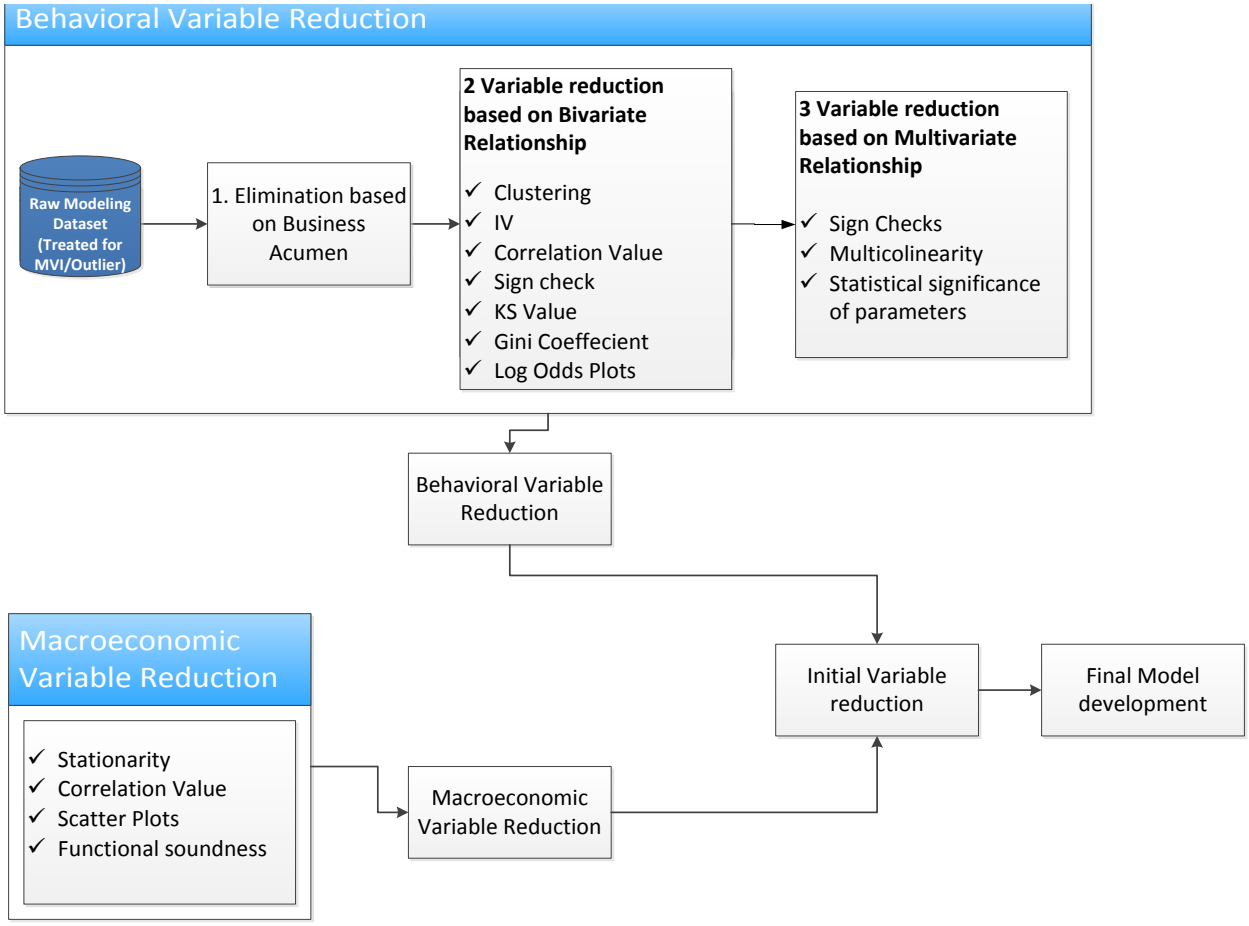
This step has been carried out as part of data quality checks to ensure that variables not meeting above mentioned criteria are dropped from the list considered for model development.

**Statistical Robustness:** Variables should (a) have coefficients whose direction (sign) are consistent with economic / business reasoning, and (b) are statistically significant and (c) do not introduce significant multi-collinearity as it may cause parameter estimates to be unstable.

**Other Aspects:** Variables should (a) have sufficient historical data availability, (b) are available in production for future use, and (c) preferably, have been in use for current and other contexts.

Following flowchart demonstrates process followed for variable selection:

Figure 39 : Variable reduction flowchart




The detailed variable selection process of the 2018 champion PD model is discussed below:

**Step 1: Compile a list of effects to be included in the model and a list of variables to represent these effects.**

They can be categorized into macro-economic effects, loan origination information, loan information at snapshot, and dynamic information.

This step involves the selection of the initial candidate variable pool of numeric and character variables. Table 6-19 lists the effects and corresponding explanatory variables for each effect.

Table 6-19 Model Effects and Illustrative Variables

| Effects | Explanatory variables |
|---|---|
| Macro effects | HPI, unemployment rate, US GDP change etc. |
| Loan origination information | Origination FICO, origination channel, product, credit line, debt to income ratio etc. |
| Loan information at snapshot | Refreshed FICO, Utilization, Balances, bureau attributes (number of trades, inquiries etc. ), Payment history, Delinquency history etc. |
| Loan dynamic information | Age of account, month since snapshot (duration), seasonality effect etc. |

Additionally, the loan origination and loan snapshot information containing variables can be categorized into several categories including Delinquency, Balances, Cash Related, Purchase, Payments, Inquiries, Fees, Bureau, and Utilization etc. The following attachment provides a detailed list and description of initial variables (corresponding to loan snapshot and loan origination effects) considered for modeling PD.

Table 6-20 Key Risk drivers and number of attributes

| Category | Count |
|---|---|
| # of Trades | 71 |
| Active | 5 |
| Balance | 74 |
| Credit Limit | 48 |
| Delinquencies | 64 |
| Derog/Public Record | 13 |
| Fees | 6 |
| GCL | 9 |
| Inquiries | 17 |
| Credit Age | 21 |
| Payment | 20 |
| Purchase/Sales | 8 |
| Utilization | 17 |
| Acquisition Variables | 83 |

5.1.1
Variable_Consolidated

## Step 2: Creation of Derived Variables

Several derived variables were created to enrich the pool of variables in consideration for selection. The objective of creation of these variables were to capture historical trends of many dimensions like delinquencies, payment behavior, balances, fees, cure rates etc. Additionally, loan origination information like product type, region, channel of acquisition etc. is captured as indicator variables. Certain policy changes which have directional impact on transitions are also identified as period indicators (for e.g. Card Act of Sept '09 etc.). The following attachment provides a detailed list and description of the derived variables created and leveraged in the model.

Derived_Vars.xlsx

## Step 3: Shortlisting variables based on descriptive statistics and Information Value

Once the PD modeling data is created, a generic descriptive statistics of the considered independent variables is obtained to assess the quality of data with respect to missing values. While assessing missing values, it is important to consider if the missing entry is system missing or condition missing. A system missing value is caused due to non-availability of any specific information corresponding to a record. For e.g. a missing Net Fraction Revolving Burden for an account can be a system missing value. However a condition missing value is one caused due to non-applicability of a condition to populate the variable corresponding to a record. For e.g. if we have a missing delinquent balance for a current account, the same can be considered as condition missing. Most condition missing values can be easily imputed. All variables with system missing values for greater than 30% of the records are dropped from the list of explanatory variables.

Subsequently, the information value of the explanatory (to be referred as independent variables henceforth) variables is obtained. The information value of a predictor X and the binary target Y is defined as,

Equation 15

$$IV = \sum_{k=1}^{k} (g_k - b_k)\,(\log \frac{g_k}{b_k})$$

Where $g_k$ and $b_k$ refers to the percentage of all goods and all bads corresponding to X = $X_k$ respectively. The IV statistic is appropriate for a predictor X with a modest number of levels,

typically under 20 with no zero cells. The IV value of the variable was used in conjunction with the proc varclus explained in the next step.

### Step 4: Shortlisting variables based on unsupervised clustering

Dimensionality reduction through unsupervised clustering is a common practice among modelers. PROC VARCLUS (algorithm in SAS) is extensively used for this purpose. The procedure helps find groups of variables that are most correlated among themselves and are as uncorrelated as possible with variables in other clusters. The algorithm is binary and divisive – all variables start as one cluster. If the second eigenvalue is above the current threshold (i.e. there is more than one dominant dimension) then the cluster is split. By default, PROC VARCLUS does a nonhierarchical version where variables can be reassigned to other clusters.

PROC VARCLUS is closely related to principal component analysis and can be used as a method for eliminating redundant dimensions. The reassignment of variables to clusters occurs in two phases. The first is a nearest component sorting phase, similar in principle to the nearest centroid sorting algorithms described by Anderberg (1973). In each iteration, the cluster components are computed and each variable is assigned to the component with which it has the highest squared correlation (SAS/STAT User's Guide, pages 1642-1643). The second phase involves a search algorithm in which each variable in turn is tested to see if assigning it to a different cluster increases the amount of variance explained. If a variable is reassigned during the search phase, the components of the two clusters involved are recomputed before the next variable is tested (SAS/STAT User's Guide, pages 1642-1643). Larger eigenvalue thresholds result in fewer clusters, and smaller thresholds (such as one or less) yield more clusters. To account for sampling variability, smaller values such as 0.7 have been suggested (Jackson 1991). In some cases where sampling variability is low, higher eigen values up to 0.9 can be considered.

To shortlist the variables after clustering, a combination of following considerations is taken into account –

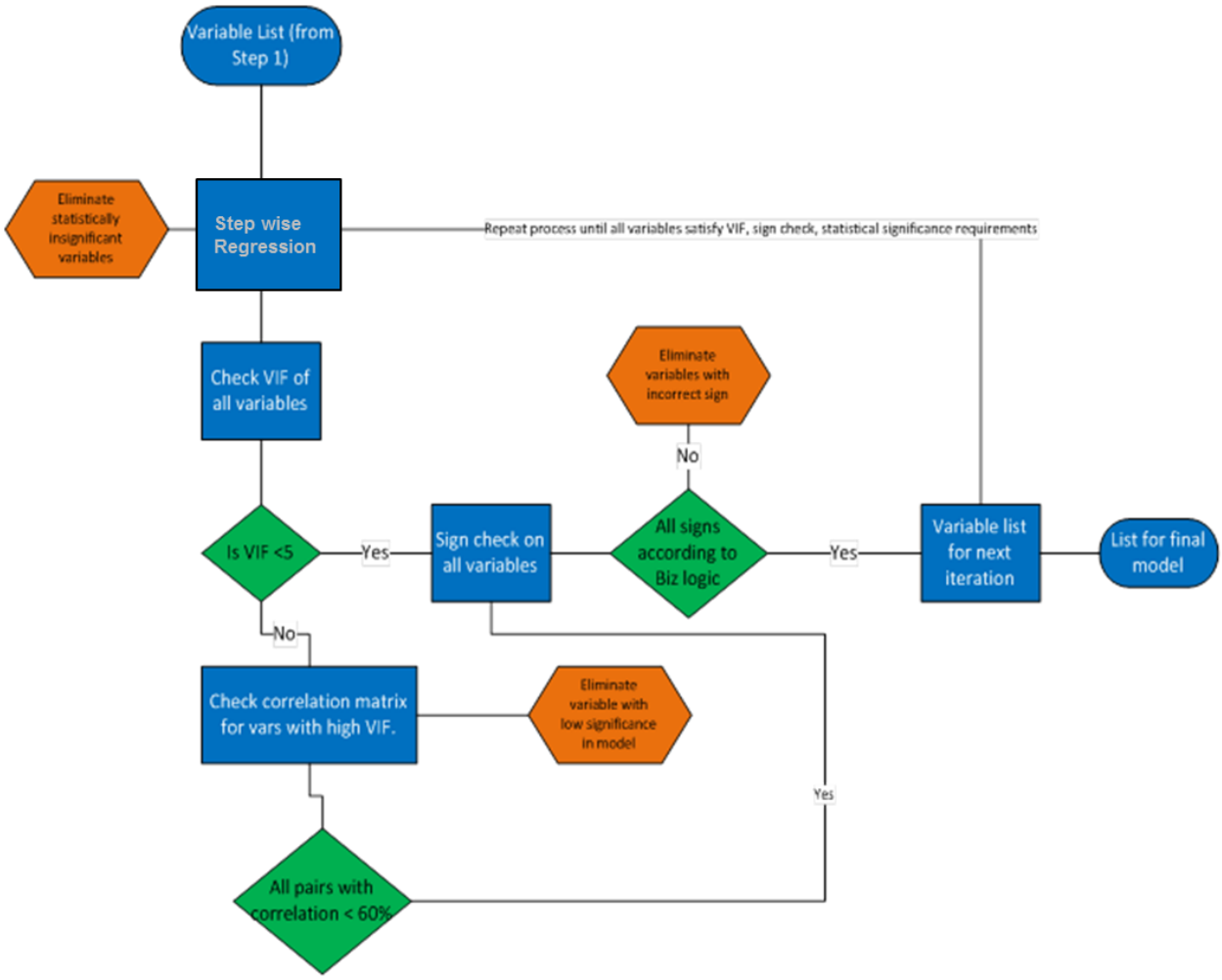
1. Variables with minimum (1-R Squared) Ratio is preferred
2. Variables with Maximum IV is preferred
3. Variables which make intuitive business / economic sense
4. Variables which are historically well populated / have been used in the past
5. High KS, Gini etc. as compared to other variables within the cluster

### Step 5: Shortlisting variables based on stepwise logistic regression

Selected variables from process mentioned till step 4 are then subjected to stepwise regression. A candidate list is shortlisted which demonstrates low multi-collinearity, high significance (p-value) in the model, correct sign with the dependent variable and business sense.

Multivariate variable selection is an iterative process. At this step number of variables is reduced to manageable number of 10-20 variables. Following flowchart summarizes the iterative process carried out to arrive at the final list of variables.

Figure 40 : Iterative process for variable reduction




## 5.1 Stepwise Regression

The variables selected after cluster and correlation analysis was used to build a logistic regression model. An iterative procedure as shown in flowchart above was followed. The process has been done separately for Quarter 1, 2 and 3-9 combined. The end result of this step was an unrefined "initial" model.

A description of key statistics and the associated mathematical construct is as below.

## 5.2 Multicollinearity Check

Multicollinearity refers to moderate to high inter correlation among the predictors. It poses a real problem for the researcher using multiple regressions for three reasons:

- It severely limits the size of R-Square, because the independent variables are going after much of the same variance on the prediction
- It makes determining the importance of a given independent variable difficult because the effects of the independent variables are confounded due to the correlation among them
- It increases the variances of the regression coefficients. The greater these variances, the more unstable will be the prediction equation

The following method was used for checking multicollinearity:

**Variance inflation Factor (VIF):**

The quantity $\frac{1}{(1-R_j^2)}$ is called the j$^{th}$ variance inflation factor, where $R_j^2$ is the squared multiple correlation for predicting the j$^{th}$ predictor from all other predictors. The variance inflation factor for a predictor indicates whether there is a strong linear association between it and all the remaining predictors. In general, any VIF in excess of 5 is an indication of high multicollinearity, and either variable deletion or an alternative to least squares estimation should be incorporated to remove multicollinearity. In the PD modeling, VIF value is maintained well below 2.5 quarterwise for majority of the attributes to minimize the impact of multicollinearity.

## 5.3 Variable coefficient significance and sign check:

In order to test statistical significance of each coefficient Wald Chi Square metric is tested. As shown in flowchart above, variables with high VIFs are identified. A cross correlation matrix is then utilized to obtain highly correlated pairs. Variable with lower Wald Chi Square value is dropped before next iteration. Description of wald chi square metric is as below:

A Wald test is used to test the statistical significance of each coefficient ($\beta$) in the model. The Wald Chi-Square test statistic is the squared ratio of the Estimate to the Standard Error (SE) of the respective predictor.

$$W = [\beta / SE_{\beta}]^2$$

Chi-Square and Pr > ChiSq are the test statistics and p-values, respectively, testing the null hypothesis that an individual predictor's regression coefficient is zero, given the other predictor variables are in the model. The Chi-Square value follows a central Chi-Square distribution with degrees of freedom given by DF, which is used to test against the alternative hypothesis that the Estimate is not equal to zero. The probability that a particular Chi-Square test statistic is as extreme as, or more so, than what has been observed under the null hypothesis is defined by Pr>ChiSq. The higher Wald Chi-Square value of a variable, the better it is.

For the PD model, the square of the t value is used as the Wald chi square value.

Further, the variables are tested for correct sign (based on business intuition) of coefficients in the logistic regression output. In case an opposite sign is observed, the same is dropped from further iterations.

Following files demonstrate iterative process carried out for selection of variable list:

PD_variable_selection .xlsx

### Step 6: Shortlisting macro-economic variables and their transformations

Macroeconomic variables in the model capture the stress impact of worsened macroeconomic conditions on portfolio losses. These economic impacts can be specified to the state level which further improves the granularity of the model and the accuracy of stress testing at the account level.

The process of variable selection began with shortlisting key macro-economic factors relevant to the business.

FRB (Federal Reserve Bureau) provides projected values for 28 variables. Sixteen of these variables capture US economic activity including asset prices, interest rates, and financial market levels. The sixteen US variables are listed below.

**Table 6-21: Variables Provided by the Federal Reserve Bank which Describe the US Economy**

| # | Variable Name | Definition |
|---|---|---|
| 1 | BBBCY | BBB Corporate Yield |
| 2 | CPI | Consumer Price Index Inflation Rate |
| 3 | CREPI | Commercial Real Estate Price Index |
| 4 | DJ | Dow Jones Market Index |
| 5 | FYTY | 5 - Year Treasury Yield |

| | | |
|---|---|---|
| 6 | HPI | Housing Price Index |
| 7 | MR | Mortgage Rate |
| 8 | MVI | Market Volatility Index |
| 9 | NDIG | Nominal Income Growth |
| 10 | NGDP | Nominal GDP growth |
| 11 | PR | Prime Rate |
| 12 | RDIG | Real disposable Income Growth |
| 13 | RGDP | Real GDP Growth |
| 14 | TMTR | 3 - month Treasury Rate |
| 15 | TYTR | 10 - Year Treasury Yield |
| 16 | UR | Unemployment Rate |

The other 12 FRB variables are composed of three different economic variables for four separate regions. The three variables are GDP, Inflation, and the US exchange rate with the region. The four regions as classified by DFAST are Euro-area, Developing Asia, Japan, and the UK. The variables which describe the combination of the regions and economic variables are listed below.

- Euro Area Real GDP Growth
- Euro Area Inflation
- Euro Area Bilateral Dollar Exchange Rate (USD/euro)
- Developing Asia Real GDP Growth
- Developing Asia Inflation
- Developing Asia Bilateral Dollar Exchange Rate (F/USD, index)
- Japan Real GDP Growth
- Japan Inflation
- Japan Bilateral Dollar Exchange Rate (yen/USD)
- U.K. Real GDP Growth
- U.K. Inflation
- U.K. Bilateral Dollar Exchange Rate (USD/pound)

Since the portfolio that we were modeling is made up of US customers, the above 12 variables were removed from consideration. The 16 variables that described the US economy were kept.

The GCRM (Global Country Risk Management) is Citi's internal macroeconomic analysis group. Similar to the FRB, the GCRM provides macroeconomic analysis for different countries and regions around the world. The information obtained from this analysis is put into variables and made available for modeling. Of the hundreds of variables provided by GCRM, the variables listed below were believed to be the variables that best captured the macroeconomic factors which were likely to affect the loss rates of the portfolio.

Table 6-22: Relevant Subset of Variables Provided by GCRM

| # | Variable | Definition |
|---|---|---|
| 1 | ConConfidence | US Consumer Confidence Index |
| 2 | ConSentiment | Monthly US Consumer Sentiment as measured by the University of Michigan |
| 3 | ConSpend | US Output Consumer Spending |
| 4 | CredBal | US Loans Credit Card Balances |
| 5 | EquitySP | US S&P Equity Price level |
| 6 | FedCredBal | US Credit Card Balances as measured by FRB |
| 7 | NewHouse | US Prices New Housing Starts |
| 8 | NonRevCred | Non Revolving US Loans Consumer Credit |
| 9 | PDI | US Income Personal Disposable Income |
| 10 | PMI | US Purchasing Managers Index |
| 11 | PersonalInc | US Income Personal Income |
| 12 | RS | US Output Retail and Food Services Sales |
| 13 | Shiller | Shiller HPI |
| 14 | Sprd10yr3yr | Spread 10 Year CMT and 3 Year CMT |
| 15 | Sprd10yr5yr | Spread 10 Year Minus 5 Year CMT |
| 16 | Sprd10yrFFund | 10 Year Treasury Rate Minus Fed Funds Rate |
| 17 | TotlCred | Total US Consumer Credit |
| 18 | UnemInsClaim | US Unemployment Insurance Claims |
| 19 | VehiSale | Number of Light Vehicle Sales in the US |

The choice of FRB variables is an important factor in building a model which has the ability to accurately predict the macroeconomic stress case, correctly captures the relationship between the probability of default and the macroeconomic environment, and is parsimonious. Using these criteria, each variable was considered, and the variables which did not meet these criteria were dropped as discussed below.

The first set of variables examined is the set of market indices. These include DJ (Dow Jones) and MVI (Market Volatility Index). Of the two market variables, MVI was removed because it was the more volatile.

**Table 6-23: Market Indices**

| Variable | Decision |
|---|---|
| MVI | Dropped |
| DJ | Kept |

The second set of variables to be examined is the set of real-estate indices: CREPI (Commercial Real Estate Price Index), and HPI (Housing Price Index). Of the two real estate variables above, HPI was determined to be the most relevant to our Cards portfolio since the real-estate which are tracked by HPI are owned by households and not businesses. Given this consideration, CREPI was removed.

**Table 6-24: Real Estate Indices**

| Variable | Decision |
|---|---|
| CREPI | Dropped |
| HPI | Kept |

MR (Mortgage rate) and (Prime Rate) were examined next. Since the portfolio being examined was a credit card portfolio and not a mortgage portfolio, MR was removed from consideration. PR (Prime Rate) was removed because it had inconsistent correlations through in-cycle data.

**Table 6-25 Prime Rate and Mortgage Rate**

| Variable | Decision |
|---|---|
| MR | Dropped |
| PR | Dropped |

The next category of variables considered was the group of bond yields and rates. This set includes BBBCY (Yield on BBB rated corporate bonds), TMTR (Three Month Treasury Rate), TYTR (Ten Year Treasury Yield) and FYTY (Five Year Treasury Yield). We begin by examining the influences of BBBCY on PD. The correlation between PD and BBBCY was low, and the mechanism by which BBBCY could be used to explain PD was indirect and in most cases had conflicting signs. As a result, BBBCY was dropped from consideration for our models. Having inconsistent correlations through in-cycle data, TMTR, TYTR, and FYTY were also removed from consideration for the model.

**Table 6-26: Bond Yield Rates**

| Variable | Decision |
|---|---|
| BBBCY | Dropped |
| TMTR | Dropped |
| TYTR | Dropped |
| FYTY | Dropped |

The next step grouped together the two disposable income variables: NDIG (Nominal Disposable Income Growth), RDIG (Real Disposable Income Growth). The difference between these two is that RDIG is adjusted to remove inflation, and NDIG isn't. In order to ensure that the impact of each variable is correctly captured, RDIG was chosen to represent income growth and NDIG was dropped. If NDIG weren't removed from the model, it wouldn't be possible to properly differentiate the impact of Disposable Income from Inflation.

**Table 6-27: Disposable Income Variables**

| Variable | Decision |
|---|---|
| NDIG | Dropped |
| RDIG | Dropped |

RGDP (Real Gross Domestic Product) and NGDP (Nominal Gross Domestic Product) are comparable in the same way that NDIG and RDIG are. With the case of RGDP and NGDP, NGDP captures inflation in its GDP measurement while RGDP removes it. So that the impact of GDP could be isolated, and its impact more accurately measured, NGDP was removed.

CPI was removed for because it had conflicting signs across different regions. Specifically, CPI correlated with improved performance in the US, while in other areas outside of the US, it was negatively correlated. Unemployment was highly correlated with PD. As a result it was kept.

**Table 6-28: Reduced List of FRB Variables**

| # | Variable | Definition |
|---|---|---|
| 1 | DowJones | Dow Jones Industrial |
| 2 | RGDP | Gross Domestic Product |
| 3 | HPI | Housing Price Index |
| 4 | RDIG | US Real Disposable Income |
| 5 | UR | US Unemployment Rate |

The variables provided by the FRB are the variables which are later used in stress testing. The FRB variables are also leveraged to create the GCRM variables. As a result, the FRB variables were preferred to GCRM variables. Therefore, where possible, the GCRM variables were be replaced by FRB variables. To facilitate this effort, a correlation matrix which compared the FRB variables with several transformations of each GCRM variable was created. The transformations used are explained below.

### Variable Transformation

For each macroeconomic variable the mean, percentage change and the ratio over a fixed time period were considered. There were five time periods ranging from 3 to 15 months that were considered for this calculation. Additionally, the lagged values of each of these transformations up to 12 months were also considered. This resulted in 75 transformations of each of the 20 variables, making the total number of variables and transformations 1500.

### Variable Reduction

For the GCRM variables, the initial reduction begins with performing a correlation analysis of each set of variables with PD. Within each set, the transformation with the highest absolute correlation with PD was kept and the other 74 variables in the set were dropped. The correlation of PD with the five FRB variables was also taken, and they are shown below along with the best correlation of each GCRM set. The first five variables in the list are the FRB variables.

**Table 6-29: Highest Absolute Correlation of GCRM Variables and FRB Variables with PD**

| # | Variable | Definition | Max |
|---|---|---|---|
| 1 | DowJones | Dow Jones Industrial | 0.84 |
| 2 | GDP | Gross Domestic Product of the US as provided by the Bureau of Economic Analysis | 0.77 |
| 3 | HPI | Housing Price Index Annualized number of residential buildings that were sold during the previous month, excluding new construction | 0.90 |
| 4 | RDI | US Real Disposable Income | 0.62 |
| 5 | UR | US Unemployment Rate | 0.90 |
| 6 | ConConfidence | US Consumer Confidence Index | 0.81 |
| 7 | ConSentiment | Monthly US Consumer Sentiment as measured by the University of Michigan | 0.77 |
| 8 | ConSpend | US Output Consumer Spending | 0.84 |
| 9 | CredBal | US Loans Credit Card Balances | 0.85 |
| 10 | EquitySP | US S&P Equity Price level | 0.86 |
| 11 | NewHouse | US Prices New Housing Starts | 0.79 |
| 12 | NonRevCred | Non Revolving US Loans Consumer Credit | 0.79 |
| 13 | PDI | US Income Personal Disposable Income | 0.61 |

| | | | |
|---|---|---|---|
| 14 | PMI | US Purchasing Managers Index | 0.71 |
| 15 | PersonalInc | US Income Personal Income | 0.67 |
| 16 | RS | US Output Retail and Food Services Sales | 0.83 |
| 17 | RateFedFunds | Fed Funds Rate | 0.76 |
| 18 | Shiller | Shiller HPI | 0.89 |
| 19 | Sprd10yr3yr | Spread 10 Year CMT and 3 Year CMT | 0.71 |
| 20 | Sprd10yr5yr | Spread 10 Year Minus 5 Year CMT | 0.78 |
| 21 | Sprd10yrFFund | 10 Year Treasury Rate Minus Fed Funds Rate | 0.71 |
| 22 | TotlCred | Total US Consumer Credit | 0.92 |
| 23 | UnemInsClaim | US Unemployment Insurance Claims | 0.94 |
| 24 | VehiSale | Number of Light Vehicle Sales in the US | 0.95 |

If a GCRM variable had a high absolute correlation with an FRB variable, above .6, the GCRM variable was dropped. The table below shows the variables dropped at this stage and the correlation with the FRB variable which could replace them.

**Table 6-30: GCRM Variables Dropped and the FRB Variable Which Could be substituted**

| # | Variable | Definition |
|---|---|---|
| 1 | ConConfidence | Highly correlated with UR (-0.75332) |
| 2 | ConSentiment | Highly correlated with UR (-0.63042) |
| 3 | ConSpend | Highly correlated with RDI (0.99909) |
| 4 | CredBal | Not Provided by Fed; Highly correlated with HPI (0.56898) |
| 5 | EquitySP | Highly correlated with DowJones (0.98967) |
| 6 | FedCredBal | Highly correlated with HPI (0.69999) |
| 7 | NonRevCred | Highly correlated with RDI (0.96123) |
| 8 | PDI | Highly correlated with RDI (0.99484) |
| 9 | PMI | Not Provided by Fed; Highly correlated with GDP (0.56391) |

| # | Variable | Definition |
|---|---|---|
| 10 | PersonalInc | Highly correlated with RDI (0.995) |
| 11 | RateFedFunds | Not Provided by Fed; Highly correlated with RatePrime (0.98722) |
| 12 | RevCred | Highly correlated with RDI (0.97291) |
| 13 | Shiller | Highly correlated with RDI (0.97291) |
| 14 | Sprd10yr3yr | Highly correlated with RatePrime (-0.80139) |
| 15 | Sprd10yr5yr | Highly correlated with RatePrime (-0.82424) |
| 16 | Sprd10yrFFund | Highly correlated with RatePrime (-0.64741) |
| 17 | TotlCred | Highly correlated with RDI (0.98934) |
| 18 | UnemInsClaim | Highly correlated with UR (0.69466) |
| 19 | VehiSale | Highly correlated with UR (-0.76921) |

For this final list of variables, NewHouse was dropped because it was similar to HPI. Though DJ was the better of the financial market variables, it was later dropped because it was too volatile. The remaining variables are up to this point is listed below.

**Table 6-31: Remaining Variables Considered**

| Definition |
|---|
| Gross Domestic Product of the US as provided by the Bureau of Economic Analysis |
| Housing Price Index Annualized number of residential buildings that were sold during the previous month, excluding new construction |
| US Real Disposable Income |
| US Unemployment Rate |

All of the above variables were tried in our models. HPI and UR were the two most significant variables. Once HPI and YR were in the model, neither RDI nor GDP improved the accuracy of the model, or improved model stability.

As per the above findings, Unemployment Rate and House Price Index were considered as candidate macro-economic variables. Based on historical evidence, known stationarity

considerations, regulatory preference and industry best practices the following transformations were tried –

1. Transformation 1 (t0) – Log average
2. Transformation 1 (t1) – Log difference
3. Transformation 1 (t2) – Average
4. Transformation 1 (t3) – Difference
5. Transformation 1 (t4) – Ratio

The above transformations were observed over particular time periods or windows. Windows of 3, 6, 9 and 12 months were considered. Hence an average transformation (t2) over a window 9 (w9) would essentially mean a variable average observed over 9 months. Further, the modelers considered the following lagged transformations

1. Lag 0 month (lo)
2. Lag 3 months (l3)
3. Lag 6 months (l6)
4. Lag 9 months (l9)
5. Lag 12 months (l12)

No further lags were considered as any more lags would have significantly reduced the impact of the macro-economic shocks on the model. Also extending the above logic of naming transformations, an HPI_l6_w9_t2 variable would mean an average transformation of HPI variable observed over 9 months when lagged by 6 months.

These transformed variables along with raw variables were independently tested for correlation with modeled transitions and a set of macro-economic variables were shortlisted. For the purpose of correlation analysis, three different time periods were considered for calculating the correlations – Overall (Jan'07 to June'16) , Recent (April'14 to June'16) and Stress (Jan'08 to Mar'11). Macroeconomic variables which had a high value of correlation along with consistent and intuitive signs were prioritized.

To further shortlist the variables, the following considerations were kept in view-

1. For comparable results, raw transformations like raw difference and averages were preferred over log transforms
2. For comparable results, contemporaneous variables were preferred
3. For HPI, only log difference (t1) and ratio (t4) was preferred as other transforms have known stationarity issues
4. For US GDP growth rate only raw average (t2) transformation is business intuitive

Hence a combination of correlation analysis and the above mentioned considerations were leveraged to arrive at the final macro-economic variable shortlist for each modeled transition. The attachment below provides results of macro-economic correlation analysis.

PD_Correlation_macro_Vars_Acct_Level.xls

The attachment below provides results of correlation of macroeconomic variable across various FICO bands and portfolio.

PD_Correlation_Interaction_Analysis.xlsx

**Step 6: Shortlisting variables based on bi-variate and multi-variate analysis**

Post the credit quality and macro-economic variable shortlist the following steps are taken to arrive to the final set of explanatory variables for each modeled transition

1. Tested candidate variables (raw / transformed) on the development sample in logistic regression.
2. Carefully reviewed the regression output and removed variables that are not statistically significant (p-value >0.05, starting from the ones with the smallest chi-square. Modelers ensured that they excluded only one insignificant variable at a time.
3. Re-estimated the logistic regression based on the remaining candidate variables and performed several iterations
4. Excluded variables that are statistically significant (p-value <0.05) but with counter-intuitive signage of the coefficients.
5. Performed bi-variate analysis of the variables and checked if the bi-variate distribution is reflective of the business intuition and in sync with co-efficient signage
6. Excluded raw variables that breach a variance inflation factor (VIF) > 5 threshold in the collinearity test (VIF test).

Tested the selected variables for stability and robustness, i.e. the signage of the coefficients should be the same if the model is re-estimated excluding 1 year of development data. Variables showing opposite effect or becoming statistically insignificant or experiencing large change of coefficients were dropped.

#### 6.1.4.2 *EAD Model variable selection*

The variable selection for the EADF models continues to draw insights from the rich body of research and from the quantitative and qualitative properties of the explanatory variables.

Branded Cards has found that a good balance between business intuitions, economic reasoning, and statistical power provides a robust guide to effective variable selection.

The variable selection process was done separately for quarter1, quarter2 and later quarters 3-13, as per the recommendation of the functional validation team. This enables the model to capture the change in predictive power of a variable overtime, which is an integral part of the discrete time framework.

The criteria for explanatory variable selection on a high level can be summarized as:

**Business Intuition / Economic Theory:** Variables should make good business sense and there should be an economic rationale for their use in credit cards loan modeling.

**Statistical Robustness:** Variables should (a) have coefficients whose direction (sign) are consistent with economic / business reasoning, and (b) are statistically significant and (c) do not introduce significant multi-collinearity as it may cause parameter estimates to be unstable.

**Other Aspects:** Variables should (a) have sufficient historical data availability, (b) are available in production for future use, and (c) preferably, have been in use for current and other contexts.

The detailed variable selection process of the 2018 champion EADF models is discussed below:

Step 1 and Step 2 for the EADF model variable selection is the same as that of the PD model and the previous section can be referred.

### Step 3: Shortlisting variables based on descriptive statistics, clustering and correlation analysis

Once the account level data is created, a generic descriptive statistics of the considered independent variables is obtained to assess the quality of data with respect to missing values, special values etc.

The EADF models are estimated using Ordinary Least Square estimation technique. Accordingly, unlike PD models, information value for variable selection is not applicable. The independent variable correlations with the dependent variable (EADF) are computed. Based on the correlation values, a shortlist of the independent variables can be obtained.

**Cluster Analysis**

Dimensionality reduction through unsupervised clustering is a common practice among modelers. PROC VARCLUS (algorithm in SAS) is extensively used for this purpose. The procedure helps find groups of variables that are most correlated among themselves and are as uncorrelated as possible with variables in other clusters. The algorithm is binary and divisive – all variables start as one cluster. If the second eigenvalue is above the current threshold (i.e. there is more than one dominant dimension) then the cluster is split. By default, PROC VARCLUS does a nonhierarchical version where variables can be reassigned to other clusters.

PROC VARCLUS is closely related to principal component analysis and can be used as an alternative method for eliminating redundant dimensions. The reassignment of variables to clusters occurs in two phases. The first is a nearest component sorting phase, similar in principle to the nearest centroid sorting algorithms described by Anderberg (1973). In each iteration, the cluster components are computed and each variable is assigned to the component with which it has the highest squared correlation (SAS/STAT User's Guide, pages 1642-1643). The second phase involves a search algorithm in which each variable in turn is tested to see if assigning it to a different cluster increases the amount of variance explained. If a variable is reassigned during the search phase, the components of the two clusters involved are recomputed before the next variable is tested (SAS/STAT User's Guide, pages 1642-1643). Larger eigenvalue thresholds result in fewer clusters, and smaller thresholds (such as one or less) yield more clusters. To account for sampling variability, smaller values such as 0.7 have been suggested (Jackson 1991). In some cases where sampling variability is low, higher eigen values up to 0.9 can be considered.

To shortlist the variables after clustering, a combination of following considerations is taken into account –

1. Variables with minimum (1-R Squared) Ratio is preferred
2. Variables with Maximum absolute correlation is preferred
3. Variables which make intuitive business / economic sense
4. Variables which are historically well populated / have been used in the past

The attached document at the end of this section provides the output of the Varclus procedure for the EADF model and highlights the shortlisted variables from this process.

***Step 4: Variable Reduction based on Multivariate Analysis***

Selected variables from univariate selection process (correlation and clustering) were then subjected to stepwise regression. A candidate is shortlisted which demonstrates low multi-collinearity, high significance (p-value) in the model, and correct sign with the dependent variable and makes business sense.

Multivariate variable selection is an iterative process. This is similar to PD, with the difference being stepwise Linear regression is done for EADF whereas, it is stepwise Logistic regression for PD.

**Stepwise Regression**

The variables selected after cluster and correlation analysis was used to build a regression model using stepwise variable selection. Stepwise regression is a semi-automated process of building a model by successively adding or removing variables based solely on the t-statistics

(explained below) of their estimated coefficients. An iterative procedure as shown in **Error! eference source not found.** above was followed. This process included multi collinearity check where the variables are checked for VIF values and all the variables with VIF higher than 5 are removed from the selection process in an iterative manner. The process has been done separately for Quarter 1, 2 and 3-9 combined. The end result of this step was an unrefined "initial" model.

**T- test:**

Coefficient significance was tested using P-values and t value tests.

The regression line that SAS calculates from the data is an estimate of a theoretical line describing the relationship between the independent variable ($X$) and the dependent variable ($Y$). A simple linear regression analysis is used to develop an equation (a linear regression line) for predicting the dependent variable given a value ($x$) of the independent variable. The regression line calculated by SAS is

$$Y = a + bx$$

Where, $a$ and $b$ are the least - squares estimates of α and β.

The null hypothesis that there is no predictive linear relationship between the two variables is that the slope of the regression equation is zero. Specifically, the hypotheses are:

$$H_0 : \beta = 0$$
$$H_a : \beta \neq 0$$

To **determine** whether the difference is statistically significant, the **t-test** calculates a **t**-value. (The p-value is obtained directly from this **t**-value.)

Equation 6.1.16

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

The probability that a particular t-test statistic is as extreme as, or more so, than what has been observed under the null hypothesis is defined by Pr > |t| value. The higher t-value of a variable, the better it is.

A low $p$ -value for this test (say, less than 0.05) indicates significant evidence to conclude that the slope of the line is not 0 — that is, knowledge of $X$ would be useful in predicting $Y$, thus not accepting the null hypothesis.

**Business intuition and Sign Checks:**

All variables which do not make business sense or do not possess the consistent sign (i.e. with the sign of their coefficients in the linear regression model) based on business intuition are removed from the model.

Following files demonstrate iterative process carried out for selection of variable list:

CCAR_Business_EADF
_Var_Reduction_2018.

### Step 5: Shortlisting macro-economic variables and their transformations

The macro- economic variable selection process is very similar to that of PD models (described earlier). The variable reduction was done for the benefit of both the EADF and PD models, however only the correlation between the macroeconomic variables and PD were considered in the variable reduction process. The reason for this is that PD is significantly more sensitive to macroeconomic shocks than EADF. Therefore, it was thought prudent to have the reduction process be guided by the relationship to PD as opposed to EADF. Any macro-economic variable that did not show adequate relevance for PD is least likely to be relevant and significant with respect to EADF.

Attached document provides the macro-economic correlations with EADF.

EADF_Correlation_Ma
cro.xlsx

### Step 6: Shortlisting variables based on bi-variate and multi-variate analysis

Following the credit quality and macro-economic variable shortlist the following steps are taken to arrive to the final set of explanatory variables for each modeled transition

1. Tested candidate variables (raw / transformed) on the development sample in OLS regression.
2. Carefully reviewed the regression output and removed variables that are not statistically significant (p-value >0.05), starting from the ones with the smallest chi-square. Modelers ensured that they excluded only one insignificant variable at a time.
3. Re-estimated the linear regression based on the remaining candidate variables and performed several iterations

4. Excluded variables that are statistically significant (p-value <0.05) but have counter-intuitive signage of the coefficients.
5. Performed bi-variate analysis of the variables and checked if the bi-variate distribution is reflective of the business intuition and in sync with co-efficient signage
6. Excluded raw variables that breach a variance inflation factor (VIF) > 5 threshold in the collinearity test (VIF test).

Tested the selected variables for all assumptions with respect to OLS models and for stability and robustness, i.e. the signage of the coefficients should be the same if the model is re-estimated excluding 1 year of development data. Variables showing opposite effect or becoming statistically insignificant or experiencing large change of coefficients were dropped.

#### 6.1.4.3 *Recovery Model variable selection*

The business is modeling only the cash collected recoveries by the recovery model. The cash collected recoveries are mostly driven by the maturation effect and the macroeconomic factors. The variables selected for a model should be acceptable from business point of view and intuitions, economic reasoning, and statistical power to provide a robust model.

The criteria for explanatory variable selection on a high level can be summarized as:

**Business Intuition / Economic Theory:** Variables should make good business sense and there should be an economic rationale for their use in credit cards loan modeling.

**Statistical Robustness:** Variables should (a) have coefficients whose direction (sign) are consistent with economic / business reasoning, and (b) are statistically significant and (c) do not introduce significant multi-collinearity as it may cause parameter estimates to be unstable.

**Other Aspects:** Variables should (a) have sufficient historical data availability, (b) are available in production for future use, and (c) preferably, have been in use for current and other contexts.

The detailed variable selection process of the 2018 challenger Recovery model is discussed below:

Step 1 and Step 2 for the recovery model variable selection is the same as that of the PD model and section 6.1.4.1 can be referred.

### Step 3: Shortlisting macro-economic variables and their transformations

The macro-economic variable selection process is very similar to that of PD models (described earlier). The variable reduction was done for the benefit of the EADF, PD and Recovery models, however only the correlation between the macroeconomic variables and PD were considered in the variable reduction process. The reason for this is that PD is significantly more sensitive to macroeconomic shocks than EADF or Recovery Rate and has a greater impact on the loss

numbers compared to the recovery model. Recovery rate is more driven by the maturation effects compared to the macroeconomic factors. Therefore, it was thought prudent to have the reduction process be guided by the relationship to PD as opposed to Recovery. Any macro-economic variable that did not show adequate relevance for PD is least likely to be relevant and significant with respect to Recovery.

The recovery rate model is estimated using Ordinary Least Square estimation technique. As a result unlike PD information value of variables is not applicable. The correlation between dependent variable and the independent variables is used for the initial shortlisting of predictor variables. Attached document provides the macro-economic correlations with Recovery rate.

In addition to having high correlation with the dependent variable, the macroeconomic variables should also pass the stationarity test for inclusion in the model.

### Step 4: Shortlisting variables based on bi-variate and multi-variate analysis

Following the credit quality and macro-economic variable shortlist the following steps are taken to arrive to the final set of explanatory variables for each modeled transition

1. Tested candidate variables (raw / transformed) on the development sample in OLS regression.
2. Carefully reviewed the regression output and removed variables that are not statistically significant (p-value >0.1), starting from the ones with the smallest t-value. Modelers ensured that they excluded only one insignificant variable at a time.
3. Re-estimated the linear regression based on the remaining candidate variables and performed several iterations
4. Excluded variables that are statistically significant (p-value <0.1) but have counter-intuitive signage of the co-efficients.
5. Excluded raw variables that breach a variance inflation factor (VIF) > 5 threshold in the collinearity test (VIF test).

Tested the selected variables for all assumptions with respect to OLS models and for stability and robustness, i.e. the signage of the coefficients should be the same if the model is re-estimated excluding 1 year of development data. Variables showing opposite effect or becoming statistically insignificant or experiencing large change of coefficients were dropped.

### Step 5: Discrete time regression modeling

Proc reg in SAS is used to run a regression analysis to establish a relationship between recovery rates and the independent variables capturing the life cycle effect, seasonality and macro-economic impact. The business has used the backward selection method to ensure that

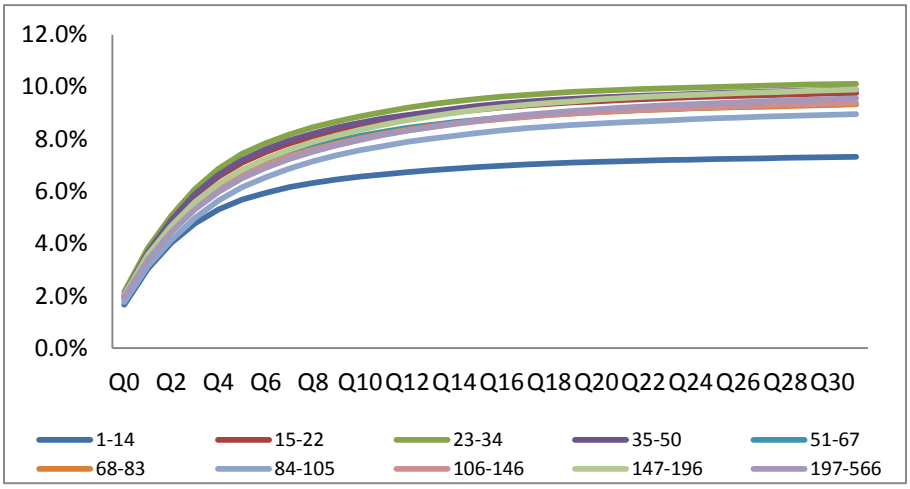
statistically significant variables are selected with confidence interval of 95%. This was done for different macro-economic transformations.

### Step 6: Additional Variable Exploration

Based on the feedback of Independent Risk during CCAR 2017 submission that the recovery model suffers from missing variable bias due to absence of behavioral attributes in the model, the model development team explored a list of behavioral attributes for the recovery model. The attributes were selected based on the learnings from existing BAU models.

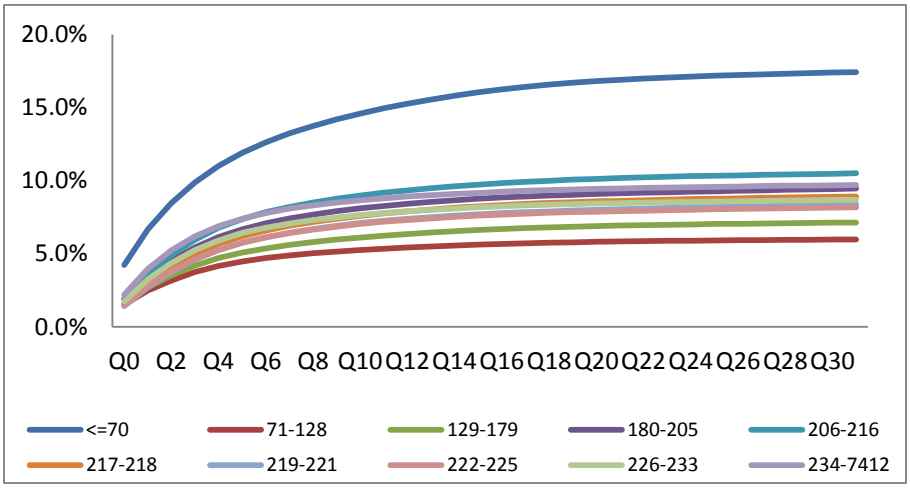
These variables were included in the model one-at-a-time to analyze the effect of each of them on model accuracy and recovery forecast. Since the recovery rate model cannot be developed at account level, bins were created for behavioral attributes such that each account falls in to one of the bins. The data was then summarized at the bin level to capture the effect of each segment. Below charts show how each bin was created for each variable.

Figure 41 Cumulative Recovery Rate by MOB at Charge-off



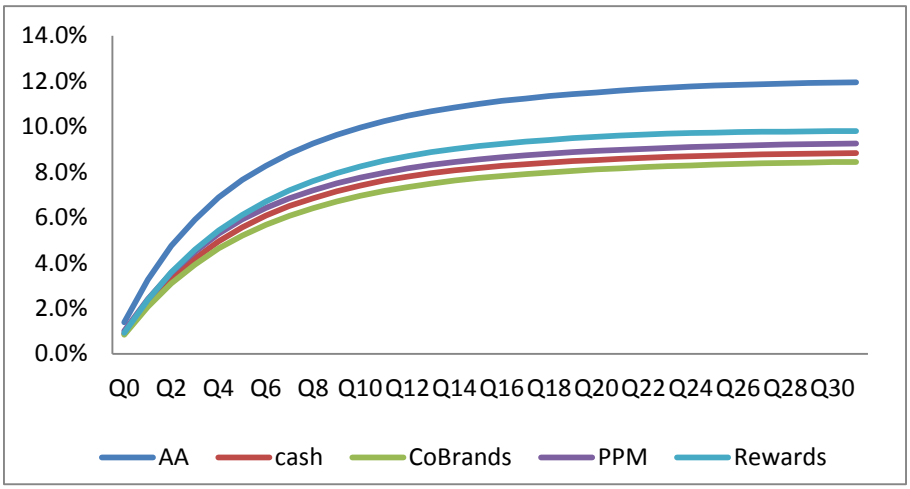

MOB at Charge-off below 14 clearly shows a clear differentiation in recovery rate performance. For modeling purpose MOB <= 12 was taken

Figure 42 Cumulative Recovery Rate by Days since last Payment




Accounts who have made last payment within 70 days prior to Charge-off have high recovery rates and the differentiation with other groups is significant. This variable has been used in BAU collections models. For modeling purpose LP_DAY<= 60 was one bin and 61+ was taken as the second bin as one group.

Figure 43 Cumulative Recovery Rate by Product




AA product has higher recovery rate which is expected based on historical trends. As a result product was grouped into two - AA and Non AA.

Model accuracy was tested for each model developed using the aforementioned variables and their respective recovery forecast was compared with the base model consisting only the macroeconomic variables, product indicator, quarterly time dummies and seasonality indicators.

Below table provides comparison of model parameters and the variables explored for inclusion in the model.

Table 23: Variables explored for Recovery Model

| Variable Description | Base Model | Behavioral Variables | | | |
|---|---|---|---|---|---|
| | | MOB at Charge-off | Credit Limit at Charge-off | Days Since Last Payment | Days Since Last Payment & MOB |
| Time Dummy | | | | | |
| Unemployment Rate | | | | | |
| House Price Index | | | | | |
| AA Product Indicator | | | | | |
| Seasonality factor | | | | | |
| MOB @ CO <= 12 Indicator | | | | | |
| On-us Credit Limit <= 20000 | | | | | |
| Days since last payment <= 60 | | | | | |

Back Test error rates were analyzed for all the models across stress, recent and holdout period for final decision making. Below tables provide the Back Test error rates.

Table 23: Back test across various models

| Stress Period (2008Q1 - 2010Q1) | Base Model | Behavioral Variables | | | |
|---|---|---|---|---|---|
| | | MOB at Charge-off | Credit Limit at Charge-off | Days Since Last Payment | Days Since Last Payment & MOB |
| Q1 | -13% | -5% | -12% | -7% | -1% |
| Q2 | -9% | 0% | -15% | -3% | 3% |
| Q3 | -6% | 7% | -9% | 2% | 5% |
| Q4 | 10% | 11% | 4% | 12% | 17% |
| Q5 | 5% | 24% | 0% | 10% | 25% |
| Q6 | 3% | 28% | 16% | 14% | 28% |
| Q7 | -4% | 20% | 31% | 7% | 14% |
| Q8 | 6% | 41% | 11% | 16% | 11% |
| Q9 | 8% | 59% | -10% | 22% | 8% |
| **Overall 9Q** | **0%** | **22%** | **2%** | **9%** | **12%** |

| Recent Period (2014Q2 - 2016Q2) | Base Model | Behavioral Variables | | | |
|---|---|---|---|---|---|
| | | MOB at Charge-off | Credit Limit at Charge-off | Days Since Last Payment | Days Since Last Payment & MOB |
| Q1 | 2% | -1% | 1% | 14% | 9% |
| Q2 | 2% | -12% | -16% | 1% | -5% |
| Q3 | 4% | -14% | 3% | 1% | -1% |
| Q4 | -5% | -13% | 3% | 4% | 0% |
| Q5 | 5% | -11% | 2% | 1% | -5% |
| Q6 | -4% | -15% | -4% | -4% | -10% |
| Q7 | -7% | -17% | -5% | -5% | -13% |
| Q8 | -9% | -19% | -6% | -5% | -13% |
| Q9 | -2% | -10% | -2% | 1% | -5% |
| **Overall 9Q** | **-1%** | **-12%** | **-3%** | **1%** | **-4%** |

| OOT (2016Q3 - 2017Q2) | Base Model | Behavioral Variables | | | |
|---|---|---|---|---|---|
| | | MOB at Charge-off | Credit Limit at Charge-off | Days Since Last Payment | Days Since Last Payment & MOB |
| Q1 | -3% | -15% | -4% | -3% | -9% |
| Q2 | 2% | -10% | 6% | 3% | -5% |
| Q3 | 0% | -5% | -5% | 3% | -2% |
| Q4 | 8% | 7% | 19% | 18% | 10% |
| **Overall 4Q** | **2%** | **-6%** | **3%** | **5%** | **-2%** |

As seen from the above tables and charts, base model is performing better than all the versions of model which include behavioral attributes.

One limitation of using behavioral attributes in the recovery rate model is that it cannot be used to forecast recoveries from future charge-off accounts in the forecasting period. It can only predict the recoveries from inventory charge-off accounts. The base model would be used to forecast the recoveries from future charge-off accounts. This would lead to increase in complexity of scoring process since two separate models would be required to forecast the future recoveries from inventory and future charge-off accounts.

Since none of the behavioral attributes were adding value to model performance and addition of these additional attributes would lead to increasing model complexity, it was decided to develop a single model including only the time dummies, macroeconomic variables, product indicator and seasonality factors.

- If multiple dependent variables were tested during development of the model, why was the specific variable used in the final model chosen over the other alternatives. If transformation is used provide rationale behind the selection of the chosen transformation and its appropriateness to the model objective?

No other dependent variable was tested for PD.

For EAD model, both the EAD dollar amount and EADF were considered. As has been described in the previous sections, EADF has been selected as the final approach for CCAR 2018 submission.

For Recovery a log transformation of the dependent variable is used. There are two reasons for using log of the dependent variable in Vintage Recovery Model

Firstly, the log of the dependent variable is used to ensure non-negative values of recovery rate in the forecasts as it is possible in a simple linear regression framework given sufficiently stress macro-economic conditions.

Secondly, during CCAR 2017 submission it was an IRMO feedback to explore having constant coefficient for macroeconomic variables through the forecasting horizon and through the lifecycle of the recovery. It is known that recoveries are significantly lower in lower quarters and hence any change in recoveries due to changing stress conditions will be lower. However, the % change in recoveries can still be similar though the lifecycle of the recoveries. A linear transformation with a fixed coefficient will not be able to capture the decreasing change in recoveries while the log transformation will be able to do so.

For example, the Recovery Rates are as follows

**Table 23: Remaining Variables Considered**

| | Recovery Rate | | | |
|---|---|---|---|---|
| Quarter | Base | Stress | Stress-Base | Stress/Base |
| Q2 | 10% | 8% | 2% | 80% |
| Q9 | 0.05% | 0.04% | 0% | 80% |

The difference in Recovery rates between the macroeconomic scenarios for Q2 is 2% and Q9 is 0.01%. However this issue is remediated when using the Log transformation of the dependent

variable and raw transformation of the independent variable as a difference in the log of recovery rates is similar to the log of the ratio of the recovery rates which is similar across the 2 quarters.

- Is the variable selection technique applied robust over time (i.e. is it invariant to the starting point)?

  Yes, it is robust over time as evident from the back-testing results and estimation over short and long sample.

- What is the rationale for applying a particular selection technique? For example, why a VarClus procedure was chosen as the variable selection technique over others?

  Variable selection for both PD and EAD models was performed over multiple steps and considering multiple techniques. Each of the underlying technique (such as Information Value, Cluster Analysis, Correlation Analysis, Bivariate Analysis etc.) has been discussed in detail in the previous sections.

  Do the independent variables make sense for use in the portfolio? In particular, was appropriate justification provided to demonstrate that all relevant variables were incorporated into the model? Was the business rationale behind all variables' relationships with historical data clearly explained and appropriately justified? (i.e., all relevant drivers are incorporated into the model and there are no variables included that have a spurious relationship with the modeled quantity)

  **Behavioral Variables (Account Level PD and EAD model):**

  Behavioral variables (Both Internal and Bureau) are key components in estimating idiosyncratic risk of account level model. Branded Cards selected a list of 250+ variables ranging across key risk drivers such as Balances, Inquiries, Delinquencies, Bureau age etc as shown in the variable list in section 5. Clustering algorithm along with correlation, KS, Gini etc was then utilized to select top variables from each of these risk driver categories. Careful attention was given to ensure proper risk drivers are captured in the final model. The same is evident through robust Back Tests.

  **Macro-economic Variables:**

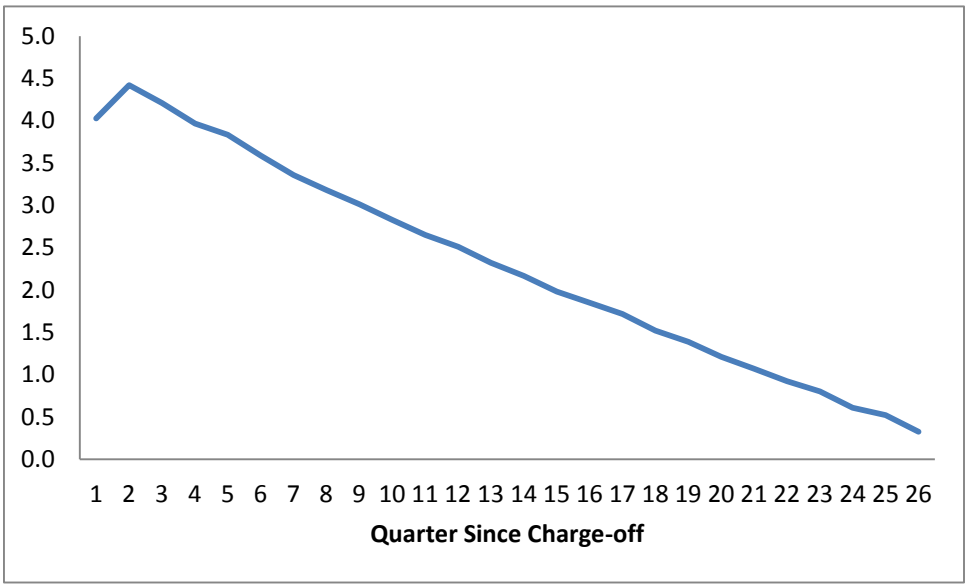
  As shown above, Branded Cards considered a comprehensive list of macroeconomic drivers. A thorough analysis of relationship between dependent variables and macroeconomic variables was then conducted to arrive at a suitable list of variables. Unemployment rate and HPI are observed to be good predictors of the credit losses which is consistent with the previous submission as well.

The macro-economic variables show a good correlation with PD, EADF and recovery rate.

It is observed that when unemployment rate increases, recovery rate declines and when the house price index increases recovery rate improves. The impact is reflected in the variable coefficient sign. The coefficient of unemployment rate is negative and that of HPI is positive.

**Maturation Effect:**

Maturation curve is the primary driver of recovery rate. As the time difference from charge-off point increases, recovery rate decreases which can be observed in the trend of coefficient of maturation curve dummies.




- Are the variable transformations meaningful?

For macro-economic variables, the variable transformations for the independent variables are necessary for them to be stationary. For Behavioral variables, missing value, outlier and special value treatment is done; which is essential for the process of data cleaning. Some behavioral variable transformations were done (only if necessary) after analyzing Bivariate Plots to ensure linearity assumption is not violated.

- What were the variables originally considered? Which ones were provided by the business? Provide answers separately for risk and business drivers

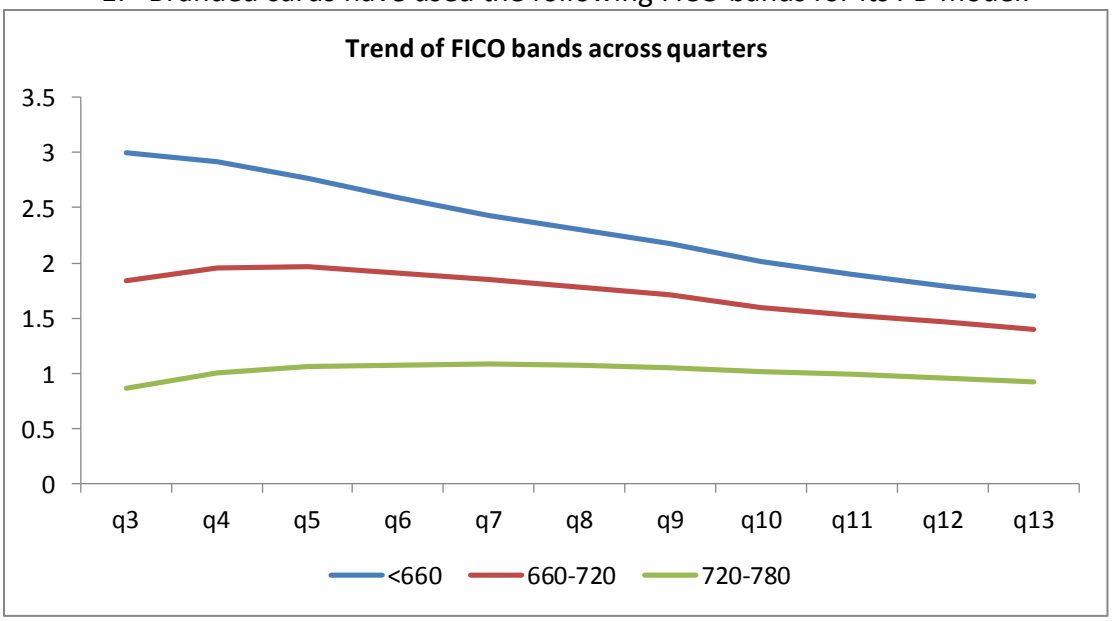
For Account level PD and EADF models, Branded Cards considered a robust list of risk drivers such as Balances, Inquiries, Delinquencies, Bureau, Credit age etc.

Further in order to capture key differences in the sub-portfolios or products, dummies have been used.

Given the nature of business i.e. Unsecured consumer lending, key macro-economic components like Unemployment rate, House Price Index, US Gross Domestic Product growth rate etc. have been considered. These macro factors are known to directly or indirectly impact the consumer behavior

Below are mentioned a few variables which were considered for modeling, however were not used in the final model. The rationale for each is provided below:

1. Branded cards have used the following FICO bands for its PD model:




FICO bands have been used for modeling since most policies used by Business are based on these FICO bands. For example the business does not differentiate between accounts with FICO score 701 and 710.

Also the odds of FICO are not consistent through the cycle. The FICO dummies capture relationship with the dependent variable better for Branded cards portfolio.

Linear piecewise FICO was explored in the model. However with the use of this variable increases under prediction in the stress period by 2%. Hence to be on a conservative side, business did not use FICO as linear piecewise.

2. Macro FICO interactions not being used for Quarters 10 to 13

Historically the impact of FICO decreases over time. Higher FICO is observed to be more stressed than lower FICO. However for a longer time span (like 10 to 13 quarters) the incremental stress is no longer observed in higher FICO bands since the impact/predictive power of FICO diminishes.

3. Inclusion of Florida dummy

Historically state of Florida observed higher stress in comparison to the other states and absence of this indicator causes under prediction of losses in the state. Hence the dummy for this variable has been included in the model.

4. Use of performance Month on book variable

To take into account the age effect of an account, the forward looking MOB was used in CCAR 2018 model. This was an enhancement identified by the team based on IRMO's feedback. This was used to capture the maturation effect of the portfolio.

- How were the drivers (both risk and business) translated (or proxied) into model variables?

  No proxy to model variables or risk drivers were used.

- Which variables were statistically significant and which were statistically insignificant? What is the evidence supporting this selection? Provide answers separately for risk and business drivers

  The model variable selection process is aimed at eliminating all statistically insignificant variables to ensure that the final list of model variables is all significant. The attached document lists all variables which were found significant and made their way into the model. Most of the important variables such as FICO, Utilization, Account age etc. were statistically significant. Among the macroeconomic variables, HPI and Unemployment Rate transformations were significant, however GDP and its transformations did not come as significant.

- How do the drivers relate to the model outputs? Please explain. Provide answers separately for risk and business drivers

  The final variables used in the PD, EAD and Recovery models are given in tables below.

**Table 6-32 : PD Model Final variables**

| PD Model Variables | Business Rationale- Relation with the Dependent (PD) Variable |
|---|---|

| PD Model Variables | Business Rationale- Relation with the Dependent (PD) Variable |
|---|---|
| US Unemployment Rate (National and Regional) | Unemployment rate has a positive correlation with PD. Increase in unemployment rate means adverse economic conditions and hence lower ability to repay debt, which leads to higher PD. Higher FICO bands show higher sensitivity with PD |
| House Price Index (National and Regional) | House Price Index has a negative correlation with PD. Since house price reflects household wealth, a fall in house prices reflect worsening economic conditions and hence higher PD. |
| FICO Scores | Branded Cards utilizes the FICO scores as the drivers of both the PD and losses of the portfolio as consumers with lower FICO would have a worse credit history, compared to consumers with higher FICO scores. Thus, consumers with lower FICO scores are expected to have a higher probability of default, and would thus have more chances of contributing to losses. |
| Seasonality | Branded Cards utilizes Seasonality Indicators as the drivers of both the PD and losses of the portfolio as the consumer spending behavior of the portfolio shows seasonal trends. For example, the portfolio usually shows highest loses in the 1st Quarter, and the lowest losses in the 3rd Quarter. |
| Snapshot Acct Delinquencies | Branded Cards utilizes snapshot delinquencies as the drivers of both the PD and losses of the portfolio as consumers who are already delinquent would have higher chances of defaulting and thus contributing to losses in the early quarters. For example, consumers in bucket 4 or above would have very high chances of defaulting in the 1st Quarter from the snapshot. |
| Snapshot Customer Delinquencies | Branded Cards utilizes snapshot customer level delinquencies as the drivers of both the PD and losses of the portfolio as consumers who are already delinquent on other Citi cards would have higher chances of defaulting and thus contributing to losses in the early quarters. For example, consumers in bucket 4 or above on a different Citi card would have very high chances of defaulting in the initial Quarters from the snapshot. |
| Product Dummies | Branded Cards utilizes Product Dummies as the drivers of both the PD and losses of the portfolio as the losses differ by products. For example, AAdvantage portfolio has lower risk than the other portfolios |
| MoB Perf | Branded Cards utilizes Months on Books (MOB) at performance for the account as the drivers of both the PD and losses of the portfolio as it allows us to capture the life cycle of an average loan better. The PD rates are observed to be increase at first then peak and flatten. |
| Regime Indicator | A forbearance policy change regime indicator has been incorporated in the model from H2'12 onwards for quarters 1-5 & 9. This is to account for the impact of changes in forbearance policy on the probability of defaulting as enrollment into forbearance programs became more restrictive |
| Utilization Snapshot | Branded Cards utilizes Snapshot Utilization of the account as the drivers of both the PD and losses of the portfolio as higher utilization shows credit hungriness of the consumer. As such, a consumer with higher utilization has higher chances of defaulting. |
| Bill Active Indicator | Branded Cards utilizes Snapshot Bill Active Indicator as the drivers of both the PD and losses of the portfolio as only consumers with some bill activity are expected to build up higher average balance over time. As such, the customers who do not have any positive balance are less likely to default and contribute to the PD and losses. |
| Inquiries in last 6 Months | Branded Cards utilizes Total number of Inquiries for the account in the last 6 months as the drivers of both the PD and losses of the portfolio as more number of inquiries show credit hunger of the consumer. As such, a consumer with higher inquiries has higher chances of defaulting. |
| Total Revolving Balance at Bureau | Branded Cards utilizes Total Revolving Balance at Bureau for the account as the drivers of both the PD and losses of the portfolio as consumers with more balance at bureau are either credit hungry or not able to pay off the balances. As such, customers with high balance at bureau have higher chance of charge off, and hence higher losses. Though a balance of 0 at bureau can be an indication of lack of credit history and is treated differently |

| PD Model Variables | Business Rationale- Relation with the Dependent (PD) Variable |
|---|---|
| Indicator for Florida | Historically state of Florida observed higher stress in comparison to the other states and absence of this indicator causes under prediction of losses in the state |
| Total balance of Bureau trades currently 60+DPD | Branded Cards utilizes total balance of trades Rated 60+DPD as the drivers of both the PD and losses of the portfolio as consumers who are already delinquent for 60+ days would have higher chances of defaulting and thus contributing to losses in the early quarters. |
| Sales active indicator | Accts that are sales active are less likely to charge off in the initial two quarters as the business would have applied authorizations to stop future sales for accounts with high likelihood of charge off. For example - accounts in delinquency may not be allowed to transact on the card |
| APR | Branded Cards utilizes APR in the model as it observes that people with APR higher than 18% have relatively higher chances to charge off in the initial quarters. Also accounts on a promo period (with APR of 0) will be less likely to charge off in the initial quarter |

**Table 6-33 : EADF Model Final variables**

| EAD Model Variables | Business Rationale- Relation with the Dependent (EADF) Variable |
|---|---|
| Portfolio Indicator | Branded Cards utilizes portfolio Indicators as the drivers of EADF and losses of the portfolio as the losses differ by products. For example, the AAdvantage card holders have lower EADF and losses than other products. |
| HPI | House Price Index has a negative correlation with EADF. Since house price reflects household wealth, a fall in house prices reflect worsening economic conditions and hence higher EADF. |
| UR | Unemployment rate has a positive correlation with EADF. This is because an increase in UR reflects adverse economic conditions, which lead to higher EADF. |
| Indicator for FICO higher than 780 | Higher Fico accounts(780+) , will have lower EADF |
| Utilization | Branded Cards utilizes Snapshot Utilization of the account as the drivers of both the EADF and losses of the portfolio as higher utilization shows credit hungriness of the consumer. As such, a consumer with higher utilization is likely to have a higher EADF. This variable is interacted with delinquency bucket in order to capture the differential impact of utilization across buckets with EADF |
| # of Purchases | Branded Cards utilizes Total Number of Purchases for the account as the drivers of both the EADF and losses of the portfolio as consumers with higher number of purchases are expected to build up higher average balance. As such, when accounts with high number of trades default, they would have higher EADF, and hence higher losses. |
| Inquiries | Branded Cards utilizes Total number of Inquiries for the account as the drivers of both the EADF and losses of the portfolio as more number of inquiries show credit hunger of the consumer. As such, a consumer with higher inquiries will default with higher EADF. |
| Net Revolving Burden Usage | The net fraction for all bank and national revolving burden using the aggregate balance. The higher the bureau utilization, the higher EADF. |
| Cash Advance Indicator | Accounts with Cash Advances will have Higher EADF |

**Table 6-34 : Recovery Model Final variables**

| Recovery Model variables | Business Rationale – Relation with dependent variable |
|---|---|
| HPI | HPI has a positive correlation with recoveries. A fall in housing prices means worse economic conditions and hence lesser recoveries |
| UR | Unemployment rate has a negative correlation with recovery rate. This is because an increase in UR reflects adverse economic conditions, which lead to lesser recoveries. |

| Recovery Model variables | Business Rationale – Relation with dependent variable |
|---|---|
| Product Dummies | Branded Cards utilizes Product Dummies as the drivers of cash recovery for the portfolio as the recovery differ by products. For example, AA has is higher recoveries than the other portfolios |
| Seasonality | Branded Cards utilizes Seasonality Indicators as the drivers of recovery in the portfolio. Recoveries in Q3 are lower than other quarters |
| Performance quarter dummy | Multiple dummies that indicate quarter since charge off from the recovery quarter to capture the lifecycle effect, this is done separately for consumer and business cards. |

### 6.1.5 Model Development Process Flow

Model development process consists of following four steps (see illustration below):

1) **Data extraction and treatment** – At this step, data is extracted from EDW. Rigorous checks as mentioned in section 5.1.2 are performed. A random sample is generated for computational ease in variable reduction. Certain attributes required for derived variable creation are then transformed. Data is then treated to perform initial variable reduction.
2) **Variable Reduction** – A rigorous process has been followed to select initial list of behavioral and macroeconomic attributes. Details on process followed to select initial variable list can be found in section above.
3) **Model development** – Candidate models are then developed using key predictors obtained from variable reduction step. Models are evaluated for model statistics such as KS, multicollinearity of attributes, significance value of coefficients. Attributes are also tested for intuitive trend of time varying coefficients considered in the model. Details on process followed to select initial variable list can be found in section above.
4) **Model Evaluation** – Candidate models satisfying criteria mentioned above are then evaluated for their performance in terms of Back Tests. Model demonstrating sufficient sensitivity to macroeconomic scenarios and possessing good Back Test is selected as the final model.

End to end approach used for modeling **PD and EAD model** can be explained using the following flowchart:

Figure 44 : Model development flowchart

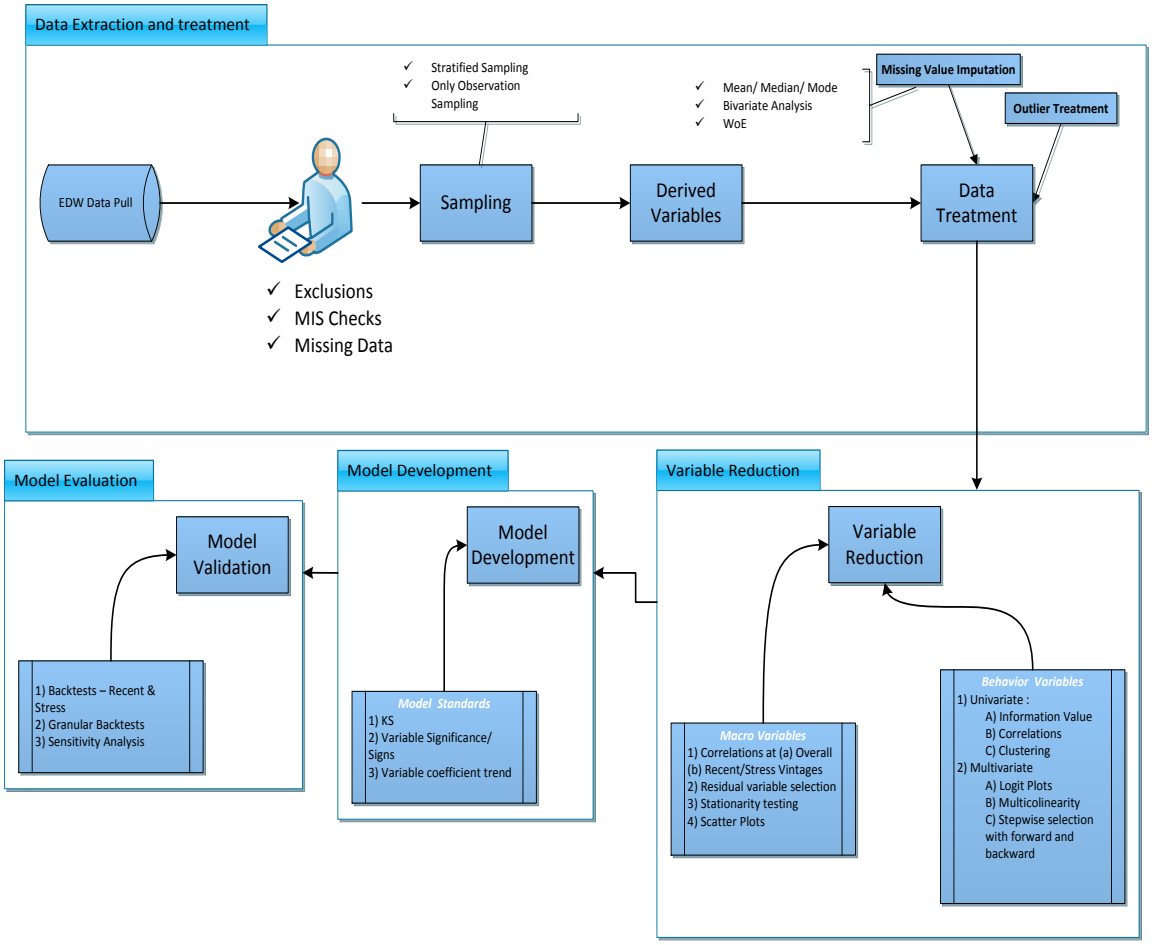
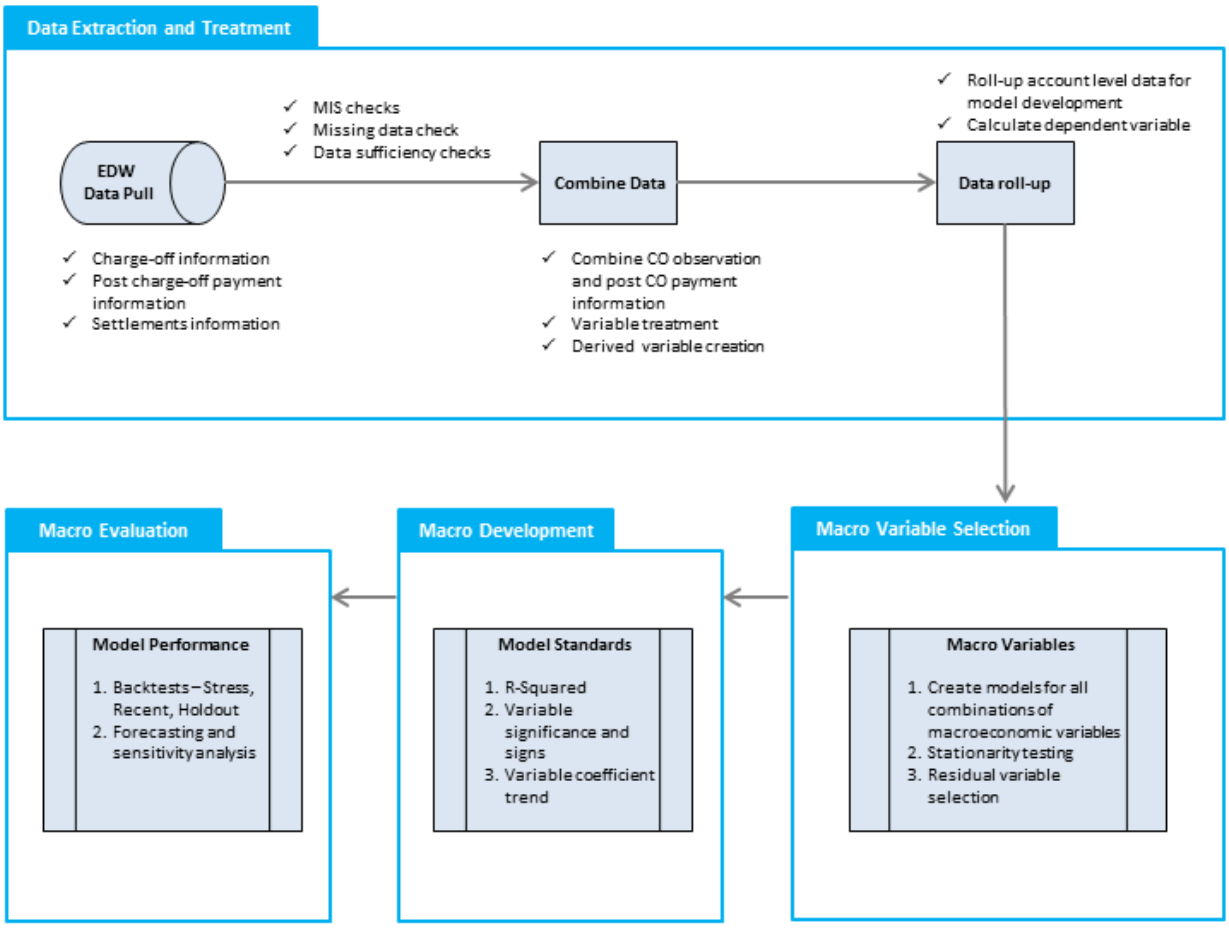


Figure 45: Model Development Flowchart - Recovery Model



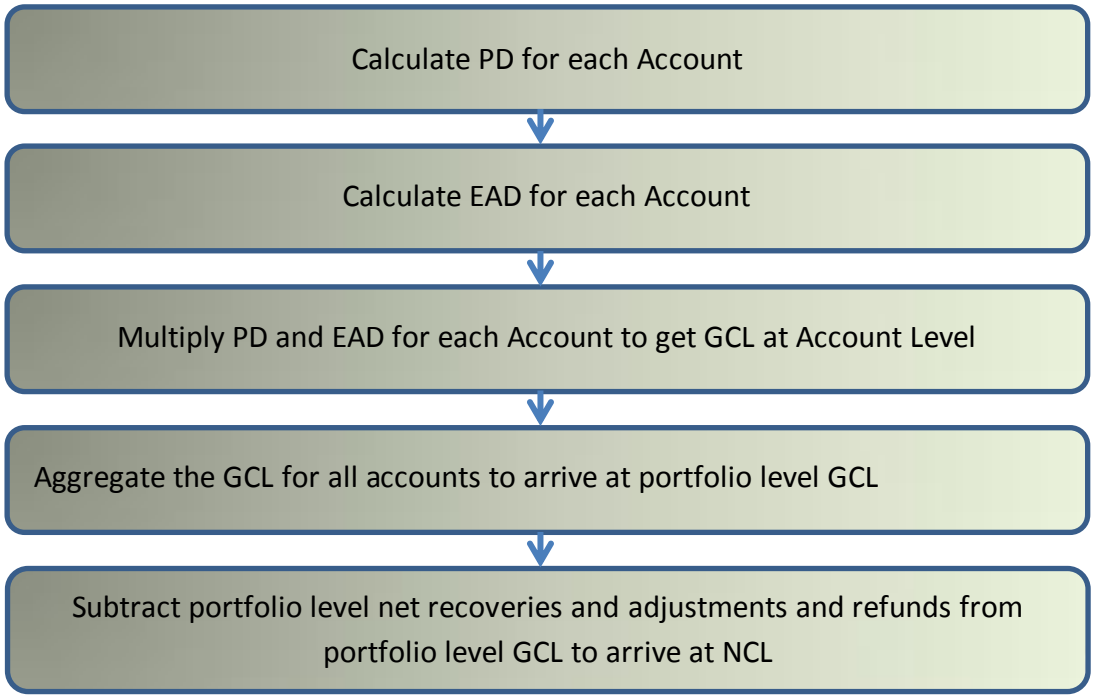

The GCL forecast from account level PD and EAD model is combined with the recovery forecast to get the NCL forecast.

## 6.2 Model Specification

### 6.2.1 Model Input, Output and Mathematical Equations

The three components (PD, EAD and Recovery) are then combined to arrive at final NCL, where, PD and EAD are output from the account level PD and EAD models respectively. Recoveries are calculated using the forecasted recovery rate and adding Asset Sales and Agency Fee components; Adjustments and refunds are subtracted to arrive at the NCL. The following diagram presents the NCL estimation process.

Figure 46 Overall $ NCL Estimation Process




The following steps outline the calculations that are performed to generate the net credit loss and cumulative net credit loss for a single hypothetical loan.

Model Inputs

For Account level PD and EAD models, representative input variables include the following categories:

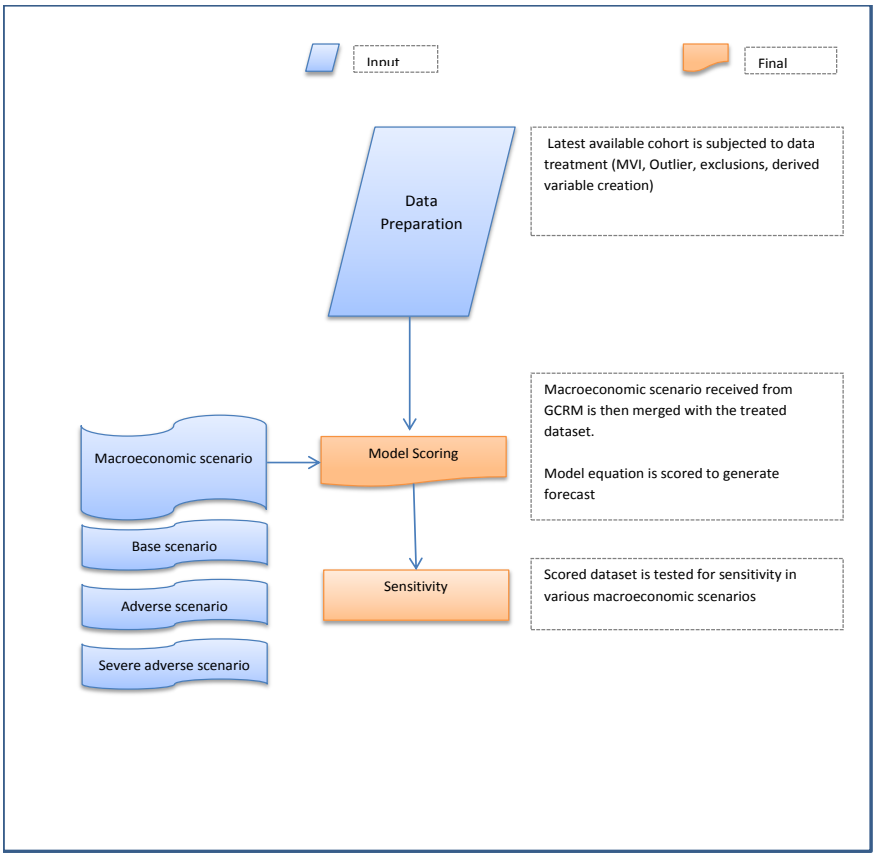
- Quarter since observation (baseline hazard)

- Account level: FICO, account age, line amount, balance, utilization, delinquency status, payment history, product etc.
- Macroeconomic: Unemployment rate, home price index, etc.
- Seasonality

Upon finalizing the model, GCL forecast is generated for supervisory scenario from previous submission using latest available cohort (Dec'16 for CCAR 2017 supervisory scenarios). Data is subjected to same data treatment as mentioned above for model development. Following flowchart demonstrates step by step process followed in generating the forecast:

For forecast of new originations in the forecast time period, model is scored for latest available originations using MOB 1 – 3 accounts. Average forecasted GCL is then weighted according to origination assumptions **received from FP&A**.

**Figure 47 : forecast generation**



For Recovery models, representative input variables include the following categories:

- Quarter since observation (Lifecycle effect)
- Account level: Product
- Macroeconomic: Unemployment rate, House Price Index, etc.
- Seasonality

- Net credit loss is further calculated as per formula below:

- **Equation 17**

- *NCL = GCL – Net Recoveries – Adjustment & Refunds*

Intermediate Calculations

***Step 1: Calculate the default hazard***

The binary logistic discrete time hazard equation is found in Equation (6.2.1.16), which we repeat here for convenience:

$$\log\left(\frac{\lambda(t_j|x_i)}{1-\lambda(t_j|x_i)}\right) = logit\ \lambda(t_j|x_i) = \alpha_j + \boldsymbol{x_i'\beta}$$

The default hazard for a specific model is calculated according to the above formula, but using the model's particular specification in terms of included variables and their coefficients.

***Step 2: Calculate the Conditional PD***

As such, the conditional PD can be expressed as:

$$Conditional\ PD_t = \frac{e^{\alpha_j + \boldsymbol{x_i'\beta}}}{1 + e^{\alpha_j + \boldsymbol{x_i'\beta}}}$$

***Step 3: Calculate the unconditional probability of default***

The next step in the process is to convert the conditional probabilities of default into unconditional probabilities. This can be done via the following equations, where the Conditional $PD_t$ notation just above is switched to the more explicit Pr(Default) notation below.

$$(Unconditional\ PD)_{\boldsymbol{t}} = S_{t-1} \times Pr\{Default\ at\ t|\ x, alive\ at\ t-1\}$$

Where, $S_t = \begin{cases} 1, & for\ t = 0 \\ S_{t-1} \times [1 - Pr\{Default\ at\ t|\ x, alive\ at\ t-1\})], & for\ t \geq 1 \end{cases}$

Model Outputs

***Step 4: Calculate the gross credit loss***

Combining the unconditional PD with the credit line at snapshot date and the EAD model output, EADF, the GCL for the current month is calculated as:

$$GCL_t = Unconditional\ PD_t * Credit\ Line_0 * EADF_t$$

***Step 5: Calculate the Net credit loss***

After the GCL is aggregated for new accounts, NCL is calculated from GCL by the following formula:

*NCL = GCL – Net Recoveries – Adjustment & Refunds*

### 6.2.2 Final Model Specification

- What business/ risk drivers should be captured?

Branded Cards has built account level PD and EAD model in order to capture both idiosyncratic risk as well as macro-economic impact on the risk level of an account. It is therefore imperative that model captures risk profile of an account captured through behavioral variables and macro-economic variables to adjust for prevailing macro-economic environment.

Branded Cards considered a robust list of risk drivers.

Further in order to capture key differences in the sub-portfolios or products, dummies has been used to capture.

Given the nature of business i.e. Unsecured consumer lending, key macro-economic components i.e. Unemployment rate, House Price Index, Gross Domestic Product of macro-economy which directly or indirectly impacts the consumer have been considered.

The final variables used in the PD, EAD and Recovery models are given in tables below.

**Table 6-35 : PD Model Final variables**

| PD Model Variables | Business Rationale- Relation with the Dependent (PD) Variable |
|---|---|
| US Unemployment Rate (National and Regional) | Unemployment rate has a positive correlation with PD. Increase in unemployment rate means adverse economic conditions and hence lower ability to repay debt, which leads to higher PD. Higher FICO bands show higher sensitivity with PD |
| House Price Index (National and Regional) | House Price Index has a negative correlation with PD. Since house price reflects household wealth, a fall in house prices reflect worsening economic conditions and hence higher PD. |
| FICO Scores | Branded Cards utilizes the FICO scores as the drivers of both the PD and losses of the portfolio as consumers with lower FICO would have a worse credit history, compared to consumers with higher FICO scores. Thus, consumers with lower FICO scores are expected to have a higher probability of default, and would thus have more chances of contributing to losses. |
| Seasonality | Branded Cards utilizes Seasonality Indicators as the drivers of both the PD and losses of the portfolio as the consumer spending behavior of the portfolio shows seasonal trends. For example, the portfolio usually shows highest loses in the 1st Quarter, and the lowest losses in the 3rd Quarter. |

| PD Model Variables | Business Rationale- Relation with the Dependent (PD) Variable |
|---|---|
| Snapshot Acct Delinquencies | Branded Cards utilizes snapshot delinquencies as the drivers of both the PD and losses of the portfolio as consumers who are already delinquent would have higher chances of defaulting and thus contributing to losses in the early quarters. For example, consumers in bucket 4 or above would have very high chances of defaulting in the 1st Quarter from the snapshot. |
| Snapshot Customer Delinquencies | Branded Cards utilizes snapshot customer level delinquencies as the drivers of both the PD and losses of the portfolio as consumers who are already delinquent on other Citi cards would have higher chances of defaulting and thus contributing to losses in the early quarters. For example, consumers in bucket 4 or above on a different Citi card would have very high chances of defaulting in the initial Quarters from the snapshot. |
| Product Dummies | Branded Cards utilizes Product Dummies as the drivers of both the PD and losses of the portfolio as the losses differ by products. For example, AAdvantage portfolio has lower risk than the other portfolios |
| MoB Perf | Branded Cards utilizes Months on Books (MOB) at performance for the account as the drivers of both the PD and losses of the portfolio as it allows us to capture the life cycle of an average loan better. The PD rates are observed to be increase at first then peak and flatten. |
| Regime Indicator | A forbearance policy change regime indicator has been incorporated in the model from H2'12 onwards for quarters 1-5 & 9. This is to account for the impact of changes in forbearance policy on the probability of defaulting as enrollment into forbearance programs became more restrictive |
| Utilization Snapshot | Branded Cards utilizes Snapshot Utilization of the account as the drivers of both the PD and losses of the portfolio as higher utilization shows credit hungriness of the consumer. As such, a consumer with higher utilization has higher chances of defaulting. |
| Bill Active Indicator | Branded Cards utilizes Snapshot Bill Active Indicator as the drivers of both the PD and losses of the portfolio as only consumers with some bill activity are expected to build up higher average balance over time. As such, the customers who do not have any positive balance are less likely to default and contribute to the PD and losses. |
| Inquiries in last 6 Months | Branded Cards utilizes Total number of Inquiries for the account in the last 6 months as the drivers of both the PD and losses of the portfolio as more number of inquiries show credit hunger of the consumer. As such, a consumer with higher inquiries has higher chances of defaulting. |
| Total Revolving Balance at Bureau | Branded Cards utilizes Total Revolving Balance at Bureau for the account as the drivers of both the PD and losses of the portfolio as consumers with more balance at bureau are either credit hungry or not able to pay off the balances. As such, customers with high balance at bureau have higher chance of charge off, and hence higher losses. Though a balance of 0 at bureau can be an indication of lack of credit history and is treated differently |
| Indicator for Florida | Historically state of Florida observed higher stress in comparison to the other states and absence of this indicator causes under prediction of losses in the state |
| Total balance of Bureau trades currently 60+DPD | Branded Cards utilizes total balance of trades Rated 60+DPD as the drivers of both the PD and losses of the portfolio as consumers who are already delinquent for 60+ days would have higher chances of defaulting and thus contributing to losses in the early quarters. |
| Sales active indicator | Accts that are sales active are less likely to charge off in the initial two quarters as the business would have applied authorizations to stop future sales for accounts with high likelihood of charge off. For example - accounts in delinquency may not be allowed to transact on the card |
| APR | Branded Cards utilizes APR in the model as it observes that people with APR higher than 18% have relatively higher chances to charge off in the initial quarters. Also accounts on a promo period (with APR of 0) will be less likely to charge off in the initial quarter |

Table 6-36 : EADF Model Final variables

| EAD Model Variables | Business Rationale- Relation with the Dependent (EADF) Variable |
|---|---|
| Portfolio Indicator | Branded Cards utilizes portfolio Indicators as the drivers of EADF and losses of the portfolio as the losses differ by products. For example, the AAdvantage card holders have lower EADF and losses than other products. |
| HPI | House Price Index has a negative correlation with EADF. Since house price reflects household wealth, a fall in house prices reflect worsening economic conditions and hence higher EADF. |
| UR | Unemployment rate has a positive correlation with EADF. This is because an increase in UR reflects adverse economic conditions, which lead to higher EADF. |
| Indicator for FICO higher than 780 | Higher Fico accounts(780+) , will have lower EADF |
| Utilization | Branded Cards utilizes Snapshot Utilization of the account as the drivers of both the EADF and losses of the portfolio as higher utilization shows credit hungriness of the consumer. As such, a consumer with higher utilization is likely to have a higher EADF. This variable is interacted with delinquency bucket in order to capture the differential impact of utilization across buckets with EADF |
| # of Purchases | Branded Cards utilizes Total Number of Purchases for the account as the drivers of both the EADF and losses of the portfolio as consumers with higher number of purchases are expected to build up higher average balance. As such, when accounts with high number of trades default, they would have higher EADF, and hence higher losses. |
| Inquiries | Branded Cards utilizes Total number of Inquiries for the account as the drivers of both the EADF and losses of the portfolio as more number of inquiries show credit hunger of the consumer. As such, a consumer with higher inquiries will default with higher EADF. |
| Net Revolving Burden Usage | The net fraction for all bank and national revolving burden using the aggregate balance. The higher the bureau utilization, the higher EADF. |
| Cash Advance Indicator | Accounts with Cash Advances will have Higher EADF |

Table 6-37 : Recovery Model Final variables

| Recovery Model variables | Business Rationale – Relation with dependent variable |
|---|---|
| HPI | HPI has a positive correlation with recoveries. A fall in housing prices means worse economic conditions and hence lesser recoveries |
| UR | Unemployment rate has a negative correlation with recovery rate. This is because an increase in UR reflects adverse economic conditions, which lead to lesser recoveries. |
| Product Dummies | Branded Cards utilizes Product Dummies as the drivers of cash recovery for the portfolio as the recovery differ by products. For example, AA has is higher recoveries than the other portfolios |
| Seasonality | Branded Cards utilizes Seasonality Indicators as the drivers of recovery in the portfolio. Recoveries in Q3 are lower than other quarters |
| Performance quarter dummy | Multiple dummies that indicate quarter since charge off from the recovery quarter to capture the lifecycle effect, this is done separately for consumer and business cards. |

Final model variable along with their relationship with PD, EAD and Recovery Rates can be found in excel below:

Model_specifications_PD.xlsx

Model_specifications_EAD.xlsx

- What changes have been implemented from previous versions (if any?)

Table 6-38 : Key differences between CCAR 2017 and CCAR 2018 account level models

| | | CCAR 2017 | CCAR 2018 | Comments |
|---|---|---|---|---|
| **Data** | **Vintages** | 2007 - 2014 | 2006 - 2015 | Larger coverage of data. Specifically pre-stress vintages |
| | **Attributes** | -> ~100 behavioral top predictors in internal model considered<br>Only snapshot variables considered | -> ~250 behavioral predictors considered<br>-> Additionally, derived variables capturing historical averages considered. Example - maximum delinquency in last 12 months | Larger coverage of behavioral attributes |
| | **Macroeconomic component** | -> Regional level variables considered<br>-> Only Log transformations considered | -> State Level variables considered<br>-> Both log and raw transformations were considered<br>-> Further, Both average and difference of UR tested in the model (For PD) | Macro-economic component has been considered at more granular level |
| **Modeling approach** | | Two step approach with behavior + ERM component | One step estimation of behavior and macro-economic component | Easier to incorporate state level macro-economy information |
| | | Business and Consumer cards were modeled separately | Business and Consumer cards were modeled together | Improvement in accuracy of Business Cards compared to CCAR 2016 models |

### 6.2.3 Manual Adjustments

There are no manual adjustments in the modeling process.

- If manual adjustments have been applied, are they algorithmic, transparent and repeatable?

  No manual adjustments have been applied

- Are the adjustments in the estimated parameters of the model justified?

  No manual adjustments have been applied.

## 6.3 Model Assumptions

The key model assumptions include the following:

1. Model also assumes no major change in regulatory environment

2. New Originations - Future originations are assumed to have similar characteristics as recent originations.

   These assumptions have been adequately tested and the results for the same are given in Section 8.4.4

   Other Framework related model assumptions are as follows:

3. The relationship between risk and key drivers (Both internal and external) is assumed to hold true in the future. Model robustness with respect to this has been tested using through the cycle Back Tests as well as out of time accuracy and stability tests

4. The models assume consistent input data definitions and quality for data obtained from a third party over time, such as original credit score. The input data's distribution/trend is examined for the model development sample to ensure no unexpected trend is observed. A similar data check will be implemented in the production process.

5. The credit quality attributes have been considered to be static at snapshot and are assumed to be able to adequately explain future events through time interactions

6. PD model has been built using logistic regression technique. Logistic regression runs on few key assumptions which have been adequately tested and have been documented below

    a. There is a linear relation between independent variables (or their transformations) and log odds ratio; the dependent and independent variables need not be linearly related

    b. No important variables have been omitted, and no extraneous variables have been included

    c. The sample size is large – reliability of estimation declines with fewer number of observations

    d. The independent variables are not a linear function of each other

    e. The observations are independent of each other

7. EADF model has been built using linear regression where following assumptions were tested

    a. There is a linear relation between independent variables (or their transformations) and response variable.

    b. Residuals follow a normal distribution

8. There is little or no multicollinearity in the data. The independent variables are not highly correlated. The residuals of the model have a constant variance.

## 6.4 Model Limitations

- What are the key limitations of the approach used?

  In the current modeling construct, recovery is considered to be independent of asset sales. However, recoveries have to be adjusted if asset sales increase.

- What are the potential weaknesses in model performance?

  No weaknesses have been identified in the model performance in through the cycle tests and out of time testing. The model Back Tests well and OOT errors are within the MRM specified limits. Performance of the model could suffer if there are significant changes in business, risk management practices or Regulatory/Legal environment.

- What are the internal and external factors used to evaluate model weaknesses?

The model has been developed and Back Tested on through the cycle data to ensure it covers a large spectrum of changes in probability of default and under a wide range of macro-economic conditions. Additionally the model has been tested for forecasting errors on an out of time sample; along with sensitivity testing on the most recent data available. Models have been thoroughly Back Tested on sub-population and the performances have been satisfactory and well within MRM thresholds.

- Are there any known limitations on the range of the model outputs outside of which the model might not perform as expected?

  The models have been developed using data of one recession time period only, so it assumes that the last recession is representative of the economic recessions in general. The model output may not project very properly given new scenarios, which are fundamentally very different from history.

- Are any potentially material risks missing?

  Branded Cards believes that there nopotential risks that have not been captured. In previous versions of the model, no potential risks were identified

- What factors, if any, mitigate the limitations of the output?

  The variables selected in the models pass all requirements and the models themselves pass all back testing and sensitivity requirements. The models Back Test very well throughout the entire economic cycle. The model should be able to pick up any potential risk that is caused by macroeconomic data shifts in the future.

# 7 Prior Versions and Related Models and Modeling Approaches

## 7.1 Regulatory Guidance/Feedback from Prior Reviews (if Applicable)

- What was the regulatory feedback regarding the model?

The Federal Reserve Bank of New York (FRBNY) supervisory letter of August 31th, 2017 issued to Citi regarding the 2017 Comprehensive Capital Analysis Review (CCAR 2017) did not call out any specific issues for the CCAR 2017 Branded Cards' Account Level models. Some past MRA/MRIAs remain open due to lingering weaknesses identified during the 2017 submission and would have an indirect effect on the model development and documentation. These include:

1) instituting an improved model validation framework, including effective challenge of models and governance over model issues and remediation; and
2) Establishing and implementing clear standards for estimation method development and use, including ongoing model performance monitoring.

The FRB also cited benchmarking as a new area for improvement and issued a new MRA related to benchmarking and a corrective action plan was developed.

Further, during the FRB CCAR 2017 Feedback Meeting, the FRB reinforced the criticality of demonstrating that:

1) the model construct's fit for purposes
2) all relevant data sources were used and sufficient effort was made to use historical data to represent the current and go-forward state of the portfolio
3) all key relevant variables were incorporated into the model, and was there a sound business rationale for the selection of all variables and transformations used in the Modeling Approach
4) model variables have a directionally appropriate relationship with the model output and that the relative strength of each variable in the final model meet business expectations
5) the portfolio segmentation is sufficiently justified given the business objective In particular, is that segmentation is justified either from a business perspective (i.e., there are clear qualitative differences between difference segments) or from a statistical perspective (i.e., different segments have different mean, standard deviation, or model drivers)
6) data exclusions due to business reasons are , was the decision to exclude the data sufficiently justified

Regulatory feedback for CCAR 2017 was received after CCAR 2018 models were developed. This document reflects enhancements related to prior regulatory feedback and limitations identified during

the technical and functional validation of prior models. Recent feedback is acknowleged and will be addressed in the development of future models.

General improvements for the Branded Cards' Account Level PD and EAD models that have been made are listed below:

| Modeling Process | Description |
|---|---|
| **Data** | • **FICO'08 purchased and used in Model development** (In CCAR 2017 aligned FICO had been used)<br>• **More than ~800 Million through-the-cycle records** used for model development compared to ~500 Million used in CCAR 2017 |
| **Design** | • Incremental Variable interactions with Product, FICO, MOB and Delinquency used in the model<br>• Use of MOB in the forecast horizon |
| **Modeling Framework** | • **EADF approach used instead of EAD approach that was used in CCAR 2017**<br>• Inclusion of age (MOB) of the account in the model |
| **Governance** | • **Regular review with Independent Risk** |
| **Feedback** | • **Time invariant coefficient of macroeconomics tested in the model** |

Vintage Recovery rate models were developed in CCAR 2017 submission and were used as challenger models. For CCAR 2018 models, there are several enhancements that have been made to CCAR 2017 models, which include the following:

| | Enhancements |
|---|---|
| Methodology | **Vintage recovery model developed as champion model** |
| | **Single coefficient of macroeconomic variables across all performance quarters** |
| Portfolio | **Included business recoveries in modeling data** |

Segmentation justification is a focused area of the feedback received. In the current submission, the Account Level PD and EAD models are developed at overall Branded Cards portfolio level. For CCAR 2017, initially, the Consumer cards and Business cards were planned to be modeled separately .However, the pre-final models of consumer and business cards were very similar and hence these portfolios were combined to get one portfolio level model for Branded Cards. Additionally, the business cards portfolio is materially much smaller compared to that of consumer portfolio.

The account level models were evaluated for accuracy by back testing for several granular sub-segments viz. Portfolio, FICO and Delinquency status. For CCAR 2017 models, all the Back Test errors were well

within the MRM thresholds. Models have been also back tested for each of the product segments and strong back test results further reinforce the fact that portfolio segmentation was not necessary

There were some opportunity areas that were recognized for sub-segments in the CCAR 2017 models. For example, In the CCAR2017 models, due to the construct of the model (no macro interactions with behavioral variables); all the FICO Bands received the same stress factor. However, it is observed historically that the higher FICO is relatively more stressed.

For CCAR2018, in both the account level PD and EADF model, macro –economic variables are interacted with FICO bands, clearly showing that higher FICO is more stressed than the others.

The CCAR 2018 account level models are evaluated for accuracy by back testing for several granular sub-segments like product and FICO. All the Back Test errors were well within the MRM thresholds.

This further reinforces the fact that portfolio segmentation is not necessary.

In order to address the Model Validation Framework challenge, In accordance with Citi's and Global Consumer Risk's Model Risk Management Policies, quarterly monitoring and annual review processes were implemented for ongoing performance reviews for this model.

Following is the performance monitoring plan for the models:

- The business plans to monitor the final model output (NCL) and the model components (PD, EAD, Recovery) every quarter for the recent 9 quarter and 13 quarters.

- Considering the strong model performance in past, the business does not expect the models to breach any MRM thresholds. However, if there are any breaches in the performance metrics thresholds the Model Sponsor will escalate the same to the MRM validation team. Post this, the appropriate adjustments and overlays will be discussed and applied.

- Also, the model will be internally monitored at the granular levels like Product and FICO bands. The inferences from performance at these granular levels will be used to recognize the improvement areas for the Model submission next year.

- What conceptual changes were applied to the model to address the regulatory feedback?

Regulatory feedback for CCAR 2017 was received after CCAR 2018 models were developed. This document reflects enhancements related to prior regulatory feedback and limitations identified during the technical and functional validation of prior models.

There were no feedbacks received on the conceptual changes methodology for the account level models in CCAR 2017. Hence, there have been no conceptual changes in the model construct in CCAR 2018.

As mentioned above, the CCAR 2018 Model does address the segmentation in a very detailed manner . The models have incorporated the interaction terms to capture the different dynamics of different sub-segments. For further details, please refer to Section 6.1.3 Model Segmentation

Also, performance monitoring is being done on a regular basis following the plan stated above from past 1 year.

- Highlight impact on model estimate, its robustness and accuracy based on the regulatory feedback

Regulatory feedback for CCAR 2017 was received after CCAR 2018 models were developed. However, recognizing the improvement areas, the segmentation justification was targeted in a very detailed manner in the current submission.

There were some opportunity areas that were recognized for sub-segments in the CCAR 2017 models. For example, In the CCAR2017 models, due to the construct of the model (no macro interactions with behavioral variables); all the FICO Bands received the same stress factor. However, it is observed historically that the higher FICO is relatively more stressed.

For CCAR2018, in both the account level PD and EADF model, macro –economic variables are interacted with FICO bands. The interaction terms show very intuitive results. Example, coefficients of Macro-economic economic variables interacted with FICO, demonstrate higher FICO being more stressed which is very intuitive. Also, The accuracy of the models was evaluated in both stress and recent time periods, across various dimensions like Product, FICO, Delinquency. It can be seen that the Back Tests results are excellent further reinforcing the fact that portfolio segmentation was not necessary.

- Explain changes to the operational aspects of model development, implementation and usage based on the regulatory feedback]

There were no changes in operational aspects of model development, implementation and usage.

## 7.2 Prior Generation/Version of Model (if Applicable)

Account level PD and EAD models were developed by Branded Cards as challenger models for CCAR 2016 submission. For the CCAR 2017 submission, the business had submitted enhanced account level models as champion models.

For CCAR 2018 models, there are several enhancements that have been made as these are the next generation models as seen in section 7.1 as above.

- Explain key principles of a prior existing model

The key principles of CCAR 2017 models are the same the principles for CCAR 2018 models:

1. **Granularity**: Model should be granular and take into account level information including FICO and state level information
2. **Accuracy**: Model should perform well in back test not just in one or two snapshots but thorughout the data used for model development. Moreover, an out-of-sample should be kept to ensure the model is robust and stable even in the data not used for modeling
3. **Sensitivity**: Model should be sensitive to macro-economic stress as seen in prior stres periods
4. **Business Intuition**: Model variable should make sense from business perspective

- Explain enhancements / changes to the prior model

The key enhancements to the prior model can be seen in the table above in Section 7.1

Further explaining the most notable enhancements as follows:

***Inclusions of the Interaction Terms***:

There were some opportunity areas that were recognized for sub-segments in the CCAR 2017 models. For example, In the CCAR2017 models, due to the construct of the model (no macro interactions with behavioral variables); all the FICO Bands received the same stress factor. However, it is observed historically that the higher FICO is relatively more stressed.

For CCAR2018, in both the account level PD and EADF model, macro –economic variables are interacted with FICO bands. The interaction terms show very intuitive results. Example, coefficients of Macro-economic economic variables interacted with FICO, demonstrate higher FICO being more stressed which is very intuitive. Also, The accuracy of the models was evaluated in both stress and recent time periods, across various dimensions like Product, FICO, Delinquency. It can be seen that the Back Tests results are excellent further reinforcing the fact that portfolio segmentation was not necessary

***Modelling Exposure using EADF*** : One of the major changes in the methodology was to model EAD using EADF and not EAD dollars as the dependent variable. For CCAR 2017, Branded Cards modeled directly the charge-off amount or EAD. After the CCAR 2017; IRMO (Independent Review Modelling Oversight) provided the feedback to evaluate modelling EADF instead of EAD (to check for stationarity, stress etc.). EAD in the form of a ratio is typically found to be more stationary. In addition, EAD ratios are be more intuitive from business judgement point of view. Hence, For CCAR 2018, Branded Cards employed the EADF approach based on the feedback from IRMO. For detailed explanation on the same, please refer to Section 5

Another enhancement has been the inclusion of ***the Regime Indicators*** in CCAR2018 PD models, the details and impact of which has been explained in Section 5.2

- Impact analysis upon model change

The impact of model improvements in terms of data and incorporating the interaction terms etc. can be seen by the strong Back Test results at the overall and granular levels. Please refer to Section 8 for details.

Please see the following table demonstrating the impact of the CCAR 2018 PD and EAD Models over the last year's model

| PD Model | EAD Model | Base GCL | Stress GCL | Sensitivity | %diff In Base | %diff In Stress |
|---|---|---|---|---|---|---|
| PD CCAR 2017 | EAD CCAR 2017 | 5,595 | 8,993 | 61% | | |
| PD CCAR 2018 | EAD CCAR 2017 | 5,687 | 9,140 | 61% | 2% | 2% |
| PD CCAR 2018 | EADF CCAR 2018 | 5,718 | 8,916 | 56% | 1% | -2% |

*The loss numbers in the table above are Cumulative 9 quarters for ECM population

In terms of impact in model forecasts (Loss numbers) ; it was seen that using EADF approach to model Exposure leads to lower Stress losses. EADF Approach assumes the current CLI practices and not the historical pre recession practices; hence leads to lower stress loss.

- Explain how the current model addresses known limitations of the prior model and validation / regular monitoring findings

For CCAR 2017, Branded Cards modeled directly the charge-off amount or EAD. After the CCAR 2017; IRMO (Independent Review Modelling Oversight) provided the feedback to evaluate modelling EADF instead of EAD (to check for stationarity, stress etc.). EAD in the form of a ratio is typically found to be more stationary. In addition, EAD ratios are be more intuitive from business judgement point of view. For CCAR 2018, Branded Cards employed the EADF approach based on the feedback from IRMO .For details , please refer to Section 1.

For the Recovery Model, there was one limitation/validation/regular monitoring findings of the prior model. IRMO provided a functional limitation as below:

*"Because the model fails to control for portfolio makeup or account characteristics, the accuracy of its forecast unduly relies on the assumption that these internal attributes are independent from economic drivers and they remain a constant. Since the assumption is unlikely to hold beyond a short horizon, IRMO concludes that the model cannot be used for CCAR purposes without consideration of a model overlay"*

The business understands the limitation and since the preference is for the Vintage based Recovery Rate model, the Recovery Rate Champion Model for 2017 will now serve as the Challenger Model to CCAR 2018 Submission. The Vintage based recovery model is the Champion Model for CCAR 2018.

For the Vintage based Recovery Rate Model, Two approaches were tried.

In Approach 1, one model was explored for both inventory and future charge offs.

In approach 2, making separate models, one for inventory based charge offs and one for future charge offs was explored. For the inventory based model, account variables such as MOB, Payment behavior before charge off, account delinquency was explored since the business has the information based on these accounts. For future charge offs only MOB information will be available. Macroeconomic variables will be used in both models.

The performance of the models and future recovery forecast were similar in both approaches as shown in Section 6 . Therefore business decided to go with a more simplistic approach 1.

This addressed the feedback of exploring account level variables in the models.

# 8 Model Testing and Outcomes Analysis

To evaluate whether model performs as intended various statistical tests and validations are performed on model inputs and outputs:

- Independent variables are tested for significance, stationarity and multicollinearity.
- To evaluate model accuracy Stress and recent back tests were performed on 9 quarter horizons.
- Sensitivity analysis to support that model output is appropriate for use under base and stress macroeconomic scenarios is carried out.

In addition, modeling assumptions are also tested. The validation results are presented in the following subsections.

## 8.1 Diagnostic and Statistical Tests

To assess the validity of models various statistical tests were conducted. Details of the statistical tests performed are discussed below.

### 8.1.1 Summary of diagnostic and statistical test results

**Statistical Significance for Independent Variables:**

A Wald Chi-Square test is used to test the statistical significance of each coefficient ($\beta$) **for PD model.**

Equation 8.1.1

$$W = [\beta / SE_{\beta}]^2$$

Chi-Square and Pr > ChiSq are the test statistics and p-values, respectively, testing the null hypothesis that an individual predictor's regression coefficient is zero, given the other predictor variables are in the model. The Chi-Square value follows a central Chi-Square distribution with degrees of freedom given by DF, which is used to test against the alternative hypothesis that the Estimate is not equal to zero. The probability that a particular Chi-Square test statistic is as extreme as, or more so, than what has been observed under the null hypothesis is defined by Pr>ChiSq. The higher Wald Chi-Square value of a variable, the better it is.

**For linear regression (EAD and Recovery models)**, Coefficient significance was tested using P-values and t-value tests.

The regression line that SAS calculates from the data is an estimate of a theoretical line describing the relationship between the independent variable (*X*) and the dependent variable ( *Y* ). A simple linear regression analysis is used to develop an equation (a linear regression line) for predicting the dependent variable given a value (*x*) of the independent variable. The regression line calculated by SAS is

**Equation 8.1.1**

$$Y = a + bx$$

Where a and *b* are the least - squares estimates of α and β.

The null hypothesis that there is no predictive linear relationship between the two variables, i.e., the slope of the regression equation is zero. Specifically, the hypotheses are:

*H* 0:β=0
*H a* : β ≠ 0

To **determine** whether the difference is statistically significant, **t**he **t**-**test** calculates a **t**-value. (The p-value is obtained directly from this **t**-value.)

**Equation 8.1.2**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

The probability that a particular t-test statistic is as extreme as, or more so, than what has been observed under the null hypothesis is defined by 'Pr>t' value. The higher t-value of a variable, the better it is.

A low *p* - value for this test (say, less than 0.05) indicates significant evidence to conclude that the slope of the line is not zero — i.e., knowledge of *X* would be useful in predicting *Y*.

**Multicollinearity**: Multicollinearity occurs in a statistical model if two or more independent variables have a strong linear relation. In case of perfect multicollinearity, we can express one independent variable as a linear combination of one (or more) independent variables i.e.

Equation 3

$$X_1 = \beta_0 + \beta_2 * X_2$$

In such a case we cannot estimate the parameters of the model uniquely. Multicollinearity leads to an increase in standard errors of the parameters and hence the estimates are not stable.

To test for the presence of multicollinearity, Variance Inflation Factor is calculated for all the variables in the model for all the quarters. VIF values less than 5 represent that there is no strong correlation (multi-collinearity) between a particular variable being considered and all of the other variables being considered in a particular model.

**Stationarity**: A time series is said to be stationary if the probability distribution of the series does not change when shifted in time. To test for stationarity, Augmented Dickey Fuller test was conducted and all macro variables were found to be stationary. In ADF test a unit root test is carried out on the model equation of the auto regressive process. The unit root test is based on the null hypothesis of whether the test statistic is less than the critical value (as calculated from Dickey-Fuller table) or not. The test statistic is defined as follows:

Equation 8.1.3

$$\text{DFt} = \frac{\gamma}{\text{SE}(\gamma)}$$

The null hypothesis is that $\gamma = 0$ while alternate hypothesis is that $\gamma < 0$. If the null hypothesis is rejected, the model equation does not have a unit root and hence the time series is stationary.

The stationarity checks were done at a confidence interval of 90%. A p-value of < 0.1 indicates stationarity of the variable.

**Statistical Significance for the Overall Model**: To test if at least one coefficient of the model is significantly different from zero, Likelihood Ratio test was conducted. Below is the test statistic for the test and the hypothesis:

Equation 8.1.4

$$-2 * LN(Likelihood\ Ratio) = -2 * \left(\frac{L_0}{L_N}\right)$$

Where, $L_0$ is the Likelihood function where all the coefficients are zero.

$L_1$ is the Likelihood for the model

**Equation 4**

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_k = 0$$

$$H_A: \beta_1 \neq 0 \; or \; \beta_2 \neq 0 \; or \; \ldots\ldots \beta_k \neq 0$$

It can be seen in the results below that null hypothesis is rejected at 1% level of significance.

**R-Squared**: The difference between SST and SSE is the improvement in prediction from the regression model, compared to the mean model. Dividing that difference by SST gives R-squared. It is the proportional improvement in prediction from the regression model, compared to the mean model. It indicates the goodness of fit of the model.

R-squared has the useful property that its scale is intuitive: it ranges from zero to one, with zero indicating that the proposed model does not improve prediction over the mean model and one indicating perfect prediction. Improvement in the regression model results in proportional increases in R-squared.

**Coefficient Of Variation (COV):** The COV is calculated as the ratio of the root mean squared error (RMSE - square root of the variance of the residuals) to the mean of the dependent variable. The COV for a model aims to describe the model fit in terms of the relative sizes of the squared residuals and outcome values. The lower the value of COV, the smaller is the residuals relative to the predicted value. This is suggestive of a good model fit.

Table 8.1.1: Diagnostic and Statistical Test Summary

| Model | Model component/ segment | Type of test | Statistical test conducted | Test results | Business justification/ Limitations | Mitigation |
|---|---|---|---|---|---|---|
| PD | All independent variables | Statistical Significance | Chi square test p-value | Satisfactory | Independent variables are significant at <10% level | Not applicable |
| | | Multi-collinearity | Variance Inflation factor | Satisfactory | VIF is less than 5 for the quarter wise Models | Not applicable |
| | Macroeconomic Variables | Stationarity | Augmented Dickey Fuller | Satisfactory | Tests were conducted on long-term series of each variable / transformation used in the model | Not applicable |
| | Model | Statistical Significance | Likelihood Ratio Test | Satisfactory | Significant at <1% significance | Not applicable |
| | Accuracy | Development in-time Validation | Back Test error rate check | Satisfactory | Not Required | Not applicable |
| EADF | Accuracy | Development in-time Validation | Back Test error rate check | Satisfactory | Not Required | Not applicable |
| | Model Fit | R-Squared | Goodness of Fit Diagnostic | Satisfactory | R-squared value is within acceptable range | Not applicable |
| | | Coefficient Of Variation | Goodness of Fit Diagnostic | Satisfactory | Low COV is preferred | Not applicable |
| | All independent variables | Contribution to model fit | Marginal R-square | Satisfactory | All variables should have positive marginal R-squared | Not applicable |
| | | Statistical Significance | t-test p-value | Satisfactory | Independent variables are significant at <10% level | Not applicable |
| | | Multi-collinearity | Variance Inflation factor | Satisfactory | VIF is less than 5 for the quarter wise Models | Not applicable |
| | Performance | Error Rates at granular level | MAPE | Satisfactory | Lower MAPE is desirable. | Not applicable |
| | | Deviation from Mean | MAD | Satisfactory | Lower MAPE is desirable | Not applicable |
| Recovery | Macro-economic variables | ADF Unit Root | Stationarity Test | Satisfactory | The series used is stationary | Not applicable |
| | Model Parameter Coefficients | t-test P Value | Test for Variable Significance | Satisfactory | Independent variables are generally significant at <5%level | Not applicable |

| | | | | | | |
|---|---|---|---|---|---|---|
| | Model Equation | Variable Inflation Factor | Multicollinearity | Satisfactory | VIF in range of 2.5 for all models | Not applicable |
| | Model Performance | Back Tests | Stress, Recent Backests | Satisfactory | The overall Back Tests are within the threshold | Not applicable |
| | Model Fit | Regress R2 | Goodness of Fit Diagnostic | Satisfactory | The Regress R2 is high for the model | Not applicable |
| | Residuals | Normality | QQ Plot | Satisfactory | The model residuals are normal | Not applicable |

### 8.1.2 Statistical Significance of Parameter Estimates

- To the extent applicable, provide evidence that the model coefficients corresponding to selected independent variables are statistically different from zero

The significance of a parameter estimate is determined by the p-value of the parameter. As a general rule, the significance is tested at 10% confidence interval. If the p-value is less than or equal to 0.1 then the parameter estimate is said to be statistically significant.

**Wald- Chi and p-value for PD Model:**

In the below table, for each variable, the first row gives the p-value and the second row gives the Wald-Chi square value.

## Figure 48: Wald-Chi Square and P-value of PD model variables

| Variable Name | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | Q13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Intercept | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 107512 | 265081 | 637874 | 840412 | 918473 | 954900 | 972827 | 980971 | 952498 | 861834 | 834702 | 804770 | 770147 |
| UTIL_RATE | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 1226 | 39149 | 85996 | 64714 | 52258 | 44129 | 37218 | 32256 | 28332 | 25731 | 22461 | 20229 | 18034 |
| TOTL_RVLV_BAL_AMT_trt | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 24649 | 66796 | 42370 | 38009 | 34155 | 29936 | 26830 | 24280 | 21223 | 17766 | 15053 | 12212 | 10582 |
| TOTL_RVLV_BAL_AMT_0_dummy | <.0001 | <.0001 | <.0001 | <.0001 | | | | | | | | | |
| | 64 | 586 | 1152 | 149 | | | | | | | | | |
| Bill_actv_ind | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 7244 | 9297 | 16983 | 13493 | 12650 | 12625 | 12764 | 13382 | 14094 | 13340 | 13463 | 13076 | 13262 |
| INQ_6MON_CT | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | | | 3710 | 4114 | 4731 | 4969 | 4872 | 4367 | 4045 | 3587 | 3292 | 2886 | 2530 |
| new_CB_BAL_DLQ2_trt | <.0001 | <.0001 | | | | | | | | | | | |
| | 12807 | 20420 | | | | | | | | | | | |
| SALES_ACTV_IND_TRT | <.0001 | <.0001 | | | | | | | | | | | |
| | 21132 | 6537 | | | | | | | | | | | |
| pre_mob_25_36 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 3149 | 3149 | 3149 | 3149 | 3149 | 3149 | 3149 | 3149 | 3149 | 1114 | 1114 | 1114 | 1114 |
| post_mob_25_36 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 3025 | 3025 | 3025 | 3025 | 3025 | 3025 | 3025 | 3025 | 3025 | 1505 | 1505 | 1505 | 1505 |
| pre_mob_37_48 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | | | | |
| | 2525 | 2525 | 2525 | 2525 | 2525 | 2525 | 2525 | 2525 | 2525 | | | | |
| post_mob_37_48 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 1001 | 1001 | 1001 | 1001 | 1001 | 1001 | 1001 | 1001 | 1001 | 727 | 727 | 727 | 727 |
| FICO_LE_660 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 15245 | 37963 | 118666 | 124334 | 119204 | 108123 | 96982 | 86412 | 76088 | 84762 | 71738 | 60069 | 50459 |
| POST_REGIME_DELQ | | | <.0001 | <.0001 | <.0001 | | | | | | | | |
| | | | 4484 | 1686 | 527 | | | | | | | | |
| FICO_661_720 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 5817 | 14527 | 44500 | 56940 | 61797 | 61499 | 59609 | 55819 | 51683 | 61564 | 54761 | 47834 | 41148 |
| POST_REGIME_BKT0 | | | | | | | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | | | | | | | | | 394 | 413 | 378 | 287 | 265 |
| FICO_721_780 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 441 | 1995 | 10343 | 16656 | 21057 | 24146 | 26160 | 26945 | 26403 | 24917 | 23229 | 21150 | 18981 |
| FICO_GT_900 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 25571 | 46898 | 62096 | 51902 | 42498 | 35318 | 31269 | 26553 | 22252 | 18480 | 14552 | 12450 | 9896 |
| Q2_dummy | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 |
| Q3_dummy | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 3421 | 3421 | 3421 | 3421 | 3421 | 3421 | 3421 | 3421 | 3421 | 3421 | 3421 | 3421 | 3421 |
| Q4_dummy | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 2257 | 2257 | 2257 | 2257 | 2257 | 2257 | 2257 | 2257 | 2257 | 2257 | 2257 | 2257 | 2257 |
| dmob_ind_0_6 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | | | | |
| | 599 | 599 | 599 | 599 | 599 | 599 | 599 | 599 | 599 | | | | |
| dmob_ind_7_12 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | | | | |
| | 2331 | 2331 | 2331 | 2331 | 2331 | 2331 | 2331 | 2331 | 2331 | | | | |
| dmob_ind_13_24 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | | | | |
| | 5013 | 5013 | 5013 | 5013 | 5013 | 5013 | 5013 | 5013 | 5013 | | | | |
| dmob_ind_49_120 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 3649 | 3649 | 3649 | 3649 | 3649 | 3649 | 3649 | 3649 | 3649 | 126 | 126 | 126 | 126 |
| FL_Dummy | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 150 | 2338 | 1692 | 2094 | 2474 | 2508 | 2731 | 2769 | 2628 | 2632 | 2713 | 2500 | 2273 |
| dmob_ind_121_144 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 1069 | 1069 | 1069 | 1069 | 1069 | 1069 | 1069 | 1069 | 1069 | 483 | 483 | 483 | 483 |
| dmob_ind_145_180 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | | | | |
| | 19 | 19 | 19 | 19 | 19 | 19 | 19 | 19 | 19 | | | | |

Figure 49: Wald-Chi Square and P-value of PD model variables

| Variable Name | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | Q13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DELQ | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | |
| | | | 66512 | 66512 | 66512 | 66512 | 66512 | 3173 | 749 | 450 | 307 | 148 | |
| bucket1_dummy | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 85984 | 492271 | 15085 | 7579 | 4601 | 5975 | 4469 | 2618 | 1151 | 662 | 336 | 197 | 60 |
| bucket2_dummy | <.0001 | <.0001 | | | | | | | | | | | |
| | 182397 | 942530 | | | | | | | | | | | |
| bucket3_dummy | <.0001 | <.0001 | | | | | | | | | | | |
| | 178202 | 977567 | | | | | | | | | | | |
| bucket4_dummy | <.0001 | <.0001 | | | | | | | | | | | |
| | 859830 | 124651 | | | | | | | | | | | |
| bucket5_dummy | <.0001 | <.0001 | | | | | | | | | | | |
| | 769024 | 48841 | | | | | | | | | | | |
| bucket6_dummy | <.0001 | <.0001 | | | | | | | | | | | |
| | 650733 | 13654 | | | | | | | | | | | |
| HPI_l1_w4_t4_CON | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | | | | |
| | | | 28948 | 28948 | 28948 | 28948 | 28948 | 28948 | 28948 | | | | |
| HPI_l1_w4_t4_biz | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | | | | |
| | | | 4268 | 4268 | 4268 | 4268 | 4268 | 4268 | 4268 | | | | |
| UR_l0_w2_t2 | <.0001 | <.0001 | | | | | | | | | | | |
| | 9964 | 16410 | | | | | | | | | | | |
| UR_l0_w2_t2_720 | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | | | | |
| | | | 50643 | 50643 | 50643 | 50643 | 50643 | 50643 | 50643 | | | | |
| UR_l0_w2_t2_900 | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | | | | |
| | | | 22243 | 22243 | 22243 | 22243 | 22243 | 22243 | 22243 | | | | |
| UR_l0_w2_t2_biz | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | | | 36841 | 36841 | 36841 | 36841 | 36841 | 36841 | 36841 | 14040 | 14040 | 14040 | 14040 |
| UR_l0_w2_t2_CON | | | | | | | | | | <.0001 | <.0001 | <.0001 | <.0001 |
| | | | | | | | | | | 20996 | 20996 | 20996 | 20996 |
| UR_l3_w4_t3 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 13813 | 13813 | 13813 | 13813 | 13813 | 13813 | 13813 | 13813 | 13813 | 5445 | 5445 | 5445 | 5445 |
| BKT0_CRM_123_IND | <.0001 | <.0001 | | | | | | | | | | | |
| | 4242 | 1782 | | | | | | | | | | | |
| BKT0_CRM_456_IND | <.0001 | <.0001 | | | | | | | | | | | |
| | 32591 | 2556 | | | | | | | | | | | |
| dec10_ind | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 8012 | 8012 | 8012 | 8012 | 8012 | 8012 | 8012 | 8012 | 8012 | 8012 | 8012 | 8012 | 8012 |
| sep09_ind | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 379 | 379 | 379 | 379 | 379 | 379 | 379 | 379 | 379 | 379 | 379 | 379 | 379 |
| sep10_ind | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 7836 | 7836 | 7836 | 7836 | 7836 | 7836 | 7836 | 7836 | 7836 | 7836 | 7836 | 7836 | 7836 |
| Indicator_AA | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| | 2438 | 6095 | 4228 | 4436 | 4311 | 4320 | 4069 | 4011 | 3820 | 3460 | 3096 | 2885 | 2440 |
| Indicator_Cobrand | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | | | | | | |
| | 146 | 191 | 1259 | 803 | 585 | 381 | 261 | | | | | | |
| Indicator_rewards | <.0001 | <.0001 | <.0001 | | | | | | | | | | |
| | 39 | 55 | 146 | | | | | | | | | | |
| PURCH_APR_LT18_IND | <.0001 | <.0001 | <.0001 | | | | | | | | | | |
| | 10445 | 3876 | 1523 | | | | | | | | | | |
| RR_4_to_CWO_POST | <.0001 | <.0001 | | | | | | | | | | | |
| | 2798 | 1313 | | | | | | | | | | | |

As seen in the above table p-value of all the variables is less than 0.1 and hence all the parameter estimates are statistically significant in the PD model.

**<u>t-value and p-value of EADF model:</u>**

In the below table, for each variable, the first row gives the t-value and the second row gives the p-value.

## Figure 50: t-value and P-value of EADF model variables

| Variable ( t value / p value) | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | Q13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Intercept | 282.37 | 282.37 | 282.37 | 282.37 | 282.37 | 282.37 | 282.37 | 282.37 | 282.37 | 282.37 | 282.37 | 282.37 | 282.37 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| AA_IND | -93.92 | -93.92 | -93.92 | -93.92 | -93.92 | -93.92 | -93.92 | -93.92 | -93.92 | -93.92 | -93.92 | -93.92 | -93.92 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| CA_AMT_ind | 59.04 | 59.04 | 59.04 | 59.04 | 59.04 | 59.04 | 59.04 | 59.04 | 59.04 | 59.04 | 59.04 | 59.04 | 59.04 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| Cash_Ind | -30.66 | -30.66 | -30.66 | -30.66 | -30.66 | -30.66 | -30.66 | -30.66 | -30.66 | -30.66 | -30.66 | -30.66 | -30.66 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| Cobrand_Ind | -74.08 | -74.08 | -74.08 | -74.08 | -74.08 | -74.08 | -74.08 | -74.08 | -74.08 | -74.08 | -74.08 | -74.08 | -74.08 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| cur_util | 417.76 | 656.53 | 512.27 | 441.08 | 559.06 | 492.81 | 426.06 | 385.08 | 345.99 | 314.66 | 283.71 | 253.71 | 230.31 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| fico_780_ind | | | -237.6 | -237.6 | -237.6 | -237.6 | -237.6 | -237.6 | -237.6 | -237.6 | -237.6 | -237.6 | -237.6 |
| | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| fico_miss | 12.35 | 12.35 | 12.35 | 12.35 | 12.35 | 12.35 | 12.35 | 12.35 | 12.35 | 12.35 | 12.35 | 12.35 | 12.35 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| HPI_Biz | | | -13.06 | -13.06 | -13.06 | -13.06 | -13.06 | -13.06 | -13.06 | -13.06 | -13.06 | -13.06 | -13.06 |
| | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| HPI_cc_FICO_IND_660 | | | -15.82 | -15.82 | -15.82 | -15.82 | -15.82 | -15.82 | -15.82 | -15.82 | -15.82 | -15.82 | -15.82 |
| | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| HPI_cc_FICO_IND_660_780 | | | -53.48 | -53.48 | -53.48 | -53.48 | -53.48 | -53.48 | -53.48 | -53.48 | -53.48 | -53.48 | -53.48 |
| | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| HPI_cc_FICO_IND_gt780 | | | -80.72 | -80.72 | -80.72 | -80.72 | -80.72 | -80.72 | -80.72 | -80.72 | -80.72 | -80.72 | -80.72 |
| | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| HPI_cons_buck1plus | | | -11.66 | -11.66 | -11.66 | -11.66 | -11.66 | -11.66 | -11.66 | -11.66 | -11.66 | -11.66 | -11.66 |
| | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| HPI_l1_w3_t4 | | -3.62 | | | | | | | | | | | |
| | | 0.0003 | | | | | | | | | | | |
| INQ_6MON_CT_mfc | | | 23.96 | 40.73 | 47.44 | 49.01 | 52.86 | 53.53 | 56.79 | 55.61 | 53.11 | 50.79 | 47.08 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| mob_ind_10_12 | 24.06 | 24.06 | 24.06 | 24.06 | 24.06 | 24.06 | 24.06 | 24.06 | 24.06 | 24.06 | 24.06 | 24.06 | 24.06 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| mob_ind_79 | 4.78 | 4.78 | 4.78 | 4.78 | 4.78 | 4.78 | 4.78 | 4.78 | 4.78 | 4.78 | 4.78 | 4.78 | 4.78 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| MOB_LP_13_360 | -316.77 | -316.77 | -316.8 | -316.8 | -316.8 | -316.8 | -316.8 | -316.8 | -316.8 | -316.8 | -316.8 | -316.8 | -316.8 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| NETFR_BONT_BURDN_NUM_mfc | | | 33.18 | 72.35 | 251.49 | 251.49 | 251.49 | 251.49 | 251.49 | 251.49 | 251.49 | 251.49 | 251.49 |
| | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| open_ind | | | 51.48 | 120.2 | 177.46 | 226.85 | 281.92 | 316.54 | 347.27 | 362.97 | 376.1 | 383.35 | 385.89 |
| | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| purch_ct_mfc | | 16.09 | 49.95 | 68.29 | 74.41 | 77.06 | 126.01 | 126.01 | 126.01 | 129.11 | 129.11 | 129.11 | 129.11 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| UR_buck1plus | | | 15.44 | 15.44 | 15.44 | 15.44 | 15.44 | 15.44 | 15.44 | 15.44 | 15.44 | 15.44 | 15.44 |
| | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| UR_cur | | | 236.25 | 236.25 | 236.25 | 236.25 | 236.25 | 236.25 | 236.25 | 236.25 | 236.25 | 236.25 | 236.25 |
| | | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| UR_l0_w3_t3 | 7.12 | 17 | | | | | | | | | | | |
| | <.0001 | <.0001 | | | | | | | | | | | |
| util_123 | 643.85 | 1292.1 | 440.71 | 355.44 | 358.06 | 320.61 | 267.01 | 217.68 | 173.35 | 144.71 | 123.17 | 98.8 | 79.57 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| util_456 | 1356.7 | 340.37 | 175.55 | 169.64 | 153.49 | 164.61 | 147.78 | 88.45 | 66.91 | 62.11 | 58.18 | 47.94 | 44.42 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |
| util0 | | 19.68 | 98.54 | 137.79 | 145.22 | 147.91 | 142.92 | 140.7 | 130.15 | 125.68 | 117.99 | 110.3 | 102.79 |
| | | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |

As seen in the above table p-value of all the variables is less than 0.1 and hence all the parameter estimates are statistically significant in the EADF model.

**t-value and p-value of Recovery Model:**

In the below table, for each variable, the first column gives the t-value and the second column gives the p-value.

Figure 51: t-value and P-value of Recovery Rate model variables

| Parameter | t Value | Pr > \|t\| |
|---|---|---|
| Intercept | -396.09 | <.0001 |
| Perf_Qtr_bus_Q0 | 79.47 | <.0001 |
| Perf_Qtr_bus_Q1 | 80.8 | <.0001 |
| Perf_Qtr_bus_Q2 | 71.53 | <.0001 |
| Perf_Qtr_bus_Q3 | 65.26 | <.0001 |
| Perf_Qtr_bus_Q4 | 59.96 | <.0001 |
| Perf_Qtr_bus_Q5 | 53.61 | <.0001 |
| Perf_Qtr_bus_Q6 | 49.16 | <.0001 |
| Perf_Qtr_bus_Q7 | 44.45 | <.0001 |
| Perf_Qtr_bus_Q8 | 40.25 | <.0001 |
| Perf_Qtr_bus_Q9 | 38.38 | <.0001 |
| Perf_Qtr_bus_Q10 | 35.81 | <.0001 |
| Perf_Qtr_bus_Q11 | 32.17 | <.0001 |
| Perf_Qtr_bus_Q12 | 28.34 | <.0001 |
| Perf_Qtr_bus_Q13 | 26.14 | <.0001 |
| Perf_Qtr_bus_Q14 | 23.45 | <.0001 |
| Perf_Qtr_bus_Q15 | 20.63 | <.0001 |
| Perf_Qtr_bus_Q16 | 19.54 | <.0001 |
| Perf_Qtr_bus_Q17 | 15.73 | <.0001 |
| Perf_Qtr_bus_Q18 | 13.89 | <.0001 |
| Perf_Qtr_bus_Q19 | 11.37 | <.0001 |
| Perf_Qtr_bus_Q20 | 10.02 | <.0001 |
| Perf_Qtr_con_Q0 | 76.86 | <.0001 |
| Perf_Qtr_con_Q1 | 84.31 | <.0001 |
| Perf_Qtr_con_Q2 | 80.27 | <.0001 |
| Perf_Qtr_con_Q3 | 75.69 | <.0001 |
| Perf_Qtr_con_Q4 | 73.14 | <.0001 |
| Perf_Qtr_con_Q5 | 68.5 | <.0001 |
| Perf_Qtr_con_Q6 | 64.12 | <.0001 |
| Perf_Qtr_con_Q7 | 60.73 | <.0001 |
| Perf_Qtr_con_Q8 | 57.53 | <.0001 |
| Perf_Qtr_con_Q9 | 53.43 | <.0001 |
| Perf_Qtr_con_Q10 | 49.44 | <.0001 |
| Perf_Qtr_con_Q11 | 46.3 | <.0001 |
| Perf_Qtr_con_Q12 | 42.34 | <.0001 |
| Perf_Qtr_con_Q13 | 39.04 | <.0001 |
| Perf_Qtr_con_Q14 | 35.2 | <.0001 |
| Perf_Qtr_con_Q15 | 32.43 | <.0001 |
| Perf_Qtr_con_Q16 | 29.68 | <.0001 |
| Perf_Qtr_con_Q17 | 25.94 | <.0001 |
| Perf_Qtr_con_Q18 | 23.32 | <.0001 |
| Perf_Qtr_con_Q19 | 20.03 | <.0001 |
| Perf_Qtr_con_Q20 | 17.43 | <.0001 |
| Perf_Qtr_con_Q21 | 14.77 | <.0001 |
| Perf_Qtr_con_Q22 | 12.6 | <.0001 |
| Perf_Qtr_con_Q23 | 9.36 | <.0001 |
| Perf_Qtr_con_Q24 | 7.85 | <.0001 |
| Perf_Qtr_con_Q25 | 4.81 | <.0001 |
| HPI_l1_w2_t4_con | 9.73 | <.0001 |
| HPI_l1_w2_t4_bus | 18.12 | <.0001 |
| UR_l2_w4_t3 | -19.45 | <.0001 |
| prod_AA_con | 10.43 | <.0001 |
| Q3_seas | -1.86 | 0.0632 |
| ind_event | -3.71 | 0.0002 |

Based on p-value all recovery rate model parameters are statistically significant.

### 8.1.3 Multi-collinearity

- To the extent applicable, provide evidence of absence of multicollinearity.

Multicollinearity is a phenomenon in which one predictor variable in a multiple regression model can be linearly predicted from the others with a substantial degree of accuracy. Multicollinearity in a model is identified by calculating the Variance Inflation Factor (VIF) for each predictor variable. VIF values less than 5 indicate absence of multicollinearity in the model. VIF values of model predictors for PD, EADF and recovery models are given in subsequent tables.

**VIF table for PD Model:**

Below table gives the quarterly VIF values of all the PD variables.

Figure 52: VIF of PD Model Variables

| Variable Name | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | Q13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *UTIL_RATE* | 2.25 | 2.20 | 2.16 | 2.14 | 2.13 | 2.11 | 2.10 | 2.09 | 2.08 | 2.05 | 2.04 | 2.03 | 2.02 |
| *TOTL_RVLV_BAL_AMT_trt* | 1.36 | 1.36 | 1.35 | 1.35 | 1.33 | 1.32 | 1.32 | 1.32 | 1.32 | 1.30 | 1.29 | 1.29 | 1.29 |
| *TOTL_RVLV_BAL_AMT_0_dummy* | 1.06 | 1.06 | 1.06 | 1.06 | | | | | | | | | |
| *Bill_actv_ind* | 2.05 | 2.06 | 1.37 | 1.35 | 1.33 | 1.33 | 1.33 | 1.33 | 1.32 | 1.32 | 1.32 | 1.31 | 1.32 |
| *INQ_6MON_CT* | | | 1.10 | 1.10 | 1.10 | 1.09 | 1.09 | 1.09 | 1.09 | 1.08 | 1.07 | 1.06 | 1.06 |
| *new_CB_BAL_DLQ2_trt* | 1.18 | 1.13 | | | | | | | | | | | |
| *SALES_ACTV_IND_TRT* | 1.70 | 1.72 | | | | | | | | | | | |
| *BKT0_CRM_123_IND* | 1.03 | 1.03 | | | | | | | | | | | |
| *BKT0_CRM_456_IND* | 1.01 | 1.01 | | | | | | | | | | | |
| *PURCH_APR_LT18_IND* | 1.16 | 1.15 | 1.14 | | | | | | | | | | |
| *FICO_LE_660* | 1.94 | 1.87 | 1.71 | 1.67 | 1.64 | 1.62 | 1.60 | 1.59 | 1.58 | 1.56 | 1.55 | 1.54 | 1.52 |
| *FICO_661_720* | 1.64 | 1.64 | 1.63 | 1.61 | 1.60 | 1.59 | 1.58 | 1.57 | 1.56 | 1.54 | 1.53 | 1.52 | 1.51 |
| *FICO_721_780* | 1.31 | 1.31 | 1.30 | 1.30 | 1.29 | 1.29 | 1.28 | 1.28 | 1.28 | 1.26 | 1.26 | 1.25 | 1.25 |
| *FICO_GT_900* | 1.17 | 1.11 | 1.15 | 1.07 | 1.08 | 1.09 | 1.12 | 1.07 | 1.08 | 1.09 | 1.13 | 1.06 | 1.06 |
| *POST_REGIME_DELQ* | | | 1.54 | 1.51 | 1.48 | | | | | | | | |
| *bucket1_dummy* | 1.05 | 1.05 | 1.54 | 1.50 | 1.47 | 1.03 | 1.03 | 1.03 | 1.05 | 1.04 | 1.04 | 1.04 | 1.04 |
| *bucket2_dummy* | 1.03 | 1.03 | | | | | | | | | | | |
| *bucket3_dummy* | 1.03 | 1.03 | | | | | | | | | | | |
| *bucket4_dummy* | 1.03 | 1.01 | | | | | | | | | | | |
| *bucket5_dummy* | 1.04 | 1.01 | | | | | | | | | | | |
| *bucket6_dummy* | 1.05 | 1.00 | | | | | | | | | | | |
| *POST_REGIME_BKT0* | | | | | | | | | | 1.80 | 1.73 | 1.62 | 1.50 |
| *DELQ* | | | 0.04 | 1.07 | 1.06 | 1.03 | 1.03 | 1.02 | 1.03 | 1.03 | 1.02 | 1.02 | . |
| *RR_4_to_CWO_POST* | 1.17 | 1.19 | | | | | | | | | | | |
| *dmob_ind_0_6* | 1.25 | | | | | | | | | | | | |
| *dmob_ind_7_12* | 1.25 | 1.31 | 1.23 | | | | | | | | | | |
| *dmob_ind_13_24* | 1.38 | 1.40 | 1.44 | 1.40 | 1.32 | 1.24 | 1.17 | | | | | | |
| *pre_mob_25_36* | 1.27 | 1.27 | 1.27 | 1.23 | 1.23 | 1.24 | 1.26 | 1.24 | 1.20 | 1.09 | 1.05 | | |
| *post_mob_25_36* | 1.10 | 1.11 | 1.10 | 1.10 | 1.10 | 1.11 | 1.12 | 1.15 | 1.16 | 1.11 | 1.11 | | |
| *pre_mob_37_48* | 1.25 | 1.25 | 1.26 | 1.23 | 1.23 | 1.22 | 1.22 | 1.20 | 1.20 | | | | |
| *post_mob_37_48* | 1.05 | 1.05 | 1.05 | 1.05 | 1.05 | 1.05 | 1.05 | 1.05 | 1.07 | 1.07 | 1.08 | 1.11 | 1.10 |
| *dmob_ind_49_120* | 1.61 | 1.61 | 1.60 | 1.56 | 1.54 | 1.53 | 1.52 | 1.46 | 1.44 | 1.14 | 1.13 | 1.11 | 1.12 |
| *dmob_ind_121_144* | 1.17 | 1.17 | 1.17 | 1.16 | 1.16 | 1.16 | 1.16 | 1.15 | 1.15 | 1.07 | 1.07 | 1.07 | 1.07 |
| *dmob_ind_145_180* | 1.22 | 1.22 | 1.21 | 1.20 | 1.20 | 1.19 | 1.18 | 1.18 | 1.17 | | | | |
| *FL_Dummy* | 1.00 | 1.00 | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 |
| *Q2_dummy* | 1.47 | 1.56 | 1.50 | 1.48 | 1.46 | 1.57 | 1.50 | 1.48 | 1.48 | 1.58 | 1.51 | 1.48 | 1.48 |
| *Q3_dummy* | 1.67 | 1.77 | 1.79 | 1.80 | 1.78 | 1.90 | 1.88 | 1.96 | 1.96 | 2.03 | 2.03 | 1.74 | 1.70 |
| *Q4_dummy* | 1.53 | 1.61 | 1.60 | 1.58 | 1.55 | 1.65 | 1.63 | 1.61 | 1.60 | 1.69 | 1.68 | 1.67 | 1.71 |
| *Indicator_AA* | 1.17 | 1.17 | 1.15 | 1.07 | 1.07 | 1.07 | 1.07 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 |
| *Indicator_Cobrand* | 1.20 | 1.20 | 1.20 | 1.10 | 1.10 | 1.10 | 1.10 | | | | | | |
| *Indicator_rewards* | 1.15 | 1.15 | 1.15 | | | | | | | | | | |
| *dec10_ind* | 1.14 | 1.14 | 1.18 | 1.19 | 1.20 | 1.20 | 1.20 | 1.20 | 1.24 | 1.25 | 1.25 | 1.24 | 1.37 |
| *sep09_ind* | 1.19 | 1.19 | 1.24 | 1.29 | 1.30 | 1.29 | 1.29 | 1.39 | 1.40 | 1.45 | 1.49 | | |
| *sep10_ind* | 1.18 | 1.18 | 1.41 | 1.44 | 1.44 | 1.44 | 1.44 | 1.53 | 1.55 | 1.55 | 1.56 | 1.53 | 1.58 |
| *UR_l0_w2_t2* | 1.15 | 1.18 | 1.48 | 1.49 | 1.51 | 1.53 | 1.57 | 1.64 | 2.16 | 2.26 | 2.25 | 2.16 | 1.99 |
| *HPI_l1_w4_t4* | | | 1.59 | 1.61 | 1.61 | 1.62 | 1.64 | 1.68 | 1.81 | 1.86 | 1.96 | 1.63 | 1.44 |
| *UR_l3_w4_t3* | | | 2.29 | 2.33 | 2.34 | 2.33 | 2.32 | 2.36 | 2.37 | 2.38 | 2.38 | 2.27 | 2.19 |

VIF of all the model predictors is below 2.5 across all quarters which imply absence of multicollinearity in the model.

**VIF table for EADF model:**

Below table gives the quarterly VIF values of all the EADF variables.

Figure 53: VIF of EADF Model Variables

| Parameter | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | Q13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AA_Ind | 1.06 | 1.07 | 1.09 | 1.10 | 1.12 | 1.12 | 1.13 | 1.14 | 1.14 | 1.15 | 1.16 | 1.18 | 1.19 |
| CA_AMT_ind | 1.01 | 1.02 | 1.03 | 1.02 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 |
| Cash_Ind | 1.05 | 1.05 | 1.05 | 1.06 | 1.06 | 1.06 | 1.06 | 1.06 | 1.06 | 1.06 | 1.06 | 1.06 | 1.07 |
| CoBrand_Ind | 1.07 | 1.07 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 |
| cur_util | 1.45 | 1.94 | 3.20 | 2.77 | 2.57 | 2.47 | 2.39 | 2.31 | 2.25 | 2.22 | 2.19 | 2.19 | 2.17 |
| fico_miss | 1.01 | 1.01 | 1.05 | 1.02 | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 |
| HPI_l1_w3_t4 | | 2.05 | | | | | | | | | | | |
| mob_ind_10_12 | 1.03 | 1.03 | 1.09 | | | | | | | | | | |
| mob_ind_79 | 1.01 | 1.02 | | | | | | | | | | | |
| MOB_LP_13_360 | 1.13 | 1.13 | 1.15 | 1.12 | 1.12 | 1.11 | 1.11 | 1.10 | 1.10 | 1.09 | 1.09 | 1.08 | 1.08 |
| PURCH_CT_mfc | | 1.06 | 1.13 | 1.13 | 1.14 | 1.14 | 1.14 | 1.15 | 1.15 | 1.16 | 1.17 | 1.18 | 1.18 |
| UR_l0_w3_t3 | 1.03 | 2.06 | | | | | | | | | | | |
| util_123 | 1.74 | 2.16 | 2.68 | 2.16 | 1.95 | 1.87 | 1.77 | 1.63 | 1.52 | 1.45 | 1.43 | 1.47 | 1.66 |
| util_456 | 2.14 | 1.20 | 1.15 | 1.16 | 1.16 | 1.19 | 1.18 | 1.08 | 1.06 | 1.07 | 1.07 | 1.06 | 1.07 |
| util0 | | 1.05 | 1.14 | 1.21 | 1.25 | 1.27 | 1.29 | 1.30 | 1.30 | 1.30 | 1.30 | 1.29 | 1.29 |
| fico_780_ind | | | 1.33 | 1.33 | 1.34 | 1.35 | 1.35 | 1.35 | 1.34 | 1.34 | 1.33 | 1.32 | 1.32 |
| HPI_Biz | | | 1.11 | 1.11 | 1.10 | 1.09 | 1.09 | 1.08 | 1.06 | 1.05 | 1.03 | 1.01 | 1.01 |
| HPI_cc_FICO_IND_660 | | | 1.58 | 1.48 | 1.41 | 1.36 | 1.31 | 1.28 | 1.24 | 1.19 | 1.13 | 1.05 | 1.03 |
| HPI_cc_FICO_IND_660_780 | | | 1.45 | 1.54 | 1.60 | 1.63 | 1.63 | 1.64 | 1.59 | 1.53 | 1.39 | 1.15 | 1.05 |
| HPI_cc_FICO_IND_gt780 | | | 1.05 | 1.06 | 1.10 | 1.11 | 1.13 | 1.13 | 1.12 | 1.12 | 1.09 | 1.07 | 1.04 |
| HPI_cons_buck1plus | | | 2.02 | 2.06 | 2.05 | 1.95 | 1.91 | 1.95 | 1.83 | 1.71 | 1.51 | 1.19 | 1.08 |
| INQ_6MON_CT_mfc | | | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.07 | 1.07 | 1.06 | 1.06 | 1.05 | 1.06 |
| NETFR_BONT_BURDN_NUM_mfc | | | 1.85 | 1.88 | 1.89 | 1.89 | 1.88 | 1.88 | 1.87 | 1.87 | 1.87 | 1.88 | 1.89 |
| OPEN_IND | | | 1.17 | 1.19 | 1.19 | 1.22 | 1.19 | 1.16 | 1.14 | 1.15 | 1.15 | 1.14 | 1.14 |
| UR_buck1plus | | | 2.00 | 2.04 | 2.02 | 1.89 | 1.83 | 1.92 | 1.81 | 1.70 | 1.51 | 1.29 | 1.40 |
| UR_cur | | | 2.08 | 2.09 | 2.09 | 2.08 | 2.06 | 2.04 | 1.94 | 1.83 | 1.61 | 1.26 | 1.11 |

VIF of most of the model predictors is below 2.5 across all the performance quarters indicating absence of multicollinearity in the model.

**VIF of Recovery Rate Model:**

Figure 54: VIF of Recovery Rate Model Variables

| Variable | VIF |
|---|---|
| Intercept | 0.00 |
| Perf_Qtr_bus_Q0 | 1.18 |
| Perf_Qtr_bus_Q1 | 1.17 |
| Perf_Qtr_bus_Q2 | 1.17 |
| Perf_Qtr_bus_Q3 | 1.16 |
| Perf_Qtr_bus_Q4 | 1.16 |
| Perf_Qtr_bus_Q5 | 1.15 |
| Perf_Qtr_bus_Q6 | 1.14 |
| Perf_Qtr_bus_Q7 | 1.14 |
| Perf_Qtr_bus_Q8 | 1.13 |
| Perf_Qtr_bus_Q9 | 1.12 |
| Perf_Qtr_bus_Q10 | 1.12 |
| Perf_Qtr_bus_Q11 | 1.11 |
| Perf_Qtr_bus_Q12 | 1.11 |
| Perf_Qtr_bus_Q13 | 1.10 |
| Perf_Qtr_bus_Q14 | 1.10 |
| Perf_Qtr_bus_Q15 | 1.09 |
| Perf_Qtr_bus_Q16 | 1.09 |
| Perf_Qtr_bus_Q17 | 1.08 |
| Perf_Qtr_bus_Q18 | 1.08 |
| Perf_Qtr_bus_Q19 | 1.08 |
| Perf_Qtr_bus_Q20 | 1.08 |

| Variable | VIF |
|---|---|
| Perf_Qtr_con_Q0 | 1.18 |
| Perf_Qtr_con_Q1 | 1.18 |
| Perf_Qtr_con_Q2 | 1.18 |
| Perf_Qtr_con_Q3 | 1.18 |
| Perf_Qtr_con_Q4 | 1.18 |
| Perf_Qtr_con_Q5 | 1.18 |
| Perf_Qtr_con_Q6 | 1.18 |
| Perf_Qtr_con_Q7 | 1.18 |
| Perf_Qtr_con_Q8 | 1.18 |
| Perf_Qtr_con_Q9 | 1.17 |
| Perf_Qtr_con_Q10 | 1.17 |
| Perf_Qtr_con_Q11 | 1.16 |
| Perf_Qtr_con_Q12 | 1.16 |
| Perf_Qtr_con_Q13 | 1.15 |
| Perf_Qtr_con_Q14 | 1.15 |
| Perf_Qtr_con_Q15 | 1.14 |
| Perf_Qtr_con_Q16 | 1.14 |
| Perf_Qtr_con_Q17 | 1.13 |
| Perf_Qtr_con_Q18 | 1.13 |
| Perf_Qtr_con_Q19 | 1.12 |
| Perf_Qtr_con_Q20 | 1.11 |
| Perf_Qtr_con_Q21 | 1.11 |
| Perf_Qtr_con_Q22 | 1.10 |
| Perf_Qtr_con_Q23 | 1.10 |
| Perf_Qtr_con_Q24 | 1.10 |
| Perf_Qtr_con_Q25 | 1.09 |
| HPI_l1_w2_t4_con | 1.23 |
| HPI_l1_w2_t4_bus | 1.28 |
| UR_l2_w4_t3 | 1.31 |
| prod_AA_con | 1.36 |
| Q3_seas | 1.00 |
| ind_event | 1.03 |

VIF of all the model predictors in recovery model is below 2 across all the performance quarters. Thus there is no issue of multicollinearity in the recovery rate model.

### 8.1.4 Stationarity

- To the extent applicable, provide evidence that the time series data are stationary. [Only applicable for time series models]

The NA Branded Cards business evaluated the national level macroeconomic variables for stationarity through the ADF (Augmented Dickey Fuller) test.

Branded Cards uses the below variables for account level PD EADF and vintage based Recovery Model development.

- Unemployment Rate (state/ region level)
- Housing Price Index (state/ region level)

There is a high degree of correlation between national and regional macro variables. Hence for consistency of variable usage across states, the national variables are used for stationarity testing.

ADF Test was run for every transformation over a period of Q1 1980 to Q2 201. The transformations are as follows:

- T2: Raw Average
- T3: Raw Difference
- T4: Percentage Difference
- Length of window is in quarters: 1,2,3,4
- Lags tested is in quarter: 0,1,2,3

The criterion for a transformation to be stationary, which is consistent across the NA Cards businesses, is:

1. That the series passes the ADF Test at 10% Level of significance.
2. If any of the ADF (Single Mean or Zero Mean) lags the p value is significant (less than 0.1), the null hypothesis of existence of unit root is rejected and the variable is accepted as a stationary transformation.

However we notice that most of the transformations are stationary, hence we are comfortable in using them in our models.

Stationarity test of UR and HPI along with different windows is conducted using augmented dickey fuller test. Non stationary transformations are dropped. The test was conducted for only those macroeconomic transformations which have been used in the models.

Following table shows the Stationarity test results of the macro economic variables used in the models.

**Table 8.1.2: Stationarity of Macroeconomic Variables**

| | | Macroeconomic Variable Description | | | | Conclusion |
|---|---|---|---|---|---|---|
| LABEL | Model | Variable | Lag (Quarter) | Window (Quarter) | Transformation | Pass/Fail |
| HPI_l1_w3_t4 | EADF | HPI | 1 | 3 | Ratio | PASS |
| HPI_l3_w4_t4 | EADF | HPI | 3 | 4 | Ratio | PASS |
| UR_l0_w3_t3 | EADF | UR | 0 | 3 | Difference | PASS |
| HPI_l1_w4_t4 | PD | HPI | 1 | 4 | Ratio | PASS |
| UR_l0_w2_t2 | PD | UR | 0 | 2 | Average | PASS |
| UR_L3_w4_t3 | PD | UR | 3 | 4 | Difference | PASS |
| HPI_l1_w2_t4 | Recovery | HPI | 1 | 2 | Ratio | PASS |
| UR_l2_w4_t3 | Recovery | UR | 2 | 4 | Difference | PASS |

Details of ADF test for different models are given below.

*PD Model:*

| HPI_l1_w4_t4 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Augmented Dickey-Fuller Unit Root Tests | | | | | | | |
| **Type** | **Lags** | **Rho** | **Pr < Rho** | **Tau** | **Pr < Tau** | **F** | **Pr > F** |
| Zero Mean | 0 | -344.28 | 0.0001 | -13.86 | <.0001 | | |
| | 1 | -492.628 | 0.0001 | -16.29 | <.0001 | | |
| | 2 | -699.856 | 0.0001 | -18.81 | <.0001 | | |
| Single Mean | 0 | -486.99 | 0.0001 | -16.22 | <.0001 | 132.07 | 0.001 |
| | 1 | -716.23 | 0.0001 | -19.33 | <.0001 | 187.17 | 0.001 |
| | 2 | -1069.88 | 0.0001 | -22.75 | <.0001 | 258.99 | 0.001 |
| Trend | 0 | -487.001 | 0.0001 | -16.22 | <.0001 | 131.63 | 0.001 |
| | 1 | -716.243 | 0.0001 | -19.33 | <.0001 | 186.77 | 0.001 |
| | 2 | -1069.88 | 0.0001 | -22.75 | <.0001 | 258.73 | 0.001 |

| UR_l0_w2_t2 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Augmented Dickey-Fuller Unit Root Tests | | | | | | | |
| **Type** | **Lags** | **Rho** | **Pr < Rho** | **Tau** | **Pr < Tau** | **F** | **Pr > F** |
| Zero Mean | 0 | -21.5582 | 0.001 | -3.28 | 0.0011 | | |
| | 1 | -43.4851 | <.0001 | -4.74 | <.0001 | | |
| | 2 | -56.7364 | <.0001 | -5.38 | <.0001 | | |
| Single Mean | 0 | -192.675 | 0.0001 | -9.87 | <.0001 | 48.74 | 0.001 |
| | 1 | -390.022 | 0.0001 | -14.03 | <.0001 | 98.48 | 0.001 |
| | 2 | -541.205 | 0.0001 | -16.25 | <.0001 | 131.99 | 0.001 |
| Trend | 0 | -194.004 | 0.0001 | -9.91 | <.0001 | 49.11 | 0.001 |
| | 1 | -392.159 | 0.0001 | -14.07 | <.0001 | 98.93 | 0.001 |
| | 2 | -544.503 | 0.0001 | -16.29 | <.0001 | 132.68 | 0.001 |

| UR_L3_w4_t3 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Augmented Dickey-Fuller Unit Root Tests | | | | | | | |
| **Type** | **Lags** | **Rho** | **Pr < Rho** | **Tau** | **Pr < Tau** | **F** | **Pr > F** |
| Zero Mean | 0 | -849.762 | 0.0001 | -21.82 | <.0001 | | |
| | 1 | -3899.3 | 0.0001 | -44.81 | <.0001 | | |
| | 2 | -2845.62 | 0.0001 | -35.18 | <.0001 | | |
| Single Mean | 0 | -854.782 | 0.0001 | -21.93 | <.0001 | 241.41 | 0.001 |
| | 1 | -3909.41 | 0.0001 | -44.87 | <.0001 | 1006.9 | 0.001 |
| | 2 | -2856.92 | 0.0001 | -35.23 | <.0001 | 620.69 | 0.001 |
| Trend | 0 | -854.781 | 0.0001 | -21.92 | <.0001 | 240.35 | 0.001 |
| | 1 | -3909.43 | 0.0001 | -44.87 | <.0001 | 1006.69 | 0.001 |
| | 2 | -2856.96 | 0.0001 | -35.23 | <.0001 | 620.54 | 0.001 |

P-value of the Tau of zero mean and single mean type root tests for lags 0 to 2 is below 0.1. Hence, all selected variables are stationary.

*EADF Model:*

| HPI_l1_w3_t4 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Augmented Dickey-Fuller Unit Root Tests | | | | | | | |
| **Type** | **Lags** | **Rho** | **Pr < Rho** | **Tau** | **Pr < Tau** | **F** | **Pr > F** |
| Zero Mean | 0 | -751.848 | 0.0001 | -20.42 | <.0001 | | |
| | 1 | -839.082 | 0.0001 | -20.94 | <.0001 | | |
| | 2 | -645.369 | 0.0001 | -18.09 | <.0001 | | |
| Single Mean | 0 | -1050.57 | 0.0001 | -24.11 | <.0001 | 290.89 | 0.001 |
| | 1 | -1225.58 | 0.0001 | -25.1 | <.0001 | 315.13 | 0.001 |
| | 2 | -968.234 | 0.0001 | -21.69 | <.0001 | 235.57 | 0.001 |
| Trend | 0 | -1050.58 | 0.0001 | -24.11 | <.0001 | 290.59 | 0.001 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | -1225.59 | 0.0001 | -25.1 | <.0001 | 314.95 | 0.001 |
| | 2 | -968.251 | 0.0001 | -21.69 | <.0001 | 235.31 | 0.001 |

| HPI_l3_w4_t4 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Augmented Dickey-Fuller Unit Root Tests | | | | | | | |
| **Type** | **Lags** | **Rho** | **Pr < Rho** | **Tau** | **Pr < Tau** | **F** | **Pr > F** |
| Zero Mean | 0 | -340.575 | 0.0001 | -13.76 | <.0001 | | |
| | 1 | -487.609 | 0.0001 | -16.19 | <.0001 | | |
| | 2 | -693.208 | 0.0001 | -18.69 | <.0001 | | |
| Single Mean | 0 | -480.738 | 0.0001 | -16.12 | <.0001 | 130.25 | 0.001 |
| | 1 | -707.075 | 0.0001 | -19.2 | <.0001 | 184.64 | 0.001 |
| | 2 | -1056.29 | 0.0001 | -22.6 | <.0001 | 255.55 | 0.001 |
| Trend | 0 | -480.748 | 0.0001 | -16.11 | <.0001 | 129.87 | 0.001 |
| | 1 | -707.087 | 0.0001 | -19.2 | <.0001 | 184.3 | 0.001 |
| | 2 | -1056.3 | 0.0001 | -22.6 | <.0001 | 255.33 | 0.001 |

| UR_l0_w3_t3 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Augmented Dickey-Fuller Unit Root Tests | | | | | | | |
| **Type** | **Lags** | **Rho** | **Pr < Rho** | **Tau** | **Pr < Tau** | **F** | **Pr > F** |
| Zero Mean | 0 | -1181.16 | 0.0001 | -26.27 | <.0001 | | |
| | 1 | -5258.17 | 0.0001 | -52.39 | <.0001 | | |
| | 2 | -2432.36 | 0.0001 | -32.94 | <.0001 | | |
| Single Mean | 0 | -1186.79 | 0.0001 | -26.39 | <.0001 | 349.98 | 0.001 |
| | 1 | -5266.71 | 0.0001 | -52.44 | <.0001 | 1375.05 | 0.001 |
| | 2 | -2440.48 | 0.0001 | -33 | <.0001 | 544.37 | 0.001 |
| Trend | 0 | -1186.8 | 0.0001 | -26.39 | <.0001 | 348.1 | 0.001 |
| | 1 | -5266.74 | 0.0001 | -52.44 | <.0001 | 1374.79 | 0.001 |
| | 2 | -2440.49 | 0.0001 | -32.99 | <.0001 | 544.31 | 0.001 |

P-value of the Tau of zero mean and single mean type root tests for lags 0 to 2 is below 0.1. Hence, all selected variables are stationary.

*Recovery Model:*

| HPI_l1_w2_t4 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Augmented Dickey-Fuller Unit Root Tests | | | | | | | |
| **Type** | **Lags** | **Rho** | **Pr < Rho** | **Tau** | **Pr < Tau** | **F** | **Pr > F** |
| Zero Mean | 0 | -1752.94 | 0.0001 | -31.61 | <.0001 | | |
| | 1 | -2247.69 | 0.0001 | -34.08 | <.0001 | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 2 | -426.878 | 0.0001 | -14.94 | <.0001 | | |
| Single Mean | 0 | -2342.53 | 0.0001 | -37.16 | <.0001 | 690.7 | 0.001 |
| | 1 | -3291.69 | 0.0001 | -41.08 | <.0001 | 843.88 | 0.001 |
| | 2 | -623.221 | 0.0001 | -17.67 | <.0001 | 156.4 | 0.001 |
| Trend | 0 | -2342.53 | 0.0001 | -37.16 | <.0001 | 690.54 | 0.001 |
| | 1 | -3291.73 | 0.0001 | -41.08 | <.0001 | 843.72 | 0.001 |
| | 2 | -623.245 | 0.0001 | -17.67 | <.0001 | 156.09 | 0.001 |

| UR_l2_w4_t3 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Augmented Dickey-Fuller Unit Root Tests | | | | | | | |
| **Type** | **Lags** | **Rho** | **Pr < Rho** | **Tau** | **Pr < Tau** | **F** | **Pr > F** |
| Zero Mean | 0 | -855.49 | 0.0001 | -21.9 | <.0001 | | |
| | 1 | -3924.43 | 0.0001 | -44.96 | <.0001 | | |
| | 2 | -2864.42 | 0.0001 | -35.3 | <.0001 | | |
| Single Mean | 0 | -860.571 | 0.0001 | -22.01 | <.0001 | 243.27 | 0.001 |
| | 1 | -3934.18 | 0.0001 | -45.02 | <.0001 | 1013.52 | 0.001 |
| | 2 | -2875.45 | 0.0001 | -35.35 | <.0001 | 624.96 | 0.001 |
| Trend | 0 | -860.571 | 0.0001 | -22 | <.0001 | 242.1 | 0.001 |
| | 1 | -3934.2 | 0.0001 | -45.02 | <.0001 | 1013.32 | 0.001 |
| | 2 | -2875.49 | 0.0001 | -35.35 | <.0001 | 624.83 | 0.001 |

P-value of the Tau of zero mean and single mean type root tests for lags 0 to 2 is below 0.1. Hence, all selected variables are stationary.

Below attached are the details (ACF, PACF, Inverse autocorrelation etc.).

SAS Report - macro_stationarity.pc

### 8.1.5 Autocorrelation

- To the extent applicable, provide evidence that residuals are not auto-correlated. [Only applicable for time series models.]

For Account level PD & EAD models, since the data is at account level. This test is not applicable.

Recovery model has been developed using discrete time linear regression technique. Hence this test is not applicable for recovery model.

Statistical testing is not applicable to NCL and GCL calculation.

### 8.1.6 Normality

- To the extent possible, provide the evidence that the residuals are normally distributed

Since EADF and Recovery rate models are linear regression models, they were checked for residual normality. Following are the results of the tests conducted on them:

**EADF Model:**

To check for normality, QQ plot of residuals is analyzed. Chart below is the QQ plot of overall model.

**Figure 55: Residual QQ plot of EADF model**

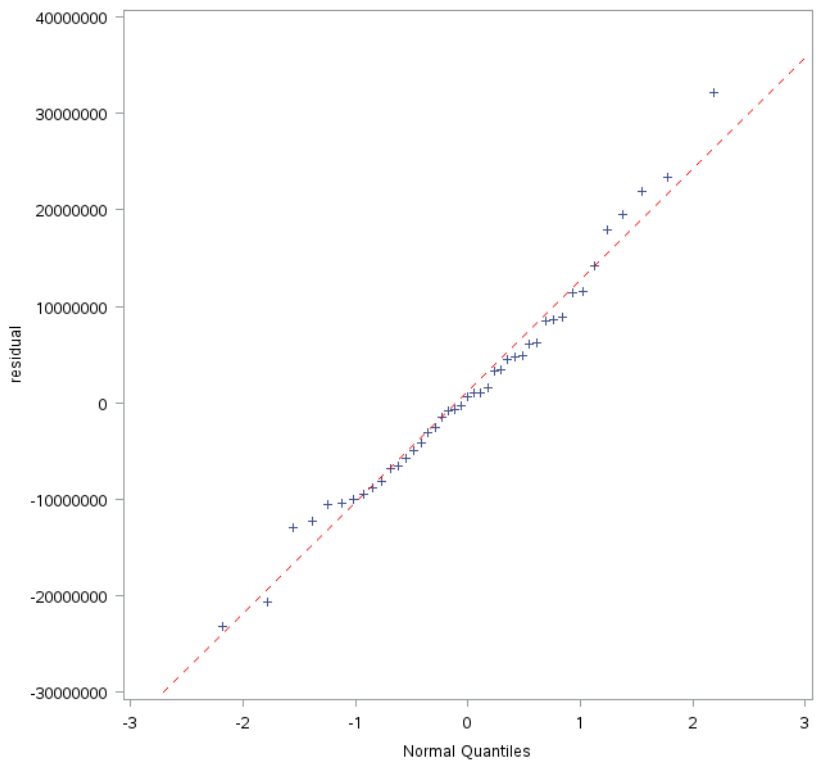
The linked image cannot be displayed. The file may have been moved, renamed, or deleted. Verify that the link points to the correct file and location.

As observed, the residual QQ plot follows a linear trend, except at tail, indicating normality of the EADF model. Quarterly QQ plots for EADF model are provided in the below attachment.

EADF_Normality_Q13.xlsx

**Recovery Rate Model:**

Figure 56: Residual QQ Plot of Recovery Rate Model




The residual QQ plot of recovery model follows a linear trend indicating that the recovery rate model passes normality test.

Normality testing is not applicable to PD model.

Statistical testing is not applicable to GCL and NCL calculations.

### 8.1.7 Heteroscedasticity

- To the extent applicable, provide evidence that there is no heteroscedasticity of residuals

As an ordinary regression model, the residuals of model are also assumed to have constant variance. Breusch-Pagen test is used to test the presence of heteroscedasticity. As shown below, p value shows <0.0001, hence It rejects the null hypothesis of no heteroscedasticity. This implies that standard errors of the parameter estimates are incorrect and, thus, any inferences derived from them may be misleading.

Table 8.1.3: Breusch-Pagen Test

| | BREUSCH-PAGEN TEST FOR HETEROSKEDASTICITY | | |
|---|---|---|---|
| **Model** | **CHI-SQUARE** | **DF** | **P-VALUE** |
| EADF | 3090.764 | 103 | 0 |
| Recovery | 652.54333 | 53 | 0 |

Hence, using the HCC Option heteroscedasticity consistent estimates were taken. However, it was seen that all the inferences drawn remain intact. Please see the sheet attached for EADF and Recovery rate models to see the comparison between the existing and heteroscedasticity consistent estimates.

8.1.7_hetero_test_EAD.xlsx

8.1.7_hetero_test_recovery.xlsx

Both EADF and Recovery Rate models pass the heteroscedasticity test based on HCC option.

This test is not applicable to PD model as it is a logistic regression model. The test is also not applicable to GCL and NCL calculations.

### 8.1.8 Model Goodness of Fit

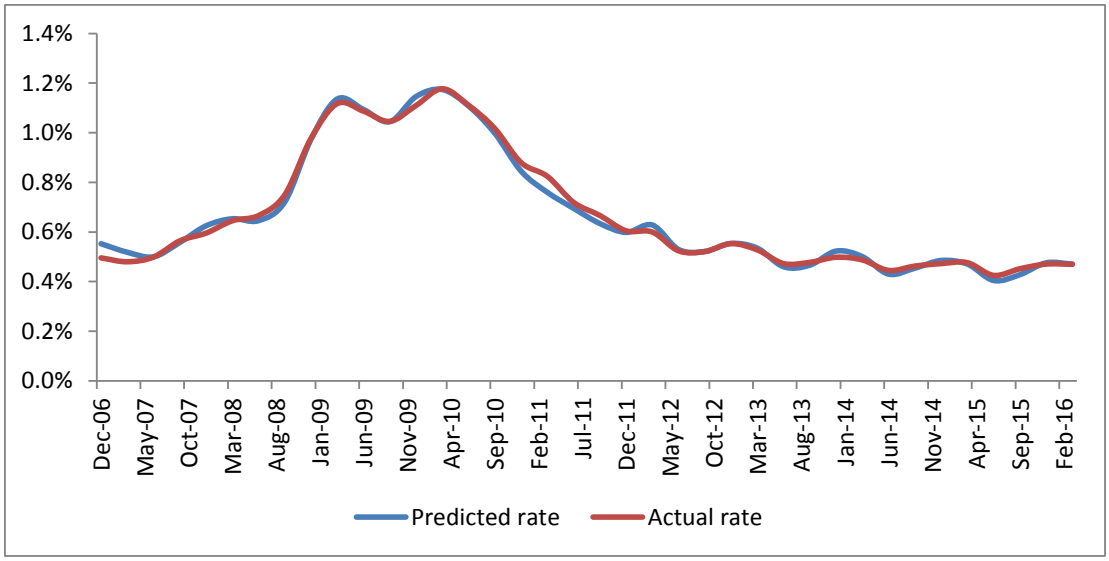
- **Predictive accuracy of the model**

Provide evidence that the predictive accuracy of the model is sufficiently high. Evaluate the accuracy and consistency of the model.

Predictive accuracy of the models was tested by analyzing the trends of actual and predicted values over the development sample. Model accuracy is high, the results of which are demonstrated in following subsections.

**PD Prediction Accuracy:**

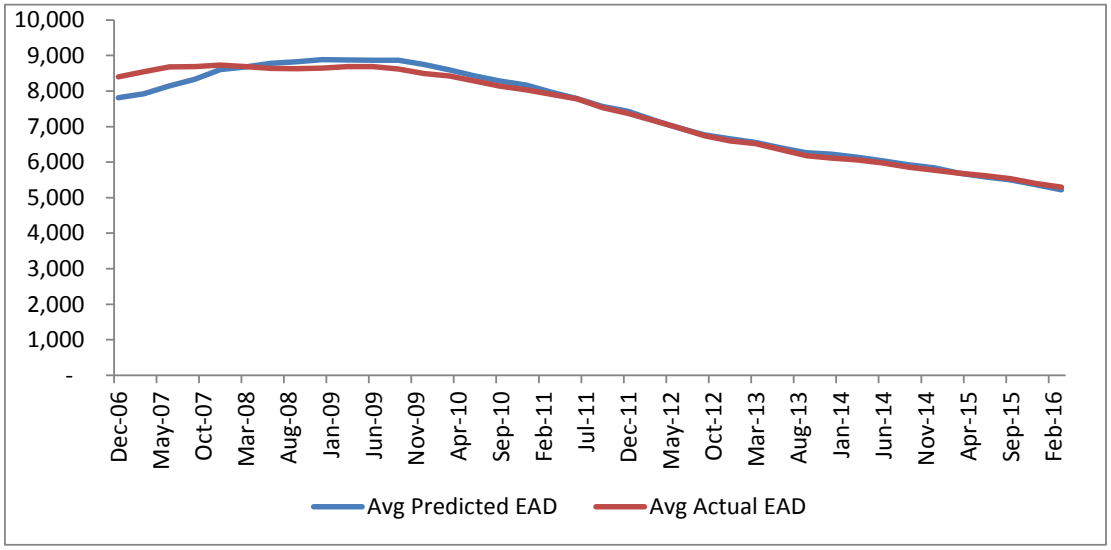
Figure 57: Actual vs Predicted PD Comparison




Predicted PD is aligned with the actual PD, resulting in error rates below +/-10% across all the observation quarters.

**<u>EAD Prediction Accuracy</u>:**

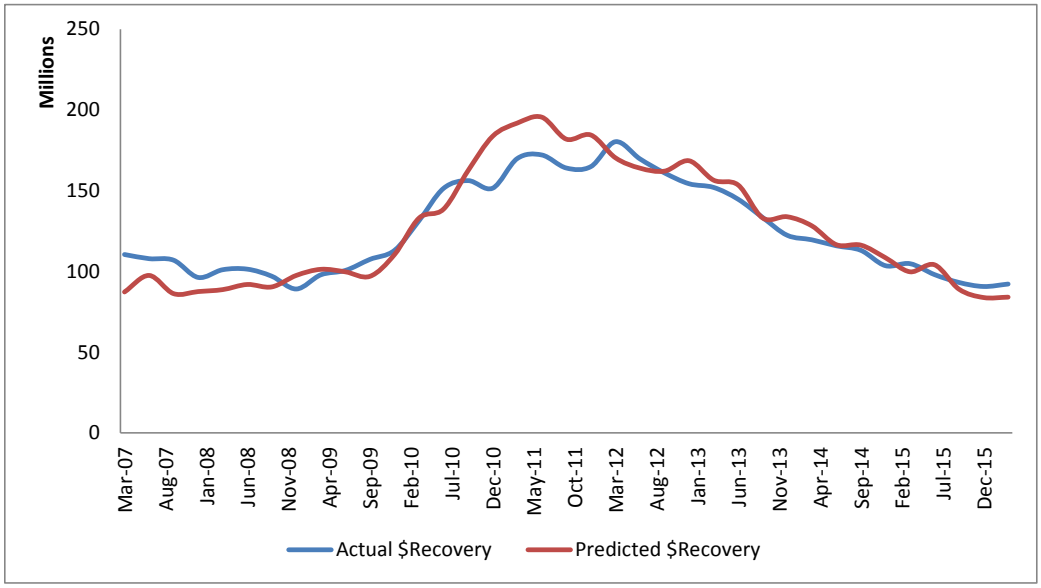
Figure 58: Actual vs Predicted EAD Comparison




Predicted EAD is aligned with the actual EAD, resulting in error rates below +/-10% across all the observation quarters.

**Recovery Rate Model Prediction Accuracy:**

Figure 59: Actual vs Predicted Recovery Comparison




The actual and predicted recoveries follow similar trends and the quarterly error rates are below the MRM threshold value of +/-25%.

The TTC model accuracy for PD, EADF and Recovery Rate models is attached in the sheet below:

8.1.8_Model_Accuracy.xlsx

- **Discriminatory power of the model**

To the extent applicable, provide evidence that the discriminatory power is sufficiently high. Evaluate the model's ability to discriminate between goods/bads or rank order outcomes.

Since EADF and Recovery rate models are developed using linear regression technique, discriminatory power tests are not applicable for them.

**PD model:**

The discriminatory power of the model has been evaluated using the following metrics:

- Concordance and Discordance

The percentage concordance, discordance and tied pairs is computed to evaluate the performance of the model.

**Percent Concordant** is defined as the Percentage of pairs where the observation with the desired outcome (event) has a higher predicted probability than the observation without the outcome (non-event).

**Percent Discordant is** defined as the Percentage of pairs where the observation with the desired outcome (event) has a lower predicted probability than the observation without the outcome (non-event).

**Percent Tied** is the Percentage of pairs where the observation with the desired outcome (event) has same predicted probability than the observation without the outcome (non-event).

In general, higher percentages of concordant pairs and lower percentages of discordant and tied pairs indicate a more desirable model.

- Gini

The Gini index is a measure of inequality of a distribution. This index describes the global quality of a scoring function. It takes values between -1 and 1. The ideal model, i.e., a scoring function that perfectly separates good and bad events, has a Gini index equal to 1. On the other hand, a model that assigns a random score to the client has a Gini index equal to 0. Negative Values correspond to a model with reversed meanings of scores.

- Gamma

Gamma is a measure of rank correlation, i.e., the similarity of the orderings of the data when ranked by each of the quantities. It measures the strength of association of the cross tabulated data when both variables are measured at the ordinal level. It makes no adjustment for either table size or ties. Values range from −1 (100% negative association, or perfect inversion) to +1 (100% positive association, or perfect agreement). A value of zero indicates the absence of association.

- Tau – a

Let's assume that we try to measure the relationship between two variables x and y. Kendall's algorithm is based on the idea that if there is a correlation among the two variables then sorting the data pairs [xi, yi] by one variable results in a more or less sorted series of values in the other variable.

In effect, Kendall's Tau-a measures the number of sorting inversions in the variable y, given that the variable x has been sorted. If there is not a single inversion in the order of y then the two variables are 100% positively correlated, if there are exclusively inversions then the variables are 100% negatively correlated.

Kendall's Tau-a is calculated according to the following formula:

**Equation 8.1.1: Formula for Tau-a**

$$\tau = \frac{4P}{n(n-1)} - 1$$

where n is the number of data pairs, and P the number of concordant pairs (i.e. pairs which do not form an inversion).

Kendall's Tau-a fails if the dataset contains a high portion of ties.

- C – statistic

When outcomes are binary, the c-statistic (equivalent to the area under the Receiver Operating Characteristic [ROC] curve) is a standard measure of the predictive accuracy of a logistic regression model.

- KS

The KS test measures the difference between two probability distributions, i.e., the distribution of 'Good' accounts and the distribution of 'Bad' accounts. KS statistic is equal to the maximum difference between the cumulative percentages of 'Goods' and 'Bads' across all score values:

**Equation 8.1.2: KS**

$$KS \equiv \underset{\substack{\text{over all score} \\ \text{values } S}}{\text{Max}} \left[ \frac{N_{\text{goods for score} \le S}}{N_{\text{total goods}}} - \frac{N_{\text{bads for score} \le S}}{N_{\text{total bads}}} \right],$$

where $N_{\text{goods for score} \le S}$ and $N_{\text{bads for score} \le S}$ are the (cumulative) numbers of 'Goods' and 'Bads' with scores $\le S$; $N_{\text{totalgoods}}$ and $N_{\text{totalbads}}$ are the total numbers of 'Goods' and 'Bads' in the sample, respectively.

As seen in the table below the model performs well across all metrics.

Table 8.1.4: KS for PD model (Quarter over Quarter)

| Quarter | % Concordant | % Discordant | % Tied | Gini | Gamma | Tau - a | c | KS |
|---|---|---|---|---|---|---|---|---|
| 1 | 96.2 | 1.6 | 2.1 | 0.946 | 0.966 | 0.013 | 0.973 | 84.4 |
| 2 | 93.3 | 2.3 | 4.3 | 0.910 | 0.951 | 0.012 | 0.955 | 80.2 |
| 3 | 87.2 | 8.2 | 4.6 | 0.791 | 0.829 | 0.010 | 0.895 | 66.7 |
| 4 | 85.1 | 9.8 | 5.0 | 0.753 | 0.793 | 0.010 | 0.877 | 62.4 |
| 5 | 83.6 | 11.0 | 5.4 | 0.725 | 0.767 | 0.009 | 0.863 | 59.0 |
| 6 | 82.1 | 12.1 | 5.8 | 0.700 | 0.743 | 0.009 | 0.850 | 56.4 |
| 7 | 79.7 | 13.6 | 6.7 | 0.677 | 0.723 | 0.008 | 0.839 | 54.6 |
| 8 | 79.4 | 13.8 | 6.8 | 0.656 | 0.704 | 0.007 | 0.828 | 52.9 |
| 9 | 78.3 | 14.4 | 7.3 | 0.638 | 0.689 | 0.007 | 0.819 | 51.4 |
| 10 | 77.0 | 15.0 | 8.0 | 0.620 | 0.674 | 0.006 | 0.810 | 50.0 |
| 11 | 75.9 | 15.4 | 8.7 | 0.604 | 0.662 | 0.006 | 0.802 | 48.8 |
| 12 | 74.6 | 15.8 | 9.6 | 0.588 | 0.650 | 0.005 | 0.794 | 47.5 |
| 13 | 73.5 | 16.1 | 10.4 | 0.574 | 0.640 | 0.005 | 0.787 | 46.5 |

The forecasting horizon for CCAR models is 13 quarters, longer than the typical scorecards that are built. It can be observed however, that KS/Gini fall as we move further into forecasting horizon. This is expected as these statistics are driven by Internal and Bureau attributes which are static across forecasting horizon. Even though the discriminatory power falls, the forecasting accuracy of the model is good as can be seen from the predictive accuracy statistics provided in the previous subsection. From the above table, it can be seen that across the thirteen quarters the discriminatory power of the model is sufficiently high, even for quarter 9 it is as high as 51.4 and for quarter 13 it is 46.5. Hence, the model discriminates sufficiently between goods and bads.

### 8.1.9 Other tests – Model-specific diagnostics

- Provide evidence of other relevant tests performed. Examples include linearity in the context of typical econometric regression models.

**Overall significance**

*PD Model:*

Likelihood ratio test is conducted to test the overall significance of the PD model.

Table 8.1.5: PD Model Diagnostics

| | Likelihood Ratio | | Wald | |
|---|---|---|---|---|
| Quarter | Chi-Square | Pr > ChiSq | Chi-Square | Pr > ChiSq |
| 1 | 2,019,231 | <.0001 | 2,436,301,127 | <.0001 |
| 2 | 4,162,898 | <.0001 | 2,542,699 | <.0001 |
| 3 | 1,771,537 | <.0001 | 1,172,242 | <.0001 |
| 4 | 1,444,634 | <.0001 | 1,022,913 | <.0001 |
| 5 | 1,222,846 | <.0001 | 901,028 | <.0001 |
| 6 | 1,042,815 | <.0001 | 803,549 | <.0001 |
| 7 | 895,652 | <.0001 | 710,175 | <.0001 |
| 8 | 765,967 | <.0001 | 612,335 | <.0001 |
| 9 | 663,593 | <.0001 | 536,586 | <.0001 |
| 10 | 570,052 | <.0001 | 471,372 | <.0001 |
| 11 | 489,142 | <.0001 | 411,128 | <.0001 |
| 12 | 418,973 | <.0001 | 356,749 | <.0001 |
| 13 | 361,022 | <.0001 | 310,230 | <.0001 |

p-value of likelihood ratio is less than 0.1 for all the quarterly models which implies that all the quarterly model are statistically significant.

*EADF Model:*

F-test is conducted to test the overall significance of OLS models.

Table 8.1.6: EADF Model Diagnostics

| | R-Square | Adj R-Sq | COV | F value | PR > F |
|---|---|---|---|---|---|
| Q1 | 98.6% | 98.6% | 1,200.35 | 3,680,150 | <.0001 |
| Q2 | 97.6% | 97.6% | 1,594.54 | 1,688,352 | <.0001 |
| Q3 | 94.6% | 94.6% | 2,454.20 | 468,861 | <.0001 |
| Q4 | 91.6% | 91.6% | 3,146.50 | 287,263 | <.0001 |
| Q5 | 89.1% | 89.1% | 3,664.42 | 203,629 | <.0001 |
| Q6 | 87.2% | 87.2% | 4,057.64 | 158,615 | <.0001 |
| Q7 | 85.2% | 85.2% | 4,446.36 | 125,248 | <.0001 |
| Q8 | 83.2% | 83.2% | 4,840.02 | 100,080 | <.0001 |
| Q9 | 81.5% | 81.5% | 5,176.91 | 82,555 | <.0001 |
| Q10 | 80.2% | 80.2% | 5,446.08 | 69,630 | <.0001 |
| Q11 | 78.6% | 78.6% | 5,760.76 | 57,541 | <.0001 |
| Q12 | 77.5% | 77.5% | 5,987.17 | 48,765 | <.0001 |
| Q13 | 75.9% | 75.9% | 6,294.36 | 40,403 | <.0001 |

EADF quarterly models have high R-squared values and F-test p-value is less than 0.1. This shows that the models are significant and model fit is good.

*Recovery Model:*

Table 8.1.7: Recovery Model Diagnostics

| | R-Square | Adj R-Sq | COV | F value | PR > F |
|---|---|---|---|---|---|
| **Recovery Model** | 91.0% | 90.9% | -6.62 | 623 | <.0001 |

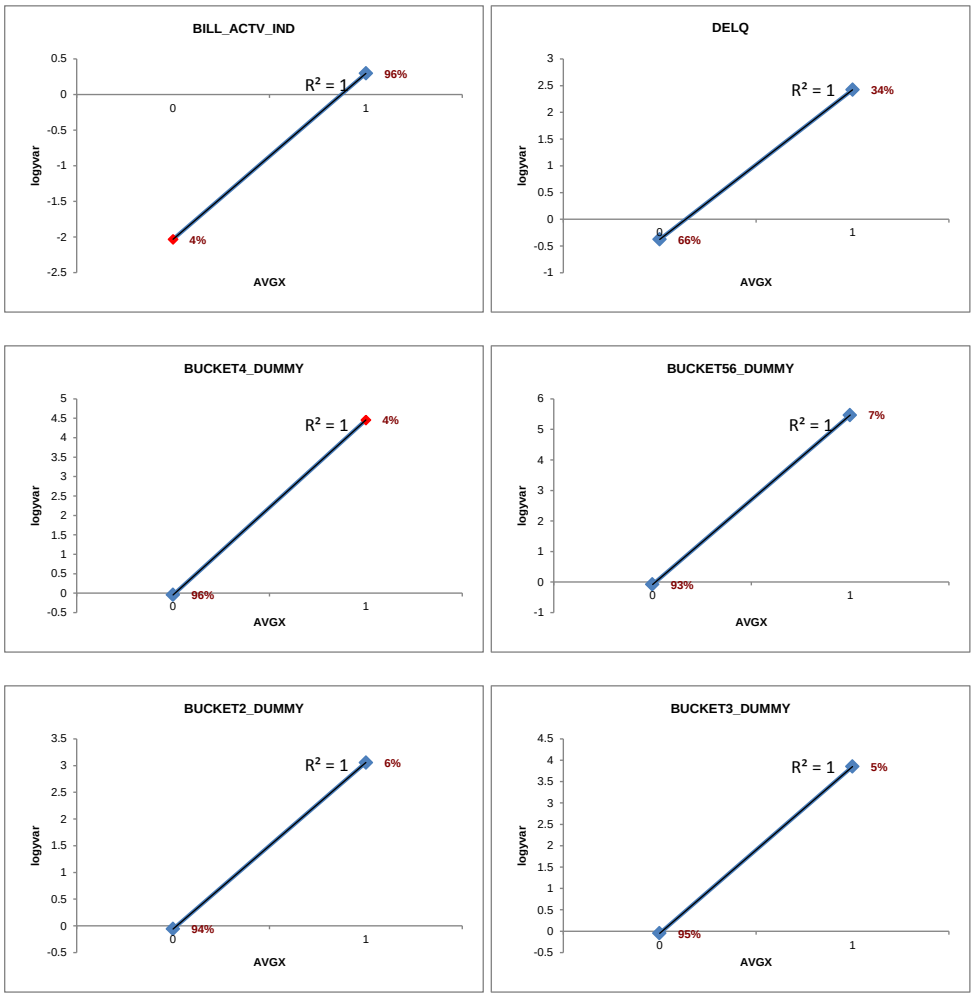
R-squared value of recovery model is more than 90% indicating very good model fit.

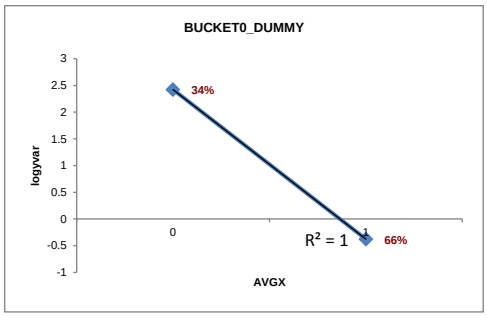
**Linearity of Independent Variables:**

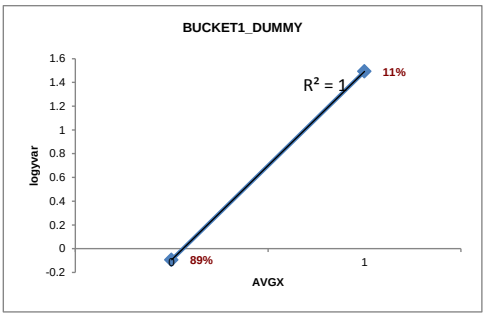
*PD Model:*

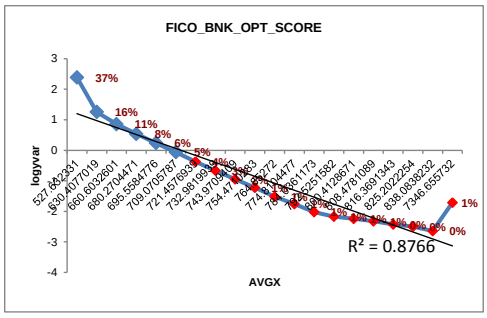
In a logistic model the independent variables should have linear relationship with the log-odds. All independent variables were tested for linearity. The bivariate plots of independent variables with log-odds are given below.

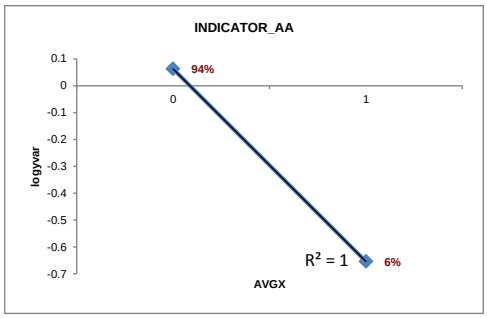
Figure 60: Bivariate Plots of PD Independent Variables with Log-Odds

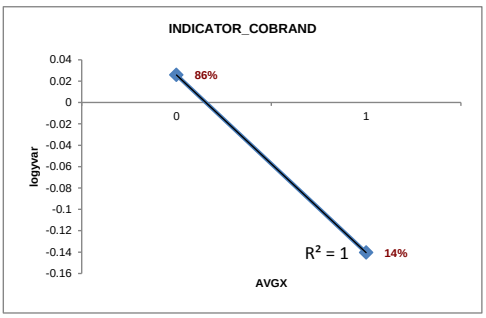


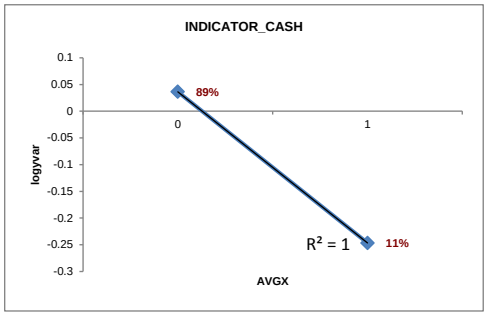

BUCKET0_DUMMY
logyvar
AVGX
34%
66%
R² = 1

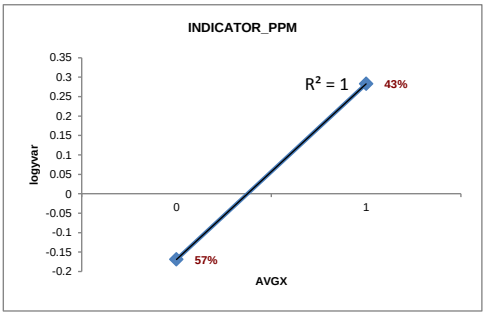

BUCKET1_DUMMY
logyvar
AVGX
R² = 1
11%
89%

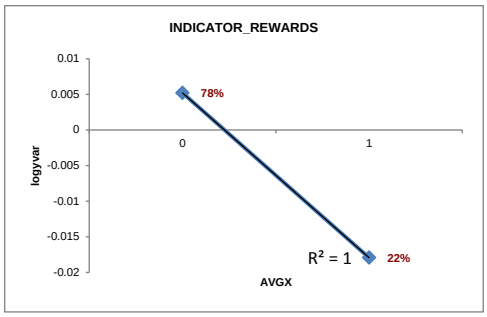

FICO_BNK_OPT_SCORE
logyvar
AVGX
37%
16%
11%
8%
6%
5%
1%
0%
R² = 0.8766

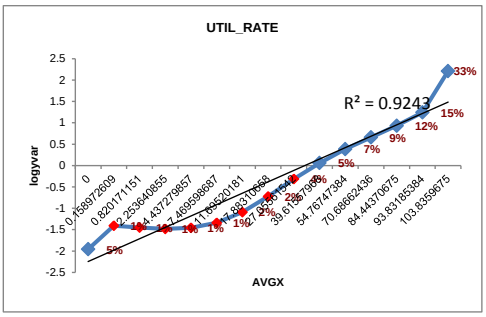

INDICATOR_AA
logyvar
AVGX
94%
6%
R² = 1

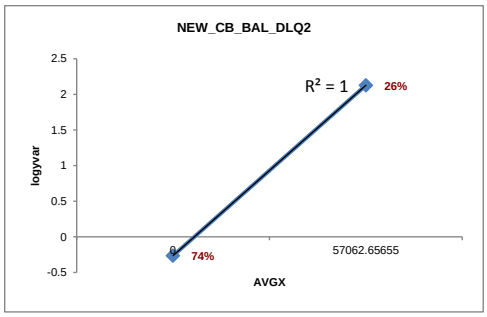

INDICATOR_COBRAND
logyvar
AVGX
86%
14%
R² = 1

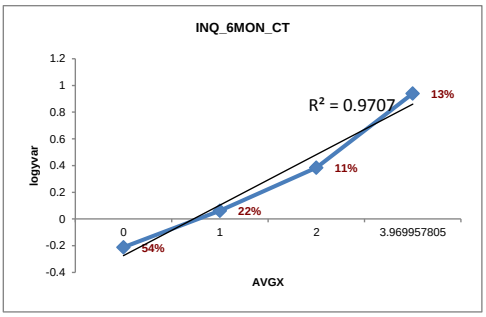

INDICATOR_CASH
logyvar
AVGX
89%
11%
R² = 1

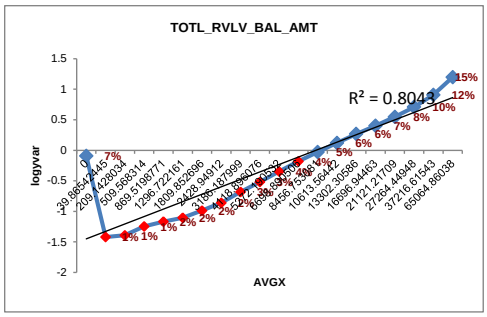

INDICATOR_PPM
logyvar
AVGX
R² = 1
43%
57%

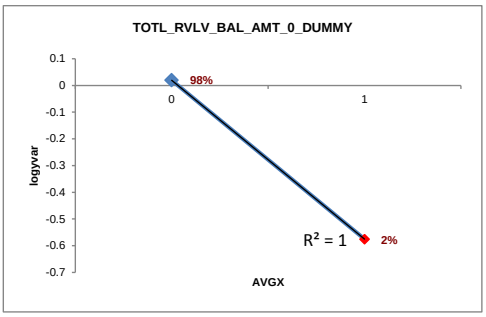

INDICATOR_REWARDS
logyvar
AVGX
R² = 1
78%
22%

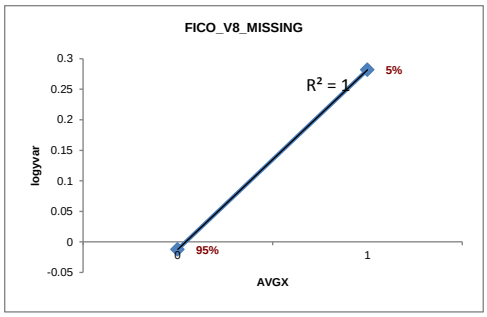

UTIL_RATE
logyvar
AVGX
R² = 0.9243
33%
15%
12%
9%
7%
5%
5%

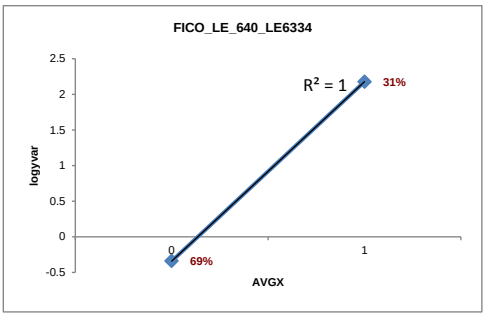

NEW_CB_BAL_DLQ2
logyvar
AVGX
R² = 1
26%
74%
57062.65655

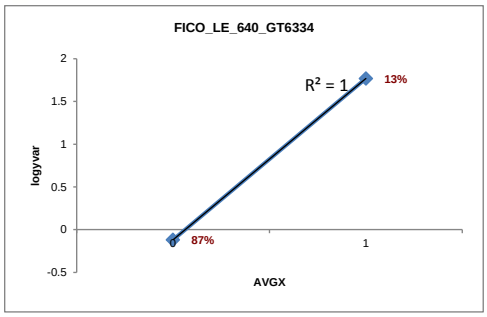

INQ_6MON_CT
logyvar
AVGX
R² = 0.9707
13%
11%
22%
54%
3.969957805

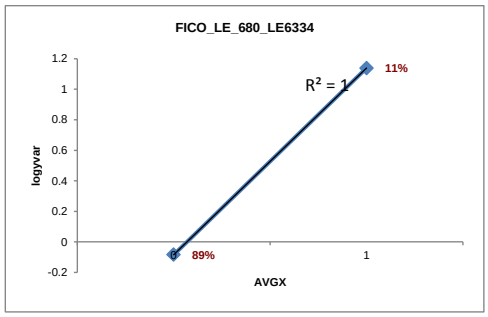

TOTL_RVLV_BAL_AMT
logyvar
AVGX
R² = 0.8049
15%
12%
10%
8%
7%
6%
6%
5%
7%
2%
1%
1%
2%
2%
2%

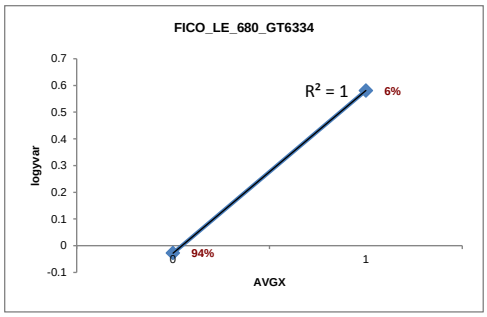

TOTL_RVLV_BAL_AMT_0_DUMMY
logyvar
AVGX
98%
R² = 1
2%

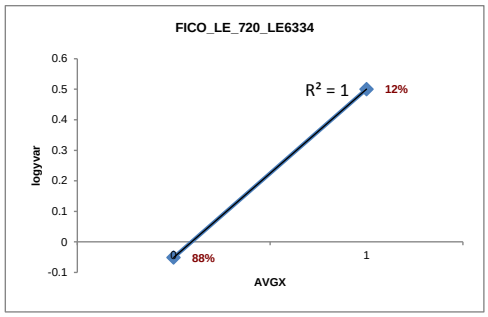

FICO_V8_MISSING
logyvar
AVGX
R² = 1
5%
95%

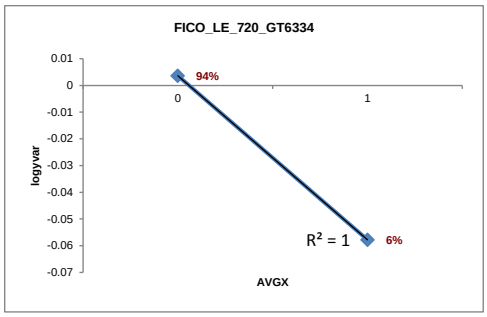

FICO_LE_640_LE6334
logyvar
AVGX
R² = 1
31%
69%



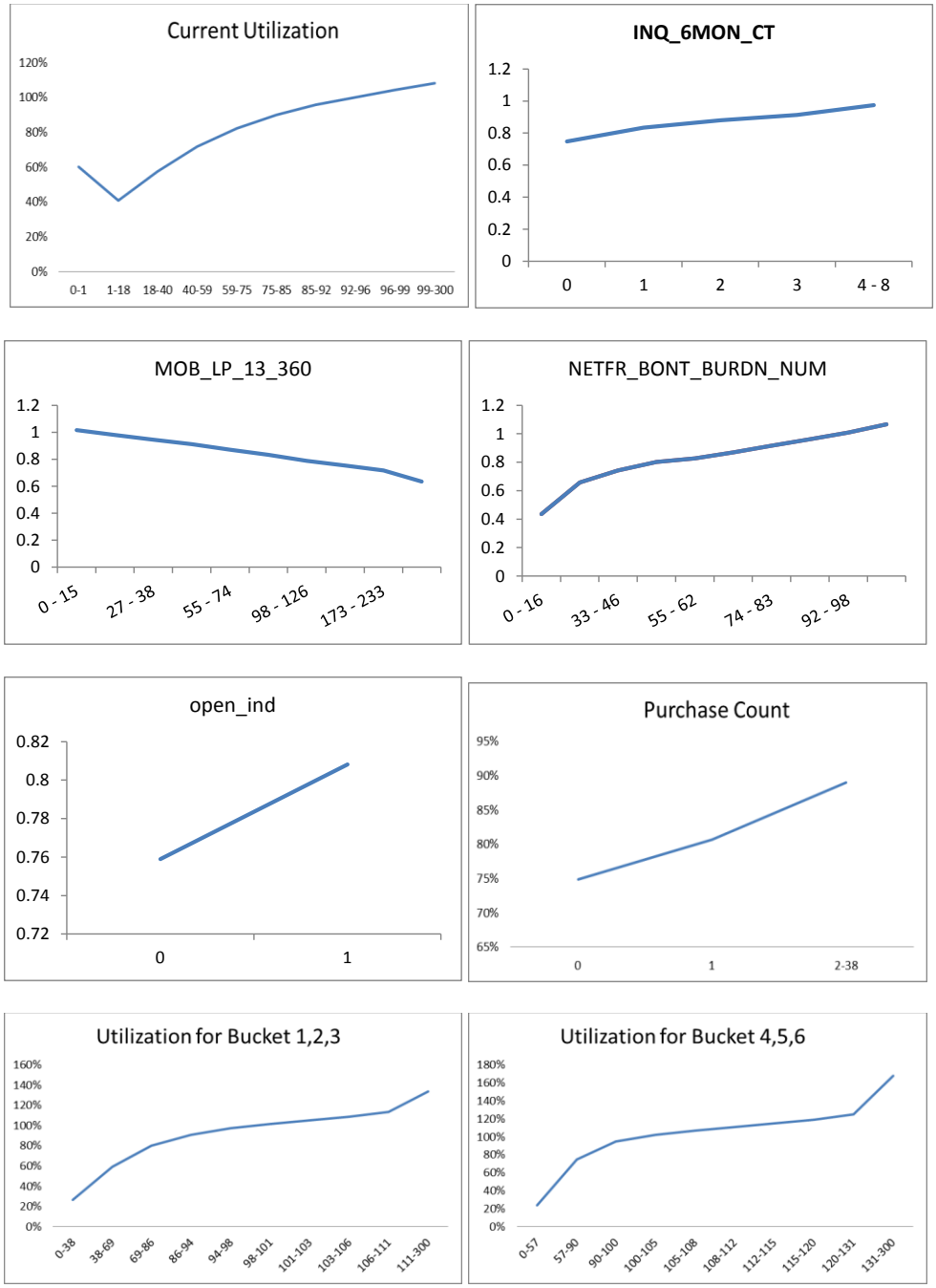




As observed from above figures, all the independent variables show linearity with the log-odds.

*<u>EADF Model:</u>*

One of the assumptions of linear regression is that there is a linear relationship between the dependent variable and the independent variables. Below chart show the relationship of all independent variables of EADF model with the dependent variable.

Figure 61: Bivariate Plots of EADF Independent Variables with EADF

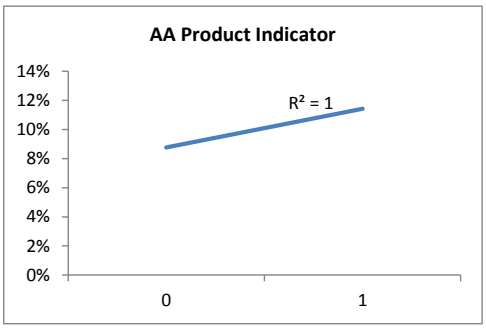



All EADF independent variables follow linear relationship with the dependent variable.

*Recovery Model:*

Recovery model has only one variable which has been created from a behavioral attribute. AA product indicator is derived from the product information of an account. The bivariate plot of AA indicator with recovery ate is given below.

Figure 62: Bivariate Plot of Recovery Rate Independent Variable with Dependent Variable




AA indicator has linear relationship with the recovery rate.

- To the extent that there are Model Assumptions listed in Section 6.3 that are not evaluated/tested elsewhere in Section 6, list in this sub-section

Please refer to above question

- Specific to Vendor Models, apply an appropriate set of validation tests to determine model performance and robustness for Citi's portfolio.

None of the models developed are vendor models. So this section is not applicable

- For vendor models, describe if additional tests are performed or additional benchmarks used to gauge the model performance, and indicate if the model output is reasonable.

None of the models developed are vendor models. So this section is not applicable

## 8.2 Model Robustness and Stability

### 8.2.1 Model Stability Design

[Please explain the Out of Sample (OOS)/ Out of Time (OOT) design. Clearly specify the OOS/OOT and the development time period. In case of OOS clearly specify the numbers of months excluded from the development sample to derive the OOS period. Refer to Model Testing Guidance for requirement on Model Stability principles and testing].

**Data Design – PD and EAD Model**

PD and EADF model are developed on the same data. Hence the data construct is same for the two models. Attachment below gives the data construct for PD and EADF models. Recent four quarters of observation data from 2016Q3 to 2017Q2 is taken as OOT sample.

8.2.2_Mod_Stbl_Dat
a_Construct_PD_EAD

PD and EADF models have been developed on observation data from 2006Q4 to 2016Q2. For model stability test two additional iterations were run – one after adding additional 4 quarters of data and the second after removing 4 quarters of data from the model development data. The stability results are given in Section 8.2.2.

**Data Design – Recovery Rate Model**

Recovery rate model predicts the recoveries for next 32 quarters. As a result the data construct is slightly different from that of PD and EADF models. The data construct is given in the below attachment. Recent four quarters of recovery information from 2016Q3 to 2017Q2 is taken as OOT sample.

8.2.2_Mod_Stbl_Dat
a_Construct_Recover

The model has been developed on recoveries from 2007Q1 to 2016Q2. For model stability test two iterations were run – first after adding 4 additional quarters of recoveries to the model development data and the second after removing 4 quarters of recoveries from the model development data. The stability result is given in section 8.2.2.

### 8.2.2 Model Stability Results

- How was model robustness and stability ensured and tested?

To test for stability of the Model coefficients has been evaluated in the following two methods:

1. The models were re-estimated on a shorter sample (excluding recent 1 year data).

2. The models were re-estimated on a longer sample (include additional 1 year (OOT) data)

Below is the construct for conducting the model stability test on the three models:

Model_Stability_Data_Construct.xlsx

Figure 63: Model Stability Check Construct

| Obs Qtr | Qtr1 | Qtr2 | Qtr3 | Qtr4 | Qtr5 | Qtr6 | Qtr7 | Qtr8 | Qtr9 | Qtr10 | Qtr11 | Qtr12 | Qtr13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dec-06 | Mar-07 | Jun-07 | Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 |
| Mar-07 | Jun-07 | Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 |
| Jun-07 | Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 |
| Sep-07 | Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 |
| Dec-07 | Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 |
| Mar-08 | Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 |
| Jun-08 | Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 |
| Sep-08 | Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 |
| Dec-08 | Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 |
| Mar-09 | Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 |
| Jun-09 | Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 |
| Sep-09 | Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 |
| Dec-09 | Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 |
| Mar-10 | Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 |
| Jun-10 | Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 |
| Sep-10 | Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 |
| Dec-10 | Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 |
| Mar-11 | Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 |
| Jun-11 | Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 |
| Sep-11 | Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 |
| Dec-11 | Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 |
| Mar-12 | Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 |
| Jun-12 | Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 |
| Sep-12 | Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 |
| Dec-12 | Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 |
| Mar-13 | Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 |
| Jun-13 | Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 |
| Sep-13 | Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 |
| Dec-13 | Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 |
| Mar-14 | Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 |
| Jun-14 | Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | |
| Sep-14 | Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | |
| Dec-14 | Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | |
| Mar-15 | Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | |
| Jun-15 | Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | | |
| Sep-15 | Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | | | |
| Dec-15 | Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | | | | |
| Mar-16 | Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | | | | | |
| Jun-16 | Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | | | | | | |
| Sep-16 | Dec-16 | Mar-17 | Jun-17 | | | | | | | | | | |
| Dec-16 | Mar-17 | Jun-17 | | | | | | | | | | | |
| Mar-17 | Jun-17 | | | | | | | | | | | | |

| | | |
|---|---|---|
| | | |
| | | |
| | | |

Development Data
Coefficient stability tested by removing one year of performance data
Coefficient stability tested by adding one year of performance data

As can be seen from results of PD, EADF and recovery rate models attached in excel below, below are the observations from the results:

- The sign of the individual coefficients does not change on shifting the data
- The shift in coefficient magnitude for the most significant predictors is small.
- Significance of the variables have not changed

Hence we can conclude that the models are stable over time.

8.2.2_Coeff_Stability_Test.xlsx

- Are the Model Stability results satisfactory? If not provide justification

As shown above, the tests are satisfactory in nature.

## 8.3 Back-Testing

To ascertain fit and accuracy of the model, back tests were performed on 9 quarter horizons for the stress and recent time period. The back testing methodology was prescribed and validated by the Model Risk Management (MRM) organization. The validation results are presented in the following subsections.

### 8.3.1 Back-testing design

- Has back-testing been conducted at various levels of the model - at the model component level (for e.g. PD, LGD, EAD), as well as final output (for e.g. , NCL, Balance, Revenue)?

The PD and EAD estimate Gross Credit Loss (for all accounts on books at the time of snapshot) for each of the nine quarters in the forecast period. GCL is computed using the PD and EADF models in conjunction with a line amount assumption, as shown in the formula below:

$$\$GCL = PD * \$Line\ Amount * EADF$$

NCL is derived using the component models (PD, EAD and Recovery). The relationship between Net Credit Loss (NCL) and GCL can be defined by:

***NCL = GCL – Net Recoveries – Adjustment & Refunds***

Net recoveries can be decomposed into three components: Cash Collected recoveries, Agency fees and Asset Sales.

1. **Cash Collected** refers to cash payments made on charged-off accounts in inventory to internal or external collection agencies. The amount of Cash Collected is driven by customer behavior and is further stimulated by the Bank's collections and recoveries practices.

2. **Agency fees** are the fees collected by a recovery agency for the recoveries they collect. They are a proportion of the recovery dollars collected by the agency.

3. **Asset Sales** are the sum of proceeds from the sale of the charged-off accounts. Asset Sales are driven by management decisions based on supply and demand of debt markets which fluctuate throughout the economic cycle.

The net recoveries are calculated by the below formula:

$$Net\ Recoveries = Cash\ Collected\ Recoveries - Agency\ fees + Asset\ Sales$$

Branded Cards has modeled Cash collected Recovery Rate since it is driven by customer behavior and is sensitive to macroeconomic changes while the other recovery components are either a constant percentage of cash collected recoveries (agency fees) or driven by policy decisions (asset sales).

For the NCL champion model, back tests have been done at component model level as well as the model output level as well. The back-testing has been performed for Probability of Default, Exposure at Default, Cash Collected Recoveries, Gross Credit Loss and Net Credit Loss.

Additionally, although EADF was the transformation used to model EAD, EAD was directly back tested. This was done because of the following reasons –

- The components to calculate the $GCL are PD and EAD. EADF is a transformation used to estimate EAD. Hence it is appropriate to back test EAD
- At an account level, given the static credit line assumption, the error rates for EAD and EADF will be the same

- At a portfolio level, back testing EAD would imply back testing a Credit Line weighted EADF, which is the appropriate approach as it is more important to be accurate for accounts with higher lines then lower lines. Hence back testing EAD is preferred.
- The back testing results can be compared across NA cards and also with the previous year champion model where business used an approach of directly modeling EAD

The Macro Economic sensitivity in these models has been captured by Unemployment Rate and House Price Index.

- Has back-testing been conducted on different time periods – recent, stress period, non-stress period (if applicable)? Do the types of back-tests conducted sufficiently cover all potential model risks within its performance and use?

Yes back-testing has been done for 9 quarters and 13 quarters for Stress and recent (or non-stress) periods. Additionally, it has been done for most recent period (9 quarters) as well which also includes 4 quarters of OOT sample. Yes the back tests cover all potential model risks.

- Describe the time frame of the back-testing and justification of the choice.

The 9 quarters and 13 quarters predicted values are compared to the Actual values on a quarter on quarter basis and on an overall level. The following set of Back Tests is performed:

**Table 8.3.1: Back testing Design**

| Period | Span | No. of Quarters |
|---|---|---|
| Stress | Jan'08 – Mar'10 | 9 |
| | Jan'08 – Mar'11 | 13 |
| Recent | Apr'14 – Jun'16 | 9 |
| | Apr'14 – Jun'17 | 13 |
| Most Recent (includes OOT*) | Jul'15 – Sep'17 | 9 |

*OOT corresponds to 2016Q3 – 2017Q2

**Table 8.3.2: Back testing Design Details**

| Quarters | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Period | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | Q13 |
| Stress | Q1-08 | Q2-08 | Q3-08 | Q4-08 | Q1-09 | Q2-09 | Q3-09 | Q4-09 | Q1-10 | Q2-10 | Q3-10 | Q4-10 | Q1-11 |

| Recent | Q2-14 | Q3-14 | Q4-14 | Q1-15 | Q2-15 | Q3-15 | Q4-15 | Q1-16 | Q2-16 | Q3-16 | Q4-16 | Q1-17 | Q2-17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Most Recent | Q3-15 | Q4-15 | Q1-16 | Q2-16 | Q3-16 | Q4-16 | Q1-17 | Q2-17 | Q3-17 | | | | |

- Has back-testing been conducted on a granular level - at the logical segments/product level?

The detailed back-testing has been done at an overall Portfolio Level as the model was developed at overall portfolio level.

However, to ensure the robustness of the Model, the model accuracy was also evaluated for various representative snapshots through the cycle in the development time period for the strategic sub-segments of the portfolio i.e. at a product level. The cumulative 13Q forecast was compared with actuals for these snapshots, which include Stress 13Q (Dec'07: Q1'08 – Q1'11), Recent 13Q (Mar'14: Q2'14 – Q2'17) and Most Recent 9Q (Jun'15: Q3'15 – Q3'17). Following are the results for the same:

PD Back Test

**Table 8.3.3: PD Back Test by Product for Stress, Recent and Most Recent**

| Period | AA | Cobrand | CASH | REWARDS | PPM | BUSINESS | Grand Total |
|---|---|---|---|---|---|---|---|
| Stress (13Q) | 0% | -9% | 9% | -3% | -1% | -4% | -1% |
| Recent (13Q) | 2% | 10% | 5% | 2% | -1% | 6% | 2% |
| Most Recent (9Q) | -5% | -2% | -1% | 4% | -6% | -1% | -4% |

**Table 8.3.4: PD Back Test by FICO for Stress, Recent and Most Recent**

| Period | <= 660 | 661-720 | 721+ | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | -2% | -6% | 10% | -1% |
| Recent (13Q) | 2% | 10% | -8% | 2% |
| Most Recent (9Q) | -3% | 3% | -16% | -4% |

**Table 8.3.5: PD Back Test by Delinquency Status for Stress, Recent and Most Recent**

| Period | Current | Delinquent | Grand Total |
|---|---|---|---|
| Stress (13Q) | -2% | 2% | -1% |
| Recent (13Q) | 3% | -1% | 2% |
| Most Recent (9Q) | -4% | -4% | -4% |

Table 8.3.6: PD Back Test by Month on Book Count for Stress, Recent and Most Recent

| Period | 1-24 | 25-60 | 61 + | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | -5% | 2% | 0% | -1% |
| Recent (13Q) | 0% | 2% | 4% | 2% |
| Most Recent (9Q) | -7% | -6% | 0% | -4% |

EAD Back Test

Table 8.3.7: EAD Back Test by Product for Stress, Recent and Most Recent

| Period | AA | Cobrand | CASH | REWARDS | PPM | BUSINESS | Grand Total |
|---|---|---|---|---|---|---|---|
| Stress (13Q) | -2% | 0% | -2% | -1% | -3% | 3% | -1% |
| Recent (13Q) | 5% | 4% | 4% | 1% | 1% | 1% | 2% |
| Most Recent (9Q) | 1% | 2% | 1% | -1% | 0% | -1% | 0% |

Table 8.3.8: EAD Back Test by FICO for Stress, Recent and Most Recent

| Period | <= 660 | 661-720 | 721+ | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | 0% | -3% | -2% | -1% |
| Recent (13Q) | 1% | 0% | 7% | 2% |
| Most Recent (9Q) | 0% | -2% | 4% | 0% |

Table 8.3.9: EAD Back Test by Delinquency Status for Stress, Recent and Most Recent

| Period | Current | Delinquent | Grand Total |
|---|---|---|---|
| Stress (13Q) | -2% | 1% | -1% |
| Recent (13Q) | 2% | 3% | 2% |
| Most Recent (9Q) | -1% | 2% | 0% |

Table 8.3.10: EAD Back Test by Month on Book Count for Stress, Recent and Most Recent

| Period | 1-24 | 25-60 | 61 + | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | -5% | -3% | 0% | -1% |
| Recent (13Q) | -2% | 1% | 3% | 2% |
| Most Recent (9Q) | -5% | 0% | 2% | 0% |

GCL Back Test

Table 8.3.11: GCL Back Test by Product for Stress, Recent and Most Recent

| Period | AA | Cobrand | CASH | REWARDS | PPM | BUSINESS | Grand Total |
|---|---|---|---|---|---|---|---|
| Stress (13Q) | 4% | -3% | -2% | -8% | -9% | -7% | -6% |
| Recent (13Q) | 14% | 9% | 10% | 7% | 5% | 10% | 8% |
| Most Recent (9Q) | 6% | 5% | 6% | 3% | -1% | 3% | 2% |

Table 8.3.12: GCL Back Test by FICO for Stress, Recent and Most Recent

| Period | <= 660 | 661-720 | 721+ | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | -4% | -12% | -2% | -6% |
| Recent (13Q) | 6% | 12% | 6% | 8% |
| Most Recent (9Q) | 1% | 6% | -2% | 2% |

Table 8.3.13: GCL Back Test by Delinquency Status for Stress, Recent and Most Recent

| Period | Current | Delinquent | Grand Total |
|---|---|---|---|
| Stress (13Q) | -7% | 0% | -6% |
| Recent (13Q) | 11% | -2% | 8% |
| Most Recent (9Q) | 5% | -4% | 2% |

Table 8.3.14: GCL Back Test by Month on Book Count for Stress, Recent and Most Recent

| Period | 1-24 | 25-60 | 61 + | Grand Total |
|---|---|---|---|---|
| Stress (13Q) | -8% | -8% | -5% | -6% |
| Recent (13Q) | 7% | 5% | 9% | 8% |
| Most Recent (9Q) | -2% | -4% | 6% | 2% |

PD, EAD and GCL Back Test error rates are within acceptable limits for stress, recent and most recent periods by product, FICO and delinquency status.

- Have the recommended back-testing metrics been used? If thresholds are breached, provide rationale for why and ensure that the model risk is mitigated by model developer. If any of these tests are not applicable, please explain why and please provide alternative analyses / tests to overcome these challenges.

The Back Test metrics are generated following the Y14-A CCAR / DFAST back-testing Guideline set by MRM. The Cumulative Percent Prediction Error (CCERPCT) is the main parameter to measure the model accuracy. It is calculated using the following formula:

$$CERRPCT(n) = {CERR(n)}/{CA(n)}, n = 1,2,3,...N$$

If the Cumulative percent prediction error for the entire back test period is less than 10%, (i.e. CEERRPCT (N) <= 10%), the model is considered as acceptable and no special actions are required. In addition, as pointed out by MRM, it's also important to track the quarterly relationship between the error rates. It is possible that models with the same CERRPCT (N) can have totally different patterns shown by the individual behavior of quarterly error rates, which can also reflect the model prediction power. Therefore, the metrics also contain a column called Percent Prediction Error calculated by the following equation:

$$ERRPCT(i) = {ERR(i)}/{A(i)}, i = 1,2,3,..., N$$

As given in the results below, none of the model components or the model outputs is breaching the threshold.

### 8.3.2 Back-testing results

The summary of Back Test results is given below:

Table 8.3.15: Summary of Back Test Result

| Model component/ segment | Type of test | Test results (breached parameter) | Business justification/ Limitations | Mitigation* |
|---|---|---|---|---|
| PD, EAD, Recoveries, GCL and NCL | Stress Back Test | No Breach | The model performs accurately on the 9 and 13 stress quarters | Not Applicable |
| PD, EAD, Recoveries, GCL and NCL | Recent Back Test | No Breach | The model performs accurately on the 9 and 13 recent quarters | Not Applicable |

| Model component/ segment | Type of test | Test results (breached parameter) | Business justification/ Limitations | Mitigation* |
|---|---|---|---|---|
| PD, EAD, Recoveries, GCL and NCL | Most Recent Back Test including OOT sample | No Breach | The model performs accurately on the 9 most recent quarters including OOT sample | Not Applicable |

None of the models breach the back testing error rate thresholds defined by MRM. The detailed Back Test results are provided in the below section.

- Provide detailed back-test results.

The Model Performance was evaluated using in-sample back-testing (Stress and Recent 9 quarters and 13 quarters) and most recent time period which also includes the OOT vintages. The Stress Period is identified as Jan'08 – Mar'11 and the Recent Period is identified as Apr'14 – Jun'17, since the recent period is non-stress this period also covers the non-stress back-test. The in-sample back-testing has been done on 10% data for Dec'07 and Mar'14 vintages. Since the defaults are already tagged in this data, survival methodology is not applied.

However, for the most recent sample which includes out of time sample testing, only 100% data has been used and scoring has been done from implementation point of view. Stress and recent vintages have also been Back Tested on 100% data from implementation point of view. Therefore, the defaulters in the future quarters have not been removed from the data and we have followed survival methodology for forecasting PD.

Below are the results for PD, EAD, Recovery, GCL and NCL back-tests below. The models perform well in both back-tests and the results are well within the MRM thresholds.

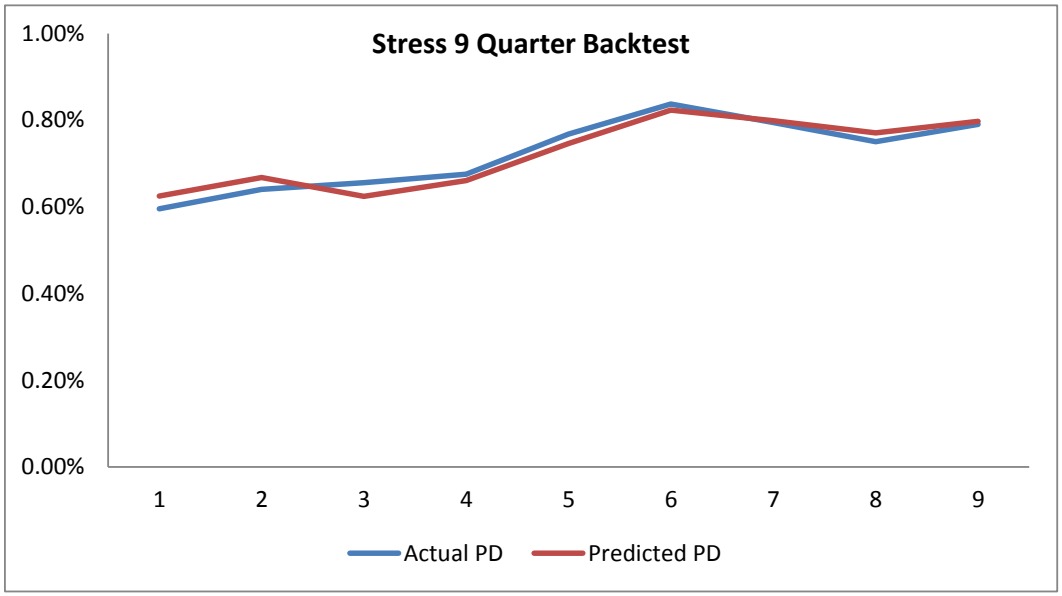
**PD Back Test Results:**

Table 8.3.16: Detailed Back-testing Results – PD (Stress 9Q - 10%)

| Stress Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Period1** | 0.60% | 0.62% | 0.03% | 4.95% | 0.60% | 0.62% | 0.03% | 4.95% | 0.03% | 0.00% | 4.95% | 4.95% |
| **Period2** | 0.64% | 0.67% | 0.03% | 4.24% | 1.24% | 1.29% | 0.06% | 4.58% | 0.03% | 0.00% | 4.24% | 4.24% |
| **Period3** | 0.66% | 0.62% | -0.03% | -4.83% | 1.89% | 1.92% | 0.02% | 1.32% | 0.03% | 0.00% | -4.83% | 4.83% |
| **Period4** | 0.68% | 0.66% | -0.01% | -2.18% | 2.57% | 2.58% | 0.01% | 0.40% | 0.01% | 0.00% | -2.18% | 2.18% |
| **Period5** | 0.77% | 0.75% | -0.02% | -2.92% | 3.33% | 3.32% | -0.01% | -0.36% | 0.02% | 0.00% | -2.92% | 2.92% |
| **Period6** | 0.84% | 0.82% | -0.01% | -1.70% | 4.17% | 4.15% | -0.03% | -0.63% | 0.01% | 0.00% | -1.70% | 1.70% |
| **Period7** | 0.79% | 0.80% | 0.00% | 0.60% | 4.97% | 4.94% | -0.02% | -0.43% | 0.00% | 0.00% | 0.60% | 0.60% |
| **Period8** | 0.75% | 0.77% | 0.02% | 2.70% | 5.72% | 5.71% | 0.00% | -0.02% | 0.02% | 0.00% | 2.70% | 2.70% |
| **Period9** | 0.79% | 0.80% | 0.01% | 0.84% | 6.51% | 6.51% | 0.01% | 0.08% | 0.01% | 0.00% | 0.84% | 0.84% |
| | | **AERR** | 0.00% | | | | | **Total** | 0.17% | 0.00% | | |
| **CERRPCT** | **0.08%** | | **MAD** | **0.02%** | **RMSE** | **0.02%** | **COV** | **2.92%** | | Max | 4.95% | 4.95% |
| | | | | | | | | | | Mean | 0.19% | 2.77% |
| | | | | | | | | | | Min | -4.83% | 0.60% |

Figure 64: Detailed Back-testing Results – PD (Stress 9Q - 10%)



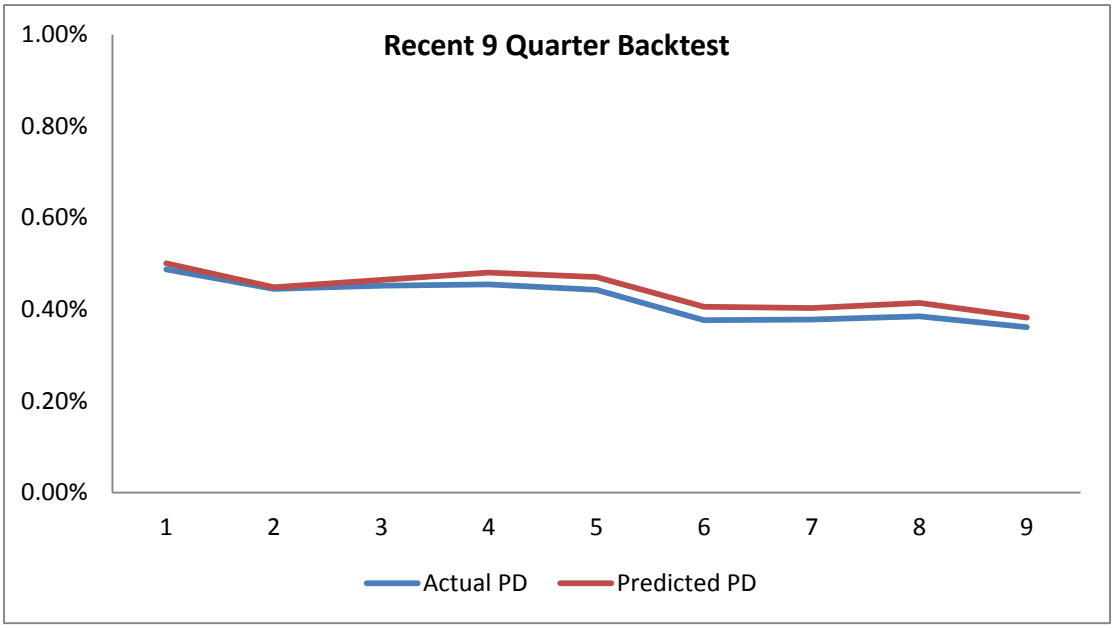

As could be observed from the chart above, the actual and predicted in the 9Q stress time period are very close which implies high model accuracy.

Table 8.3.17: Detailed Back-testing Results – PD (Recent 9Q - 10%)

| Recent Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |

| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Period1** | 0.49% | 0.50% | 0.01% | 2.76% | 0.49% | 0.50% | 0.01% | 2.76% | 0.01% | 0.00% | 2.76% | 2.76% |
| **Period2** | 0.45% | 0.45% | 0.00% | 0.68% | 0.93% | 0.95% | 0.02% | 1.76% | 0.00% | 0.00% | 0.68% | 0.68% |
| **Period3** | 0.45% | 0.46% | 0.01% | 2.69% | 1.38% | 1.41% | 0.03% | 2.06% | 0.01% | 0.00% | 2.69% | 2.69% |
| **Period4** | 0.45% | 0.48% | 0.03% | 5.75% | 1.84% | 1.89% | 0.05% | 2.97% | 0.03% | 0.00% | 5.75% | 5.75% |
| **Period5** | 0.44% | 0.47% | 0.03% | 6.27% | 2.28% | 2.36% | 0.08% | 3.61% | 0.03% | 0.00% | 6.27% | 6.27% |
| **Period6** | 0.38% | 0.41% | 0.03% | 7.69% | 2.66% | 2.77% | 0.11% | 4.19% | 0.03% | 0.00% | 7.69% | 7.69% |
| **Period7** | 0.38% | 0.40% | 0.03% | 6.72% | 3.04% | 3.17% | 0.14% | 4.51% | 0.03% | 0.00% | 6.72% | 6.72% |
| **Period8** | 0.38% | 0.41% | 0.03% | 7.63% | 3.42% | 3.59% | 0.17% | 4.86% | 0.03% | 0.00% | 7.63% | 7.63% |
| **Period9** | 0.36% | 0.38% | 0.02% | 5.91% | 3.78% | 3.97% | 0.19% | 4.96% | 0.02% | 0.00% | 5.91% | 5.91% |
| | | **AERR** | 0.02% | | | | | **Total** | 0.19% | 0.00% | | |
| **CERRPCT** | **4.96%** | | **MAD** | **0.02%** | **RMSE** | **0.02%** | **COV** | **5.37%** | | Max | 7.69% | 7.69% |
| | | | | | | | | | | Mean | 5.12% | 5.12% |
| | | | | | | | | | | Min | 0.68% | 0.68% |

Figure 65: Detailed Back-testing Results – PD (Recent 9Q - 10%)

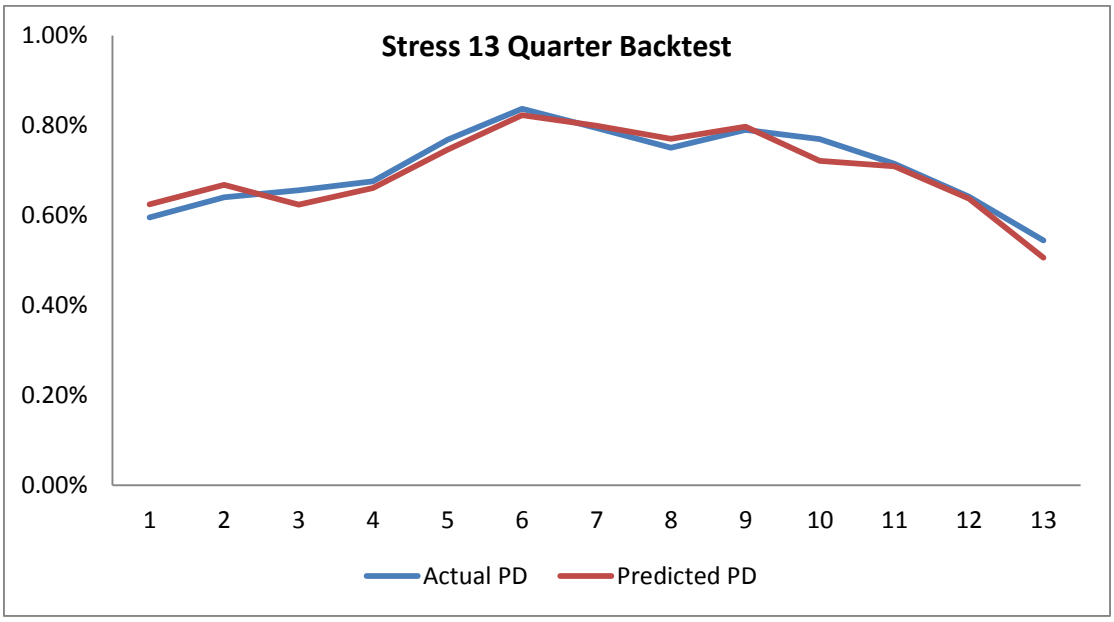



As observed from the chart above, the actual and predicted in the 9Q recent time period on 10% model development data are very close which implies good model accuracy.

Table 8.3.18: Detailed Back-testing Results – PD (Stress 13Q - 10%)

| Stress Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| Period1 | 0.60% | 0.62% | 0.03% | 4.95% | 0.60% | 0.62% | 0.03% | 4.95% | 0.03% | 0.00% | 4.95% | 4.95% |
| Period2 | 0.64% | 0.67% | 0.03% | 4.24% | 1.24% | 1.29% | 0.06% | 4.58% | 0.03% | 0.00% | 4.24% | 4.24% |
| Period3 | 0.66% | 0.62% | -0.03% | -4.83% | 1.89% | 1.92% | 0.02% | 1.32% | 0.03% | 0.00% | -4.83% | 4.83% |
| Period4 | 0.68% | 0.66% | -0.01% | -2.18% | 2.57% | 2.58% | 0.01% | 0.40% | 0.01% | 0.00% | -2.18% | 2.18% |
| Period5 | 0.77% | 0.75% | -0.02% | -2.92% | 3.33% | 3.32% | -0.01% | -0.36% | 0.02% | 0.00% | -2.92% | 2.92% |
| Period6 | 0.84% | 0.82% | -0.01% | -1.70% | 4.17% | 4.15% | -0.03% | -0.63% | 0.01% | 0.00% | -1.70% | 1.70% |
| Period7 | 0.79% | 0.80% | 0.00% | 0.60% | 4.97% | 4.94% | -0.02% | -0.43% | 0.00% | 0.00% | 0.60% | 0.60% |
| Period8 | 0.75% | 0.77% | 0.02% | 2.70% | 5.72% | 5.71% | 0.00% | -0.02% | 0.02% | 0.00% | 2.70% | 2.70% |
| Period9 | 0.79% | 0.80% | 0.01% | 0.84% | 6.51% | 6.51% | 0.01% | 0.08% | 0.01% | 0.00% | 0.84% | 0.84% |
| Period10 | 0.77% | 0.72% | -0.05% | -6.29% | 7.28% | 7.23% | -0.04% | -0.59% | 0.05% | 0.00% | -6.29% | 6.29% |
| Period11 | 0.71% | 0.71% | -0.01% | -0.79% | 7.99% | 7.94% | -0.05% | -0.61% | 0.01% | 0.00% | -0.79% | 0.79% |
| Period12 | 0.64% | 0.64% | 0.00% | -0.65% | 8.63% | 8.58% | -0.05% | -0.61% | 0.00% | 0.00% | -0.65% | 0.65% |
| Period13 | 0.54% | 0.51% | -0.04% | -7.02% | 9.18% | 9.08% | -0.09% | -0.99% | 0.04% | 0.00% | -7.02% | 7.02% |
| | | AERR | -0.01% | | | | | Total | 0.27% | 0.00% | | |
| CERRPCT | -0.99% | | MAD | 0.03% | RMSE | 0.03% | COV | 4.18% | | Max | 4.95% | 7.02% |
| | | | | | | | | | | Mean | -1.00% | 3.05% |
| | | | | | | | | | | Min | -7.02% | 0.60% |

Figure 66: Detailed Back-testing Results – PD (Stress 13Q - 10%)

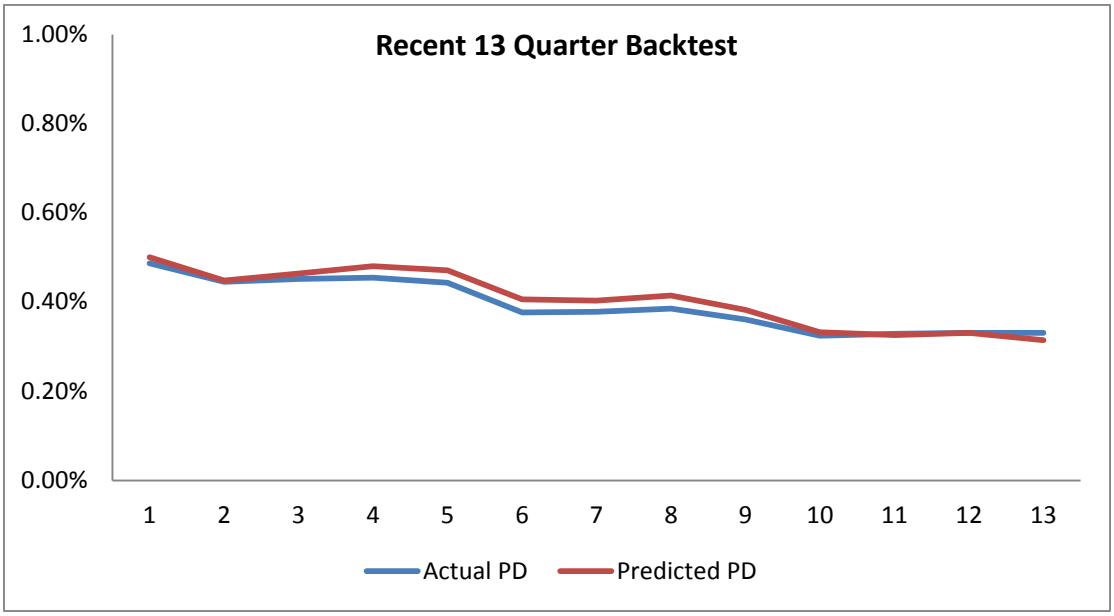



13Q stress Back Test performance of the model is also good.

Table 8.3.19: Detailed Back-testing Results – PD (Recent 13Q - 10%)

| Recent Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 0.49% | 0.50% | 0.01% | 2.76% | 0.49% | 0.50% | 0.01% | 2.76% | 0.01% | 0.00% | 2.76% | 2.76% |
| **Period2** | 0.45% | 0.45% | 0.00% | 0.68% | 0.93% | 0.95% | 0.02% | 1.76% | 0.00% | 0.00% | 0.68% | 0.68% |
| **Period3** | 0.45% | 0.46% | 0.01% | 2.69% | 1.38% | 1.41% | 0.03% | 2.06% | 0.01% | 0.00% | 2.69% | 2.69% |
| **Period4** | 0.45% | 0.48% | 0.03% | 5.75% | 1.84% | 1.89% | 0.05% | 2.97% | 0.03% | 0.00% | 5.75% | 5.75% |
| **Period5** | 0.44% | 0.47% | 0.03% | 6.27% | 2.28% | 2.36% | 0.08% | 3.61% | 0.03% | 0.00% | 6.27% | 6.27% |
| **Period6** | 0.38% | 0.41% | 0.03% | 7.69% | 2.66% | 2.77% | 0.11% | 4.19% | 0.03% | 0.00% | 7.69% | 7.69% |
| **Period7** | 0.38% | 0.40% | 0.03% | 6.72% | 3.04% | 3.17% | 0.14% | 4.51% | 0.03% | 0.00% | 6.72% | 6.72% |
| **Period8** | 0.38% | 0.41% | 0.03% | 7.63% | 3.42% | 3.59% | 0.17% | 4.86% | 0.03% | 0.00% | 7.63% | 7.63% |
| **Period9** | 0.36% | 0.38% | 0.02% | 5.91% | 3.78% | 3.97% | 0.19% | 4.96% | 0.02% | 0.00% | 5.91% | 5.91% |
| **Period10** | 0.32% | 0.33% | 0.01% | 2.52% | 4.11% | 4.30% | 0.20% | 4.77% | 0.01% | 0.00% | 2.52% | 2.52% |
| **Period11** | 0.33% | 0.33% | 0.00% | -0.83% | 4.44% | 4.63% | 0.19% | 4.35% | 0.00% | 0.00% | -0.83% | 0.83% |
| **Period12** | 0.33% | 0.33% | 0.00% | -0.01% | 4.77% | 4.96% | 0.19% | 4.05% | 0.00% | 0.00% | -0.01% | 0.01% |
| **Period13** | 0.33% | 0.31% | -0.02% | -4.81% | 5.10% | 5.27% | 0.18% | 3.47% | 0.02% | 0.00% | -4.81% | 4.81% |
| | | **AERR** | 0.01% | | | | | **Total** | 0.21% | 0.00% | | |
| **CERRPCT** | **3.47%** | | **MAD** | **0.02%** | **RMSE** | **0.02%** | **COV** | **5.96%** | | Max | 7.69% | 7.69% |
| | | | | | | | | | | Mean | 3.30% | 4.17% |
| | | | | | | | | | | Min | -4.81% | 0.01% |

Figure 67: Detailed Back-testing Results – PD (Recent 13Q - 10%)

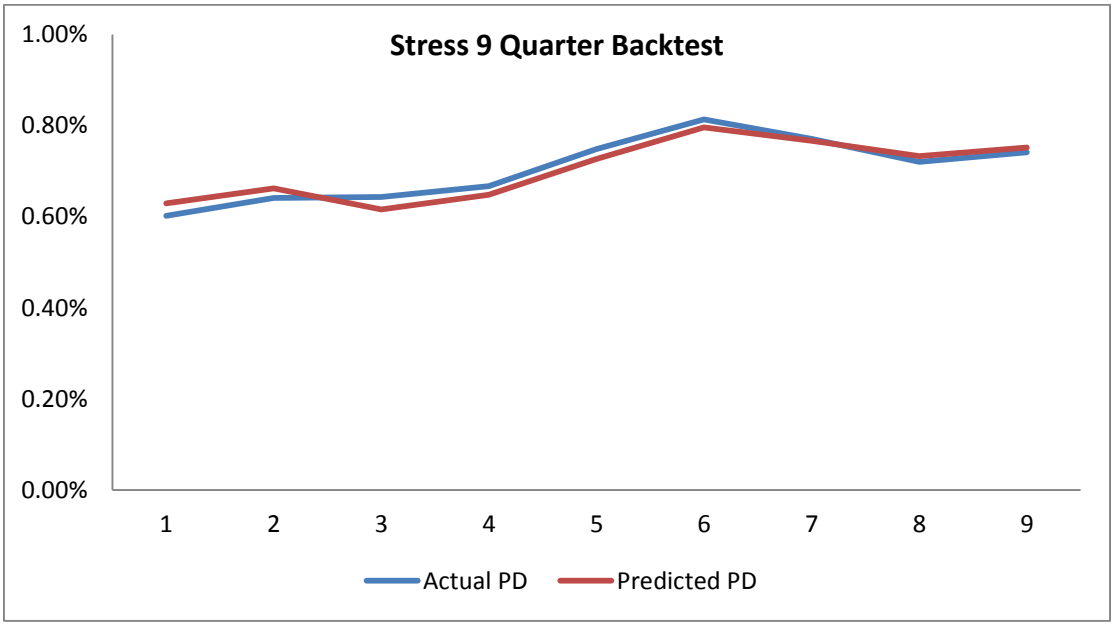



Model is performing well in the recent 13Q of Back Test.

Table 8.3.20: Detailed Back-testing Results – PD (Stress 9Q - 100%)

| Stress Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| Period1 | 0.60% | 0.63% | 0.03% | 4.55% | 0.60% | 0.63% | 0.03% | 4.55% | 0.03% | 0.00% | 4.55% | 4.55% |
| Period2 | 0.64% | 0.66% | 0.02% | 3.28% | 1.24% | 1.29% | 0.05% | 3.89% | 0.02% | 0.00% | 3.28% | 3.28% |
| Period3 | 0.64% | 0.62% | -0.03% | -4.26% | 1.89% | 1.91% | 0.02% | 1.11% | 0.03% | 0.00% | -4.26% | 4.26% |
| Period4 | 0.67% | 0.65% | -0.02% | -2.88% | 2.55% | 2.55% | 0.00% | 0.07% | 0.02% | 0.00% | -2.88% | 2.88% |
| Period5 | 0.75% | 0.73% | -0.02% | -2.87% | 3.30% | 3.28% | -0.02% | -0.60% | 0.02% | 0.00% | -2.87% | 2.87% |
| Period6 | 0.81% | 0.80% | -0.02% | -2.09% | 4.11% | 4.08% | -0.04% | -0.89% | 0.02% | 0.00% | -2.09% | 2.09% |
| Period7 | 0.77% | 0.77% | 0.00% | -0.56% | 4.88% | 4.84% | -0.04% | -0.84% | 0.00% | 0.00% | -0.56% | 0.56% |
| Period8 | 0.72% | 0.73% | 0.01% | 1.80% | 5.60% | 5.58% | -0.03% | -0.50% | 0.01% | 0.00% | 1.80% | 1.80% |
| Period9 | 0.74% | 0.75% | 0.01% | 1.50% | 6.34% | 6.33% | -0.02% | -0.27% | 0.01% | 0.00% | 1.50% | 1.50% |
| | | AERR | 0.00% | | | | | Total | 0.16% | 0.00% | | |
| CERRPCT | -0.27% | | MAD | 0.02% | RMSE | 0.02% | COV | 2.74% | | Max | 4.55% | 4.55% |
| | | | | | | | | | | Mean | -0.17% | 2.64% |
| | | | | | | | | | | Min | -4.26% | 0.56% |

Figure 68: Detailed Back-testing Results – PD (Stress 9Q - 100%)

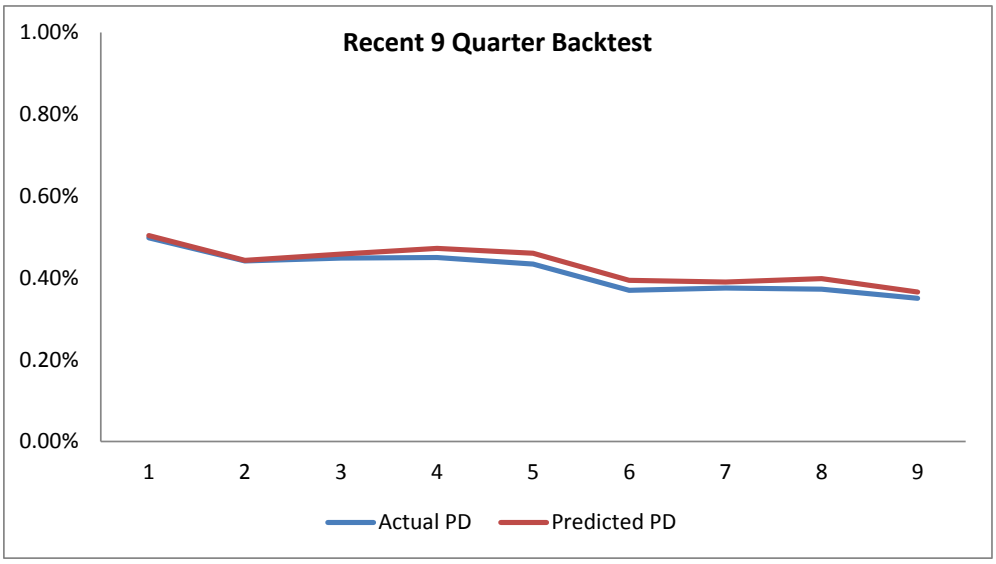


The 100% stress data was scored using implementation methodology and Back Test was performed. The actual and predicted PDs are close in the 9Q Back Test indicating good model accuracy.

Table 8.3.21: Detailed Back-testing Results – PD (Recent 9Q - 100%)

| Recent Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| Period1 | 0.50% | 0.50% | 0.01% | 1.17% | 0.50% | 0.50% | 0.01% | 1.17% | 0.01% | 0.00% | 1.17% | 1.17% |
| Period2 | 0.44% | 0.44% | 0.00% | 0.29% | 0.94% | 0.95% | 0.01% | 0.76% | 0.00% | 0.00% | 0.29% | 0.29% |
| Period3 | 0.45% | 0.46% | 0.01% | 2.19% | 1.39% | 1.40% | 0.02% | 1.22% | 0.01% | 0.00% | 2.19% | 2.19% |
| Period4 | 0.45% | 0.47% | 0.02% | 4.87% | 1.84% | 1.88% | 0.04% | 2.11% | 0.02% | 0.00% | 4.87% | 4.87% |
| Period5 | 0.43% | 0.46% | 0.03% | 6.11% | 2.27% | 2.34% | 0.07% | 2.88% | 0.03% | 0.00% | 6.11% | 6.11% |
| Period6 | 0.37% | 0.39% | 0.02% | 6.54% | 2.64% | 2.73% | 0.09% | 3.39% | 0.02% | 0.00% | 6.54% | 6.54% |
| Period7 | 0.38% | 0.39% | 0.01% | 3.89% | 3.02% | 3.12% | 0.10% | 3.45% | 0.01% | 0.00% | 3.89% | 3.89% |
| Period8 | 0.37% | 0.40% | 0.03% | 6.89% | 3.39% | 3.52% | 0.13% | 3.83% | 0.03% | 0.00% | 6.89% | 6.89% |
| Period9 | 0.35% | 0.37% | 0.02% | 4.51% | 3.74% | 3.88% | 0.15% | 3.89% | 0.02% | 0.00% | 4.51% | 4.51% |
| | | AERR | 0.02% | | | | | Total | 0.15% | 0.00% | | |
| CERRPCT | 3.89% | | MAD | 0.02% | RMSE | 0.02% | COV | 4.41% | | Max | 6.89% | 6.89% |
| | | | | | | | | | | Mean | 4.05% | 4.05% |
| | | | | | | | | | | Min | 0.29% | 0.29% |

gure 69: Detailed Back-testing Results – PD (Recent 9Q - 100%)

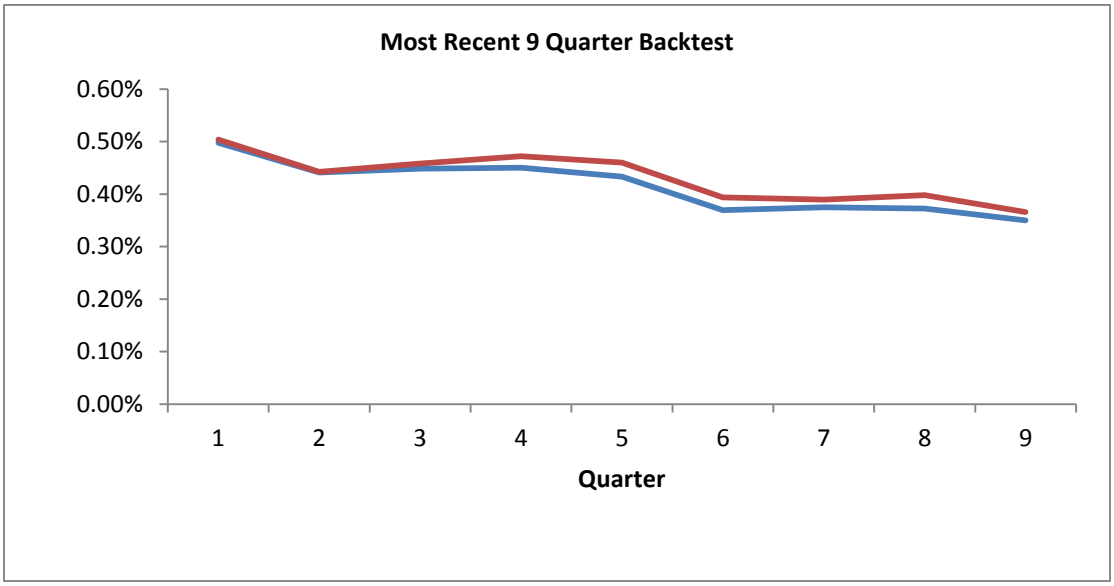


9Q Back Test on 100% recent sample is also satisfactory.

**Table 8.3.22: Detailed Back-testing Results – PD (Most Recent/OOT 9Q - 100%)**

| Most recent back-test/OOT (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT (i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 0.43% | 0.40% | -0.02% | -5.82% | 0.43% | 0.40% | -0.02% | -5.82% | 0.02% | 0.00% | -5.82% | 5.82% |
| **Period2** | 0.45% | 0.42% | -0.04% | -8.36% | 0.88% | 0.82% | -0.06% | -7.13% | 0.04% | 0.00% | -8.36% | 8.36% |
| **Period3** | 0.47% | 0.47% | 0.00% | -0.36% | 1.35% | 1.29% | -0.06% | -4.79% | 0.00% | 0.00% | -0.36% | 0.36% |
| **Period4** | 0.45% | 0.46% | 0.01% | 3.19% | 1.80% | 1.75% | -0.05% | -2.80% | 0.01% | 0.00% | 3.19% | 3.19% |
| **Period5** | 0.40% | 0.40% | 0.00% | 0.89% | 2.20% | 2.15% | -0.05% | -2.13% | 0.00% | 0.00% | 0.89% | 0.89% |
| **Period6** | 0.42% | 0.40% | -0.02% | -5.18% | 2.62% | 2.55% | -0.07% | -2.62% | 0.02% | 0.00% | -5.18% | 5.18% |
| **Period7** | 0.43% | 0.41% | -0.01% | -3.01% | 3.04% | 2.96% | -0.08% | -2.68% | 0.01% | 0.00% | -3.01% | 3.01% |
| **Period8** | 0.42% | 0.40% | -0.02% | -4.22% | 3.46% | 3.36% | -0.10% | -2.86% | 0.02% | 0.00% | -4.22% | 4.22% |
| **Period9** | 0.38% | 0.33% | -0.05% | -13.20% | 3.84% | 3.69% | -0.15% | -3.88% | 0.05% | 0.00% | -13.20% | 13.20% |
| | | **AERR** | -0.02% | | | | | **Total** | 0.18% | 0.00% | | |
| **CERRPCT** | **-3.88%** | | **MAD** | **0.02%** | **RMSE** | **0.03%** | **COV** | **5.91%** | | Max | 3.19% | 13.20% |
| | | | | | | | | | | Mean | -4.01% | 4.91% |
| | | | | | | | | | | Min | -13.20% | 0.36% |

Figure 70: Detailed Back-testing Results – PD (Most Recent/OOT 9Q - 100%)

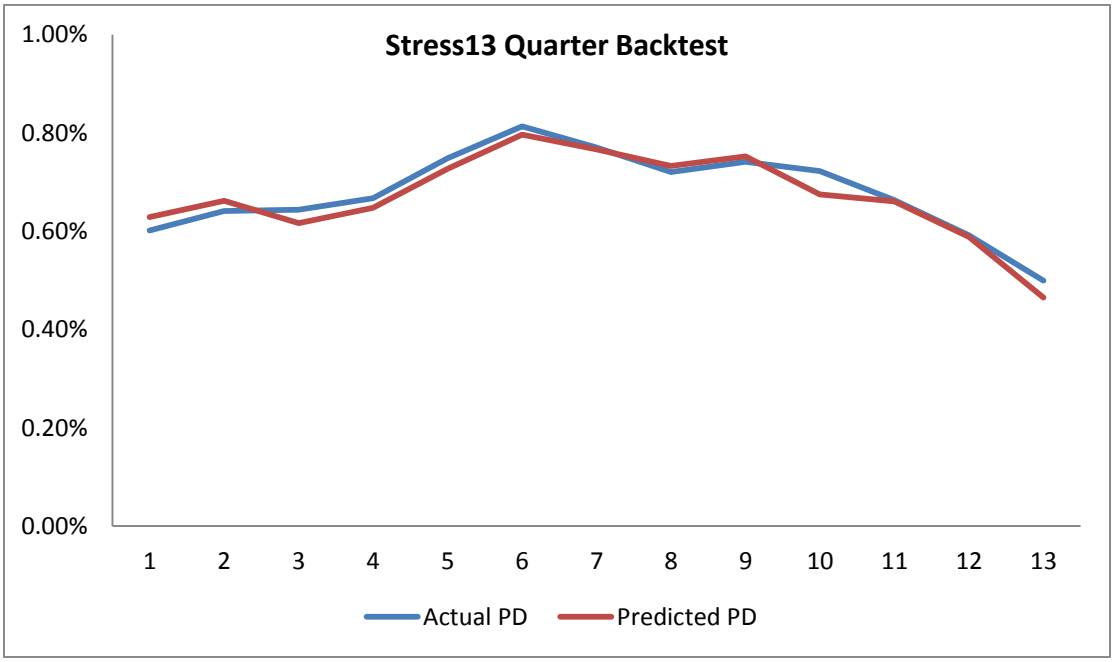



9Q model Back Test was also performed on 100% most recent data which also includes 4 OOT vintages. Model accuracy is good across all the nine quarters.

Table 8.3.23: Detailed Back-testing Results – PD (Stress 13Q - 100%)

| Stress Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| Period1 | 0.60% | 0.63% | 0.03% | 4.55% | 0.60% | 0.63% | 0.03% | 4.55% | 0.03% | 0.00% | 4.55% | 4.55% |
| Period2 | 0.64% | 0.66% | 0.02% | 3.28% | 1.24% | 1.29% | 0.05% | 3.89% | 0.02% | 0.00% | 3.28% | 3.28% |
| Period3 | 0.64% | 0.62% | -0.03% | -4.26% | 1.89% | 1.91% | 0.02% | 1.11% | 0.03% | 0.00% | -4.26% | 4.26% |
| Period4 | 0.67% | 0.65% | -0.02% | -2.88% | 2.55% | 2.55% | 0.00% | 0.07% | 0.02% | 0.00% | -2.88% | 2.88% |
| Period5 | 0.75% | 0.73% | -0.02% | -2.87% | 3.30% | 3.28% | -0.02% | -0.60% | 0.02% | 0.00% | -2.87% | 2.87% |
| Period6 | 0.81% | 0.80% | -0.02% | -2.09% | 4.11% | 4.08% | -0.04% | -0.89% | 0.02% | 0.00% | -2.09% | 2.09% |
| Period7 | 0.77% | 0.77% | 0.00% | -0.56% | 4.88% | 4.84% | -0.04% | -0.84% | 0.00% | 0.00% | -0.56% | 0.56% |
| Period8 | 0.72% | 0.73% | 0.01% | 1.80% | 5.60% | 5.58% | -0.03% | -0.50% | 0.01% | 0.00% | 1.80% | 1.80% |
| Period9 | 0.74% | 0.75% | 0.01% | 1.50% | 6.34% | 6.33% | -0.02% | -0.27% | 0.01% | 0.00% | 1.50% | 1.50% |
| Period10 | 0.72% | 0.67% | -0.05% | -6.52% | 7.07% | 7.00% | -0.06% | -0.91% | 0.05% | 0.00% | -6.52% | 6.52% |
| Period11 | 0.66% | 0.66% | 0.00% | -0.45% | 7.73% | 7.66% | -0.07% | -0.87% | 0.00% | 0.00% | -0.45% | 0.45% |
| Period12 | 0.59% | 0.59% | 0.00% | -0.47% | 8.32% | 8.25% | -0.07% | -0.84% | 0.00% | 0.00% | -0.47% | 0.47% |
| Period13 | 0.50% | 0.46% | -0.03% | -6.87% | 8.82% | 8.72% | -0.10% | -1.18% | 0.03% | 0.00% | -6.87% | 6.87% |
| | | AERR | -0.01% | | | | | Total | 0.25% | 0.00% | | |
| CERRPCT | -1.18% | | MAD | 0.03% | RMSE | 0.03% | COV | 4.04% | | Max | 4.55% | 6.87% |
| | | | | | | | | | | Mean | -1.22% | 2.93% |
| | | | | | | | | | | Min | -6.87% | 0.45% |

Figure 71: Detailed Back-testing Results – PD (Stress 13Q - 100%)

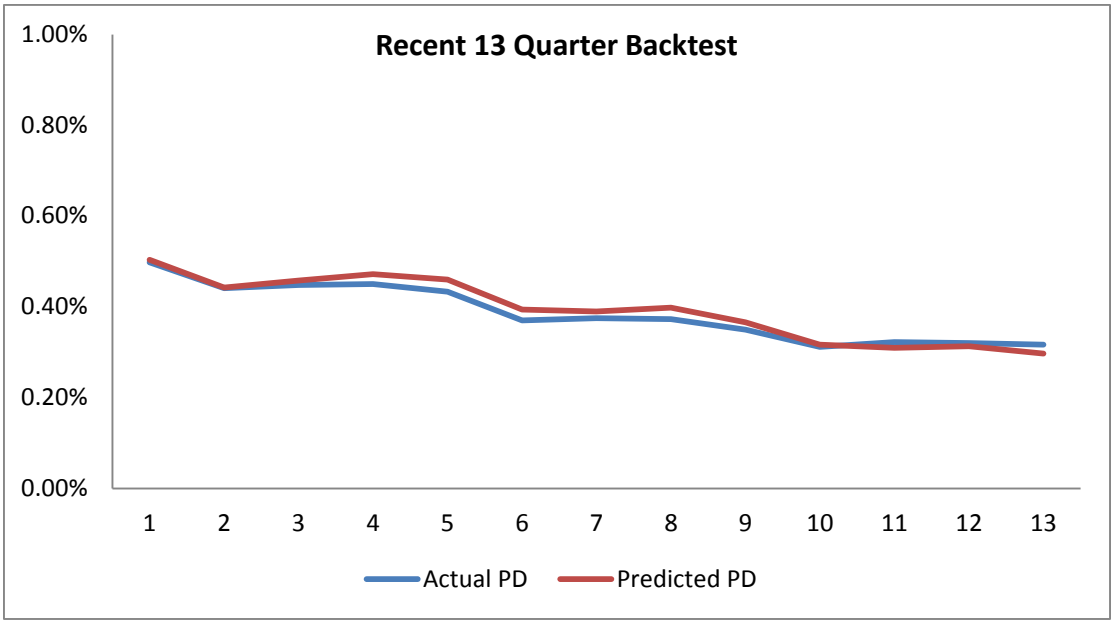



Model accuracy on 13Q stress Back Test on 100% data is within MRM thresholds which signifies good model accuracy.

Table 8.3.24: Detailed Back-testing Results – PD (Recent 13Q - 100%)

| Recent Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 0.50% | 0.50% | 0.01% | 1.17% | 0.50% | 0.50% | 0.01% | 1.17% | 0.01% | 0.00% | 1.17% | 1.17% |
| **Period2** | 0.44% | 0.44% | 0.00% | 0.29% | 0.94% | 0.95% | 0.01% | 0.76% | 0.00% | 0.00% | 0.29% | 0.29% |
| **Period3** | 0.45% | 0.46% | 0.01% | 2.19% | 1.39% | 1.40% | 0.02% | 1.22% | 0.01% | 0.00% | 2.19% | 2.19% |
| **Period4** | 0.45% | 0.47% | 0.02% | 4.87% | 1.84% | 1.88% | 0.04% | 2.11% | 0.02% | 0.00% | 4.87% | 4.87% |
| **Period5** | 0.43% | 0.46% | 0.03% | 6.11% | 2.27% | 2.34% | 0.07% | 2.88% | 0.03% | 0.00% | 6.11% | 6.11% |
| **Period6** | 0.37% | 0.39% | 0.02% | 6.54% | 2.64% | 2.73% | 0.09% | 3.39% | 0.02% | 0.00% | 6.54% | 6.54% |
| **Period7** | 0.38% | 0.39% | 0.01% | 3.89% | 3.02% | 3.12% | 0.10% | 3.45% | 0.01% | 0.00% | 3.89% | 3.89% |
| **Period8** | 0.37% | 0.40% | 0.03% | 6.89% | 3.39% | 3.52% | 0.13% | 3.83% | 0.03% | 0.00% | 6.89% | 6.89% |
| **Period9** | 0.35% | 0.37% | 0.02% | 4.51% | 3.74% | 3.88% | 0.15% | 3.89% | 0.02% | 0.00% | 4.51% | 4.51% |
| **Period10** | 0.31% | 0.32% | 0.01% | 1.69% | 4.05% | 4.20% | 0.15% | 3.72% | 0.01% | 0.00% | 1.69% | 1.69% |
| **Period11** | 0.32% | 0.31% | -0.01% | -3.89% | 4.37% | 4.51% | 0.14% | 3.16% | 0.01% | 0.00% | -3.89% | 3.89% |
| **Period12** | 0.32% | 0.31% | -0.01% | -2.25% | 4.69% | 4.82% | 0.13% | 2.79% | 0.01% | 0.00% | -2.25% | 2.25% |
| **Period13** | 0.32% | 0.30% | -0.02% | -6.32% | 5.01% | 5.12% | 0.11% | 2.22% | 0.02% | 0.00% | -6.32% | 6.32% |
| | | **AERR** | 0.01% | | | | | **Total** | 0.19% | 0.00% | | |
| **CERRPCT** | **2.22%** | | **MAD** | **0.02%** | **RMSE** | **0.02%** | **COV** | **5.23%** | | Max | 6.89% | 6.89% |
| | | | | | | | | | | Mean | 1.98% | 3.89% |
| | | | | | | | | | | Min | -6.32% | 0.29% |

Figure 72: Detailed Back-testing Results – PD (Recent 13Q - 100%)




The model is performing well in 100% recent 13Q Back Test.

Below attached sheet contains the Back Test results of EAD model.

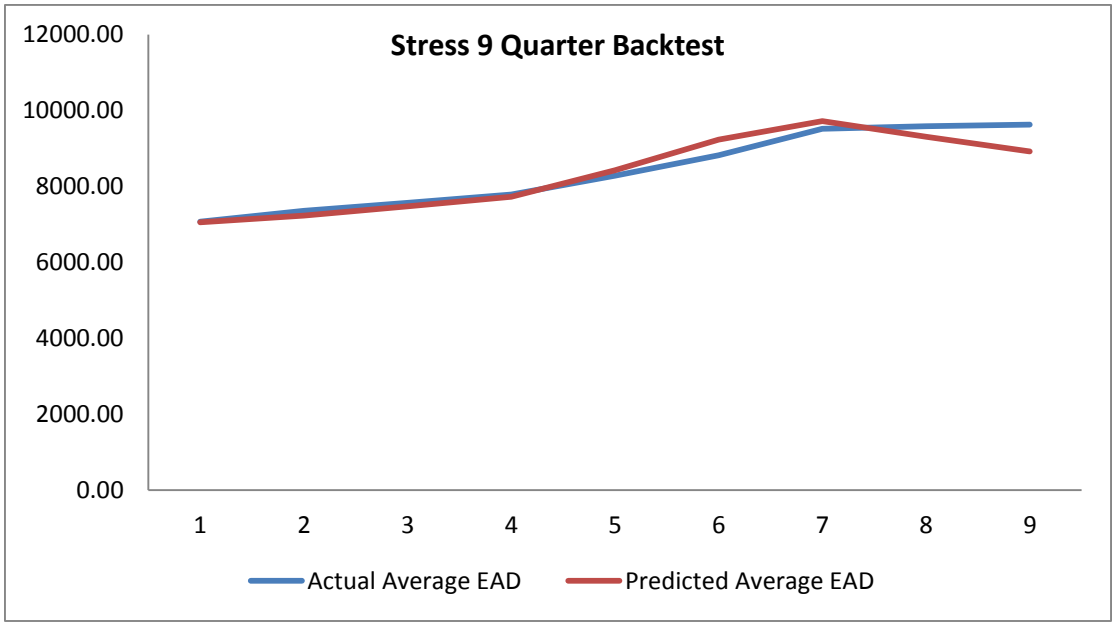
8.3.2_backtest_results_PD.xlsx

## EAD Back Test Results:

Table 8.3.25: Detailed Back-testing Results – EAD (Stress 9Q - 10%)

| Stress Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 7070.49 | 7055.64 | -14.85 | -0.21% | 7,070.5 | 7,055.6 | -14.8 | -0.21% | 14.85 | 220.43 | -0.21% | 0.21% |
| **Period2** | 7351.22 | 7224.40 | -126.81 | -1.73% | 14,421.7 | 14,280.0 | -141.7 | -0.98% | 126.81 | 16081.93 | -1.73% | 1.73% |
| **Period3** | 7564.31 | 7475.25 | -89.06 | -1.18% | 21,986.0 | 21,755.3 | -230.7 | -1.05% | 89.06 | 7932.41 | -1.18% | 1.18% |
| **Period4** | 7785.49 | 7727.61 | -57.88 | -0.74% | 29,771.5 | 29,482.9 | -288.6 | -0.97% | 57.88 | 3349.65 | -0.74% | 0.74% |
| **Period5** | 8282.33 | 8419.32 | 136.99 | 1.65% | 38,053.8 | 37,902.2 | -151.6 | -0.40% | 136.99 | 18766.61 | 1.65% | 1.65% |
| **Period6** | 8815.96 | 9227.85 | 411.89 | 4.67% | 46,869.8 | 47,130.1 | 260.3 | 0.56% | 411.89 | 169652.32 | 4.67% | 4.67% |
| **Period7** | 9517.50 | 9720.03 | 202.52 | 2.13% | 56,387.3 | 56,850.1 | 462.8 | 0.82% | 202.52 | 41016.16 | 2.13% | 2.13% |
| **Period8** | 9579.93 | 9303.48 | -276.44 | -2.89% | 65,967.2 | 66,153.6 | 186.4 | 0.28% | 276.44 | 76421.44 | -2.89% | 2.89% |
| **Period9** | 9623.39 | 8914.61 | -708.78 | -7.37% | 75,590.6 | 75,068.2 | -522.4 | -0.69% | 708.78 | 502362.22 | -7.37% | 7.37% |
| | | **AERR** | -58.05 | | | | | **Total** | 2025.23 | 835803.16 | | |
| **CERRPCT** | **-0.69%** | | **MAD** | 225.03 | **RMSE** | 304.74 | **COV** | **3.63%** | | Max | 4.67% | 7.37% |
| | | | | | | | | | | Mean | -0.63% | 2.51% |
| | | | | | | | | | | Min | -7.37% | 0.21% |

Figure 73: Detailed Back-testing Results – EAD (Stress 9Q - 10%)

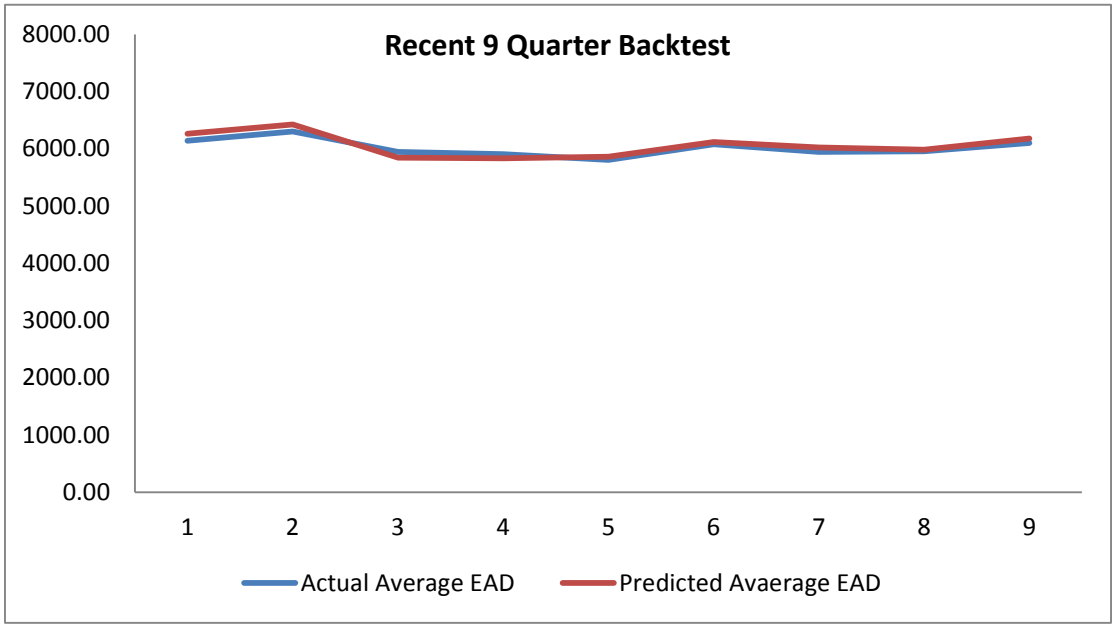



As seen from above chart, the actual and predicted are aligned to each other in the 9Q horizon of stress period implying high model accuracy

Table 8.3.26: Detailed Back-testing Results – EAD (Recent 9Q - 10%)

| Recent Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT (i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR (n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 6141.96 | 6264.80 | 122.84 | 2.00% | 6,142.0 | 6,264.8 | 122.8 | 2.00% | 122.84 | 15090.57 | 2.00% | 2.00% |
| **Period2** | 6303.71 | 6423.79 | 120.08 | 1.90% | 12,445.7 | 12,688.6 | 242.9 | 1.95% | 120.08 | 14418.09 | 1.90% | 1.90% |
| **Period3** | 5944.78 | 5843.01 | -101.76 | -1.71% | 18,390.4 | 18,531.6 | 141.2 | 0.77% | 101.76 | 10355.63 | -1.71% | 1.71% |
| **Period4** | 5906.39 | 5832.83 | -73.56 | -1.25% | 24,296.8 | 24,364.4 | 67.6 | 0.28% | 73.56 | 5411.36 | -1.25% | 1.25% |
| **Period5** | 5807.31 | 5864.16 | 56.85 | 0.98% | 30,104.1 | 30,228.6 | 124.4 | 0.41% | 56.85 | 3231.87 | 0.98% | 0.98% |
| **Period6** | 6076.78 | 6116.65 | 39.87 | 0.66% | 36,180.9 | 36,345.2 | 164.3 | 0.45% | 39.87 | 1590.00 | 0.66% | 0.66% |
| **Period7** | 5944.13 | 6025.08 | 80.94 | 1.36% | 42,125.1 | 42,370.3 | 245.3 | 0.58% | 80.94 | 6551.96 | 1.36% | 1.36% |
| **Period8** | 5955.58 | 5986.91 | 31.34 | 0.53% | 48,080.6 | 48,357.2 | 276.6 | 0.58% | 31.34 | 981.92 | 0.53% | 0.53% |
| **Period9** | 6104.22 | 6177.89 | 73.67 | 1.21% | 54,184.8 | 54,535.1 | 350.3 | 0.65% | 73.67 | 5427.32 | 1.21% | 1.21% |
| | | **AERR** | 38.92 | | | | | **Total** | 700.92 | 63058.71 | | |
| **CERRPCT** | **0.65%** | | **MAD** | 77.88 | **RMSE** | 83.70 | **COV** | **1.39%** | | Max | 2.00% | 2.00% |
| | | | | | | | | | | Mean | 0.63% | 1.29% |
| | | | | | | | | | | Min | -1.71% | 0.53% |

Figure 74: Detailed Back-testing Results – EAD (Recent 9Q - 10%)

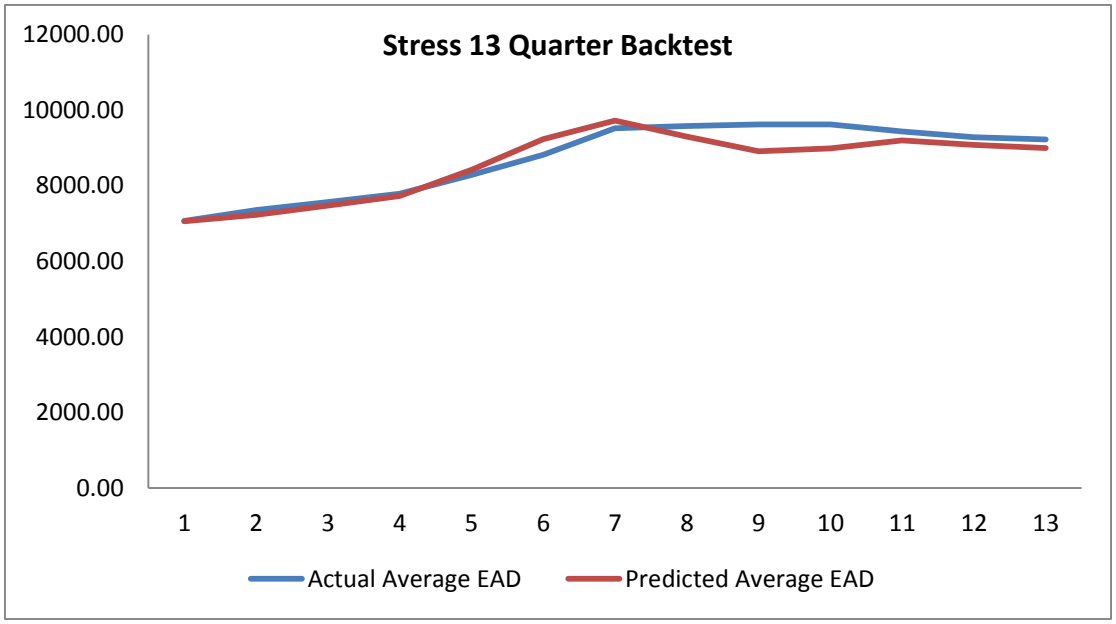



The actual and predicted are very close in the 9Q recent period indicating high model accuracy.

Table 8.3.27: Detailed Back-testing Results – EAD (Stress 13Q - 10%)

| Stress Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 7070.49 | 7055.64 | -14.85 | -0.21% | 7070.49 | 7055.64 | -14.85 | -0.21% | 14.85 | 220.43 | -0.21% | 0.21% |
| **Period2** | 7351.22 | 7224.40 | -126.81 | -1.73% | 14421.71 | 14280.05 | -141.66 | -0.98% | 126.81 | 16081.93 | -1.73% | 1.73% |
| **Period3** | 7564.31 | 7475.25 | -89.06 | -1.18% | 21986.02 | 21755.30 | -230.73 | -1.05% | 89.06 | 7932.41 | -1.18% | 1.18% |
| **Period4** | 7785.49 | 7727.61 | -57.88 | -0.74% | 29771.51 | 29482.91 | -288.60 | -0.97% | 57.88 | 3349.65 | -0.74% | 0.74% |
| **Period5** | 8282.33 | 8419.32 | 136.99 | 1.65% | 38053.84 | 37902.23 | -151.61 | -0.40% | 136.99 | 18766.61 | 1.65% | 1.65% |
| **Period6** | 8815.96 | 9227.85 | 411.89 | 4.67% | 46869.80 | 47130.08 | 260.28 | 0.56% | 411.89 | 169652.32 | 4.67% | 4.67% |
| **Period7** | 9517.50 | 9720.03 | 202.52 | 2.13% | 56387.31 | 56850.11 | 462.80 | 0.82% | 202.52 | 41016.16 | 2.13% | 2.13% |
| **Period8** | 9579.93 | 9303.48 | -276.44 | -2.89% | 65967.23 | 66153.59 | 186.36 | 0.28% | 276.44 | 76421.44 | -2.89% | 2.89% |
| **Period9** | 9623.39 | 8914.61 | -708.78 | -7.37% | 75590.62 | 75068.20 | -522.42 | -0.69% | 708.78 | 502362.22 | -7.37% | 7.37% |
| **Period10** | 9623.59 | 8988.07 | -635.53 | -6.60% | 85214.21 | 84056.27 | -1157.95 | -1.36% | 635.53 | 403897.39 | -6.60% | 6.60% |
| **Period11** | 9431.49 | 9198.97 | -232.53 | -2.47% | 94645.71 | 93255.23 | -1390.47 | -1.47% | 232.53 | 54068.10 | -2.47% | 2.47% |
| **Period12** | 9285.04 | 9079.55 | -205.49 | -2.21% | 103930.75 | 102334.78 | -1595.96 | -1.54% | 205.49 | 42226.53 | -2.21% | 2.21% |
| **Period13** | 9219.42 | 8992.41 | -227.01 | -2.46% | 113150.16 | 111327.20 | -1822.97 | -1.61% | 227.01 | 51531.65 | -2.46% | 2.46% |
| | | AERR | -140.23 | | | | | **Total** | 3325.78 | 1387526.84 | | |
| **CERRPCT** | **-1.61%** | | **MAD** | 369.53 | **RMSE** | 392.64 | **COV** | **4.51%** | | Max | 4.67% | 7.37% |
| | | | | | | | | | | Mean | -1.49% | 2.79% |
| | | | | | | | | | | Min | -7.37% | 0.21% |

Figure 75: Detailed Back-testing Results – EAD (Stress 13Q - 10%)

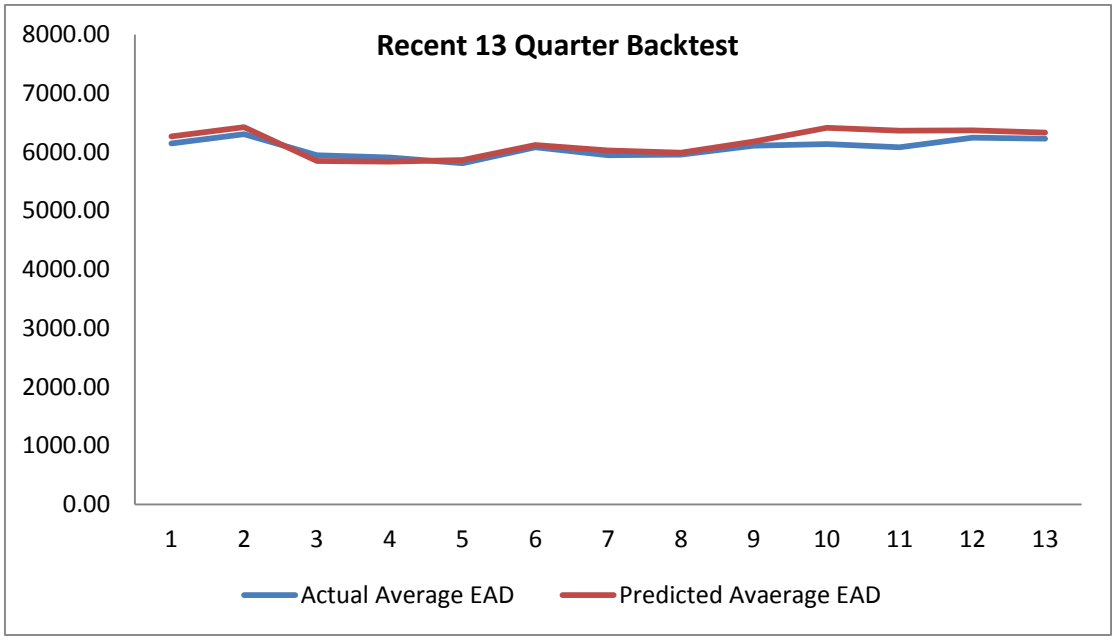



Actual and predicted EAD are close in 13Q of stress period which implies high model accuracy.

Table 8.3.28: Detailed Back-testing Results – EAD (Recent 13Q - 10%)

| Recent Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 6141.96 | 6264.80 | 122.84 | 2.00% | 6141.96 | 6264.80 | 122.84 | 2.00% | 122.84 | 15090.57 | 2.00% | 2.00% |
| **Period2** | 6303.71 | 6423.79 | 120.08 | 1.90% | 12445.67 | 12688.59 | 242.92 | 1.95% | 120.08 | 14418.09 | 1.90% | 1.90% |
| **Period3** | 5944.78 | 5843.01 | -101.76 | -1.71% | 18390.45 | 18531.61 | 141.16 | 0.77% | 101.76 | 10355.63 | -1.71% | 1.71% |
| **Period4** | 5906.39 | 5832.83 | -73.56 | -1.25% | 24296.84 | 24364.43 | 67.59 | 0.28% | 73.56 | 5411.36 | -1.25% | 1.25% |
| **Period5** | 5807.31 | 5864.16 | 56.85 | 0.98% | 30104.15 | 30228.59 | 124.44 | 0.41% | 56.85 | 3231.87 | 0.98% | 0.98% |
| **Period6** | 6076.78 | 6116.65 | 39.87 | 0.66% | 36180.92 | 36345.24 | 164.32 | 0.45% | 39.87 | 1590.00 | 0.66% | 0.66% |
| **Period7** | 5944.13 | 6025.08 | 80.94 | 1.36% | 42125.06 | 42370.32 | 245.26 | 0.58% | 80.94 | 6551.96 | 1.36% | 1.36% |
| **Period8** | 5955.58 | 5986.91 | 31.34 | 0.53% | 48080.63 | 48357.23 | 276.60 | 0.58% | 31.34 | 981.92 | 0.53% | 0.53% |
| **Period9** | 6104.22 | 6177.89 | 73.67 | 1.21% | 54184.85 | 54535.12 | 350.27 | 0.65% | 73.67 | 5427.32 | 1.21% | 1.21% |
| **Period10** | 6135.43 | 6409.11 | 273.68 | 4.46% | 60320.28 | 60944.23 | 623.95 | 1.03% | 273.68 | 74899.33 | 4.46% | 4.46% |
| **Period11** | 6079.54 | 6359.49 | 279.95 | 4.60% | 66399.82 | 67303.72 | 903.90 | 1.36% | 279.95 | 78371.87 | 4.60% | 4.60% |
| **Period12** | 6241.63 | 6367.20 | 125.57 | 2.01% | 72641.46 | 73670.92 | 1029.46 | 1.42% | 125.57 | 15767.17 | 2.01% | 2.01% |
| **Period13** | 6226.35 | 6328.45 | 102.10 | 1.64% | 78867.80 | 79999.37 | 1131.57 | 1.43% | 102.10 | 10425.26 | 1.64% | 1.64% |
| | | AERR | 87.04 | | | | | **Total** | 1482.22 | 242522.33 | | |
| **CERRPCT** | **1.43%** | | **MAD** | 164.69 | **RMSE** | 164.16 | **COV** | **2.71%** | | Max | 4.60% | 4.60% |
| | | | | | | | | | | Mean | 1.41% | 1.87% |
| | | | | | | | | | | Min | -1.71% | 0.53% |

Figure 76: Detailed Back-testing Results – EAD (Recent 13Q - 10%)

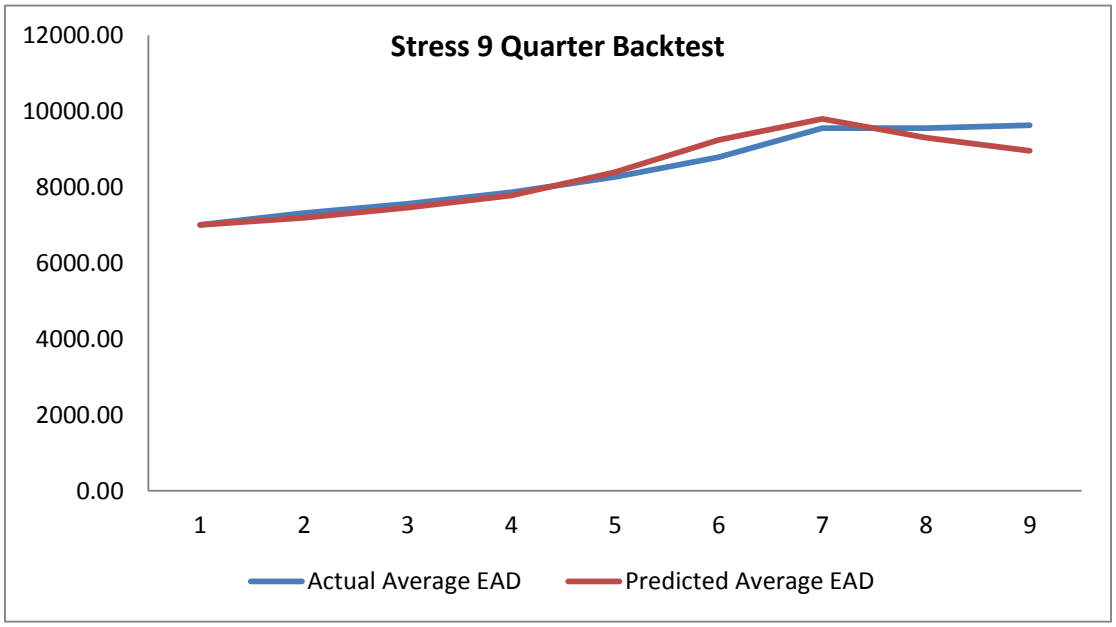



EAD prediction is very close to the actual EAD in recent 13 quarters indicating high model accuracy.

Table 8.3.29: Detailed Back-testing Results – EAD (Stress 9Q - 100%)

| Stress Back Tests | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 7004.30 | 7004.44 | 0.14 | 0.00% | 7004.30 | 7004.44 | 0.14 | 0.00% | 0.14 | 0.02 | 0.00% | 0.00% |
| **Period2** | 7309.35 | 7189.73 | -119.62 | -1.64% | 14313.65 | 14194.17 | -119.48 | -0.83% | 119.62 | 14308.47 | -1.64% | 1.64% |
| **Period3** | 7556.71 | 7454.89 | -101.82 | -1.35% | 21870.36 | 21649.07 | -221.29 | -1.01% | 101.82 | 10366.88 | -1.35% | 1.35% |
| **Period4** | 7855.24 | 7778.23 | -77.01 | -0.98% | 29725.60 | 29427.29 | -298.31 | -1.00% | 77.01 | 5930.73 | -0.98% | 0.98% |
| **Period5** | 8260.88 | 8392.10 | 131.23 | 1.59% | 37986.48 | 37819.40 | -167.08 | -0.44% | 131.23 | 17220.23 | 1.59% | 1.59% |
| **Period6** | 8786.83 | 9235.31 | 448.48 | 5.10% | 46773.31 | 47054.71 | 281.40 | 0.60% | 448.48 | 201133.72 | 5.10% | 5.10% |
| **Period7** | 9549.00 | 9796.61 | 247.60 | 2.59% | 56322.31 | 56851.32 | 529.00 | 0.94% | 247.60 | 61306.42 | 2.59% | 2.59% |
| **Period8** | 9546.61 | 9293.14 | -253.47 | -2.66% | 65868.92 | 66144.46 | 275.53 | 0.42% | 253.47 | 64245.35 | -2.66% | 2.66% |
| **Period9** | 9623.25 | 8948.70 | -674.54 | -7.01% | 75492.17 | 75093.16 | -399.01 | -0.53% | 674.54 | 455010.76 | -7.01% | 7.01% |
| | | **AERR** | -44.33 | | | | | **Total** | 2053.91 | 829522.59 | | |
| **CERRPCT** | **-0.53%** | | **MAD** | 228.21 | **RMSE** | 303.59 | **COV** | **3.62%** | | Max | 5.10% | 7.01% |
| | | | | | | | | | | Mean | -0.48% | 2.55% |
| | | | | | | | | | | Min | -7.01% | 0.00% |

Figure 77: Detailed Back-testing Results – EAD (Stress 9Q - 100%)

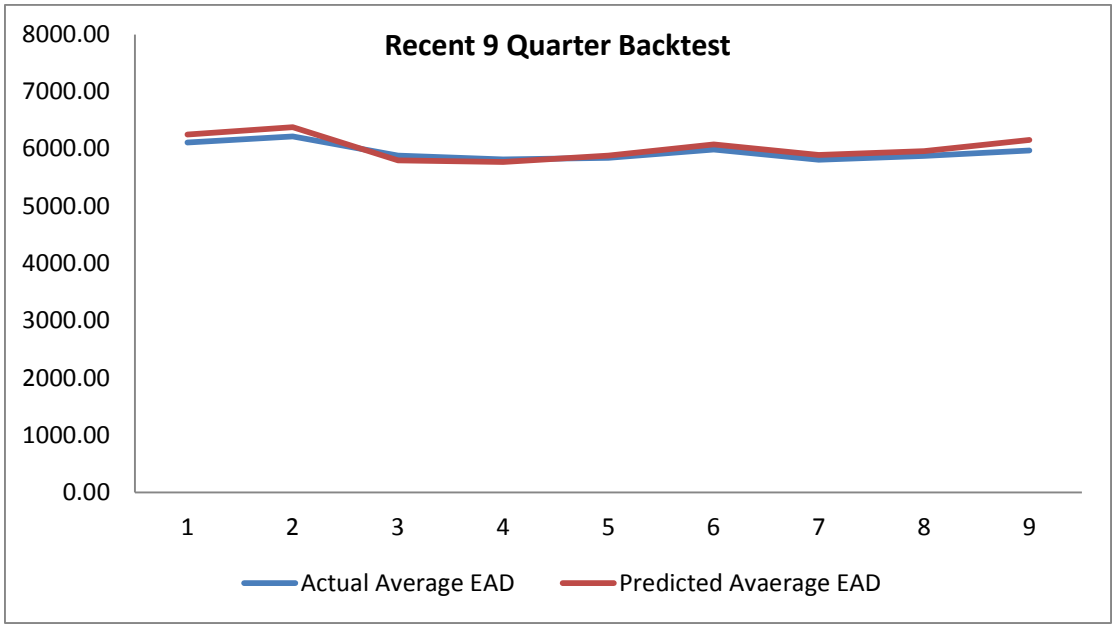


Model accuracy is high in stress 9Q forecast using 100% data using implementation forecasting methodology.

**Table 8.3.30: Detailed Back-testing Results – EAD (Recent 9Q - 100%)**

| Recent Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 6115.08 | 6250.21 | 135.13 | 2.21% | 6115.08 | 6250.21 | 135.13 | 2.21% | 135.13 | 18259.89 | 2.21% | 2.21% |
| **Period2** | 6220.14 | 6383.21 | 163.06 | 2.62% | 12335.23 | 12633.42 | 298.19 | 2.42% | 163.06 | 26590.01 | 2.62% | 2.62% |
| **Period3** | 5886.04 | 5800.89 | -85.15 | -1.45% | 18221.26 | 18434.31 | 213.04 | 1.17% | 85.15 | 7250.89 | -1.45% | 1.45% |
| **Period4** | 5817.39 | 5772.59 | -44.79 | -0.77% | 24038.65 | 24206.90 | 168.25 | 0.70% | 44.79 | 2006.25 | -0.77% | 0.77% |
| **Period5** | 5843.20 | 5883.97 | 40.77 | 0.70% | 29881.85 | 30090.87 | 209.02 | 0.70% | 40.77 | 1662.26 | 0.70% | 0.70% |
| **Period6** | 5990.09 | 6081.18 | 91.10 | 1.52% | 35871.94 | 36172.05 | 300.12 | 0.84% | 91.10 | 8298.51 | 1.52% | 1.52% |
| **Period7** | 5813.62 | 5896.55 | 82.94 | 1.43% | 41685.55 | 42068.61 | 383.05 | 0.92% | 82.94 | 6878.38 | 1.43% | 1.43% |
| **Period8** | 5880.87 | 5961.81 | 80.94 | 1.38% | 47566.42 | 48030.42 | 464.00 | 0.98% | 80.94 | 6551.97 | 1.38% | 1.38% |
| **Period9** | 5973.62 | 6157.57 | 183.95 | 3.08% | 53540.04 | 54187.98 | 647.95 | 1.21% | 183.95 | 33836.78 | 3.08% | 3.08% |
| | | **AERR** | 71.99 | | | | | **Total** | 907.83 | 111334.94 | | |
| **CERRPCT** | **1.21%** | | **MAD** | 100.87 | **RMSE** | 111.22 | **COV** | **1.87%** | | Max | 3.08% | 3.08% |
| | | | | | | | | | | Mean | 1.19% | 1.68% |
| | | | | | | | | | | Min | -1.45% | 0.70% |

Figure 78: Detailed Back-testing Results – EAD (Recent 9Q - 100%)

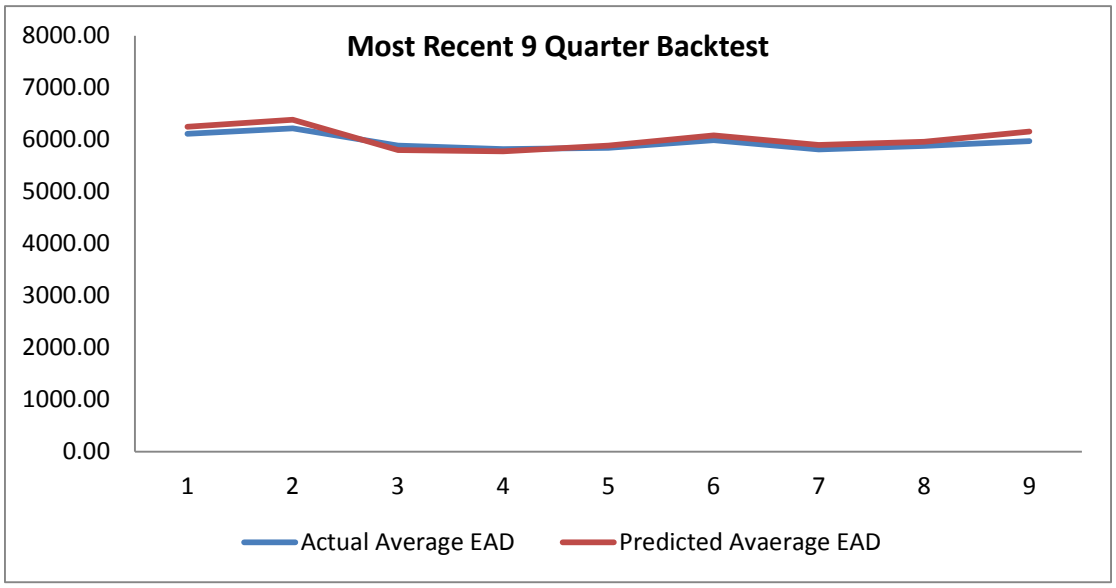



Model accuracy is high in recent 9Q forecast on 100% data using forecasting methodology.

Table 8.3.31: Detailed Back-testing Results – EAD (Most Recent/OOT 9Q - 100%)

| Most recent back-test/OOT (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 5727.17 | 5869.79 | 142.62 | 2.49% | 5727.17 | 5869.79 | 142.62 | 2.49% | 142.62 | 20339.34 | 2.49% | 2.49% |
| **Period2** | 5463.21 | 5575.53 | 112.32 | 2.06% | 11190.37 | 11445.31 | 254.94 | 2.28% | 112.32 | 12616.37 | 2.06% | 2.06% |
| **Period3** | 5465.88 | 5382.17 | -83.72 | -1.53% | 16656.26 | 16827.48 | 171.22 | 1.03% | 83.72 | 7008.46 | -1.53% | 1.53% |
| **Period4** | 5529.54 | 5506.44 | -23.10 | -0.42% | 22185.80 | 22333.92 | 148.12 | 0.67% | 23.10 | 533.61 | -0.42% | 0.42% |
| **Period5** | 5710.59 | 5690.87 | -19.72 | -0.35% | 27896.40 | 28024.80 | 128.40 | 0.46% | 19.72 | 388.90 | -0.35% | 0.35% |
| **Period6** | 5531.90 | 5599.26 | 67.36 | 1.22% | 33428.30 | 33624.06 | 195.76 | 0.59% | 67.36 | 4537.80 | 1.22% | 1.22% |
| **Period7** | 5668.99 | 5572.56 | -96.43 | -1.70% | 39097.29 | 39196.62 | 99.34 | 0.25% | 96.43 | 9298.34 | -1.70% | 1.70% |
| **Period8** | 5677.37 | 5586.31 | -91.06 | -1.60% | 44774.66 | 44782.93 | 8.28 | 0.02% | 91.06 | 8291.78 | -1.60% | 1.60% |
| **Period9** | 5841.13 | 5783.42 | -57.71 | -0.99% | 50615.79 | 50566.36 | -49.43 | -0.10% | 57.71 | 3330.29 | -0.99% | 0.99% |
| | | **AERR** | -5.49 | | | | | **Total** | 694.03 | 66344.90 | | |
| **CERRPCT** | **-0.10%** | | **MAD** | 77.11 | **RMSE** | 85.86 | **COV** | **1.53%** | | Max | 2.49% | 2.49% |
| | | | | | | | | | | Mean | -0.09% | 1.37% |
| | | | | | | | | | | Min | -1.70% | 0.35% |

Figure 79: Detailed Back-testing Results – EAD (Most Recent/OOT 9Q - 100%)

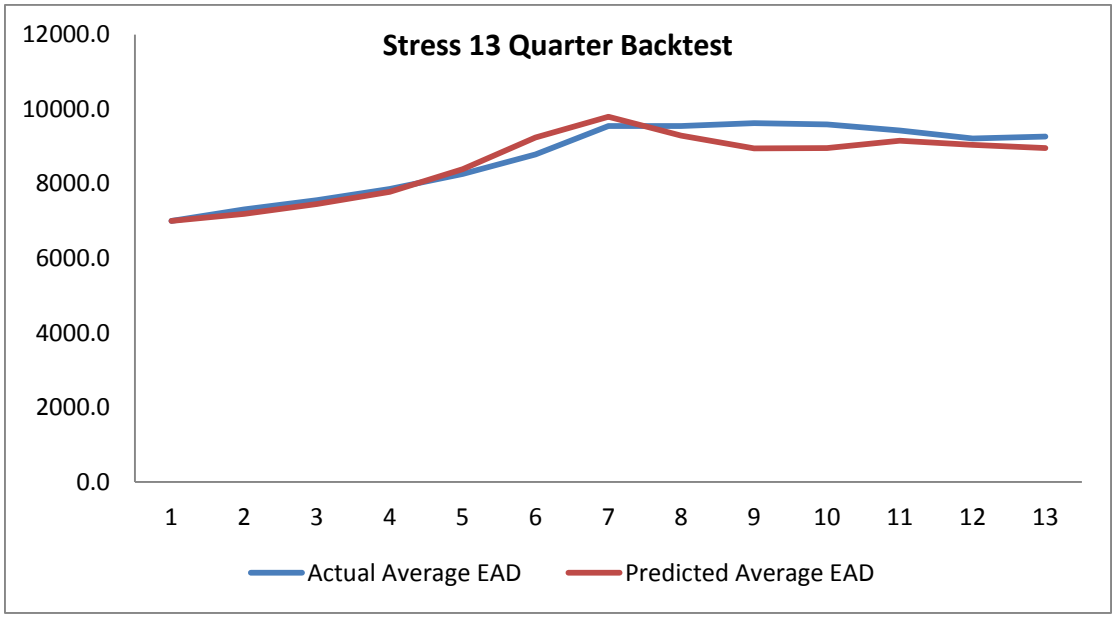


Actual and predicted EAD are very close in the most recent 9Q on 100% data which implies high model accuracy in the most recent period.

**Table 8.3.32: Detailed Back-testing Results – EAD (Stress 13Q - 100%)**

| Stress Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 7004.3 | 7004.4 | 0.1 | 0.00% | 7004.3 | 7004.4 | 0.1 | 0.00% | 0.1 | 0.0 | 0.00% | 0.00% |
| **Period2** | 7309.4 | 7189.7 | -119.6 | -1.64% | 14313.6 | 14194.2 | -119.5 | -0.83% | 119.6 | 14308.5 | -1.64% | 1.64% |
| **Period3** | 7556.7 | 7454.9 | -101.8 | -1.35% | 21870.4 | 21649.1 | -221.3 | -1.01% | 101.8 | 10366.9 | -1.35% | 1.35% |
| **Period4** | 7855.2 | 7778.2 | -77.0 | -0.98% | 29725.6 | 29427.3 | -298.3 | -1.00% | 77.0 | 5930.7 | -0.98% | 0.98% |
| **Period5** | 8260.9 | 8392.1 | 131.2 | 1.59% | 37986.5 | 37819.4 | -167.1 | -0.44% | 131.2 | 17220.2 | 1.59% | 1.59% |
| **Period6** | 8786.8 | 9235.3 | 448.5 | 5.10% | 46773.3 | 47054.7 | 281.4 | 0.60% | 448.5 | 201133.7 | 5.10% | 5.10% |
| **Period7** | 9549.0 | 9796.6 | 247.6 | 2.59% | 56322.3 | 56851.3 | 529.0 | 0.94% | 247.6 | 61306.4 | 2.59% | 2.59% |
| **Period8** | 9546.6 | 9293.1 | -253.5 | -2.66% | 65868.9 | 66144.5 | 275.5 | 0.42% | 253.5 | 64245.3 | -2.66% | 2.66% |
| **Period9** | 9623.2 | 8948.7 | -674.5 | -7.01% | 75492.2 | 75093.2 | -399.0 | -0.53% | 674.5 | 455010.8 | -7.01% | 7.01% |
| **Period10** | 9586.9 | 8952.0 | -634.9 | -6.62% | 85079.0 | 84045.1 | -1033.9 | -1.22% | 634.9 | 403064.9 | -6.62% | 6.62% |
| **Period11** | 9422.7 | 9155.8 | -266.9 | -2.83% | 94501.7 | 93201.0 | -1300.7 | -1.38% | 266.9 | 71215.4 | -2.83% | 2.83% |
| **Period12** | 9215.2 | 9038.7 | -176.5 | -1.91% | 103716.9 | 102239.7 | -1477.2 | -1.42% | 176.5 | 31138.4 | -1.91% | 1.91% |
| **Period13** | 9265.2 | 8957.7 | -307.6 | -3.32% | 112982.1 | 111197.4 | -1784.8 | -1.58% | 307.6 | 94596.2 | -3.32% | 3.32% |
| | | AERR | -137.3 | | | | | **Total** | 3439.7 | 1429537.5 | | |
| **CERRPCT** | **-1.58%** | | **MAD** | 382.19 | **RMSE** | 398.54 | **COV** | **4.59%** | | Max | 5.10% | 7.01% |
| | | | | | | | | | | Mean | -1.46% | 2.89% |
| | | | | | | | | | | Min | -7.01% | 0.00% |

Figure 80: Detailed Back-testing Results – EAD (Stress 13Q - 100%)



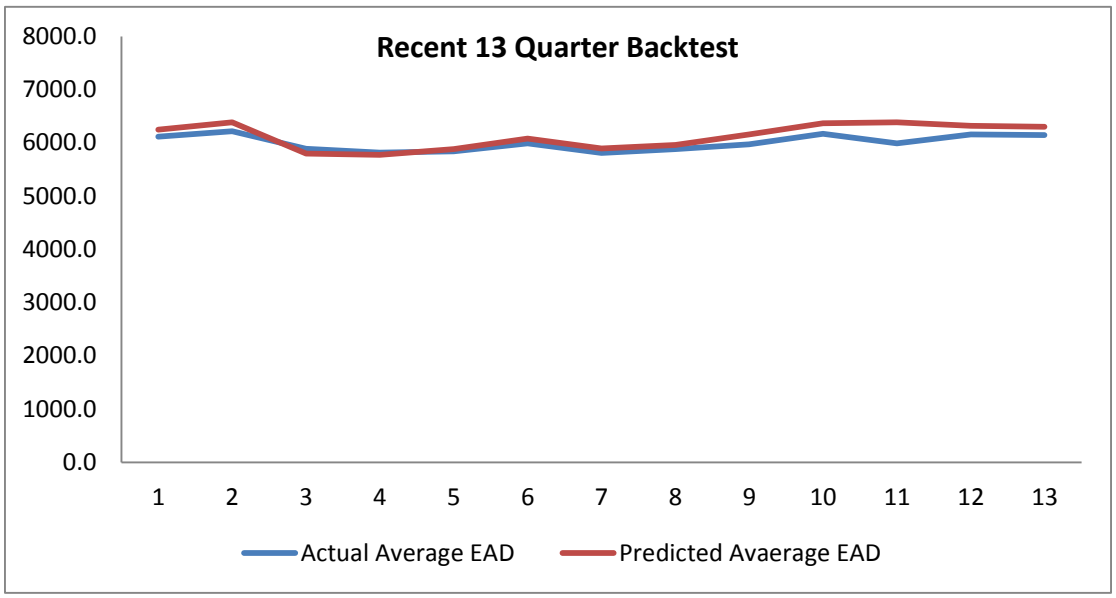

Actual and predicted EAD are close in 100% stress 13Q data implying high model accuracy.

Table 8.3.33: Detailed Back-testing Results – EAD (Recent 13Q - 100%)

| Recent Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | 6115.1 | 6250.2 | 135.1 | 2.21% | 6115.1 | 6250.2 | 135.1 | 2.21% | 135.1 | 18259.9 | 2.21% | 2.21% |
| **Period2** | 6220.1 | 6383.2 | 163.1 | 2.62% | 12335.2 | 12633.4 | 298.2 | 2.42% | 163.1 | 26590.0 | 2.62% | 2.62% |
| **Period3** | 5886.0 | 5800.9 | -85.2 | -1.45% | 18221.3 | 18434.3 | 213.0 | 1.17% | 85.2 | 7250.9 | -1.45% | 1.45% |
| **Period4** | 5817.4 | 5772.6 | -44.8 | -0.77% | 24038.6 | 24206.9 | 168.3 | 0.70% | 44.8 | 2006.2 | -0.77% | 0.77% |
| **Period5** | 5843.2 | 5884.0 | 40.8 | 0.70% | 29881.9 | 30090.9 | 209.0 | 0.70% | 40.8 | 1662.3 | 0.70% | 0.70% |
| **Period6** | 5990.1 | 6081.2 | 91.1 | 1.52% | 35871.9 | 36172.1 | 300.1 | 0.84% | 91.1 | 8298.5 | 1.52% | 1.52% |
| **Period7** | 5813.6 | 5896.6 | 82.9 | 1.43% | 41685.6 | 42068.6 | 383.1 | 0.92% | 82.9 | 6878.4 | 1.43% | 1.43% |
| **Period8** | 5880.9 | 5961.8 | 80.9 | 1.38% | 47566.4 | 48030.4 | 464.0 | 0.98% | 80.9 | 6552.0 | 1.38% | 1.38% |
| **Period9** | 5973.6 | 6157.6 | 183.9 | 3.08% | 53540.0 | 54188.0 | 647.9 | 1.21% | 183.9 | 33836.8 | 3.08% | 3.08% |
| **Period10** | 6171.1 | 6370.2 | 199.1 | 3.23% | 59711.1 | 60558.2 | 847.1 | 1.42% | 199.1 | 39647.2 | 3.23% | 3.23% |
| **Period1** | 5988.5 | 6383.0 | 394.5 | 6.59% | 65699. | 66941. | 1241.5 | 1.89% | 394.5 | 155617.6 | 6.59% | 6.59% |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | | | | | 6 | 2 | | | | | | |
| **Period12** | 6157.1 | 6319.1 | 162.0 | 2.63% | 71856.8 | 73260.3 | 1403.5 | 1.95% | 162.0 | 26237.9 | 2.63% | 2.63% |
| **Period13** | 6144.5 | 6304.4 | 159.8 | 2.60% | 78001.3 | 79564.6 | 1563.3 | 2.00% | 159.8 | 25541.1 | 2.60% | 2.60% |
| | | AERR | 120.3 | | | | | **Total** | 1823.2 | 358378.8 | | |
| **CERRPCT** | **2.00%** | | **MAD** | 202.58 | **RMSE** | 199.55 | **COV** | **3.33%** | | Max | 6.59% | 6.59% |
| | | | | | | | | | | Mean | 1.98% | 2.32% |
| | | | | | | | | | | Min | -1.45% | 0.70% |

Figure 81: Detailed Back-testing Results – EAD (Recent 9Q - 100%)




The model is performing well in the 100% recent 13Q Back Test for EAD model.

Below attached sheet contains the Back Test results of EAD model.

8.3.2_backtest_results_EAD.xlsx

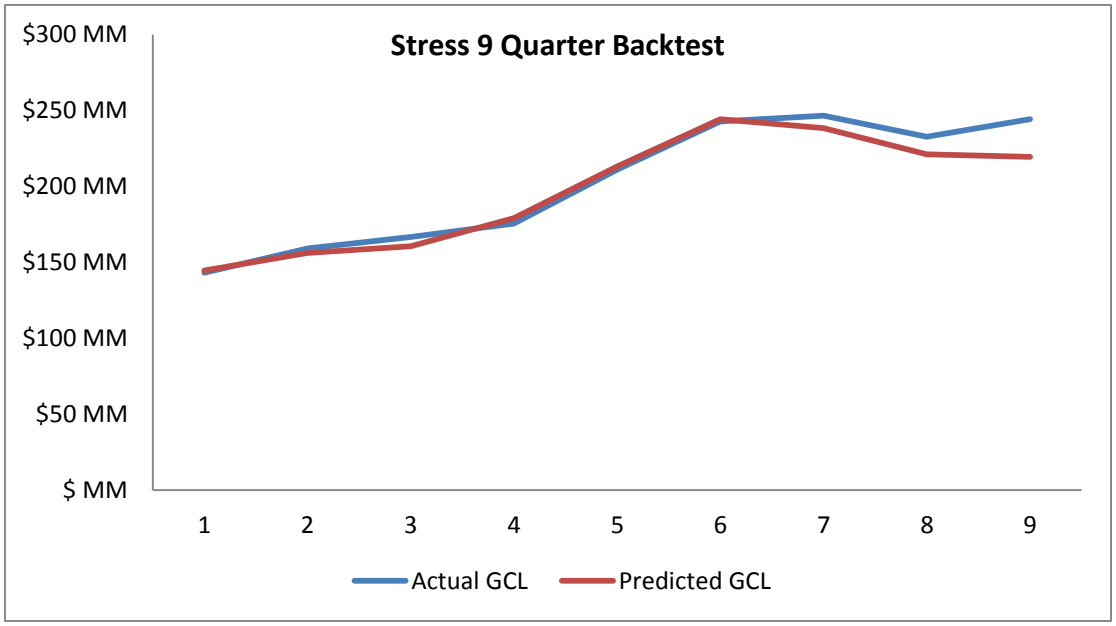
## GCL back-tests

The GCL back-tests were also carried out for recent and stress 9 quarters. It is observed that all errors for the model are within the specified GMO thresholds.

Table 8.3.34: Detailed Back-testing Results – GCL (Stress 9Q - 10%)

| Stress Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | $143 MM | $145 MM | $1 MM | 1.00% | $143 MM | $145 MM | $1 MM | 1.00% | $1 MM | $2,059,654 MM | 1.00% | 1.00% |
| **Period2** | $159 MM | $156 MM | -$3 MM | -1.91% | $302 MM | $301 MM | -$2 MM | -0.53% | $3 MM | $9,246,046 MM | -1.91% | 1.91% |
| **Period3** | $167 MM | $161 MM | -$6 MM | -3.60% | $469 MM | $461 MM | -$8 MM | -1.62% | $6 MM | $36,048,889 MM | -3.60% | 3.60% |
| **Period4** | $176 MM | $179 MM | $4 MM | 2.00% | $644 MM | $640 MM | -$4 MM | -0.64% | $4 MM | $12,330,033 MM | 2.00% | 2.00% |
| **Period5** | $211 MM | $213 MM | $2 MM | 1.06% | $855 MM | $853 MM | -$2 MM | -0.22% | $2 MM | $4,963,297 MM | 1.06% | 1.06% |
| **Period6** | $243 MM | $244 MM | $1 MM | 0.62% | $1,098 MM | $1,098 MM | $ MM | -0.03% | $1 MM | $2,249,797 MM | 0.62% | 0.62% |
| **Period7** | $247 MM | $238 MM | -$8 MM | -3.32% | $1,345 MM | $1,336 MM | -$9 MM | -0.64% | $8 MM | $66,863,354 MM | -3.32% | 3.32% |
| **Period8** | $233 MM | $221 MM | -$11 MM | -4.91% | $1,577 MM | $1,557 MM | -$20 MM | -1.27% | $11 MM | $130,429,300 MM | -4.91% | 4.91% |
| **Period9** | $244 MM | $219 MM | -$25 MM | -10.16% | $1,821 MM | $1,777 MM | -$45 MM | -2.46% | $25 MM | $615,494,545 MM | -10.16% | 10.16% |
| | | **AERR** | -$5 MM | | | | | **Total** | $62 MM | $879,684,915 MM | | |
| **CERRPCT** | **-2.46%** | | **MAD** | $7 MM | **RMSE** | $10 MM | **COV** | **4.88%** | | Max | 2.00% | 10.16% |
| | | | | | | | | | | Mean | -2.14% | 3.18% |
| | | | | | | | | | | Min | -10.16% | 0.62% |

Figure 82: Detailed Back-testing Results – GCL (Stress 9Q - 10%)

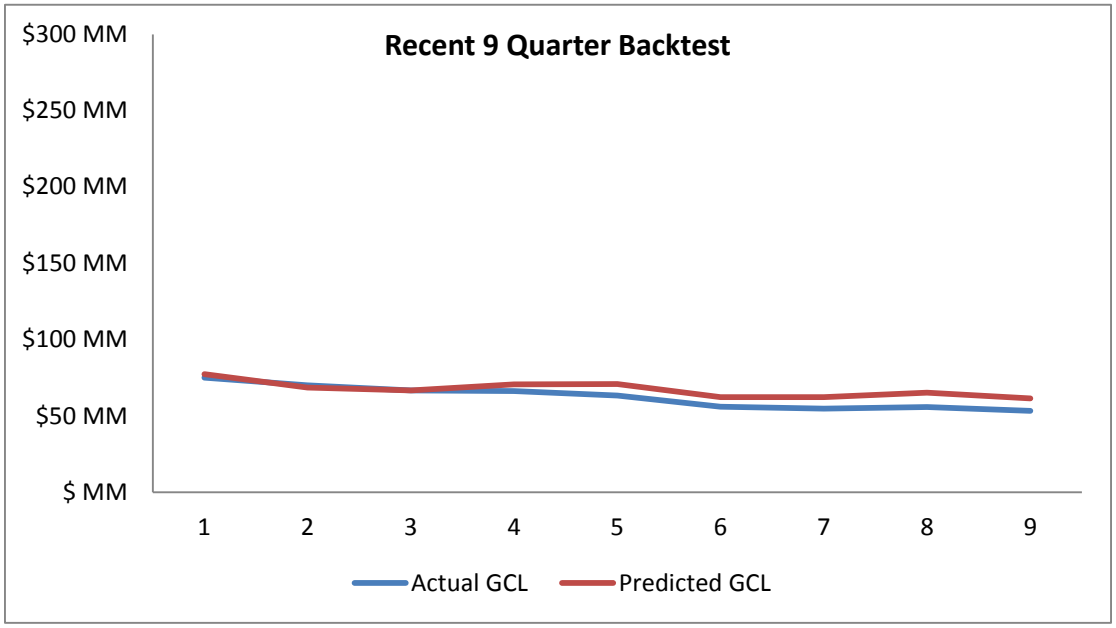



Model accuracy is good for 9Q GCL prediction in stress period.

Table 8.3.35: Detailed Back-testing Results – GCL (Recent 9Q - 10%)

| Recent Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | $75 MM | $77 MM | $2 MM | 3.10% | $75 MM | $77 MM | $2 MM | 3.10% | $2 MM | $5,410,025 MM | 3.10% | 3.10% |
| **Period2** | $70 MM | $69 MM | -$1 MM | -1.93% | $145 MM | $146 MM | $1 MM | 0.67% | $1 MM | $1,822,940 MM | -1.93% | 1.93% |
| **Period3** | $67 MM | $67 MM | $ MM | -0.01% | $212 MM | $213 MM | $1 MM | 0.46% | $ MM | $53 MM | -0.01% | 0.01% |
| **Period4** | $66 MM | $71 MM | $4 MM | 6.32% | $278 MM | $284 MM | $5 MM | 1.85% | $4 MM | $17,607,741 MM | 6.32% | 6.32% |
| **Period5** | $63 MM | $71 MM | $8 MM | 11.86% | $342 MM | $355 MM | $13 MM | 3.71% | $8 MM | $56,536,026 MM | 11.86% | 11.86% |
| **Period6** | $56 MM | $62 MM | $6 MM | 10.91% | $398 MM | $417 MM | $19 MM | 4.73% | $6 MM | $37,552,568 MM | 10.91% | 10.91% |
| **Period7** | $55 MM | $62 MM | $8 MM | 13.76% | $453 MM | $479 MM | $26 MM | 5.82% | $8 MM | $57,058,521 MM | 13.76% | 13.76% |
| **Period8** | $56 MM | $65 MM | $9 MM | 16.90% | $509 MM | $545 MM | $36 MM | 7.04% | $9 MM | $89,027,205 MM | 16.90% | 16.90% |
| **Period9** | $53 MM | $62 MM | $8 MM | 15.31% | $562 MM | $606 MM | $44 MM | 7.82% | $8 MM | $66,916,082 MM | 15.31% | 15.31% |
| | | **AERR** | $5 MM | | | | | **Total** | $47 MM | $331,931,161 MM | | |
| **CERRPCT** | **7.82%** | | **MAD** | $5 MM | **RMSE** | $6 MM | **COV** | **9.72%** | | Max | 16.90% | 16.90% |
| | | | | | | | | | | Mean | 8.47% | 8.90% |
| | | | | | | | | | | Min | -1.93% | 0.01% |

Figure 83: Detailed Back-testing Results – GCL (Recent 9Q - 10%)




In the recent period, the model is slightly over predicting in the later performance quarters. However, the error rates are within the thresholds prescribed by MRM. Predicted GCL follows the trend of actual GCL.

Table 8.3.36: Detailed Back-testing Results – GCL (Stress 13Q - 10%)

| Stress Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | $143 MM | $145 MM | $1 MM | 1.00% | $143 MM | $145 MM | $1 MM | 1.00% | $1 MM | $2,059,654 MM | 1.00% | 1.00% |
| **Period2** | $159 MM | $156 MM | -$3 MM | -1.91% | $302 MM | $301 MM | -$2 MM | -0.53% | $3 MM | $9,246,046 MM | -1.91% | 1.91% |
| **Period3** | $167 MM | $161 MM | -$6 MM | -3.60% | $469 MM | $461 MM | -$8 MM | -1.62% | $6 MM | $36,048,889 MM | -3.60% | 3.60% |
| **Period4** | $176 MM | $179 MM | $4 MM | 2.00% | $644 MM | $640 MM | -$4 MM | -0.64% | $4 MM | $12,330,033 MM | 2.00% | 2.00% |
| **Period5** | $211 MM | $213 MM | $2 MM | 1.06% | $855 MM | $853 MM | -$2 MM | -0.22% | $2 MM | $4,963,297 MM | 1.06% | 1.06% |
| **Period6** | $243 MM | $244 MM | $1 MM | 0.62% | $1,098 MM | $1,098 MM | $ MM | -0.03% | $1 MM | $2,249,797 MM | 0.62% | 0.62% |
| **Period7** | $247 MM | $238 MM | -$8 MM | -3.32% | $1,345 MM | $1,336 MM | -$9 MM | -0.64% | $8 MM | $66,863,354 MM | -3.32% | 3.32% |
| **Period8** | $233 MM | $221 MM | -$11 MM | -4.91% | $1,577 MM | $1,557 MM | -$20 MM | -1.27% | $11 MM | $130,429,300 MM | -4.91% | 4.91% |
| **Period9** | $244 MM | $219 MM | -$25 MM | -10.16% | $1,821 MM | $1,777 MM | -$45 MM | -2.46% | $25 MM | $615,494,545 MM | -10.16% | 10.16% |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Period10** | $236 MM | $192 MM | -$44 MM | -18.67% | $2,057 MM | $1,969 MM | -$89 MM | -4.32% | $44 MM | $1,939,671,289 MM | -18.67% | 18.67% |
| **Period11** | $213 MM | $187 MM | -$26 MM | -12.39% | $2,271 MM | $2,155 MM | -$115 MM | -5.07% | $26 MM | $697,119,974 MM | -12.39% | 12.39% |
| **Period12** | $187 MM | $166 MM | -$20 MM | -10.95% | $2,457 MM | $2,322 MM | -$136 MM | -5.52% | $20 MM | $419,382,724 MM | -10.95% | 10.95% |
| **Period13** | $156 MM | $132 MM | -$24 MM | -15.53% | $2,614 MM | $2,454 MM | -$160 MM | -6.12% | $24 MM | $590,632,985 MM | -15.53% | 15.53% |
| | | AERR | -$12 MM | | | | | **Total** | $177 MM | $4,526,491,886 MM | | |
| **CERRPCT** | **-6.12%** | | **MAD** | $20 MM | **RMSE** | $22 MM | **COV** | **11.15%** | | Max | 2.00% | 18.67% |
| | | | | | | | | | | Mean | -5.91% | 6.62% |
| | | | | | | | | | | Min | -18.67% | 0.62% |

The model is slightly under-predicting in the later quarters of stress period, but within the acceptable thresholds.

Table 8.3.37: Detailed Back-testing Results – GCL (Recent 13Q - 10%)

| Recent Back Tests (10% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | $75 MM | $77 MM | $2 MM | 3.10% | $75 MM | $77 MM | $2 MM | 3.10% | $2 MM | $5,410,025 MM | 3.10% | 3.10% |
| **Period2** | $70 MM | $69 MM | -$1 MM | -1.93% | $145 MM | $146 MM | $1 MM | 0.67% | $1 MM | $1,822,940 MM | -1.93% | 1.93% |
| **Period3** | $67 MM | $67 MM | $ MM | -0.01% | $212 MM | $213 MM | $1 MM | 0.46% | $ MM | $53 MM | -0.01% | 0.01% |
| **Period4** | $66 MM | $71 MM | $4 MM | 6.32% | $278 MM | $284 MM | $5 MM | 1.85% | $4 MM | $17,607,741 MM | 6.32% | 6.32% |
| **Period5** | $63 MM | $71 MM | $8 MM | 11.86% | $342 MM | $355 MM | $13 MM | 3.71% | $8 MM | $56,536,026 MM | 11.86 % | 11.86% |
| **Period6** | $56 MM | $62 MM | $6 MM | 10.91% | $398 MM | $417 MM | $19 MM | 4.73% | $6 MM | $37,552,568 MM | 10.91 % | 10.91% |
| **Period7** | $55 MM | $62 MM | $8 MM | 13.76% | $453 MM | $479 MM | $26 MM | 5.82% | $8 MM | $57,058,521 MM | 13.76 % | 13.76% |
| **Period8** | $56 MM | $65 MM | $9 MM | 16.90% | $509 MM | $545 MM | $36 MM | 7.04% | $9 MM | $89,027,205 MM | 16.90 % | 16.90% |
| **Period9** | $53 MM | $62 MM | $8 MM | 15.31% | $562 MM | $606 MM | $44 MM | 7.82% | $8 MM | $66,916,082 MM | 15.31 % | 15.31% |
| **Period10** | $48 MM | $54 MM | $6 MM | 12.53% | $610 MM | $660 MM | $50 MM | 8.19% | $6 MM | $36,276,522 MM | 12.53 % | 12.53% |
| **Period11** | $48 MM | $54 MM | $5 MM | 11.37% | $658 MM | $714 MM | $55 MM | 8.43% | $5 MM | $29,940,114 MM | 11.37 % | 11.37% |
| **Period12** | $50 MM | $55 MM | $5 MM | 10.33% | $708 MM | $769 MM | $61 MM | 8.56% | $5 MM | $26,227,376 MM | 10.33 % | 10.33% |
| **Period13** | $49 MM | $52 MM | $3 MM | 5.34% | $757 MM | $820 MM | $63 MM | 8.35% | $3 MM | $6,904,662 MM | 5.34% | 5.34% |
| | | AERR | $5 MM | | | | | **Total** | $66 MM | $431,279,835 MM | | |
| **CERRPCT** | **8.35%** | | **MAD** | $7 MM | **RMSE** | $7 MM | **COV** | **11.88%** | | Max | 16.90 % | 16.90% |
| | | | | | | | | | | Mean | 8.91% | 9.20% |
| | | | | | | | | | | Min | -1.93% | 0.01% |

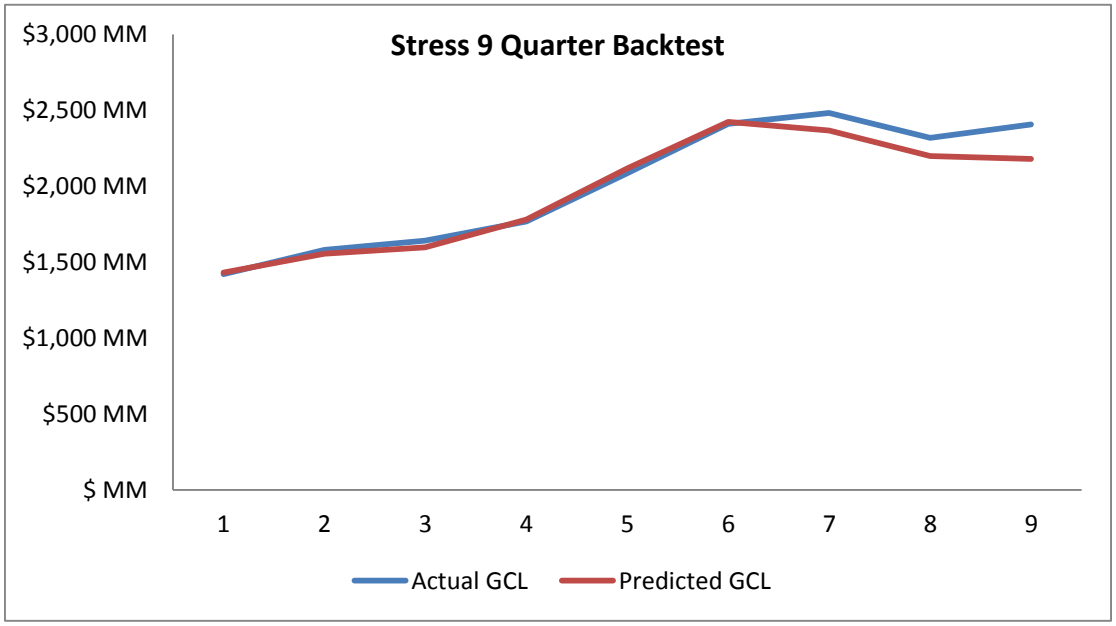
Predicted GCL is slightly higher than the actual GCL in 13Q Back Test of 100% recent sample.

Table 8.3.38: Detailed Back-testing Results – GCL (Stress 9Q - 100%)

| Stress Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | $1,421 MM | $1,431 MM | $10 MM | 0.71% | $1,421 MM | $1,431 MM | $10 MM | 0.71% | $10 MM | $102,635,850 MM | 0.71 % | 0.71% |
| **Period2** | $1,580 MM | $1,555 MM | -$25 MM | -1.60% | $3,001 MM | $2,986 MM | -$15 MM | -0.50% | $25 MM | $637,610,776 MM | -1.60 % | 1.60% |
| **Period3** | $1,640 MM | $1,598 MM | -$42 MM | -2.59% | $4,641 MM | $4,584 MM | -$58 MM | -1.24% | $42 MM | $1,801,557,855 MM | -2.59 % | 2.59% |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Period4** | $1,767 MM | $1,779 MM | $12 MM | 0.68% | $6,409 MM | $6,363 MM | -$46 MM | -0.71% | $12 MM | $144,536,033 MM | 0.68 % | 0.68% |
| **Period5** | $2,085 MM | $2,116 MM | $31 MM | 1.49% | $8,493 MM | $8,479 MM | -$14 MM | -0.17% | $31 MM | $967,787,489 MM | 1.49 % | 1.49% |
| **Period6** | $2,410 MM | $2,423 MM | $13 MM | 0.54% | $10,904 MM | $10,902 MM | -$2 MM | -0.01% | $13 MM | $167,155,905 MM | 0.54 % | 0.54% |
| **Period7** | $2,482 MM | $2,367 MM | -$116 MM | -4.66% | $13,386 MM | $13,269 MM | -$117 MM | -0.88% | $116 MM | $13,400,057,665 MM | -4.66 % | 4.66% |
| **Period8** | $2,319 MM | $2,198 MM | -$121 MM | -5.22% | $15,704 MM | $15,466 MM | -$238 MM | -1.52% | $121 MM | $14,656,629,186 MM | -5.22 % | 5.22% |
| **Period9** | $2,405 MM | $2,180 MM | -$225 MM | -9.37% | $18,110 MM | $17,646 MM | -$464 MM | -2.56% | $225 MM | $50,756,652,724 MM | -9.37 % | 9.37% |
| | | **AERR** | -$52 MM | | | | | **Total** | $596 MM | $82,634,623,483 MM | | |
| **CERRPCT** | **-2.56%** | | **MAD** | $66 MM | **RMSE** | $96 MM | **COV** | **4.76%** | | Max | 1.49 % | 9.37% |
| | | | | | | | | | | Mean | -2.22 % | 2.98% |
| | | | | | | | | | | Min | -9.37 % | 0.54% |

Figure 84: Detailed Back-testing Results – GCL (Stress 9Q - 100%)

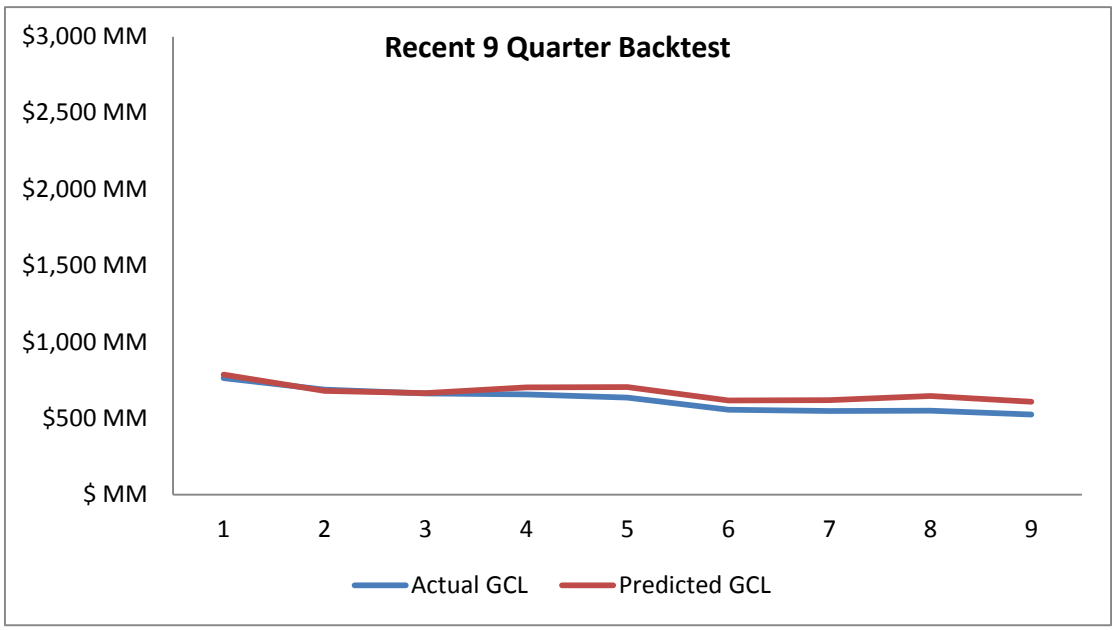



Actual and predicted GCL are close in 9Q Back Test results of 100% stress data.

Table 8.3.39: Detailed Back-testing Results – GCL (Recent 9Q - 100%)

| Recent Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | $764 MM | $787 MM | $23 MM | 2.95% | $764 MM | $787 MM | $23 MM | 2.95% | $23 MM | $506,647,404 MM | 2.95% | 2.95% |
| **Period2** | $689 MM | $681 MM | -$8 MM | -1.23% | $1,454 MM | $1,468 MM | $14 MM | 0.97% | $8 MM | $71,802,392 MM | -1.23% | 1.23% |
| **Period3** | $663 MM | $666 MM | $3 MM | 0.46% | $2,116 MM | $2,133 MM | $17 MM | 0.81% | $3 MM | $9,106,960 MM | 0.46% | 0.46% |
| **Period4** | $657 MM | $704 MM | $46 MM | 7.07% | $2,774 MM | $2,837 MM | $64 MM | 2.29% | $46 MM | $2,160,136,314 MM | 7.07% | 7.07% |
| **Period5** | $636 MM | $706 MM | $70 MM | 10.95% | $3,409 MM | $3,543 MM | $133 MM | 3.91% | $70 MM | $4,849,354,030 MM | 10.95% | 10.95% |
| **Period6** | $556 MM | $618 MM | $62 MM | 11.20% | $3,966 MM | $4,161 MM | $195 MM | 4.93% | $62 MM | $3,879,484,045 MM | 11.20% | 11.20% |
| **Period7** | $548 MM | $619 MM | $72 MM | 13.11% | $4,513 MM | $4,780 MM | $267 MM | 5.92% | $72 MM | $5,152,628,267 MM | 13.11% | 13.11% |
| **Period8** | $550 MM | $646 MM | $96 MM | 17.39% | $5,063 MM | $5,426 MM | $363 MM | 7.17% | $96 MM | $9,155,807,597 MM | 17.39% | 17.39% |
| **Period9** | $525 MM | $609 MM | $84 MM | 16.00% | $5,588 MM | $6,035 MM | $447 MM | 8.00% | $84 MM | $7,046,771,170 MM | 16.00% | 16.00% |
| | | **AERR** | $50 MM | | | | | **Total** | $464 MM | $32,831,738,179 MM | | |
| **CERRPCT** | **8.00%** | | **MAD** | $52 MM | **RMSE** | $60 MM | **COV** | **9.73%** | | Max | 17.39% | 17.39% |
| | | | | | | | | | | Mean | 8.65% | 8.93% |
| | | | | | | | | | | Min | -1.23% | 0.46% |

Figure 85: Detailed Back-testing Results – GCL (Recent 9Q - 100%)

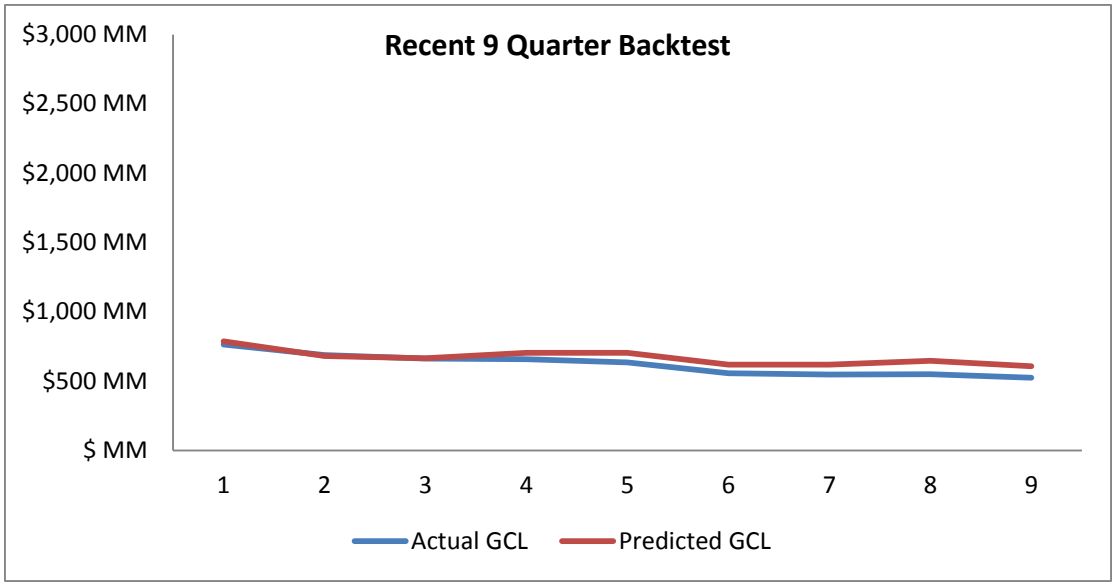


Over prediction is observed in the later quarters of 100% recent data. The quarterly and cumulative error rates are within the thresholds prescribed by MRM.

**Table 8.3.40: Detailed Back-testing Results – GCL (Most Recent/OOT 9Q - 100%)**

| Most recent back-test/OOT (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | $592 MM | $571 MM | -$20 MM | -3.45% | $592 MM | $571 MM | -$20 MM | -3.45% | $20 MM | $417,207,291 MM | -3.45% | 3.45% |
| **Period2** | $599 MM | $568 MM | -$31 MM | -5.23% | $1,191 MM | $1,139 MM | -$52 MM | -4.35% | $31 MM | $981,631,116 MM | -5.23% | 5.23% |
| **Period3** | $615 MM | $630 MM | $14 MM | 2.32% | $1,806 MM | $1,769 MM | -$37 MM | -2.08% | $14 MM | $203,455,743 MM | 2.32% | 2.32% |
| **Period4** | $594 MM | $648 MM | $53 MM | 9.00% | $2,401 MM | $2,417 MM | $16 MM | 0.67% | $53 MM | $2,860,713,783 MM | 9.00% | 9.00% |
| **Period5** | $552 MM | $586 MM | $34 MM | 6.14% | $2,953 MM | $3,003 MM | $50 MM | 1.69% | $34 MM | $1,148,862,679 MM | 6.14% | 6.14% |
| **Period6** | $563 MM | $590 MM | $27 MM | 4.81% | $3,516 MM | $3,592 MM | $77 MM | 2.19% | $27 MM | $731,388,011 MM | 4.81% | 4.81% |
| **Period7** | $581 MM | $617 MM | $35 MM | 6.06% | $4,097 MM | $4,209 MM | $112 MM | 2.74% | $35 MM | $1,241,856,744 MM | 6.06% | 6.06% |
| **Period8** | $573 MM | $604 MM | $31 MM | 5.36% | $4,670 MM | $4,813 MM | $143 MM | 3.06% | $31 MM | $941,266,254 MM | 5.36% | 5.36% |
| **Period9** | $531 MM | $504 MM | -$27 MM | -5.17% | $5,201 MM | $5,316 MM | $115 MM | 2.22% | $27 MM | $755,637,866 MM | -5.17% | 5.17% |
| | | **AERR** | $13 MM | | | | | **Total** | $274 MM | $9,282,019,486 MM | | |
| **CERRPCT** | **2.22%** | | **MAD** | $30 MM | **RMSE** | $32 MM | **COV** | **5.56%** | | Max | 9.00% | 9.00% |
| | | | | | | | | | | Mean | 2.20% | 5.28% |
| | | | | | | | | | | Min | -5.23% | 2.32% |

Figure 86: Detailed Back-testing Results – GCL (Most Recent/OOT 9Q - 100%)




The model is slightly over-predicting in the most recent 9Q of 100% data. The cumulative error rate is 2.22%.

Table 8.3.41: Detailed Back-testing Results – GCL (Stress 13Q - 100%)

| Stress Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| Period1 | $1,421 MM | $1,431 MM | $10 MM | 0.71% | $1,421 MM | $1,431 MM | $10 MM | 0.71% | $10 MM | $102,635,850 MM | 0.71% | 0.71% |
| Period2 | $1,580 MM | $1,555 MM | -$25 MM | -1.60% | $3,001 MM | $2,986 MM | -$15 MM | -0.50% | $25 MM | $637,610,776 MM | -1.60% | 1.60% |
| Period3 | $1,640 MM | $1,598 MM | -$42 MM | -2.59% | $4,641 MM | $4,584 MM | -$58 MM | -1.24% | $42 MM | $1,801,557,855 MM | -2.59% | 2.59% |
| Period4 | $1,767 MM | $1,779 MM | $12 MM | 0.68% | $6,409 MM | $6,363 MM | -$46 MM | -0.71% | $12 MM | $144,536,033 MM | 0.68% | 0.68% |
| Period5 | $2,085 MM | $2,116 MM | $31 MM | 1.49% | $8,493 MM | $8,479 MM | -$14 MM | -0.17% | $31 MM | $967,787,489 MM | 1.49% | 1.49% |
| Period6 | $2,410 MM | $2,423 MM | $13 MM | 0.54% | $10,904 MM | $10,902 MM | -$2 MM | -0.01% | $13 MM | $167,155,905 MM | 0.54% | 0.54% |
| Period7 | $2,482 MM | $2,367 MM | -$116 MM | -4.66% | $13,386 MM | $13,269 MM | -$117 MM | -0.88% | $116 MM | $13,400,057,665 MM | -4.66% | 4.66% |
| Period8 | $2,319 MM | $2,198 MM | -$121 MM | -5.22% | $15,704 MM | $15,466 MM | -$238 MM | -1.52% | $121 MM | $14,656,629,186 MM | -5.22% | 5.22% |
| Period9 | $2,405 MM | $2,180 MM | -$225 MM | -9.37% | $18,110 MM | $17,646 MM | -$464 MM | -2.56% | $225 MM | $50,756,652,724 MM | -9.37% | 9.37% |
| Period10 | $2,335 MM | $1,906 MM | -$429 MM | -18.38% | $20,445 MM | $19,552 MM | -$893 MM | -4.37% | $429 MM | $184,234,938,673 MM | -18.38% | 18.38% |
| Period11 | $2,108 MM | $1,862 MM | -$246 MM | -11.69% | $22,553 MM | $21,414 MM | -$1,139 MM | -5.05% | $246 MM | $60,729,425,651 MM | -11.69% | 11.69% |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Period12** | $1,838 MM | $1,661 MM | -$177 MM | -9.64% | $24,391 MM | $23,075 MM | -$1,317 MM | -5.40% | $177 MM | $31,440,024,621 MM | -9.64% | 9.64% |
| **Period13** | $1,559 MM | $1,320 MM | -$239 MM | -15.33% | $25,951 MM | $24,395 MM | -$1,556 MM | -5.99% | $239 MM | $57,127,963,351 MM | -15.33% | 15.33% |
| | | AERR | -$120 MM | | | | | **Total** | $1,688 MM | $416,166,975,778 MM | | |
| **CERRPCT** | **-5.99%** | | **MAD** | $188 MM | **RMSE** | $215 MM | **COV** | **10.77%** | | Max | 1.49% | 18.38% |
| | | | | | | | | | | Mean | -5.77% | 6.30% |
| | | | | | | | | | | Min | -18.38% | 0.54% |

Actual and predicted GCL are close in13Q Back Test of 100% stress data. Slight under-prediction is observed in the later quarters. However, the quarterly and cumulative error rates are within the MRM thresholds.

**Table 8.3.42: Detailed Back-testing Results – GCL (Recent 13Q - 100%)**

| Recent Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| **Quarter** | **Actuals A(i)** | **Model Output P(i)** | **ERR(i) = P(i) - A(i)** | **ERRPCT(i) = ERR(i) / A(i)** | **CA(n)** | **CP(n)** | **CERR(n) = CP(n) - CA(n)** | **CERRPCT(n) = CERR(n) / CA(n)** | **Abs(ERR(i))** | **(ERR(i))^2** | **Error** | **Abs ERROR** |
| **Period1** | $764 MM | $787 MM | $23 MM | 2.95% | $764 MM | $787 MM | $23 MM | 2.95% | $23 MM | $506,647,404 MM | 2.95% | 2.95% |
| **Period2** | $689 MM | $681 MM | -$8 MM | -1.23% | $1,454 MM | $1,468 MM | $14 MM | 0.97% | $8 MM | $71,802,392 MM | -1.23% | 1.23% |
| **Period3** | $663 MM | $666 MM | $3 MM | 0.46% | $2,116 MM | $2,133 MM | $17 MM | 0.81% | $3 MM | $9,106,960 MM | 0.46% | 0.46% |
| **Period4** | $657 MM | $704 MM | $46 MM | 7.07% | $2,774 MM | $2,837 MM | $64 MM | 2.29% | $46 MM | $2,160,136,314 MM | 7.07% | 7.07% |
| **Period5** | $636 MM | $706 MM | $70 MM | 10.95% | $3,409 MM | $3,543 MM | $133 MM | 3.91% | $70 MM | $4,849,354,030 MM | 10.95% | 10.95% |
| **Period6** | $556 MM | $618 MM | $62 MM | 11.20% | $3,966 MM | $4,161 MM | $195 MM | 4.93% | $62 MM | $3,879,484,045 MM | 11.20% | 11.20% |
| **Period7** | $548 MM | $619 MM | $72 MM | 13.11% | $4,513 MM | $4,780 MM | $267 MM | 5.92% | $72 MM | $5,152,628,267 MM | 13.11% | 13.11% |
| **Period8** | $550 MM | $646 MM | $96 MM | 17.39% | $5,063 MM | $5,426 MM | $363 MM | 7.17% | $96 MM | $9,155,807,597 MM | 17.39% | 17.39% |
| **Period9** | $525 MM | $609 MM | $84 MM | 16.00% | $5,588 MM | $6,035 MM | $447 MM | 8.00% | $84 MM | $7,046,771,170 MM | 16.00% | 16.00% |
| **Period10** | $483 MM | $534 MM | $51 MM | 10.61% | $6,071 MM | $6,569 MM | $498 MM | 8.20% | $51 MM | $2,624,018,305 MM | 10.61% | 10.61% |
| **Period11** | $484 MM | $529 MM | $45 MM | 9.19% | $6,555 MM | $7,098 MM | $543 MM | 8.28% | $45 MM | $1,980,660,409 MM | 9.19% | 9.19% |
| **Period12** | $495 MM | $539 MM | $44 MM | 8.92% | $7,050 MM | $7,637 MM | $587 MM | 8.32% | $44 MM | $1,950,751,970 MM | 8.92% | 8.92% |
| **Period13** | $489 MM | $511 MM | $22 MM | 4.56% | $7,539 MM | $8,148 MM | $609 MM | 8.08% | $22 MM | $497,415,533 MM | 4.56% | 4.56% |
| | | AERR | $47 MM | | | | | **Total** | $626 MM | $39,884,584,396 MM | | |
| **CERRPCT** | **8.08%** | | **MAD** | $70 MM | **RMSE** | $67 MM | **COV** | **11.48%** | | Max | 17.39% | 17.39% |
| | | | | | | | | | | Mean | 8.55% | 8.74% |
| | | | | | | | | | | Min | -1.23% | 0.46% |

Predicted GCL follows the trend of actual GCL. Over-prediction is observed in the 13Q Back Test of 100% recent sample. The cumulative error rate is within the MRM threshold of +/- 10%.

Below sheet contains the GCL Back Test results on 10% and 100% data on stress, recent and most recent samples.

8.3.2_backtest_results_GCL.xlsx

## Recovery back-tests

Table 8.3.43: Detailed Back-testing Results – Recovery (Stress 9Q - 100%)

| Stress Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| Period 1 | $101 MM | $89 MM | -12328860.45 | -12.21% | $101 MM | $89 MM | -$12 MM | -12.21% | 12328860.45 | 152000799995574.00 | -12.21% | 12.21% |
| Period 2 | $101 MM | $92 MM | -9501480.90 | -9.38% | $202 MM | $181 MM | -$22 MM | -10.79% | 9501480.90 | 90278139293064.90 | -9.38% | 9.38% |
| Period 3 | $97 MM | $90 MM | -6623838.88 | -6.83% | $299 MM | $271 MM | -$28 MM | -9.51% | 6623838.88 | 43875241508199.80 | -6.83% | 6.83% |
| Period 4 | $89 MM | $97 MM | 8416525.54 | 9.45% | $388 MM | $368 MM | -$20 MM | -5.16% | 8416525.54 | 70837902165472.20 | 9.45% | 9.45% |
| Period 5 | $98 MM | $101 MM | 3383637.24 | 3.46% | $486 MM | $470 MM | -$17 MM | -3.43% | 3383637.24 | 11449000971914.80 | 3.46% | 3.46% |
| Period 6 | $101 MM | $100 MM | -873946.89 | -0.87% | $587 MM | $569 MM | -$18 MM | -2.99% | 873946.89 | 763783166540.67 | -0.87% | 0.87% |
| Period 7 | $107 MM | $97 MM | -10492830.67 | -9.77% | $694 MM | $666 MM | -$28 MM | -4.04% | 10492830.67 | 110099495469293.00 | -9.77% | 9.77% |
| Period 8 | $113 MM | $110 MM | -2600128.67 | -2.30% | $807 MM | $776 MM | -$31 MM | -3.79% | 2600128.67 | 6760669100555.98 | -2.30% | 2.30% |
| Period 9 | $131 MM | $133 MM | 1601106.45 | 1.22% | $938 MM | $909 MM | -$29 MM | -3.09% | 1601106.45 | 2563541864231.56 | 1.22% | 1.22% |
| | | AERR | -3224424 | | | | | Total | 55822356 | 488628573534847 | | |
| CERRPCT | -3.09% | | MAD | $6 MM | RMSE | $7 MM | COV | 7.07% | | Max | 9.45% | 12.21% |
| | | | | | | | | | | Mean | -3.03% | 6.17% |
| | | | | | | | | | | Min | -12.21% | 0.87% |

Figure 87: Detailed Back-testing Results – Recovery (Stress 9Q - 100%)

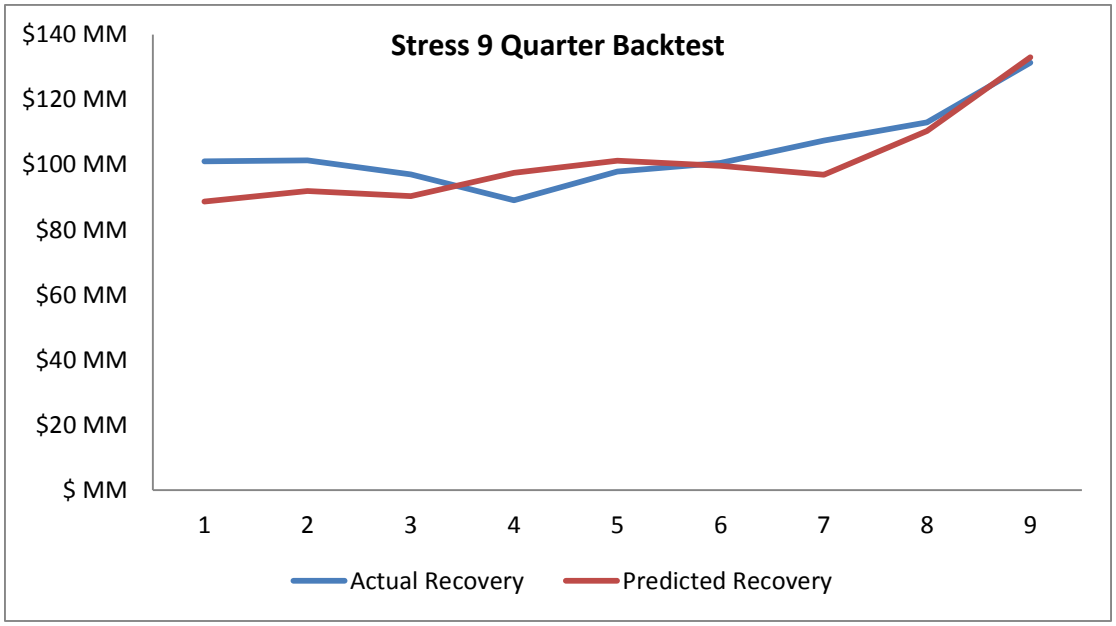
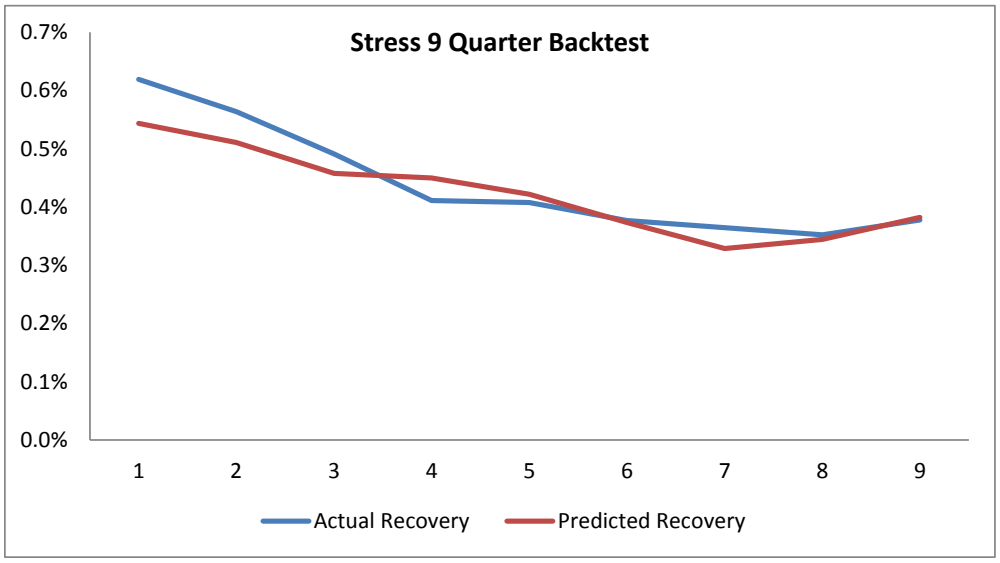



Figure 88: Detailed Back-testing Results – Marginal Recovery Rate (Stress 9Q - 100%)




Actual and predicted recovery dollar amounts and recovery rates are close in the 100% 9Q stress which implies high model accuracy.

The recovery rate in stress is higher than recent and most recent periods because the model has been developed using historical GCL from 2005Q1 charge-off quarter. So very few data

points were available for stress vintage. However, the actuals and predicted are aligned across the performance quarters. The lack of data points is not a limitation since this is a weighted recovery rate.

**Table 8.3.44: Detailed Back-testing Results – Recovery (Recent 9Q - 100%)**

| Recent Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period 1** | $116 MM | $116 MM | 681887.21 | 0.59% | $116 MM | $116 MM | $1 MM | 0.59% | 681887.21 | 464970167161.60 | 0.59% | 0.59% |
| **Period 2** | $113 MM | $116 MM | 3227454.39 | 2.86% | $229 MM | $233 MM | $4 MM | 1.71% | 3227454.39 | 10416461839530.30 | 2.86% | 2.86% |
| **Period 3** | $103 MM | $108 MM | 4939896.87 | 4.77% | $332 MM | $341 MM | $9 MM | 2.66% | 4939896.87 | 24402581086235.80 | 4.77% | 4.77% |
| **Period 4** | $105 MM | $100 MM | -5013309.27 | -4.79% | $437 MM | $441 MM | $4 MM | 0.88% | 5013309.27 | 25133269836668.00 | -4.79% | 4.79% |
| **Period 5** | $98 MM | $104 MM | 6071056.81 | 6.20% | $535 MM | $545 MM | $10 MM | 1.85% | 6071056.81 | 36857730790247.40 | 6.20% | 6.20% |
| **Period 6** | $93 MM | $89 MM | -4180332.27 | -4.49% | $628 MM | $634 MM | $6 MM | 0.91% | 4180332.27 | 17475177887603.30 | -4.49% | 4.49% |
| **Period 7** | $91 MM | $84 MM | -6840244.23 | -7.55% | $719 MM | $718 MM | -$1 MM | -0.15% | 6840244.23 | 46788941126048.40 | -7.55% | 7.55% |
| **Period 8** | $92 MM | $84 MM | -8115416.50 | -8.81% | $811 MM | $802 MM | -$9 MM | -1.14% | 8115416.50 | 65859984968472.20 | -8.81% | 8.81% |
| **Period 9** | $84 MM | $83 MM | -1512917.07 | -1.80% | $895 MM | $884 MM | -$11 MM | -1.20% | 1512917.07 | 2288918060697.36 | -1.80% | 1.80% |
| | | **AERR** | -1193547 | | | | | **Total** | 40582515 | 229688035762664 | | |
| **CERRPCT** | **-1.20%** | | **MAD** | $5 MM | **RMSE** | $5 MM | **COV** | **5.08%** | | Max | 6.20% | 8.81% |
| | | | | | | | | | | Mean | -1.45% | 4.65% |
| | | | | | | | | | | Min | -8.81% | 0.59% |

Figure 89: Detailed Back-testing Results – Recovery (Recent 9Q - 100%)

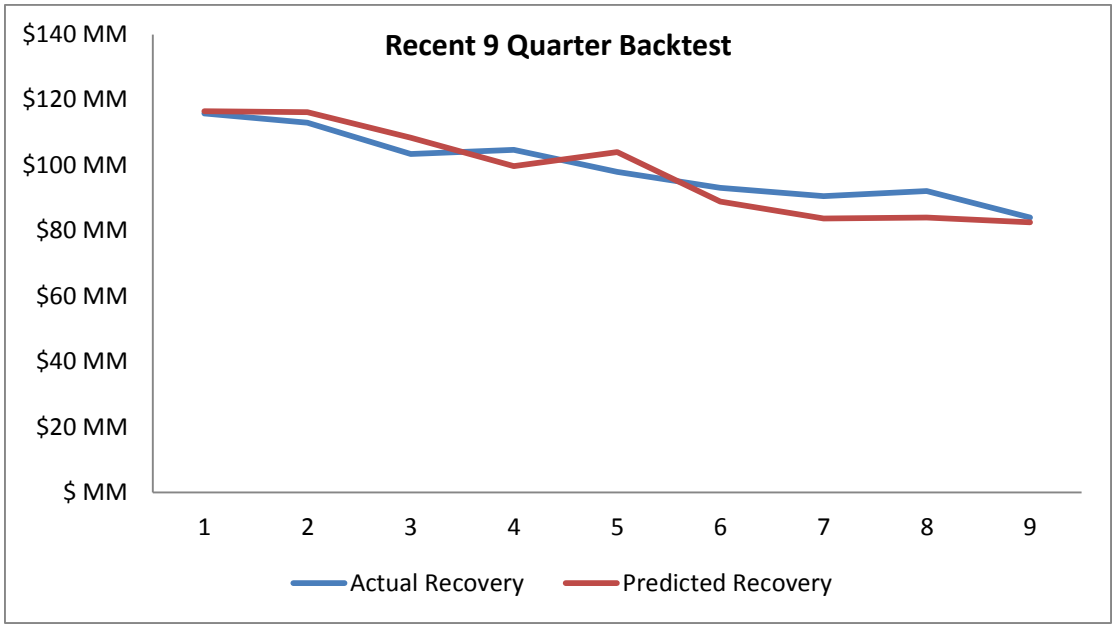
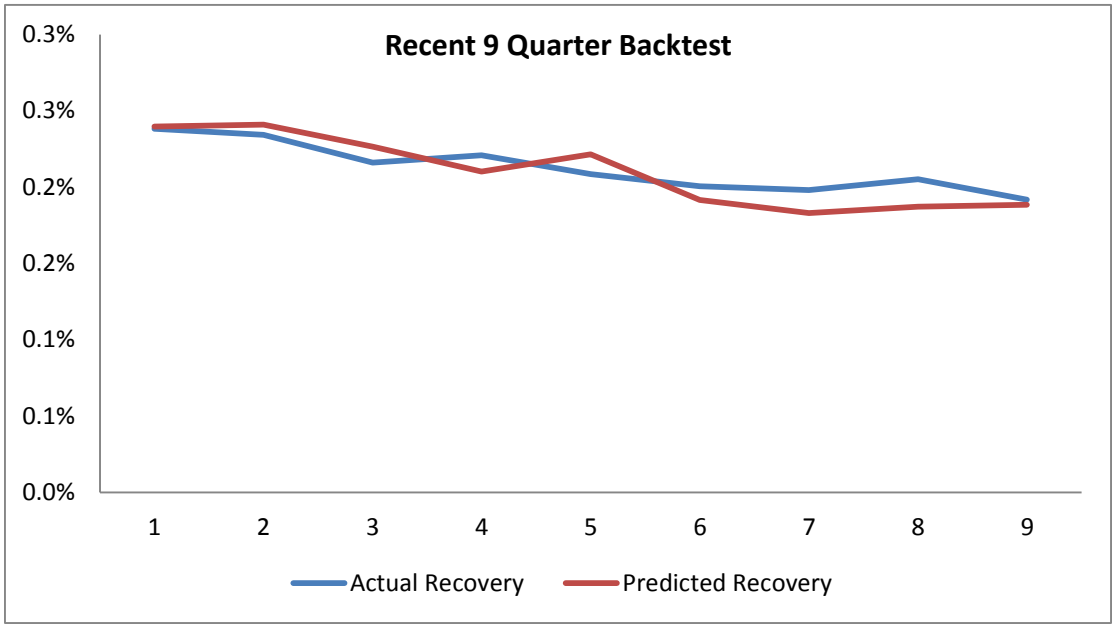



Figure 90: Detailed Back-testing Results – Marginal Recovery Rate (Recent 9Q - 100%)




Model is performing well in the 100% 9Q recent Back Test with cumulative error rate within +/- 5%.

Table 8.3.45: Detailed Back-testing Results – Recovery (Most Recent/OOT 9Q - 100%)

| Most recent back-test/OOT (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period 1** | $93 MM | $89 MM | -4180332.27 | -4.49% | $93 MM | $89 MM | -$4 MM | -4.49% | 4180332.27 | 17475177887603.30 | -4.49% | 4.49% |
| **Period 2** | $91 MM | $84 MM | -6840244.23 | -7.55% | $184 MM | $173 MM | -$11 MM | -6.00% | 6840244.23 | 46788941126048.40 | -7.55% | 7.55% |
| **Period 3** | $92 MM | $84 MM | -8115416.50 | -8.81% | $276 MM | $257 MM | -$19 MM | -6.94% | 8115416.50 | 65859984968472.20 | -8.81% | 8.81% |
| **Period 4** | $84 MM | $83 MM | -1512917.07 | -1.80% | $360 MM | $339 MM | -$21 MM | -5.74% | 1512917.07 | 2288918060697.36 | -1.80% | 1.80% |
| **Period 5** | $80 MM | $77 MM | -3073212.94 | -3.85% | $440 MM | $416 MM | -$24 MM | -5.40% | 3073212.94 | 9444637774583.43 | -3.85% | 3.85% |
| **Period 6** | $73 MM | $74 MM | 1074514.13 | 1.48% | $512 MM | $490 MM | -$23 MM | -4.42% | 1074514.13 | 1154580615569.65 | 1.48% | 1.48% |
| **Period 7** | $76 MM | $75 MM | -655561.84 | -0.87% | $588 MM | $565 MM | -$23 MM | -3.96% | 655561.84 | 429761326064.19 | -0.87% | 0.87% |
| **Period 8** | $68 MM | $73 MM | 4749558.03 | 6.98% | $656 MM | $637 MM | -$19 MM | -2.83% | 4749558.03 | 22558301480337.50 | 6.98% | 6.98% |
| **Period 9** | $60 MM | $71 MM | 11426399.89 | 19.20% | $716 MM | $708 MM | -$7 MM | -1.00% | 11426399.89 | 130562614446192.00 | 19.20% | 19.20% |
| | | **AERR** | -791913 | | | | | **Total** | 41628157 | 296562917685568 | | |
| **CERRPCT** | **-1.00%** | | **MAD** | $5 MM | **RMSE** | $6 MM | **COV** | **7.22%** | | Max | 19.20% | 19.20% |
| | | | | | | | | | | Mean | 0.03% | 6.11% |
| | | | | | | | | | | Min | -8.81% | 0.87% |

Figure 91: Detailed Back-testing Results – Recovery (Most Recent/OOT 9Q - 100%)

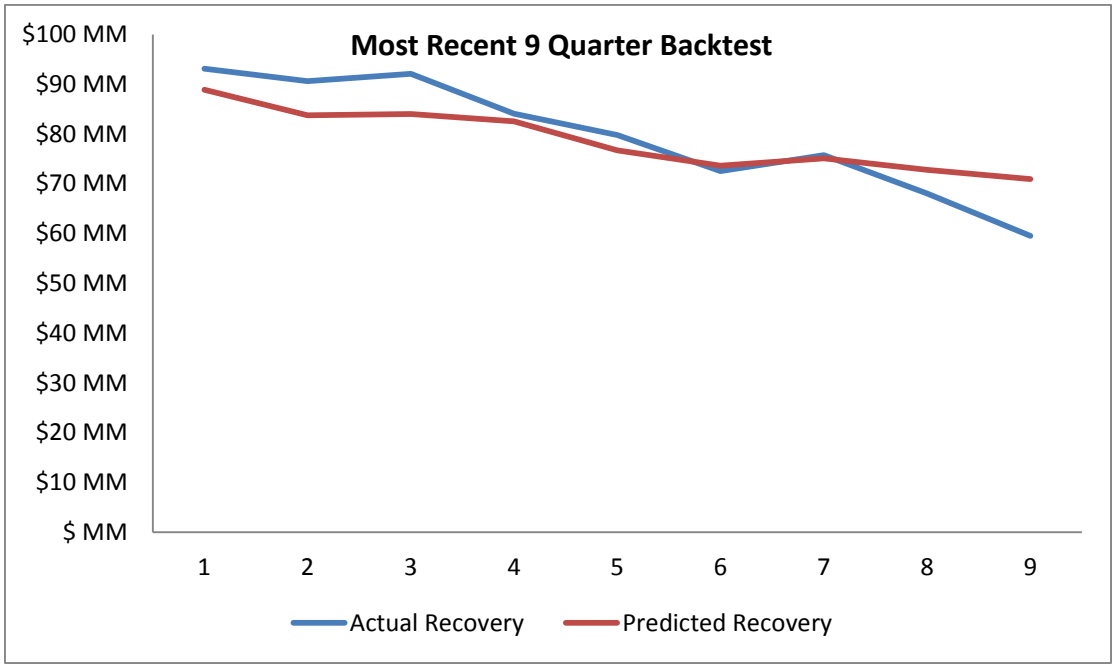
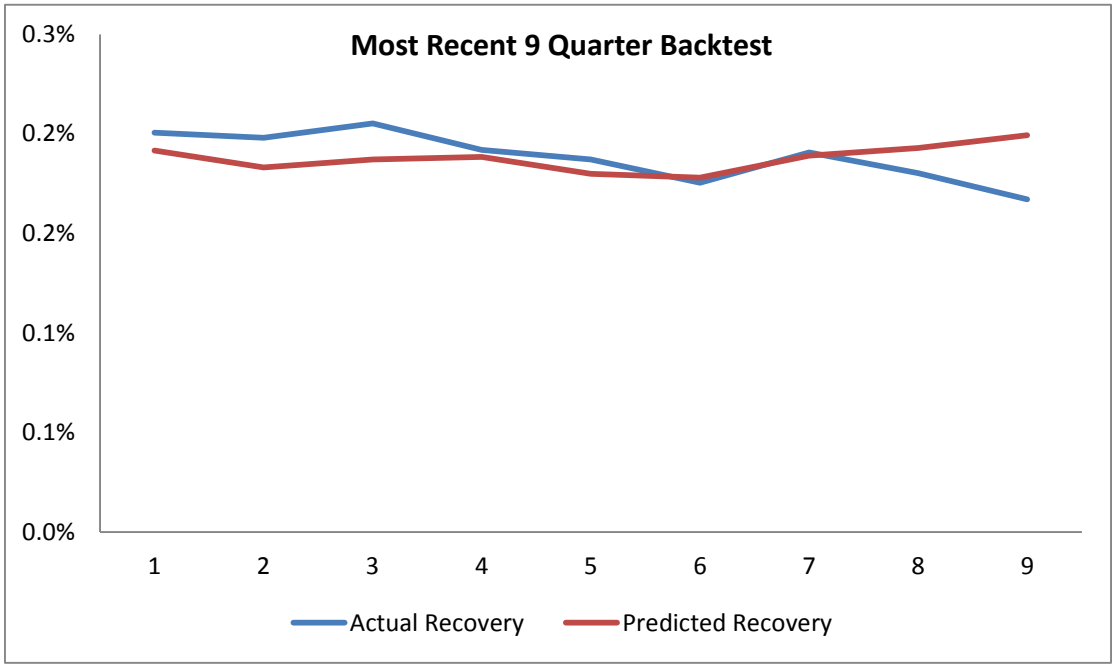



Figure 92: Detailed Back-testing Results – Marginal Recovery Rate (Most Recent/OOT 9Q - 100%)

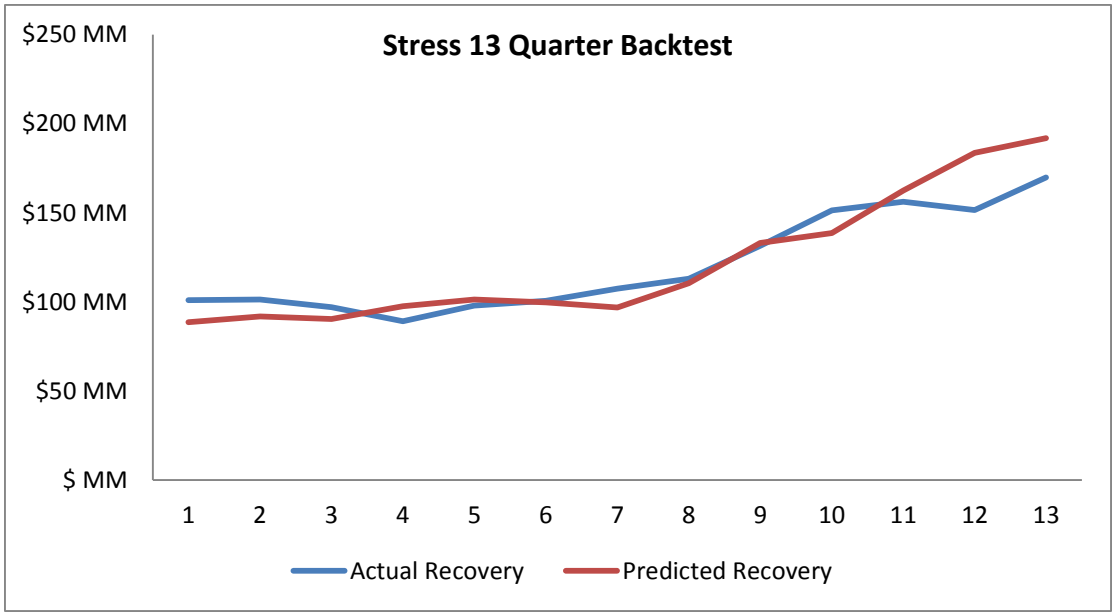


Actual and predicted recovery dollar amounts are close in the 100% 9Q of the most recent sample Back Test which also includes the OOT vintages. This indicates that the model is performance s good in the most recent period.

**Table 8.3.46: Detailed Back-testing Results – Recovery (Stress 13Q - 100%)**

| Stress Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | $101 MM | $89 MM | -$12 MM | -12.21% | $101 MM | $89 MM | -$12 MM | -12.21% | 12328860 | 152000799995574 | -12.21% | 12.21% |
| **Period2** | $101 MM | $92 MM | -$10 MM | -9.38% | $202 MM | $181 MM | -$22 MM | -10.79% | 9501481 | 90278139293065 | -9.38% | 9.38% |
| **Period3** | $97 MM | $90 MM | -$7 MM | -6.83% | $299 MM | $271 MM | -$28 MM | -9.51% | 6623839 | 43875241508200 | -6.83% | 6.83% |
| **Period4** | $89 MM | $97 MM | $8 MM | 9.45% | $388 MM | $368 MM | -$20 MM | -5.16% | 8416526 | 70837902165472 | 9.45% | 9.45% |
| **Period5** | $98 MM | $101 MM | $3 MM | 3.46% | $486 MM | $470 MM | -$17 MM | -3.43% | 3383637 | 11449000971915 | 3.46% | 3.46% |
| **Period6** | $101 MM | $100 MM | -$1 MM | -0.87% | $587 MM | $569 MM | -$18 MM | -2.99% | 873947 | 763783166541 | -0.87% | 0.87% |
| **Period7** | $107 MM | $97 MM | -$10 MM | -9.77% | $694 MM | $666 MM | -$28 MM | -4.04% | 10492831 | 110099495469293 | -9.77% | 9.77% |
| **Period8** | $113 MM | $110 MM | -$3 MM | -2.30% | $807 MM | $776 MM | -$31 MM | -3.79% | 2600129 | 6760669100556 | -2.30% | 2.30% |
| **Period9** | $131 MM | $133 MM | $2 MM | 1.22% | $938 MM | $909 MM | -$29 MM | -3.09% | 1601106 | 2563541864232 | 1.22% | 1.22% |
| **Period10** | $151 MM | $139 MM | -$13 MM | -8.51% | $1,090 MM | $1,048 MM | -$42 MM | -3.84% | 12879896 | 165891732047527 | -8.51% | 8.51% |
| **Period11** | $156 MM | $162 MM | $6 MM | 4.00% | $1,246 MM | $1,210 MM | -$36 MM | -2.86% | 6246469 | 39018372219515 | 4.00% | 4.00% |
| **Period12** | $151 MM | $184 MM | $32 MM | 21.25% | $1,398 MM | $1,394 MM | -$3 MM | -0.25% | 32183727 | 1035792315150580 | 21.25% | 21.25% |
| **Period13** | $170 MM | $192 MM | $22 MM | 12.94% | $1,567 MM | $1,586 MM | $19 MM | 1.18% | 21978475 | 483053379589697 | 12.94% | 12.94% |
| | | AERR | 1423766.00 | | | | | **Total** | 129110924 | 2212384372542170 | | |
| **CERRPCT** | **1.18%** | | **MAD** | $14 MM | **RMSE** | $16 MM | **COV** | **13.00%** | | Max | 21.25% | 21.25% |
| | | | | | | | | | | Mean | 0.19% | 7.86% |
| | | | | | | | | | | Min | -12.21% | 0.87% |

Figure 93: Detailed Back-testing Results – Recovery (Stress 13Q - 100%)



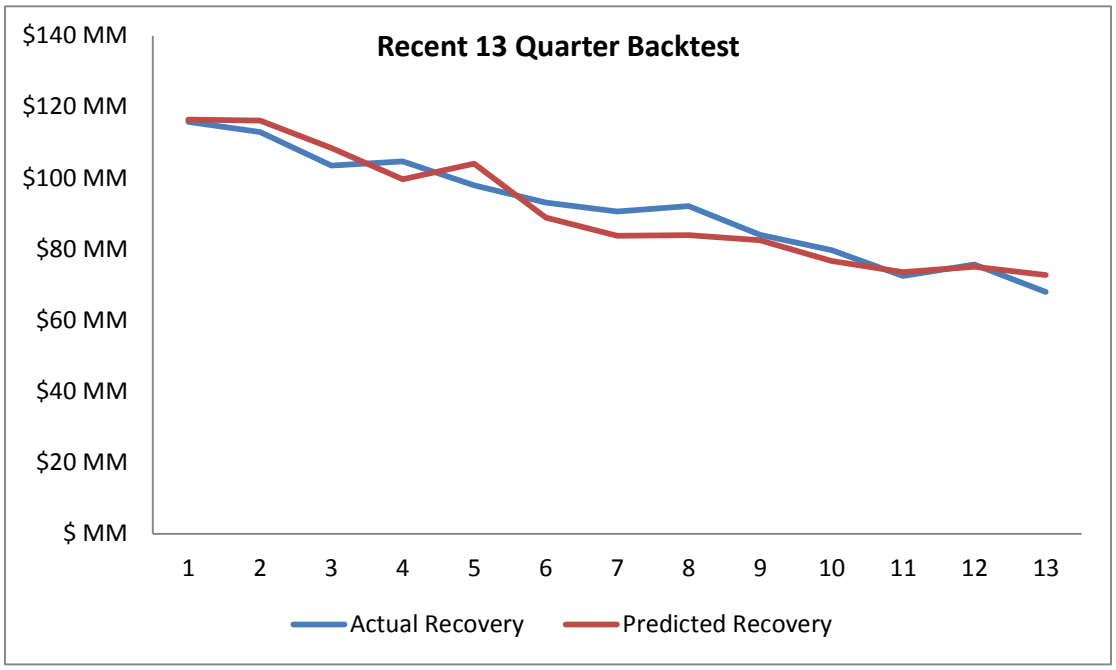

Actual and predicted recovery dollar amounts are close in the 100% 13Q stress Back Test. The last 4 quarters have slightly lower predicted recoveries, but the error rates are very low.

Table 8.3.47: Detailed Back-testing Results – Recovery (Recent 13Q - 100%)

| Recent Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | $116 MM | $116 MM | $1 MM | 0.59% | $116 MM | $116 MM | $1 MM | 0.59% | 681887 | 464970167162 | 0.59 % | 0.59% |
| **Period2** | $113 MM | $116 MM | $3 MM | 2.86% | $229 MM | $233 MM | $4 MM | 1.71% | 3227454 | 10416461839530 | 2.86 % | 2.86% |
| **Period3** | $103 MM | $108 MM | $5 MM | 4.77% | $332 MM | $341 MM | $9 MM | 2.66% | 4939897 | 24402581086236 | 4.77 % | 4.77% |
| **Period4** | $105 MM | $100 MM | -$5 MM | -4.79% | $437 MM | $441 MM | $4 MM | 0.88% | 5013309 | 25133269836668 | -4.79 % | 4.79% |
| **Period5** | $98 MM | $104 MM | $6 MM | 6.20% | $535 MM | $545 MM | $10 MM | 1.85% | 6071057 | 36857730790247 | 6.20 % | 6.20% |
| **Period6** | $93 MM | $89 MM | -$4 MM | -4.49% | $628 MM | $634 MM | $6 MM | 0.91% | 4180332 | 17475177887603 | -4.49 % | 4.49% |
| **Period7** | $91 MM | $84 MM | -$7 MM | -7.55% | $719 MM | $718 MM | -$1 MM | -0.15% | 6840244 | 46788941126048 | -7.55 % | 7.55% |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Period8** | $92 MM | $84 MM | -$8 MM | -8.81% | $811 MM | $802 MM | -$9 MM | -1.14% | 8115417 | 6585998496847 2 | -8.81 % | 8.81% |
| **Period9** | $84 MM | $83 MM | -$2 MM | -1.80% | $895 MM | $884 MM | -$11 MM | -1.20% | 1512917 | 2288918060697 | -1.80 % | 1.80% |
| **Period1 0** | $80 MM | $77 MM | -$3 MM | -3.85% | $975 MM | $961 MM | -$14 MM | -1.42% | 3073213 | 9444637774583 | -3.85 % | 3.85% |
| **Period1 1** | $73 MM | $74 MM | $1 MM | 1.48% | $1,04 7 MM | $1,034 MM | -$13 MM | -1.22% | 1074514 | 1154580615570 | 1.48 % | 1.48% |
| **Period1 2** | $76 MM | $75 MM | -$1 MM | -0.87% | $1,12 3 MM | $1,110 MM | -$13 MM | -1.19% | 655562 | 429761326064 | -0.87 % | 0.87% |
| **Period1 3** | $68 MM | $73 MM | $5 MM | 6.98% | $1,19 1 MM | $1,182 MM | -$9 MM | -0.73% | 4749558 | 2255830148033 8 | 6.98 % | 6.98% |
| | | AERR | -66512 5 | | | | | **Total** | 50135362 | 2632753169592 19 | | |
| **CERRPC T** | **-0.73%** | | **MAD** | $6 MM | **RMSE** | $5 MM | **COV** | **5.90%** | | Max | 6.98 % | 8.81% |
| | | | | | | | | | | Mean | -0.71 % | 4.23% |
| | | | | | | | | | | Min | -8.81 % | 0.59% |

Figure 94: Detailed Back-testing Results – Recovery (Recent 13Q - 100%)




Model is performing well in the 100% 13Q recent Back Test with slight over prediction in later performance quarters. The cumulative error rate is within +/-5% indicating high model accuracy.

Below sheet contains the back testing details for recovery model.

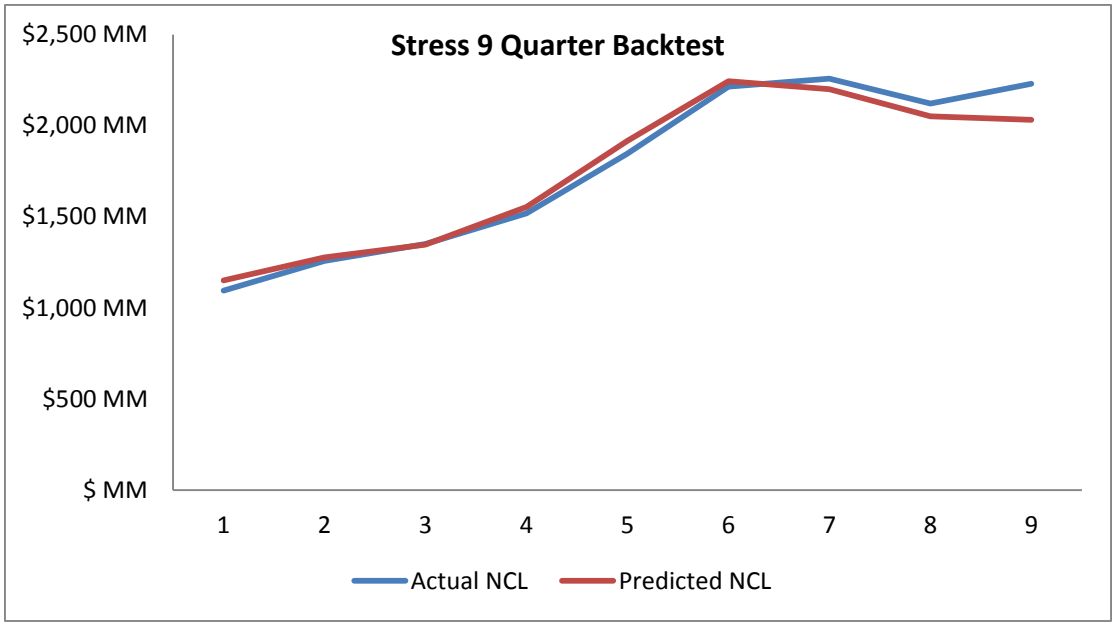
8.3.2_Backtest_Results Recovery_v2.xls

## NCL Back Test Results:

Table 8.3.48: Detailed Back-testing Results – NCL (Stress 9Q - 100%)

| Stress Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | $1,094 MM | $1,150 MM | $55 MM | 5.07% | $1,094 MM | $1,150 MM | $55 MM | 5.07% | $55 MM | 3075350621751260.00 | 5.07% | 5.07% |
| **Period2** | $1,258 MM | $1,277 MM | $19 MM | 1.51% | $2,352 MM | $2,427 MM | $74 MM | 3.16% | $19 MM | 358835588075702.00 | 1.51% | 1.51% |
| **Period3** | $1,351 MM | $1,346 MM | -$4 MM | -0.32% | $3,703 MM | $3,773 MM | $70 MM | 1.89% | $4 MM | 18950640307426.40 | -0.32% | 0.32% |
| **Period4** | $1,518 MM | $1,553 MM | $36 MM | 2.37% | $5,221 MM | $5,327 MM | $106 MM | 2.03% | $36 MM | 1293184557886770.00 | 2.37% | 2.37% |
| **Period5** | $1,846 MM | $1,916 MM | $70 MM | 3.78% | $7,067 MM | $7,243 MM | $176 MM | 2.49% | $70 MM | 4865978916026220.00 | 3.78% | 3.78% |
| **Period6** | $2,214 MM | $2,243 MM | $28 MM | 1.28% | $9,281 MM | $9,486 MM | $204 MM | 2.20% | $28 MM | 803110903793006.00 | 1.28% | 1.28% |
| **Period7** | $2,257 MM | $2,200 MM | -$57 MM | -2.54% | $11,539 MM | $11,685 MM | $147 MM | 1.27% | $57 MM | 3290607460340050.00 | -2.54% | 2.54% |
| **Period8** | $2,122 MM | $2,051 MM | -$71 MM | -3.32% | $13,660 MM | $13,737 MM | $76 MM | 0.56% | $71 MM | 4971856598179620.00 | -3.32% | 3.32% |
| **Period9** | $2,229 MM | $2,031 MM | -$198 MM | -8.88% | $15,889 MM | $15,767 MM | -$122 MM | -0.77% | $198 MM | 39171922847103700.00 | -8.88% | 8.88% |
| | | **AERR** | -$14 MM | | | | | **Total** | $539 MM | 57849798133463700 | | |
| **CERRPCT** | **-0.77%** | | **MAD** | $60 MM | **RMSE** | $80 MM | **COV** | **4.54%** | | Max | 5.07% | 8.88% |
| | | | | | | | | | | Mean | -0.12% | 3.23% |
| | | | | | | | | | | Min | -8.88% | 0.32% |

Figure 95: Detailed Back-testing Results – NCL (Stress 9Q - 100%)

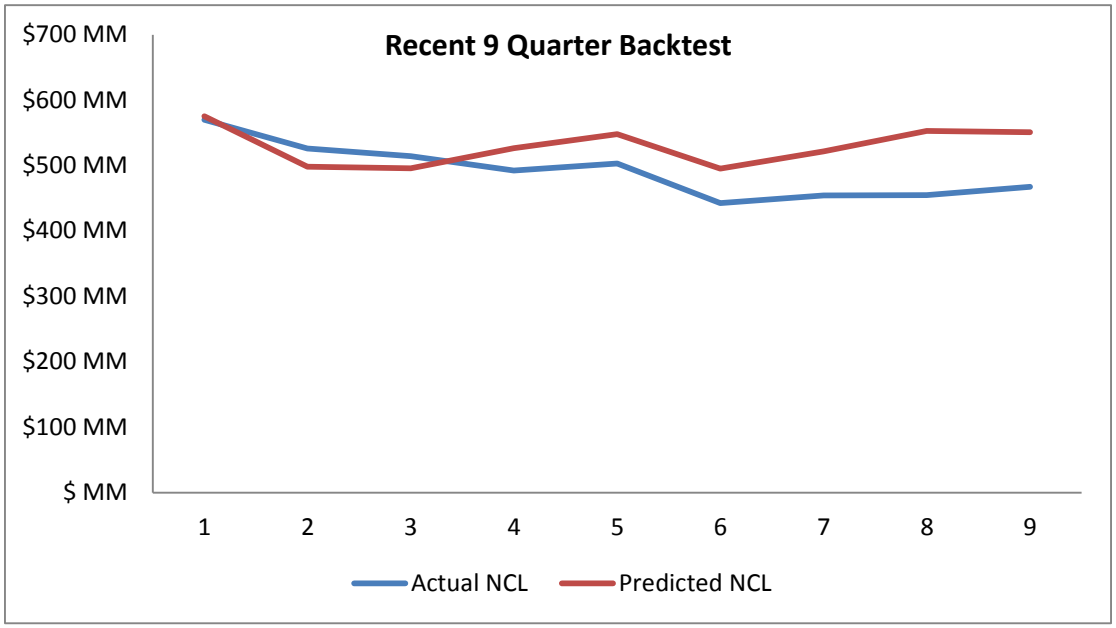



Actual and predicted NCL are very close in 9Q Stress period which implies high accuracy of the model.

Table 8.3.49: Detailed Back-testing Results – NCL (Recent 9Q - 100%)

| Recent Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period 1** | $570 MM | $575 MM | $5 MM | 0.92% | $570 MM | $575 MM | $5 MM | 0.92% | $5 MM | 27371942839869.80 | 0.92% | 0.92% |
| **Period 2** | $526 MM | $498 MM | -$28 MM | -5.24% | $1,096 MM | $1,074 MM | -$22 MM | -2.03% | $28 MM | 758158808079719.00 | -5.24% | 5.24% |
| **Period 3** | $514 MM | $496 MM | -$19 MM | -3.64% | $1,611 MM | $1,570 MM | -$41 MM | -2.55% | $19 MM | 350303451908087.00 | -3.64% | 3.64% |
| **Period 4** | $492 MM | $526 MM | $34 MM | 6.95% | $2,103 MM | $2,096 MM | -$7 MM | -0.32% | $34 MM | 1168751755109690.00 | 6.95% | 6.95% |
| **Period 5** | $503 MM | $548 MM | $45 MM | 8.90% | $2,606 MM | $2,644 MM | $38 MM | 1.46% | $45 MM | 2003665428342370.00 | 8.90% | 8.90% |
| **Period 6** | $443 MM | $495 MM | $53 MM | 11.86% | $3,049 MM | $3,139 MM | $90 MM | 2.97% | $53 MM | 2758904646588090.00 | 11.86% | 11.86% |
| **Period 7** | $454 MM | $522 MM | $68 MM | 14.86% | $3,503 MM | $3,661 MM | $158 MM | 4.51% | $68 MM | 4558732254659100.00 | 14.86% | 14.86% |
| **Period 8** | $455 MM | $553 MM | $98 MM | 21.47% | $3,958 MM | $4,214 MM | $256 MM | 6.46% | $98 MM | 9546698347171390.00 | 21.47% | 21.47% |
| **Period 9** | $467 MM | $551 MM | $83 MM | 17.81% | $4,426 MM | $4,764 MM | $339 MM | 7.66% | $83 MM | 6934952191240150.00 | 17.81% | 17.81% |
| | | **AERR** | $38 MM | | | | | **Total** | $431 MM | 28107538825938500 | | |
| **CERRPCT** | **7.66%** | | **MAD** | $48 MM | **RMSE** | $56 MM | **COV** | **11.36%** | | Max | 21.47% | 21.47% |
| | | | | | | | | | | Mean | 8.21% | 10.18% |
| | | | | | | | | | | Min | -5.24% | 0.92% |

Figure 96: Detailed Back-testing Results – NCL (Recent 9Q - 100%)

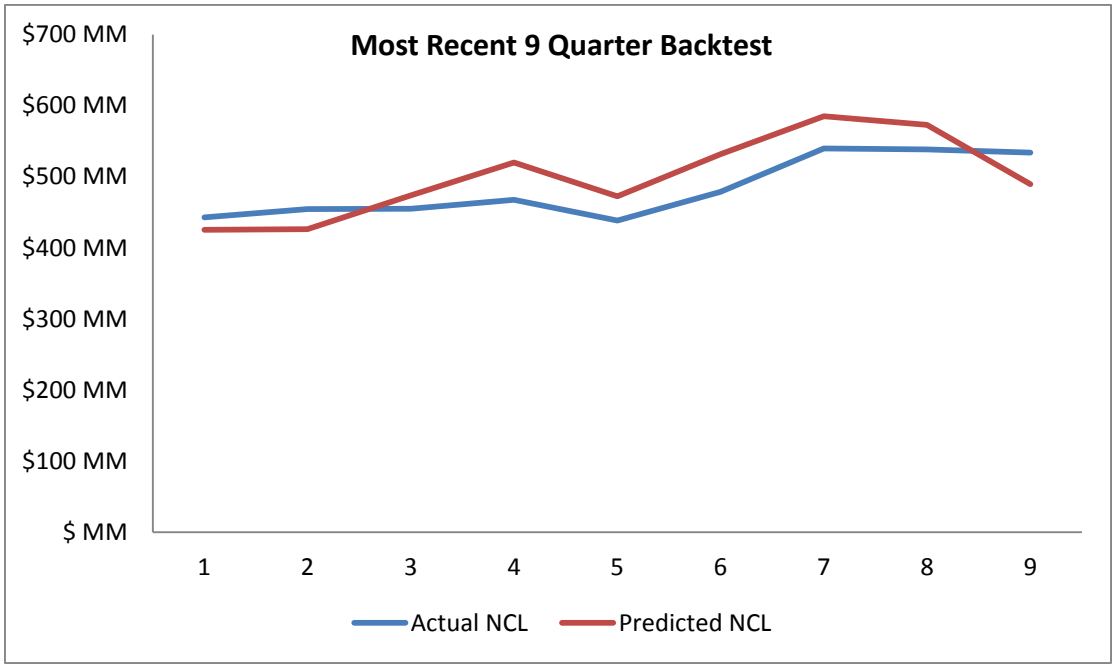


Over-prediction is observed in later quarters of stress period. The cumulative error rate is below the MRM threshold.

**Table 8.3.50: Detailed Back-testing Results – NCL (Most Recent/OOT 9Q - 100%)**

| Most recent back-test/OOT (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **7** | $443 MM | $425 MM | -$18 MM | -3.96% | $443 MM | $425 MM | -$18 MM | -3.96% | $18 MM | 308085479606502.00 | -3.96% | 3.96% |
| **Period 2** | $454 MM | $426 MM | -$28 MM | -6.19% | $897 MM | $851 MM | -$46 MM | -5.09% | $28 MM | 791439097470468.00 | -6.19% | 6.19% |
| **Period 3** | $455 MM | $474 MM | $19 MM | 4.07% | $1,352 MM | $1,325 MM | -$27 MM | -2.01% | $19 MM | 342569501751486.00 | 4.07% | 4.07% |
| **Period 4** | $467 MM | $520 MM | $52 MM | 11.20% | $1,820 MM | $1,845 MM | $25 MM | 1.38% | $52 MM | 2743320248834810.00 | 11.20% | 11.20% |
| **Period 5** | $438 MM | $472 MM | $34 MM | 7.77% | $2,258 MM | $2,317 MM | $59 MM | 2.62% | $34 MM | 1159014867368550.00 | 7.77% | 7.77% |
| **Period 6** | $479 MM | $531 MM | $53 MM | 10.97% | $2,737 MM | $2,848 MM | $112 MM | 4.08% | $53 MM | 2760865034278590.00 | 10.97% | 10.97% |
| **Period 7** | $540 MM | $585 MM | $45 MM | 8.40% | $3,276 MM | $3,433 MM | $157 MM | 4.79% | $45 MM | 2052690286911480.00 | 8.40% | 8.40% |
| **Period 8** | $538 MM | $573 MM | $35 MM | 6.45% | $3,814 MM | $4,006 MM | $192 MM | 5.03% | $35 MM | 1204934818973270.00 | 6.45% | 6.45% |
| **Period 9** | $534 MM | $489 MM | -$44 MM | -8.31% | $4,348 MM | $4,496 MM | $147 MM | 3.39% | $44 MM | 1965696308639940.00 | -8.31% | 8.31% |
| | | **AERR** | $16 MM | | | | | **Total** | $328 MM | 13328615643835100 | | |
| **CERRPCT** | **3.39%** | | MAD | $36 MM | **RMSE** | $38 MM | **COV** | **7.97%** | | Max | 11.20% | 11.20% |
| | | | | | | | | | | Mean | 3.38% | 7.48% |
| | | | | | | | | | | Min | -8.31% | 3.96% |

Figure 97: Detailed Back-testing Results – NCL (Most Recent/OOT 9Q - 100%)




Over-prediction is observed in the most recent period. The cumulative error rate is below the MRM threshold.

Table 8.3.51: Detailed Back-testing Results – NCL (Stress 13Q - 100%)

| Stress Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period 1** | $1,094 MM | $1,150 MM | $55 MM | 5.07% | $1,094 MM | $1,150 MM | $55 MM | 5.07% | $55 MM | 3075350621751260 | 5.07% | 5.07% |
| **Period 2** | $1,258 MM | $1,277 MM | $19 MM | 1.51% | $2,352 MM | $2,427 MM | $74 MM | 3.16% | $19 MM | 358835588075702 | 1.51% | 1.51% |
| **Period 3** | $1,351 MM | $1,346 MM | -$4 MM | -0.32% | $3,703 MM | $3,773 MM | $70 MM | 1.89% | $4 MM | 18950640307426 | -0.32% | 0.32% |
| **Period 4** | $1,518 MM | $1,553 MM | $36 MM | 2.37% | $5,221 MM | $5,327 MM | $106 MM | 2.03% | $36 MM | 1293184557886770 | 2.37% | 2.37% |
| **Period 5** | $1,846 MM | $1,916 MM | $70 MM | 3.78% | $7,067 MM | $7,243 MM | $176 MM | 2.49% | $70 MM | 4865978916026220 | 3.78% | 3.78% |
| **Period 6** | $2,214 MM | $2,243 MM | $28 MM | 1.28% | $9,281 MM | $9,486 MM | $204 MM | 2.20% | $28 MM | 803110903793006 | 1.28% | 1.28% |
| **Period 7** | $2,257 MM | $2,200 MM | -$57 MM | -2.54% | $11,539 MM | $11,685 MM | $147 MM | 1.27% | $57 MM | 3290607460340050 | -2.54% | 2.54% |
| **Period 8** | $2,122 MM | $2,051 MM | -$71 MM | -3.32% | $13,660 MM | $13,737 MM | $76 MM | 0.56% | $71 MM | 4971856598179620 | -3.32% | 3.32% |
| **Period 9** | $2,229 MM | $2,031 MM | -$198 MM | -8.88% | $15,889 MM | $15,767 MM | -$122 MM | -0.77% | $198 MM | 39171922847103700 | -8.88% | 8.88% |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Period 10** | $2,190 MM | $1,746 MM | -$445 MM | -20.30% | $18,079 MM | $17,513 MM | -$566 MM | -3.13% | $445 MM | 197697144393523000 | -20.30% | 20.30% |
| **Period 11** | $2,001 MM | $1,695 MM | -$306 MM | -15.27% | $20,081 MM | $19,209 MM | -$872 MM | -4.34% | $306 MM | 93435582187753300 | -15.27% | 15.27% |
| **Period 12** | $1,746 MM | $1,498 MM | -$248 MM | -14.23% | $21,827 MM | $20,706 MM | -$1,120 MM | -5.13% | $248 MM | 61695316872117600 | -14.23% | 14.23% |
| **Period 13** | $1,422 MM | $1,168 MM | -$254 MM | -17.87% | $23,249 MM | $21,874 MM | -$1,374 MM | -5.91% | $254 MM | 64576350327812800 | -17.87% | 17.87% |
| | | AERR | -$106 MM | | | | | **Total** | $1,791 MM | 475254191914670000 | | |
| **CERRPCT** | **-5.91%** | | **MAD** | $199 MM | **RMSE** | $230 MM | **COV** | **12.85%** | | Max | 5.07% | 20.30% |
| | | | | | | | | | | Mean | -5.29% | 7.44% |
| | | | | | | | | | | Min | -20.30% | 0.32% |

Under-prediction is observed in later quarters of stress period. However, the cumulative error rate is below the MRM threshold.

**Table 8.3.52: Detailed Back-testing Results – NCL (Recent 13Q - 100%)**

| Recent Back Tests (100% Sample) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| Quarter | Actuals A(i) | Model Output P(i) | ERR(i) = P(i) - A(i) | ERRPCT(i) = ERR(i) / A(i) | CA(n) | CP(n) | CERR(n) = CP(n) - CA(n) | CERRPCT(n) = CERR(n) / CA(n) | Abs(ERR(i)) | (ERR(i))^2 | Error | Abs ERROR |
| **Period1** | $570 MM | $575 MM | $5 MM | 0.92% | $570 MM | $575 MM | $5 MM | 0.92% | $5 MM | 27371942839870 | 0.92% | 0.92% |
| **Period2** | $526 MM | $498 MM | -$28 MM | -5.24% | $1,096 MM | $1,074 MM | -$22 MM | -2.03% | $28 MM | 758158808079719 | -5.24% | 5.24% |
| **Period3** | $514 MM | $496 MM | -$19 MM | -3.64% | $1,611 MM | $1,570 MM | -$41 MM | -2.55% | $19 MM | 350303451908087 | -3.64% | 3.64% |
| **Period4** | $492 MM | $526 MM | $34 MM | 6.95% | $2,103 MM | $2,096 MM | -$7 MM | -0.32% | $34 MM | 1168751755109690 | 6.95% | 6.95% |
| **Period5** | $503 MM | $548 MM | $45 MM | 8.90% | $2,606 MM | $2,644 MM | $38 MM | 1.46% | $45 MM | 2003665428342370 | 8.90% | 8.90% |
| **Period6** | $443 MM | $495 MM | $53 MM | 11.86% | $3,049 MM | $3,139 MM | $90 MM | 2.97% | $53 MM | 2758904646588090 | 11.86% | 11.86% |
| **Period7** | $454 MM | $522 MM | $68 MM | 14.86% | $3,503 MM | $3,661 MM | $158 MM | 4.51% | $68 MM | 4558732254659100 | 14.86% | 14.86% |
| **Period8** | $455 MM | $553 MM | $98 MM | 21.47% | $3,958 MM | $4,214 MM | $256 MM | 6.46% | $98 MM | 9546698347171390 | 21.47% | 21.47% |
| **Period9** | $467 MM | $551 MM | $83 MM | 17.81% | $4,426 MM | $4,764 MM | $339 MM | 7.66% | $83 MM | 6934952191240150 | 17.81% | 17.81% |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Period10** | $438 MM | $492 MM | $54 MM | 12.28% | $4,864 MM | $5,257 MM | $393 MM | 8.08% | $54 MM | 289666905866778 0 | 12.28% | 12.28% |
| **Period11** | $479 MM | $557 MM | $78 MM | 16.31% | $5,343 MM | $5,814 MM | $471 MM | 8.81% | $78 MM | 6101385875752390 | 16.31% | 16.31% |
| **Period12** | $540 MM | $601 MM | $61 MM | 11.36% | $5,882 MM | $6,414 MM | $532 MM | 9.05% | $61 MM | 3757619651478030 | 11.36% | 11.36% |
| **Period13** | $538 MM | $568 MM | $30 MM | 5.62% | $6,420 MM | $6,983 MM | $562 MM | 8.76% | $30 MM | 915277873535044 | 5.62% | 5.62% |
| | | AERR | $43 MM | | | | | **Total** | $655 MM | 41778491285371700 | | |
| **CERRPCT** | **8.76%** | | **MAD** | $73 MM | **RMSE** | $68 MM | **COV** | **13.80%** | | Max | 21.47% | 21.47% |
| | | | | | | | | | | Mean | 9.19% | 10.56% |
| | | | | | | | | | | Min | -5.24% | 0.92% |

Over-prediction is observed in later quarters. However, the cumulative error rate is below the MRM threshold.

Below sheet contains the Back Test results on NCL, GCL Rate and NCL Rate.

8.3.2_Backtest_Results NCL.xlsx

All errors of PD, EAD, Recovery, GCL and NCL are well below the prescribed cut-offs. The models are performing accurately across tress, recent and most recent sample.

The overall back testing results for PD, EAD, GCL, Recovery and NCL are available in following attached file:

Backtest_Template_Lead.xlsm | Backtest_Template_Recovery.xlsm | Backtest_Template_NCL.xlsm | Section8_BACKTEST_TEMPLATE_PD.xlsm | Section8_BACKTEST_TEMPLATE_GCL.xlsm

## 8.4 Model Sensitivity

### 8.4.1 Sensitivity of model results under baseline vs stress macro scenarios

- Have sensitivity tests been performed at various levels of the model - at the model component level (for e.g. PD, LGD, EAD), as well as final output (for e.g. NCL/balance)?

Yes, the sensitivity tests have been performed at model component level (PD, EAD, and Recovery) as well as at the model output level (GCL and NCL). All macroeconomic variables in final models are used for sensitivity analysis. These include Unemployment Rate (UR) and House Price Index (HPI).

The scenarios that were evaluated were the Supervisory Base, Adverse and Severe Adverse from the 2018 CCAR Submission as those gave the most realistic expected scenario for the future and the most stressed scenario. This was done using the Dec'2016 vintage.

The MCST Scenarios (base, adverse and severe adverse) were also analyzed using June'2017 vintage

- If the model output (for e.g. Net Credit Loss ($NCL)) based on cumulative percent change between severe stress and base runs less than threshold, provide justification and explain why you still consider the model to be sufficiently responsive to macro environment – is the path variation between severe stress and base scenario sufficient

As shown in subsequent sub sections below, the sensitivity of NCL is greater than 20%.

- If the dependent variable (for e.g. PD) and forecasted variable (for e.g. Net Credit Loss ($NCL)/Balance/Revenue) based on cumulative percent change between severely stress and base is less than threshold as per the Model Testing Guidance, provide justification and explain why you still consider the model to be sufficiently responsive to macro environment – is the model sufficiently responsive to macroeconomic stress?

As shown in table below, the sensitivity of NCL which is the final model output is more than the threshold of 20%. Sensitivity of individual models is also within their expected range.

**Table 8.4.1: Model Sensitivity - FRB Scenario**

| | PD | EAD | GCL ($MM) | Recovery ($MM) | NCL ($MM) |
|---|---|---|---|---|---|
| **Supervisory Base** | 5.79% | $5,788 | $8,428 MM | $886 MM | $7,737 MM |
| **Supervisory Adverse** | 7.37% | $6,043 | $11,197 MM | $970 MM | $10,762 MM |
| **Supervisory Severely Adverse** | 9.27% | $6,294 | $14,665 MM | $771 MM | $14,516 MM |
| **Adverse Stress %** | 27% | 4% | 33% | 9% | 39% |
| **Severely Adverse Stress %** | 60% | 9% | 74% | -13% | 88% |

**Table 8.4.2: Model Sensitivity - MCST Scenario**

| | PD | EAD | GCL ($MM) | Recovery ($MM) | NCL ($MM) |
|---|---|---|---|---|---|
| **Supervisory Base** | 5.43% | $5,745 | $7,925 MM | $878 MM | $7,213 MM |

| | | | | | |
|---|---|---|---|---|---|
| **Supervisory Adverse** | 6.83% | $6,016 | $10,439 MM | $847 MM | $10,039 MM |
| **Supervisory Severely Adverse** | 7.88% | $6,203 | $12,417 MM | $727 MM | $12,122 MM |
| **Adverse Stress %** | 26% | 5% | 32% | -4% | 39% |
| **Severely Adverse Stress %** | 45% | 8% | 57% | -17% | 68% |

- What are the sensitivity test results?

Branded Cards assessed the sensitivity of losses to macroeconomic variables as per guidelines from MRM. All macroeconomic variables in final model are used for sensitivity analysis. These include Unemployment Rate (UR) and House Price Index (HPI). Univariate approach is used for sensitivity analysis of each macroeconomic variable.

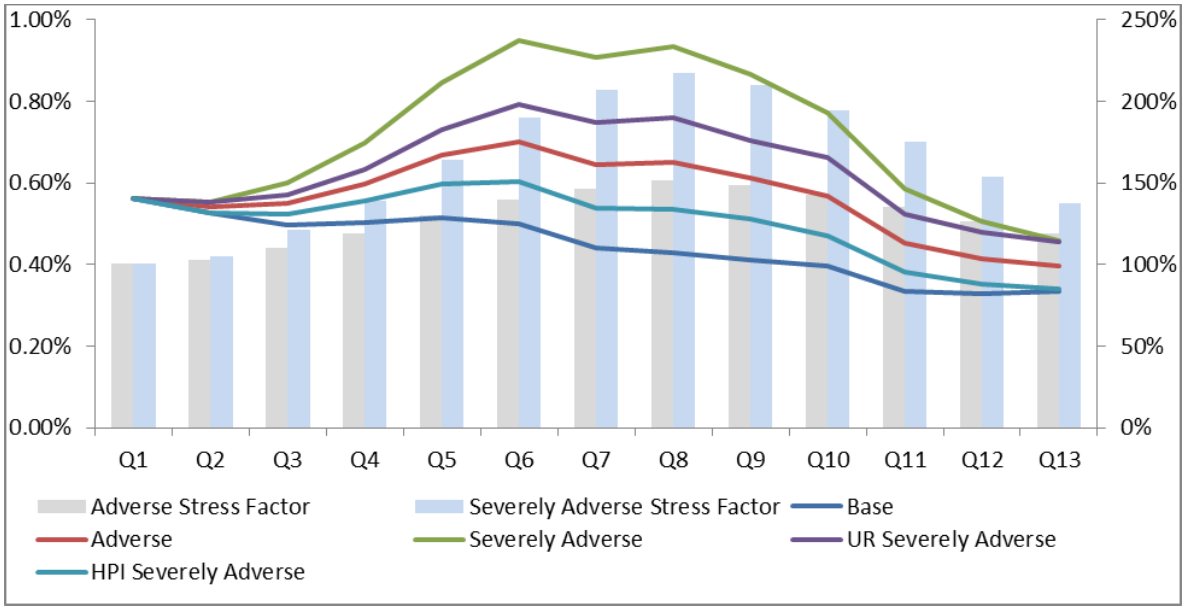
The stress for PD, EAD, GCL, Recovery and NCL for FRB and MCST scenarios is given below:

*PD Sensitivity:*

**Table 8.4.3: PD rates for FRB Scenarios (Jan'17-Mar'20)**

| | PD Rates Stress FRB Scenarios | | | | |
|---|---|---|---|---|---|
| | **Base** | **Adverse** | **Severely Adverse** | **UR Severely Adverse** | **HPI Severely Adverse** |
| Q1 | 0.56% | 0.56% | 0.56% | 0.56% | 0.56% |
| Q2 | 0.53% | 0.54% | 0.55% | 0.55% | 0.53% |
| Q3 | 0.50% | 0.55% | 0.60% | 0.57% | 0.52% |
| Q4 | 0.50% | 0.60% | 0.70% | 0.63% | 0.55% |
| Q5 | 0.51% | 0.67% | 0.84% | 0.73% | 0.60% |
| Q6 | 0.50% | 0.70% | 0.95% | 0.79% | 0.60% |
| Q7 | 0.44% | 0.64% | 0.91% | 0.75% | 0.54% |
| Q8 | 0.43% | 0.65% | 0.94% | 0.76% | 0.53% |
| Q9 | 0.41% | 0.61% | 0.87% | 0.70% | 0.51% |
| Q10 | 0.40% | 0.57% | 0.78% | 0.67% | 0.47% |
| Q11 | 0.34% | 0.46% | 0.59% | 0.53% | 0.38% |
| Q12 | 0.33% | 0.42% | 0.51% | 0.48% | 0.36% |
| Q13 | 0.34% | 0.40% | 0.46% | 0.46% | 0.34% |
| **%PD (9Q)** | 4.38% | 5.52% | 6.92% | 6.05% | 4.95% |
| **%Sensitivity (9Q)** | | 26% | 58% | 38% | 13% |
| **%PD (13Q)** | 5.79% | 7.37% | 9.27% | 8.19% | 6.50% |
| **%Sensitivity (13Q)** | | 27% | 60% | 41% | 12% |

Figure 98: PD Sensitivity for Dec'16 under different FRB Scenarios




As can be observed from the above charts:

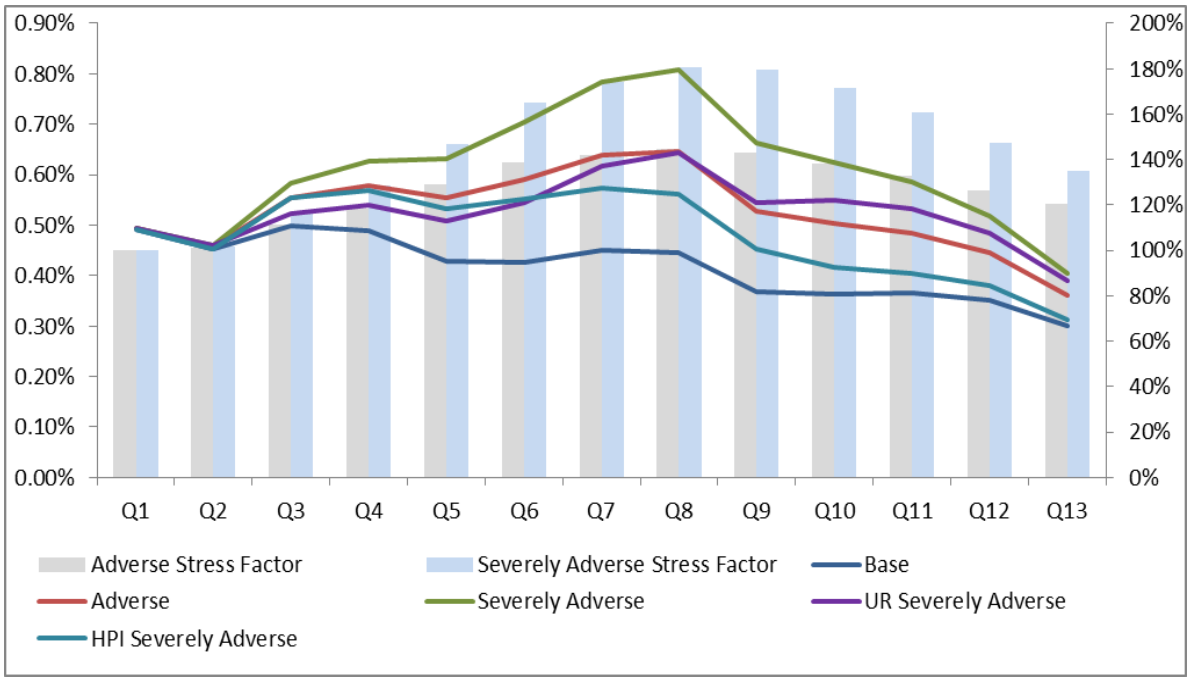
- Under FRB Severely Adverse Scenario PD shows a stress of 60% in 13Q
- Under FRB Adverse Scenario PD shows a stress of 27% in 13Q

Table 8.4.4: PD rates for MCST Scenarios (Jul'17-Sep'20)

| | PD Rates Stress MCST Scenarios | | | | |
|---|---|---|---|---|---|
| | Base | Adverse | Severely Adverse | UR Severely Adverse | HPI Severely Adverse |
| Q1 | 0.49% | 0.49% | 0.49% | 0.49% | 0.49% |
| Q2 | 0.45% | 0.46% | 0.46% | 0.46% | 0.45% |
| Q3 | 0.50% | 0.56% | 0.58% | 0.52% | 0.56% |
| Q4 | 0.49% | 0.58% | 0.63% | 0.54% | 0.57% |
| Q5 | 0.43% | 0.55% | 0.63% | 0.51% | 0.53% |
| Q6 | 0.43% | 0.59% | 0.70% | 0.55% | 0.55% |
| Q7 | 0.45% | 0.64% | 0.78% | 0.62% | 0.57% |
| Q8 | 0.45% | 0.65% | 0.81% | 0.64% | 0.56% |
| Q9 | 0.37% | 0.53% | 0.66% | 0.54% | 0.45% |
| Q10 | 0.36% | 0.50% | 0.62% | 0.55% | 0.42% |
| Q11 | 0.37% | 0.48% | 0.59% | 0.53% | 0.41% |
| Q12 | 0.35% | 0.44% | 0.52% | 0.48% | 0.38% |
| Q13 | 0.30% | 0.36% | 0.41% | 0.39% | 0.31% |

| | | | | | |
|---|---|---|---|---|---|
| %PD (9Q) | 4.05% | 5.04% | 5.75% | 4.87% | 4.74% |
| %Sensitivity (9Q) | | 24% | 42% | 20% | 17% |
| %PD (13Q) | 5.43% | 6.83% | 7.88% | 6.83% | 6.25% |
| %Sensitivity (13Q) | | 26% | 45% | 26% | 15% |

Figure 99: PD Sensitivity for Jun'17 under different MCST Scenarios




As can be observed from the above charts:

- Under MCST Severely Adverse Scenario PD shows a stress of 45% in 13Q
- Under MCST Adverse Scenario PD shows a stress of 26% in 13Q

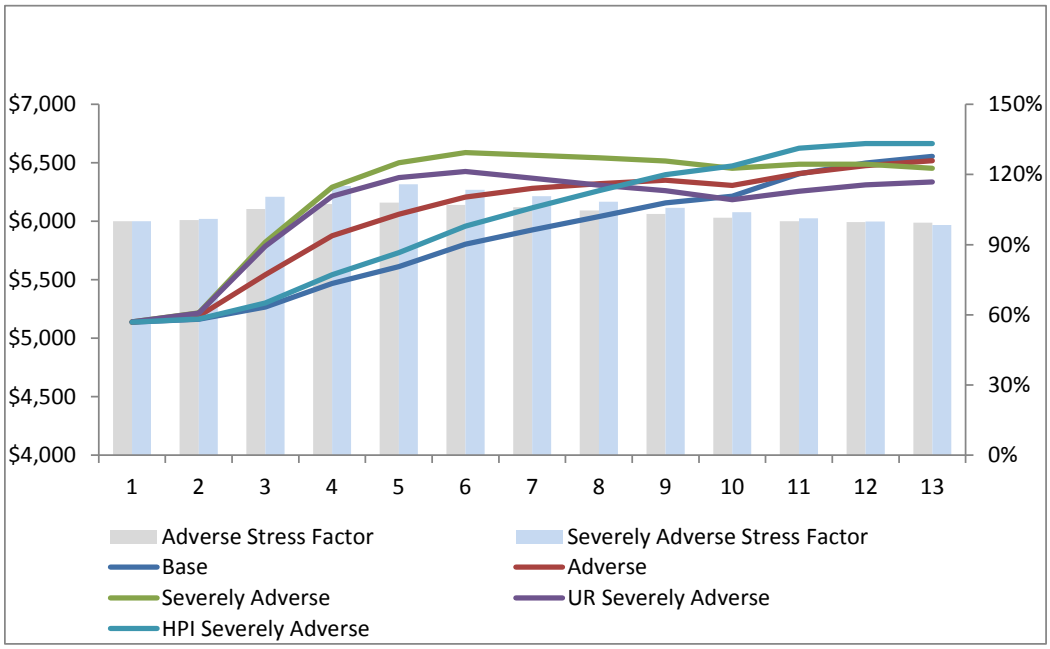
*EAD Sensitivity:*

Table 8.4.5: Average EAD for FRB Scenarios (Jan'17-Mar'20)

| | EAD $ Sensitivity FRB scenarios | | | | |
|---|---|---|---|---|---|
| | Base | Adverse | Severely Adverse | UR Severely Adverse | HPI Severely Adverse |
| Q1 | $5,138 | $5,138 | $5,139 | $5,139 | $5,138 |
| Q2 | $5,162 | $5,190 | $5,218 | $5,216 | $5,164 |
| Q3 | $5,267 | $5,542 | $5,822 | $5,788 | $5,301 |

| | | | | | |
|---|---|---|---|---|---|
| Q4 | $5,469 | $5,875 | $6,291 | $6,215 | $5,542 |
| Q5 | $5,613 | $6,061 | $6,500 | $6,375 | $5,733 |
| Q6 | $5,803 | $6,208 | $6,588 | $6,427 | $5,957 |
| Q7 | $5,927 | $6,281 | $6,565 | $6,369 | $6,115 |
| Q8 | $6,040 | $6,322 | $6,543 | $6,310 | $6,263 |
| Q9 | $6,158 | $6,352 | $6,515 | $6,261 | $6,400 |
| Q10 | $6,215 | $6,308 | $6,453 | $6,185 | $6,472 |
| Q11 | $6,406 | $6,410 | $6,488 | $6,258 | $6,626 |
| Q12 | $6,497 | $6,477 | $6,488 | $6,311 | $6,664 |
| Q13 | $6,556 | $6,517 | $6,455 | $6,338 | $6,664 |
| **Total (9Q)** | $5,588 | $5,917 | $6,235 | $6,066 | $5,733 |
| **%Sensitivity (9Q)** | | 6% | 12% | 9% | 3% |
| **Total (13Q)** | $5,788 | $6,043 | $6,294 | $6,118 | $5,939 |
| **%Sensitivity (13Q)** | | 4% | 9% | 6% | 3% |

**Figure 100: EAD Sensitivity for Dec'16 under different FRB Scenarios**



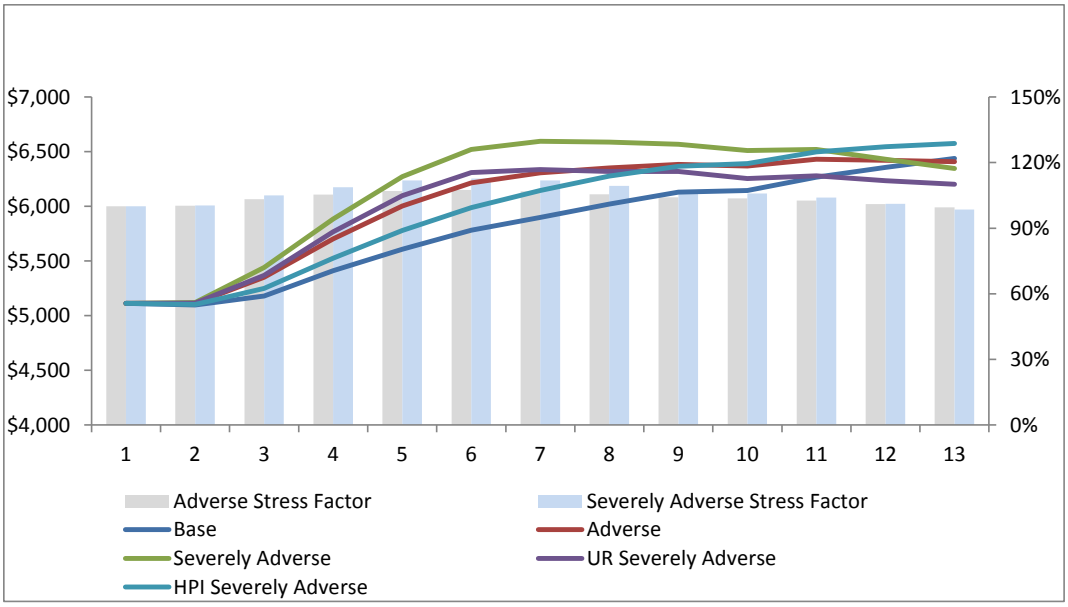

As can be observed from the above charts:

- Under FRB Severely Adverse Scenario EAD shows a stress of 9% in 13Q
- Under FRB Adverse Scenario EAD shows a stress of 4% in 13Q

Table 8.4.6: Average EAD for MCST Scenarios (Jul'17-Sep'20)

| | EAD $ Sensitivity MCST scenarios | | | | |
|---|---|---|---|---|---|
| | Base | Adverse | Severely Adverse | UR Severely Adverse | HPI Severely Adverse |
| Q1 | $5,112 | $5,112 | $5,113 | $5,113 | $5,112 |
| Q2 | $5,099 | $5,113 | $5,119 | $5,115 | $5,103 |
| Q3 | $5,180 | $5,350 | $5,440 | $5,369 | $5,250 |
| Q4 | $5,411 | $5,703 | $5,884 | $5,766 | $5,527 |
| Q5 | $5,608 | $6,002 | $6,272 | $6,097 | $5,779 |
| Q6 | $5,783 | $6,214 | $6,520 | $6,308 | $5,987 |
| Q7 | $5,898 | $6,306 | $6,595 | $6,337 | $6,146 |
| Q8 | $6,020 | $6,351 | $6,587 | $6,318 | $6,276 |
| Q9 | $6,129 | $6,385 | $6,567 | $6,318 | $6,365 |
| Q10 | $6,146 | $6,368 | $6,510 | $6,254 | $6,391 |
| Q11 | $6,267 | $6,431 | $6,519 | $6,278 | $6,498 |
| Q12 | $6,357 | $6,420 | $6,431 | $6,236 | $6,545 |
| Q13 | $6,438 | $6,409 | $6,346 | $6,203 | $6,576 |
| Total (9Q) | $5,558 | $5,877 | $6,106 | $5,898 | $5,737 |
| %Sensitivity (9Q) | | 6% | 10% | 6% | 3% |
| Total (13Q) | $5,745 | $6,016 | $6,203 | $5,998 | $5,921 |
| %Sensitivity (13Q) | | 5% | 8% | 4% | 3% |

Figure 101: EAD Sensitivity for Jun'17 under different MCST Scenarios




As can be observed from the above charts:

- Under MCST Severely Adverse Scenario EAD shows a stress of 8% in 13Q
- Under MCST Adverse Scenario EAD shows a stress of 5% in 13Q

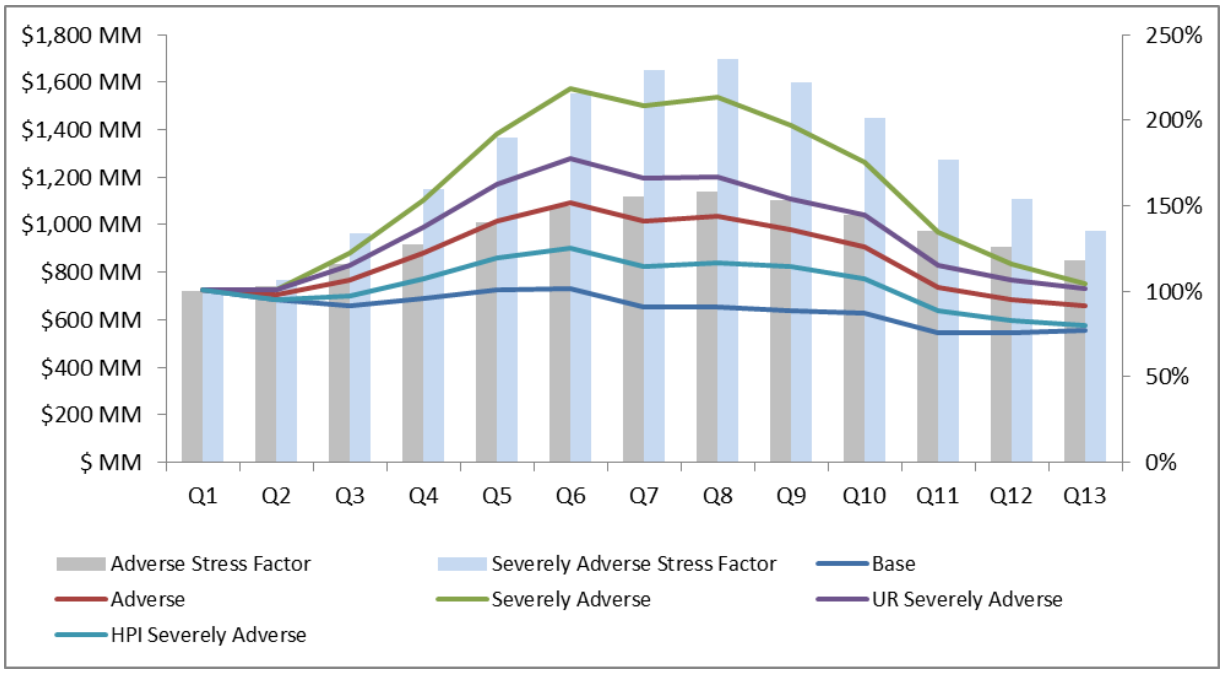
*GCL Sensitivity:*

Table 8.4.7: GCL for FRB Scenarios (Jan'17-Mar'20)

| | GCL $ Sensitivity FRB scenarios | | | | |
|---|---|---|---|---|---|
| | Base | Adverse | Severely Adverse | UR Severely Adverse | HPI Severely Adverse |
| Q1 | $724 MM | $726 MM | $727 MM | $727 MM | $724 MM |
| Q2 | $684 MM | $706 MM | $727 MM | $726 MM | $684 MM |
| Q3 | $658 MM | $765 MM | $880 MM | $830 MM | $699 MM |
| Q4 | $691 MM | $881 MM | $1,105 MM | $990 MM | $772 MM |
| Q5 | $726 MM | $1,017 MM | $1,380 MM | $1,169 MM | $860 MM |
| Q6 | $730 MM | $1,092 MM | $1,571 MM | $1,278 MM | $902 MM |
| Q7 | $654 MM | $1,017 MM | $1,500 MM | $1,196 MM | $826 MM |
| Q8 | $652 MM | $1,034 MM | $1,538 MM | $1,203 MM | $841 MM |
| Q9 | $638 MM | $977 MM | $1,419 MM | $1,109 MM | $824 MM |
| Q10 | $628 MM | $908 MM | $1,265 MM | $1,040 MM | $771 MM |

| | | | | | |
|---|---|---|---|---|---|
| Q11 | $545 MM | $737 MM | $966 MM | $831 MM | $640 MM |
| Q12 | $543 MM | $682 MM | $835 MM | $766 MM | $597 MM |
| Q13 | $555 MM | $657 MM | $751 MM | $733 MM | $574 MM |
| **Total (9Q)** | $6,157 MM | $8,213 MM | $10,848 MM | $9,230 MM | $7,131 MM |
| **%Sensitivity (9Q)** | | 33% | 76% | 50% | 16% |
| **Total (13Q)** | $8,428 MM | $11,197 MM | $14,665 MM | $12,599 MM | $9,712 MM |
| **%Sensitivity (13Q)** | | 33% | 74% | 49% | 15% |

Figure 102: GCL Sensitivity for Dec'16 under different FRB Scenarios




It can be seen from the above charts:

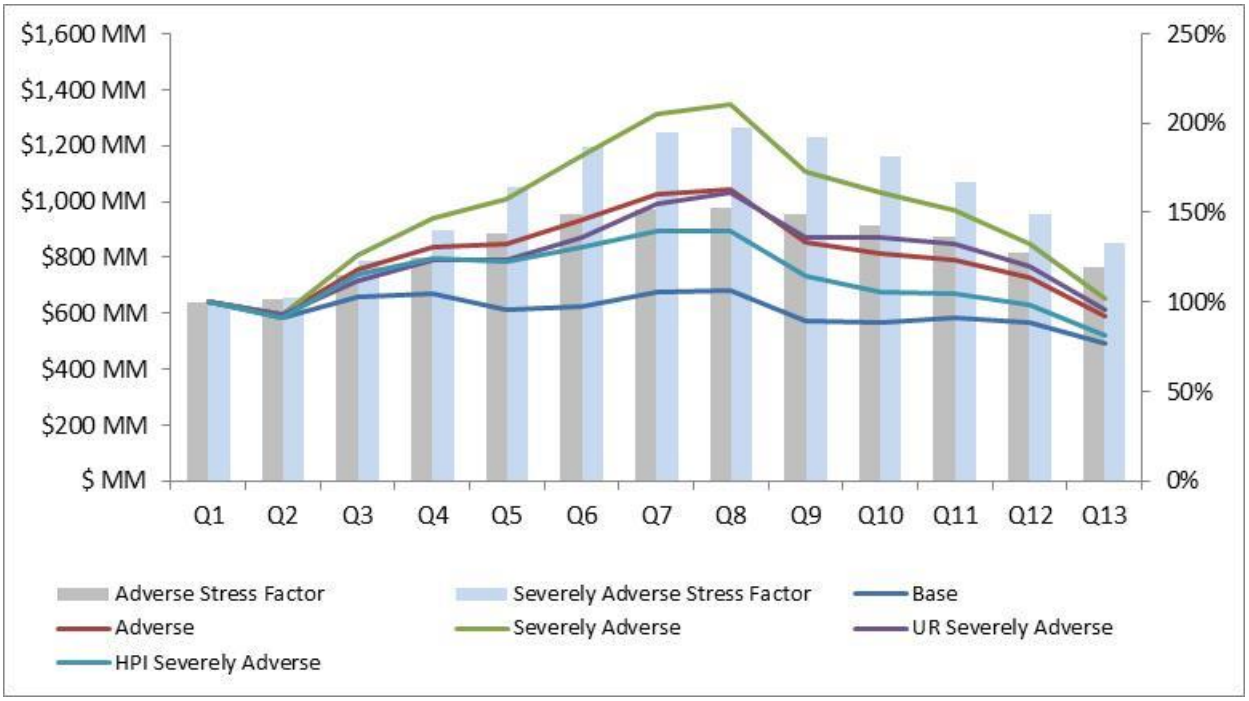
- Under FRB Severely Adverse Scenario GCL shows a stress of 74% in 13Q
- Under FRB Adverse Scenario GCL shows a stress of 33% in 13Q

Table 8.4.8: GCL for MCST Scenarios (Jul'17-Sep'20)

| | GCL $ Sensitivity MCST scenarios | | | | |
|---|---|---|---|---|---|
| | **Base** | **Adverse** | **Severely Adverse** | **UR Severely Adverse** | **HPI Severely Adverse** |
| Q1 | $639 MM | $640 MM | $640 MM | $640 MM | $639 MM |
| Q2 | $585 MM | $594 MM | $597 MM | $596 MM | $586 MM |

| | | | | | |
|---|---|---|---|---|---|
| Q3 | $656 MM | $755 MM | $805 MM | $714 MM | $740 MM |
| Q4 | $671 MM | $837 MM | $937 MM | $789 MM | $798 MM |
| Q5 | $612 MM | $846 MM | $1,006 MM | $789 MM | $782 MM |
| Q6 | $625 MM | $933 MM | $1,165 MM | $873 MM | $838 MM |
| Q7 | $674 MM | $1,024 MM | $1,311 MM | $992 MM | $895 MM |
| Q8 | $683 MM | $1,041 MM | $1,350 MM | $1,033 MM | $896 MM |
| Q9 | $575 MM | $856 MM | $1,104 MM | $872 MM | $731 MM |
| Q10 | $568 MM | $813 MM | $1,032 MM | $871 MM | $677 MM |
| Q11 | $581 MM | $791 MM | $970 MM | $847 MM | $669 MM |
| Q12 | $567 MM | $725 MM | $847 MM | $766 MM | $630 MM |
| Q13 | $490 MM | $587 MM | $653 MM | $614 MM | $523 MM |
| **Total (9Q)** | $5,718 MM | $7,524 MM | $8,916 MM | $7,299 MM | $6,904 MM |
| **%Sensitivity (9Q)** | | 32% | 56% | 28% | 21% |
| **Total (13Q)** | $7,925 MM | $10,439 MM | $12,417 MM | $10,397 MM | $9,403 MM |
| **%Sensitivity (13Q)** | | 32% | 57% | 31% | 19% |

Figure 103: GCL Sensitivity for Jun'17 under different MCST Scenarios




It can be seen from the above charts:

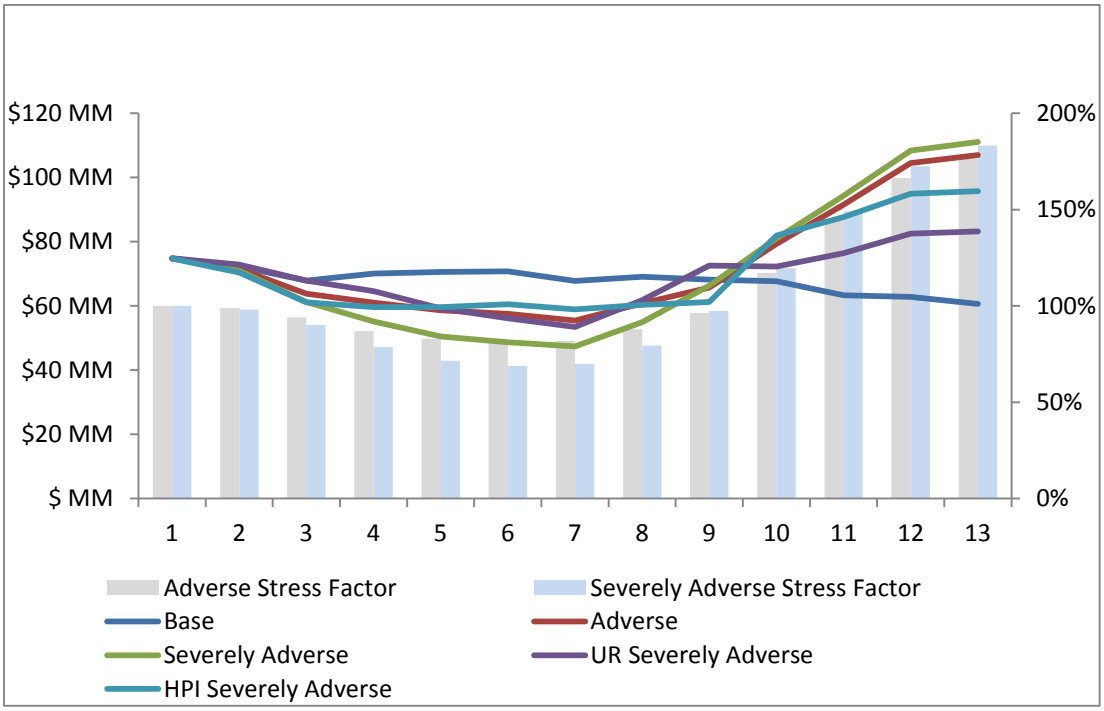
- Under MCST Severely Adverse Scenario GCL shows a stress of 57% in 13Q
- Under MCST Adverse Scenario GCL shows a stress of 32% in 13Q

*Recovery Sensitivity:*

Table 8.4.9: Recovery for FRB Scenarios (Jan'17-Mar'20)

| | Recovery $ Sensitivity FRB scenarios | | | | |
|---|---|---|---|---|---|
| | Base | Adverse | Severely Adverse | UR Severely Adverse | HPI Severely Adverse |
| Q1 | $75 MM | $75 MM | $75 MM | $75 MM | $75 MM |
| Q2 | $72 MM | $71 MM | $71 MM | $73 MM | $70 MM |
| Q3 | $68 MM | $64 MM | $61 MM | $68 MM | $61 MM |
| Q4 | $70 MM | $61 MM | $55 MM | $65 MM | $60 MM |
| Q5 | $71 MM | $59 MM | $51 MM | $59 MM | $60 MM |
| Q6 | $71 MM | $58 MM | $49 MM | $56 MM | $61 MM |
| Q7 | $68 MM | $55 MM | $47 MM | $53 MM | $59 MM |
| Q8 | $69 MM | $61 MM | $55 MM | $62 MM | $60 MM |
| Q9 | $68 MM | $66 MM | $66 MM | $73 MM | $61 MM |
| Q10 | $68 MM | $79 MM | $81 MM | $72 MM | $82 MM |
| Q11 | $63 MM | $92 MM | $94 MM | $76 MM | $88 MM |
| Q12 | $63 MM | $104 MM | $108 MM | $82 MM | $95 MM |
| Q13 | $61 MM | $107 MM | $111 MM | $83 MM | $96 MM |
| Total (9Q) | $631 MM | $569 MM | $530 MM | $583 MM | $567 MM |
| %Sensitivity (9Q) | | -10% | -16% | -8% | -10% |
| Total (13Q) | $886 MM | $951 MM | $925 MM | $898 MM | $927 MM |
| %Sensitivity (13Q) | | 7% | 4% | 1% | 5% |

Figure 104: Recovery Sensitivity for Dec'16 under different FRB Scenarios

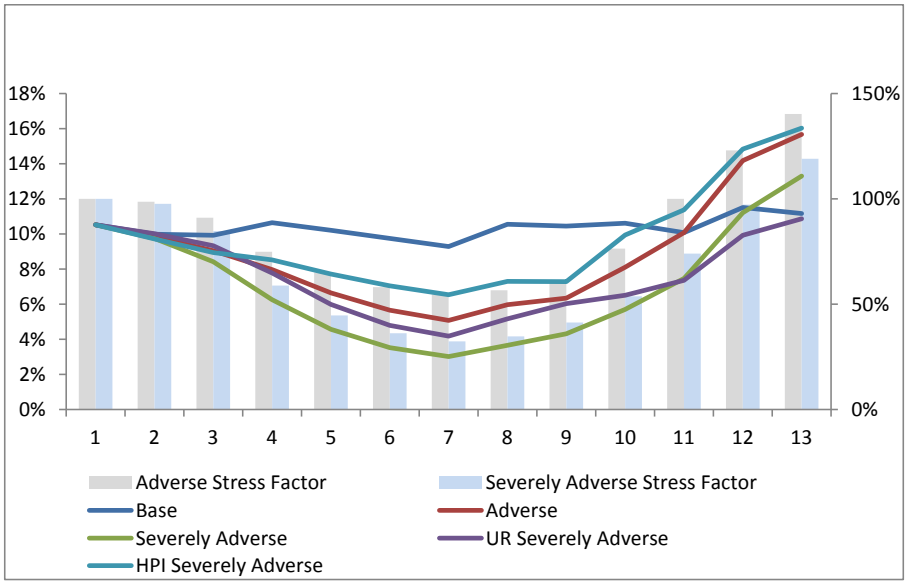



It can be seen from the above charts:

- Under FRB Severely Adverse Scenario Recovery shows a stress of -13% in 13Q
- Under FRB Adverse Scenario Recovery shows a stress of 9% in 13Q

The recovery amount increases in stress scenarios post Q9 because the stress peaks at Q8-Q9 and the GCL is maximum at this point. Post Q9 the stress scenario starts approaching the base scenario due to which the recoveries increase from increased GCL of the peak stress period.

Figure 105: Recovery Rate Sensitivity for Dec'16 under different FRB Scenarios



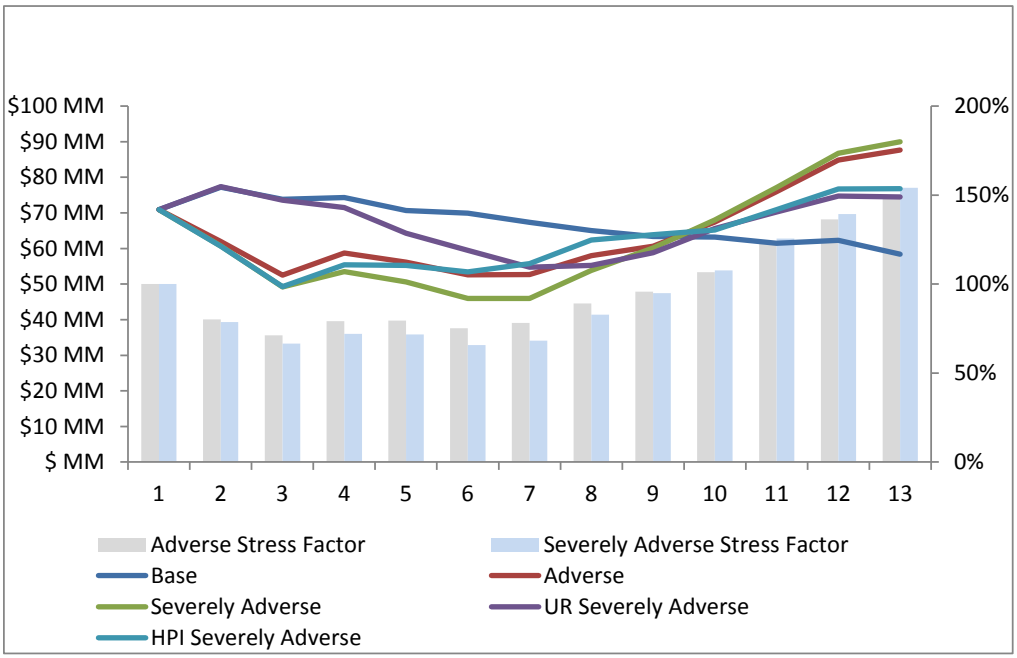

The sensitivity of weighted recovery rate for FRB scenario follows the expected trend. Post Q9 the recovery rate increases due to increased recoveries from peak stress period GCL.

Table 8.4.10: Recovery for MCST Scenarios (Jul'17-Sep'20)

| | Recovery $ Sensitivity MCST scenarios | | | | |
|---|---|---|---|---|---|
| | Base | Adverse | Severely Adverse | UR Severely Adverse | HPI Severely Adverse |
| Q1 | $71 MM | $71 MM | $71 MM | $71 MM | $71 MM |
| Q2 | $77 MM | $62 MM | $61 MM | $77 MM | $61 MM |
| Q3 | $74 MM | $53 MM | $49 MM | $74 MM | $49 MM |
| Q4 | $74 MM | $59 MM | $53 MM | $71 MM | $55 MM |
| Q5 | $71 MM | $56 MM | $51 MM | $64 MM | $55 MM |
| Q6 | $70 MM | $53 MM | $46 MM | $59 MM | $53 MM |
| Q7 | $67 MM | $53 MM | $46 MM | $55 MM | $56 MM |
| Q8 | $65 MM | $58 MM | $54 MM | $55 MM | $62 MM |
| Q9 | $63 MM | $61 MM | $60 MM | $59 MM | $64 MM |
| Q10 | $63 MM | $67 MM | $68 MM | $66 MM | $65 MM |
| Q11 | $61 MM | $76 MM | $77 MM | $70 MM | $71 MM |
| Q12 | $62 MM | $85 MM | $87 MM | $75 MM | $77 MM |
| Q13 | $58 MM | $88 MM | $90 MM | $74 MM | $77 MM |
| Total (9Q) | $633 MM | $524 MM | $491 MM | $586 MM | $527 MM |
| %Sensitivity (9Q) | | -17% | -22% | -7% | -17% |
| Total (13Q) | $878 MM | $840 MM | $813 MM | $871 MM | $817 MM |

| | | | | | |
|---|---|---|---|---|---|
| %Sensitivity (13Q) | | -4% | -7% | -1% | -7% |

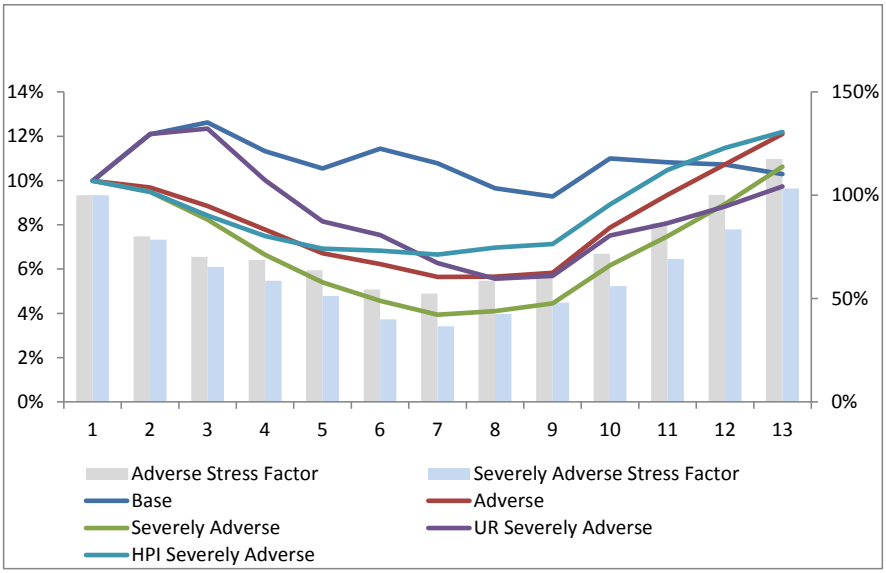
Figure 106: Recovery Sensitivity for Jun'17 under different MCST Scenarios




It can be seen from the above charts:

- Under MCST Severely Adverse Scenario Recovery shows a stress of -17% in 13Q
- Under MCST Adverse Scenario Recovery shows a stress of -4% in 13Q

Figure 107: Recovery Rate Sensitivity for Jun'17 under different MCST Scenarios



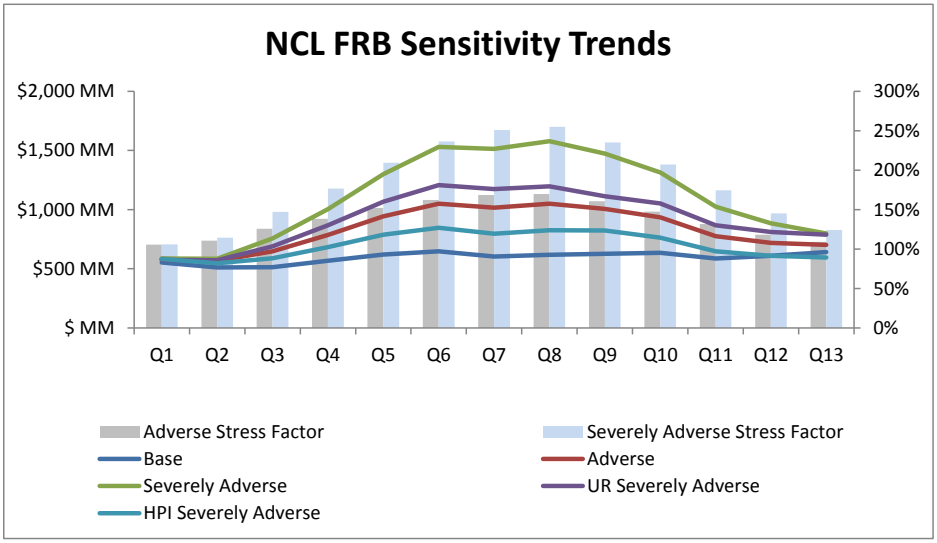

Q2 and Q3 recovery dollar amount and recovery rates are higher in base and UR SA due to

*NCL Sensitivity:*

Table 8.4.11: NCL for FRB Scenarios (Jan'17-Mar'20)

| | NCL $ Sensitivity FRB scenarios | | | | |
|---|---|---|---|---|---|
| | Base | Adverse | Severely Adverse | UR Severely Adverse | HPI Severely Adverse |
| Q1 | $553 MM | $584 MM | $586 MM | $574 MM | $582 MM |
| Q2 | $511 MM | $565 MM | $585 MM | $571 MM | $546 MM |
| Q3 | $514 MM | $647 MM | $756 MM | $689 MM | $588 MM |
| Q4 | $568 MM | $786 MM | $1,003 MM | $868 MM | $684 MM |
| Q5 | $621 MM | $944 MM | $1,300 MM | $1,067 MM | $787 MM |
| Q6 | $647 MM | $1,050 MM | $1,529 MM | $1,208 MM | $847 MM |
| Q7 | $603 MM | $1,016 MM | $1,514 MM | $1,174 MM | $797 MM |
| Q8 | $619 MM | $1,050 MM | $1,578 MM | $1,197 MM | $825 MM |
| Q9 | $627 MM | $1,006 MM | $1,474 MM | $1,114 MM | $824 MM |
| Q10 | $635 MM | $935 MM | $1,315 MM | $1,052 MM | $762 MM |
| Q11 | $588 MM | $774 MM | $1,025 MM | $868 MM | $647 MM |
| Q12 | $609 MM | $720 MM | $884 MM | $811 MM | $610 MM |
| Q13 | $642 MM | $701 MM | $798 MM | $788 MM | $595 MM |
| Total (9Q) | $5,262 MM | $7,647 MM | $10,325 MM | $8,461 MM | $6,481 MM |
| %Sensitivity (9Q) | | 45% | 96% | 61% | 23% |
| Total (13Q) | $7,737 MM | $10,778 MM | $14,348 MM | $11,980 MM | $9,096 MM |
| %Sensitivity (13Q) | | 39% | 85% | 55% | 18% |

Figure 108: NCL Sensitivity for Dec'16 under different FRB Scenarios



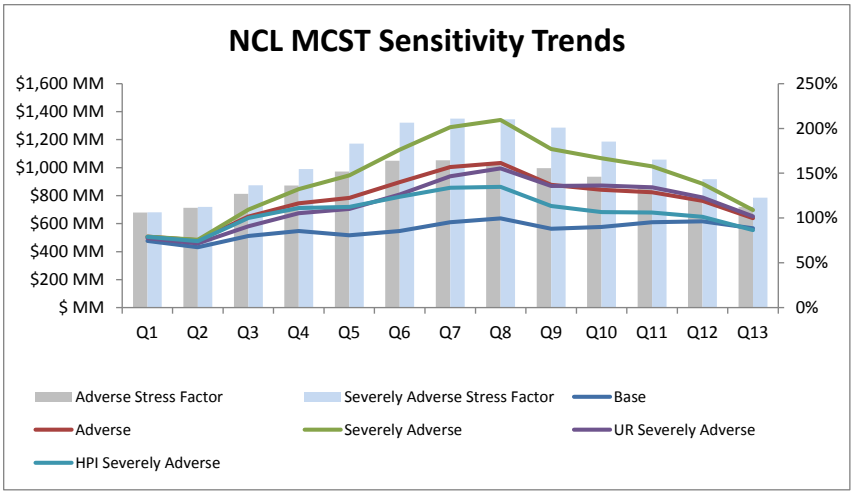
It can be seen from the above charts:

- Under FRB Severely Adverse Scenario NCL shows a stress of 85% in 13Q
- Under FRB Adverse Scenario NCL shows a stress of 39% in 13Q

Table 8.4.12: NCL for MCST Scenarios (Jul'17-Sep'20)

| | NCL $ Sensitivity MCST scenarios | | | | |
|---|---|---|---|---|---|
| | Base | Adverse | Severely Adverse | UR Severely Adverse | HPI Severely Adverse |
| Q1 | $477 MM | $506 MM | $507 MM | $497 MM | $506 MM |
| Q2 | $432 MM | $481 MM | $485 MM | $458 MM | $475 MM |
| Q3 | $512 MM | $650 MM | $700 MM | $581 MM | $640 MM |
| Q4 | $547 MM | $746 MM | $846 MM | $675 MM | $711 MM |
| Q5 | $516 MM | $784 MM | $944 MM | $705 MM | $720 MM |
| Q6 | $547 MM | $897 MM | $1,129 MM | $808 MM | $792 MM |
| Q7 | $611 MM | $1,005 MM | $1,289 MM | $939 MM | $856 MM |
| Q8 | $638 MM | $1,033 MM | $1,341 MM | $995 MM | $863 MM |
| Q9 | $564 MM | $878 MM | $1,133 MM | $869 MM | $725 MM |
| Q10 | $576 MM | $841 MM | $1,067 MM | $873 MM | $683 MM |
| Q11 | $610 MM | $824 MM | $1,008 MM | $859 MM | $680 MM |
| Q12 | $617 MM | $763 MM | $885 MM | $788 MM | $648 MM |
| Q13 | $567 MM | $638 MM | $696 MM | $653 MM | $554 MM |
| Total (9Q) | $4,843 MM | $6,979 MM | $8,372 MM | $6,526 MM | $6,289 MM |
| %Sensitivity (9Q) | | 44% | 73% | 35% | 30% |
| Total (13Q) | $7,213 MM | $10,046 MM | $12,030 MM | $9,697 MM | $8,854 MM |
| %Sensitivity (13Q) | | 39% | 67% | 34% | 23% |

Figure 109: NCL Sensitivity for Jun'17 under different MCST Scenarios




It can be seen from the above charts:

- Under MCST Severely Adverse Scenario NCL shows a stress of 67% in 13Q
- Under MCST Adverse Scenario NCL shows a stress of 39% in 13Q

It is observed from above results that the models show stress to Macro-economic variables. The sensitivity results of PD, GCL, EAD, Recovery and NCL for Dec'16 and June'17 under FRB and MCST scenarios have been attached in the excel sheet.

8.4.1_Model_Sensitivity_All_Models.xlsx

8.4.1_FRB_PD_Sensitivity_Detail_final.xlsx

8.4.1_MCST_PD_Sensitivity_Detail_final.xl

8.4.1_FRB_EAD_Sensitivity_Detail.xlsx

8.4.1_MCST_EAD_Sensitivity_Detail.xlsx

8.4.1_FRB_GCL_Sensitivity_Detail_final.xl

8.4.1_MCST_GCL_Sensitivity_Detail_final.xy

8.4.1_FRB_Recovery_Sensitivity_Detail.xry

8.4.1_MCST_Recovery_Sensitivity_Detail.

8.4.1_FRB_NCL_Sensitivity_Detail.xlsx

8.4.1_MCST_NCL_Sensitivity_Detail.xlsx

Table 8.4.1.5: Summary of Sensitivity Testing Results - Baseline vs. Stress

| Model component | Test / shock description | Test results (metric applied and result) | Business interpretation | Mitigation / limitation if performance is not deemed sufficient |
|---|---|---|---|---|
| PD | Fed Scenarios and MCST Scenarios | Thirteen Quarter Cumulative Stress Ratio | PD shows sufficient sensitivity to Macro factors. | Not Applicable |
| GCL | Fed Scenarios and MCST Scenarios | Thirteen Quarter Cumulative Stress Ratio | GCL shows sufficient sensitivity to Macro factors. | Not Applicable |
| EAD | Fed Scenarios and MCST Scenarios | Thirteen Quarter Cumulative Stress Ratio | EAD sensitivity less than threshold | Historically, EAD has lower sensitivity to macroeconomic factors |
| Recovery | Fed Scenarios and MCST Scenarios | Thirteen Quarter Cumulative Stress Ratio | Recovery shows sufficient sensitivity to Macro factors. | Not Applicable |
| NCL | Fed Scenarios and MCST Scenarios | Thirteen Quarter Cumulative Stress Ratio | NCL shows sufficient sensitivity to Macro factors. | Not Applicable |

### 8.4.2 Sensitivity of model results under stress macro scenarios vs historical stress episode

- Have sensitivity tests been performed at various levels of the model - at the model component level (for e.g. PD, LGD, EAD), as well as final output (for e.g. NCL, Balance)?

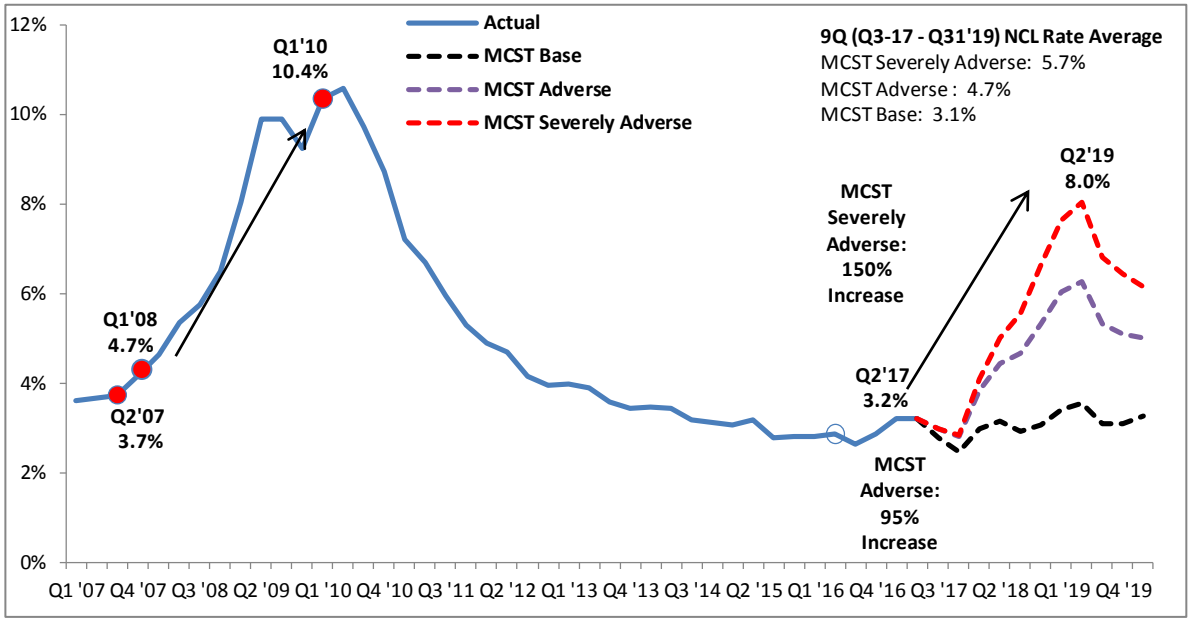
  Yes, the sensitivity tests have been performed at model component level (PD, EAD, and Recovery) as well as at the model output level (GCL and NCL).

- What are the sensitivity test results?

The Following chart demonstrates historical NCL sensitivity and forecasted MCST CCAR 2017 scenarios of

Branded Cards portfolio:

Figure 110: MCST Sensitivity of NCL for historical time period




Due to improvement in the credit quality of the portfolio, our starting point for NCL rate is significantly lower today as compared to the start of the last recession (3.2% today vs 4.7% in 1Q08). Compared to the last recession, the MCST Severely Adverse Scenario is more stressed (150% vs. 126%). To be consistent with 9 quarter timeframe, 1Q08 was used as start point for last recession. However, NCL started to increase as of 2Q07.

Table 8.4.2.1: Summary of Sensitivity Testing Results - Historical Stress Episode

| Model component | Test / shock description | Test results (metric applied and result) | Business interpretation | Mitigation / limitation if performance is not deemed sufficient |
|---|---|---|---|---|
| NCL | Stress Ratio comparison between FRB Scenarios and Historical Stress | The stress ratio is comparable to 2008 recession | The stress ratio is comparable to 2008 recession | Not Applicable |

### 8.4.3 Sensitivity of model results to changes in underlying macro variables (univariate and multivariate analysis)

- Have sensitivity tests been performed at various levels of the model - at the model component level (for e.g. PD, LGD, EAD), as well as final output (for e.g. NCL, Balance)?

  Yes, the sensitivity tests have been performed at model component level (PD, EAD, and Recovery) as well as at the model output level (GCL and NCL).

- Have all univariate and multivariate tests been performed?

  Yes, all univariate and multivariate tests have been performed.

- What are effects of the tested macro factors on the model output (linear, non-linear (concave, convex, spline), additive across macro variables etc.)?

  The sensitivity tested on the macro outputs is non-additive in nature. The stress in NCL Model is a function of the stress from the component models (PD, EAD and Recovery) and the transformations of macro-economic factors. For each of the component model, the shape of the stress curve will depend on structural model and the transformations of Macro Economic Variables. For instance for the PD model, predicted PD does not have a linear relationship with the macro-factors or any of the other predictors. It is rather the logit of PD which has a liner relationship with the input variables. In case of EAD, even though the structural model is linear, the transformations of macroeconomic variables can lead to non-additive stress. The stress obtained from UR and HPI does not add up to the total stress observed in the stress scenarios.

- What variables and values were tested?

Unemployment Rate and House Price Index were tested for the Supervisory Base, Adverse and Severe Adverse scenarios provided by the Federal Reserve Bank.

Unemployment Rate and House Price Index were tested for the MCST Base, Adverse and Severe Adverse scenarios provided by the BHC.

- What are the sensitivity test results?

Branded Cards assessed the sensitivity of losses to macroeconomic variables as per guidelines from MRM. All macroeconomic variables in final model are used for sensitivity analysis. These include Unemployment Rate (UR), and House Price Index (HPI). Univariate approach is used for sensitivity analysis of each variable as all Branded Cards models are two variables models. Under this approach one variable is subjected to stress condition in all models having that specific macroeconomic variable. Other variables are kept at base scenario levels for all applicable models. CCAR 2017 Supervisory Base and Supervisory Severely Adverse scenarios are used for testing sensitivity. MCST 2017 Base and Severely Adverse scenarios are used for testing sensitivity. Below table summarizes the tests:

Table 8.4.3.1: Summary of Sensitivity Testing Results - Macro Variables Sensitivity

| Model component | Test / shock description | Test results (metric applied and result) | Business interpretation | Mitigation / limitation if performance is not deemed sufficient |
|---|---|---|---|---|
| PD | Univariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to UR | Not Applicable |
| PD | Univariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to HPI | Not Applicable |
| PD | Multivariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to both UR and HPI | Not Applicable |
| EAD | Univariate Sensitivity Analysis | Satisfactory | EAD model has lower sensitivity to UR | Historically, EAD has lower sensitivity to macroeconomic factors |
| EAD | Univariate Sensitivity Analysis | Satisfactory | EAD model has lower sensitivity to HPI | Historically, EAD has lower sensitivity to macroeconomic factors |
| EAD | Multivariate Sensitivity Analysis | Satisfactory | EAD model has lower sensitivity to both UR and HPI | Historically, EAD has lower sensitivity to macroeconomic factors |
| Recovery | Univariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to UR | Not Applicable |
| Recovery | Univariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to HPI | Not Applicable |
| Recovery | Multivariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to both UR and HPI | Not Applicable |
| GCL | Univariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to UR | Not Applicable |
| GCL | Univariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to HPI | Not Applicable |
| GCL | Multivariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to both UR and HPI | Not Applicable |

| Model component | Test / shock description | Test results (metric applied and result) | Business interpretation | Mitigation / limitation if performance is not deemed sufficient |
|---|---|---|---|---|
| NCL | Univariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to UR | Not Applicable |
| NCL | Univariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to HPI | Not Applicable |
| NCL | Multivariate Sensitivity Analysis | Satisfactory | Model is sufficiently sensitive to both UR and HPI | Not Applicable |

Univariate approach is used for sensitivity analysis of each variable. Under this approach one variable is subjected to stress condition in all models having that specific macroeconomic variable. Other variables are kept at base scenario levels for all applicable models. CCAR 2017 Supervisory Base and Supervisory Severely Adverse scenarios are used for testing sensitivity.

To analyze the impact, Branded Cards evaluated the impact of the two macroeconomic variables used in modeling - Unemployment Rate and House Price Index to see how each separately as well as combined would impact the overall expected PD and GCL for the Branded Cards business. The scenarios that were evaluated were the FRB Base and FRB Severe Adverse from the 2017 CCAR Submission.

*PD Model Sensitivity:*

**Figure 111: Sensitivity Analysis: PD (FRB scenario)**

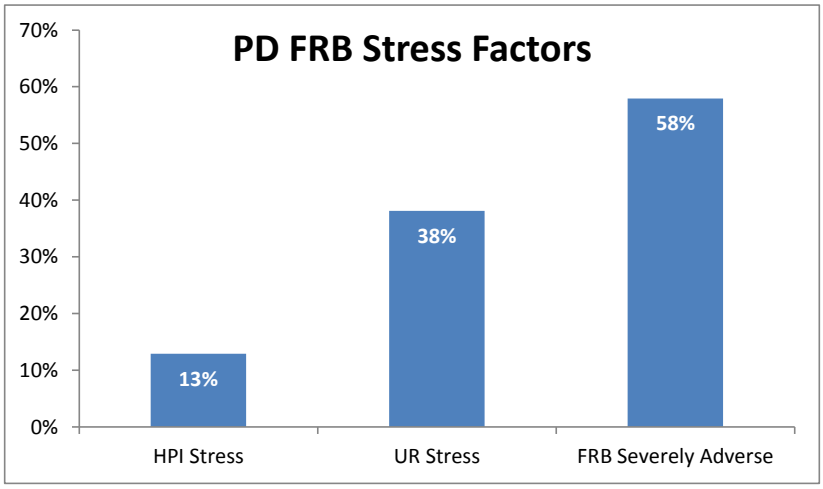


Figure 112: Stress contribution of macro variables for PD (FRB scenario)

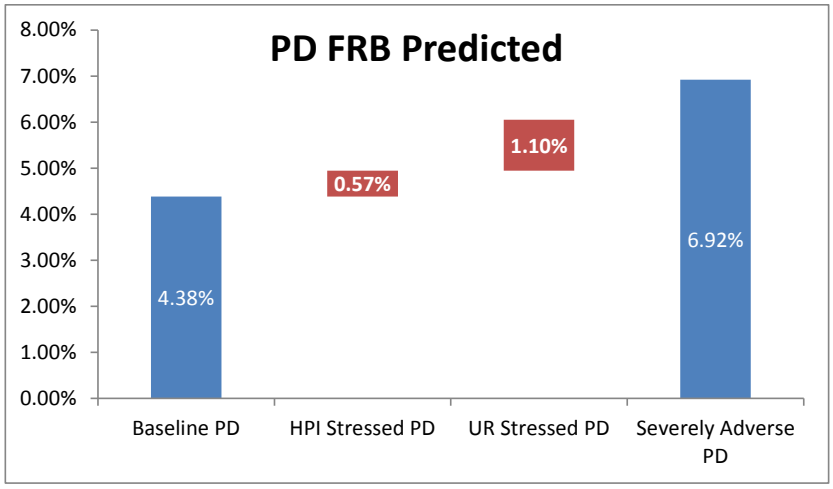



Figure 113: Sensitivity Analysis: PD (MCST scenario)

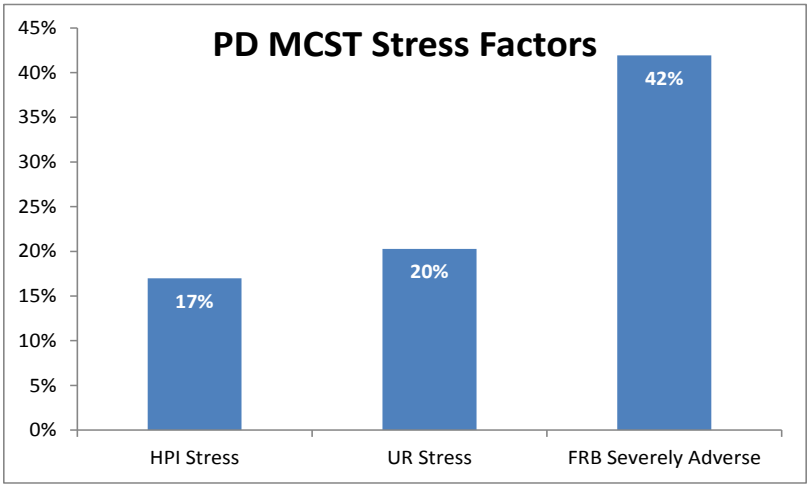
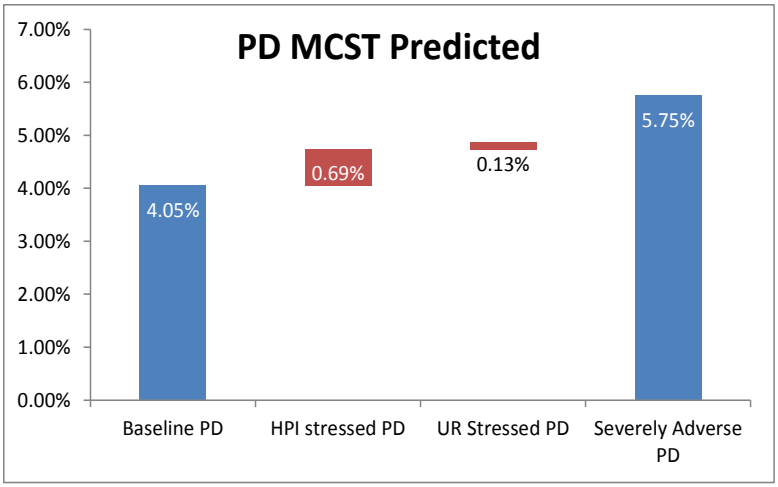


Figure 114: Stress contribution of macro variables for PD (MCST scenario)




As can be seen from the results above, PD is sufficiently stressed to both UR and HPI. UR contributes more towards the sensitivity. This is in line with business expectation.

*EAD Model Sensitivity:*

Figure 115: Sensitivity Analysis: EAD (FRB scenario)

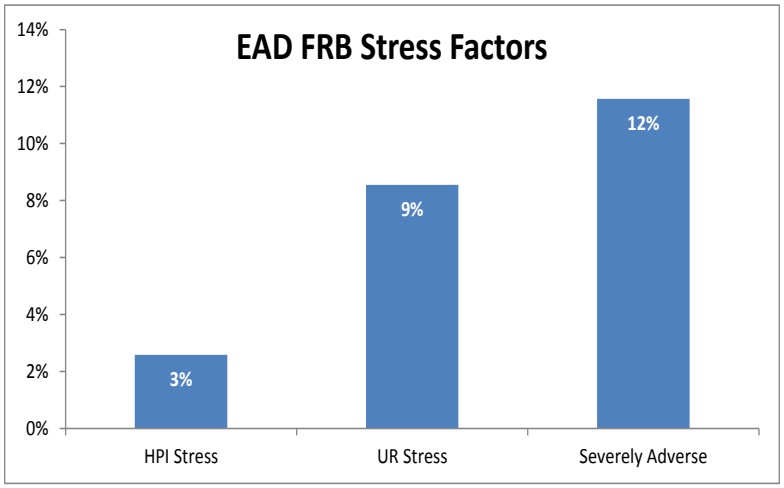


Figure 116: Stress contribution of macro variables for EAD (FRB scenario)

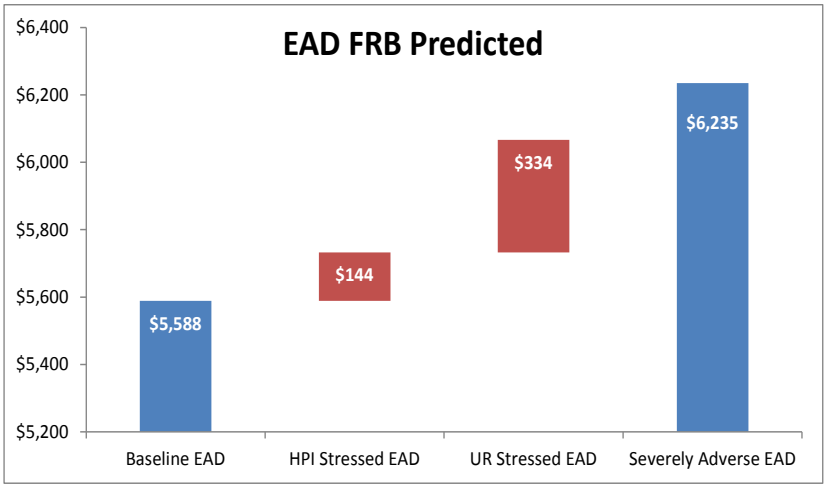



Figure 117: Sensitivity Analysis: EAD (MCST scenario)

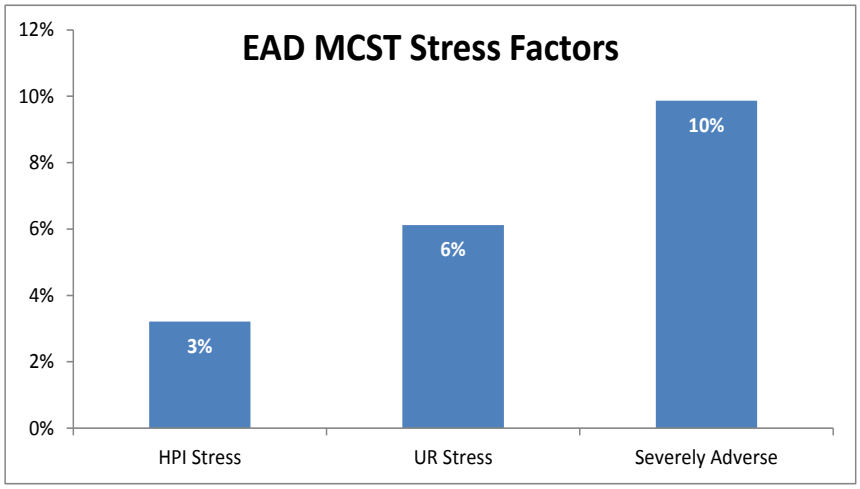
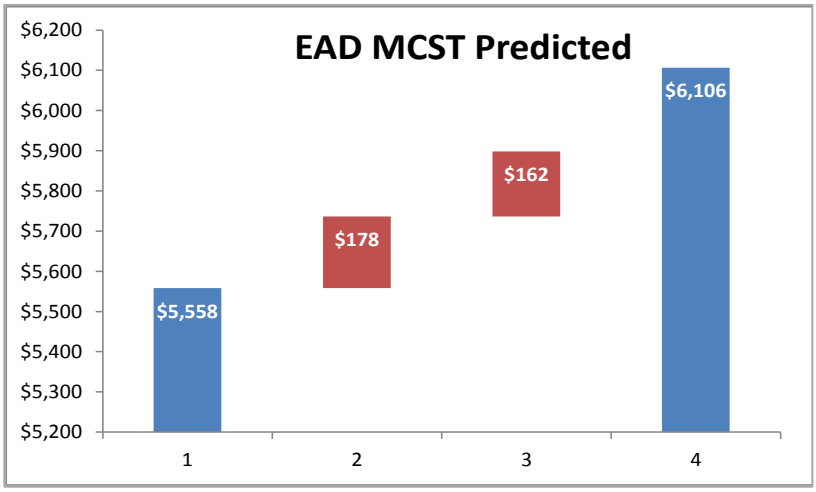


Figure 118: Stress contribution of macro variables for EAD (MCST scenario)




As can be seen from the results above, EAD is sensitive to both UR and HPI. UR contributes more towards the sensitivity. This is in line with business expectation.

*GCL Sensitivity:*

Figure 119: Sensitivity Analysis: GCL (FRB scenario)

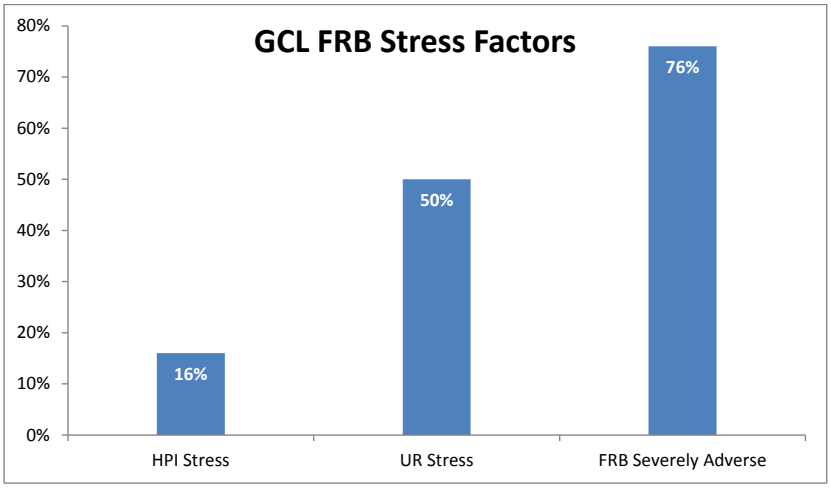


Figure 120: Stress contribution of macro variables for GCL (FRB scenario)

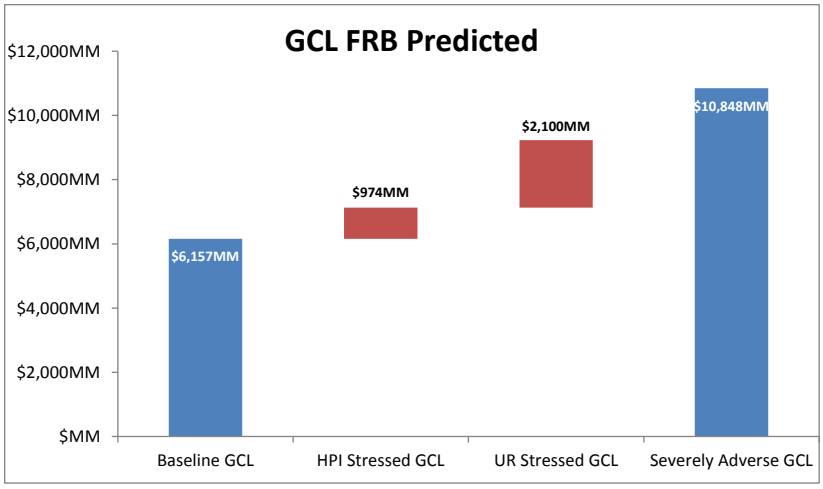



Figure 121: Sensitivity Analysis: GCL (MCST scenario)

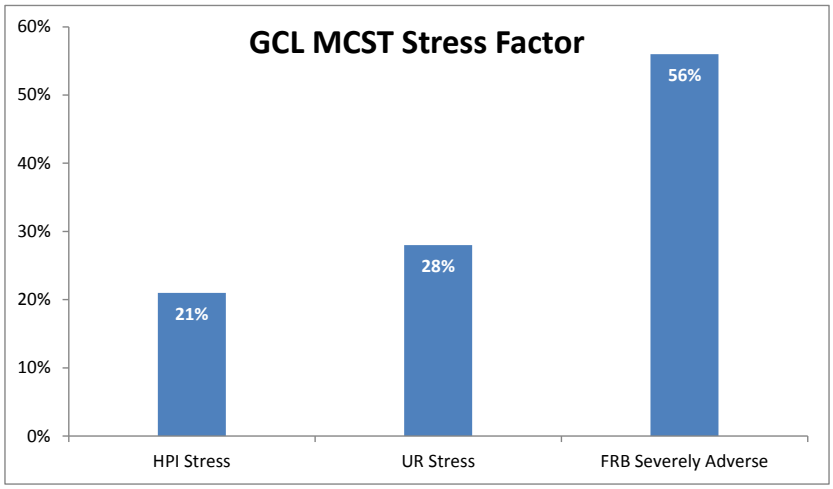
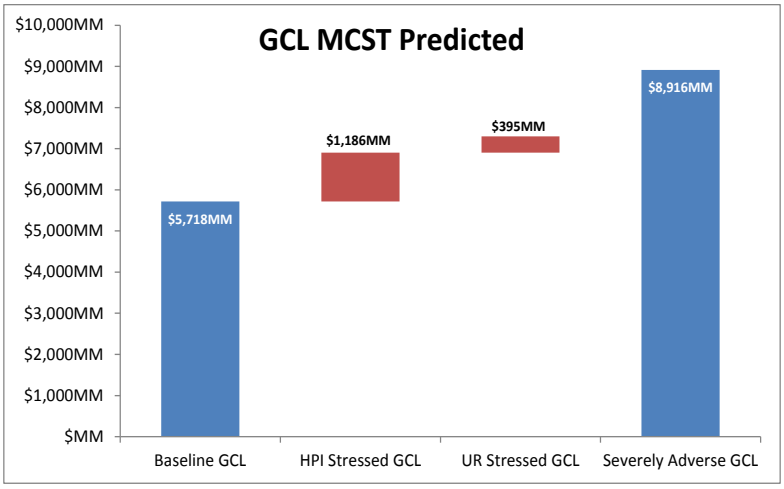


Figure 122: Stress contribution of macro variables for GCL (MCST scenario)




As can be seen from the results above, GCL is sufficiently stressed to both UR and HPI. UR contributes more towards the sensitivity. This is in line with business expectation.

*Recovery Sensitivity:*

Figure 123: Sensitivity Analysis: Recovery (FRB scenario)

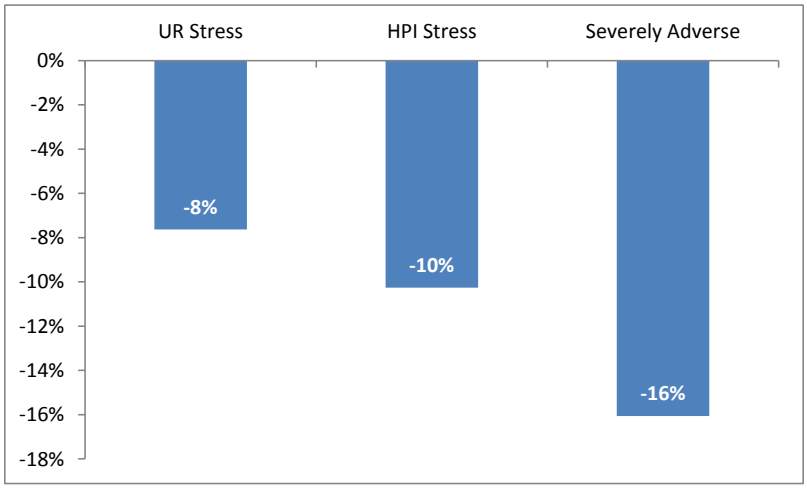


Figure 124: Stress contribution of macro variables for Recovery (FRB scenario)

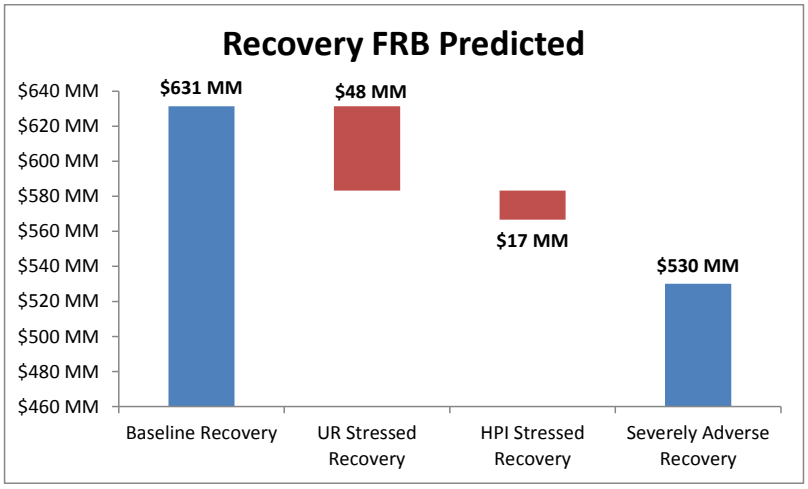



Figure 125: Sensitivity Analysis: Recovery (MCST scenario)

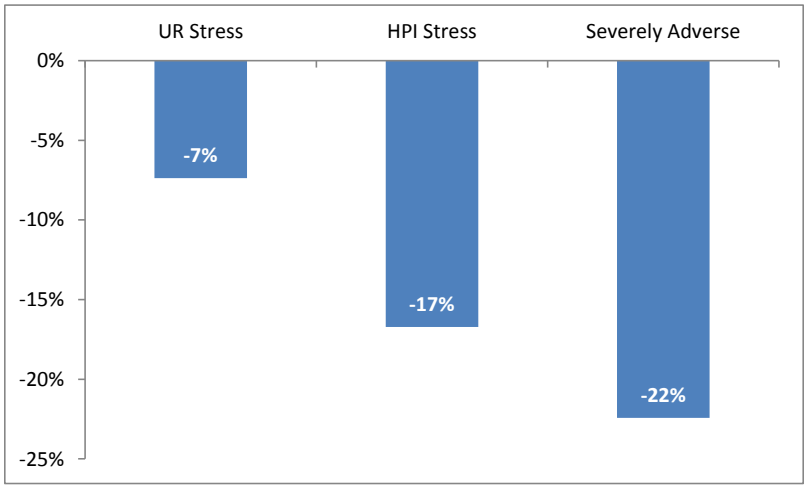
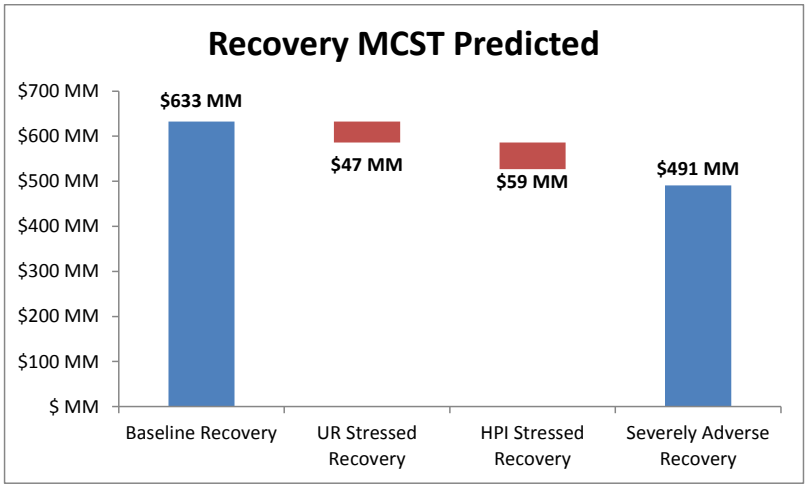


Figure 126: Stress contribution of macro variables for Recovery (MCST scenario)




As can be seen from the results above, Recovery is sufficiently stressed to both UR and HPI. HPI contributes more towards the sensitivity in Recovery. This is in line with business expectation.

As can be seen in the charts above, all models are adequately sensitive to both unemployment rate and House Price Index. The sensitivity is relatively more for unemployment rate.

This is in line with business intuition as unemployment rate may impact a wider set of people with varying degrees as compared to HPI. Unemployment rate may also capture economic stress more generally and across sectors as compared to HPI. HPI, however, continues to remain an important indicator.

*NCL sensitivity:*

Figure 127: Sensitivity Analysis: NCL (FRB scenario)

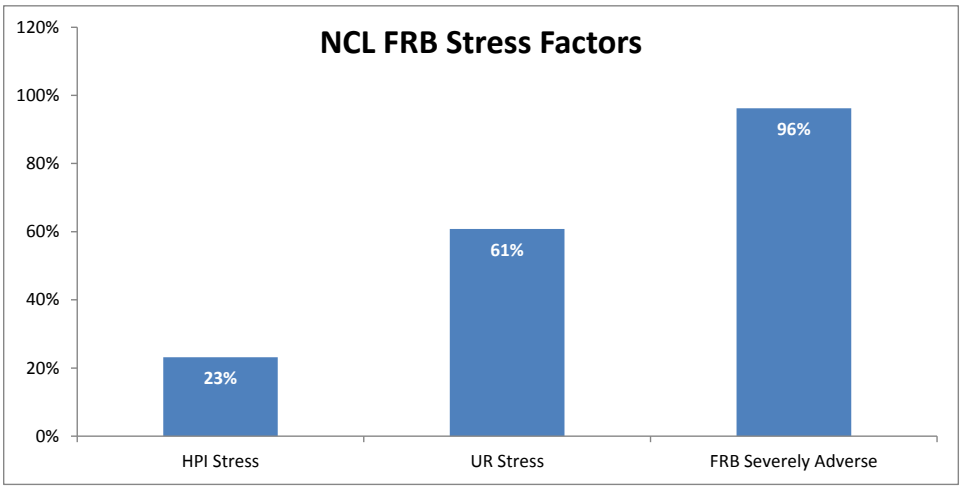



Figure 128: Stress contribution of macro variables for NCL (FRB scenario)

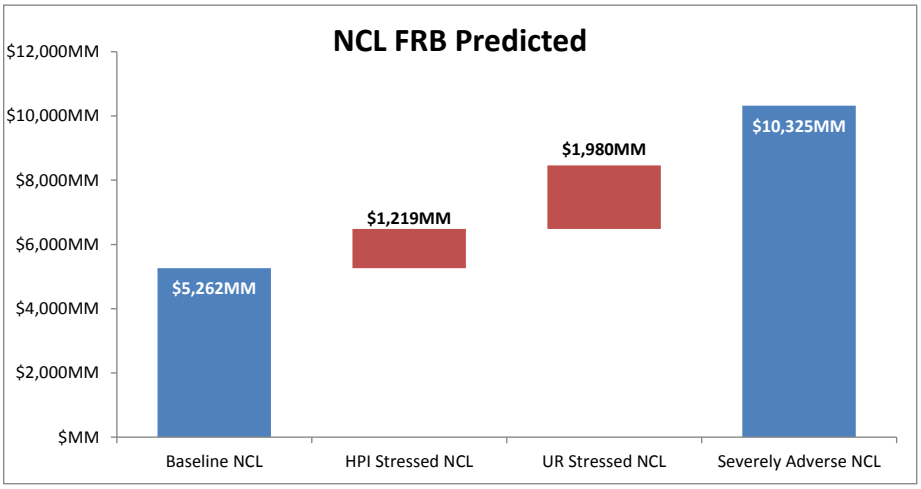


Figure 129: Sensitivity Analysis: NCL (MCST scenario)

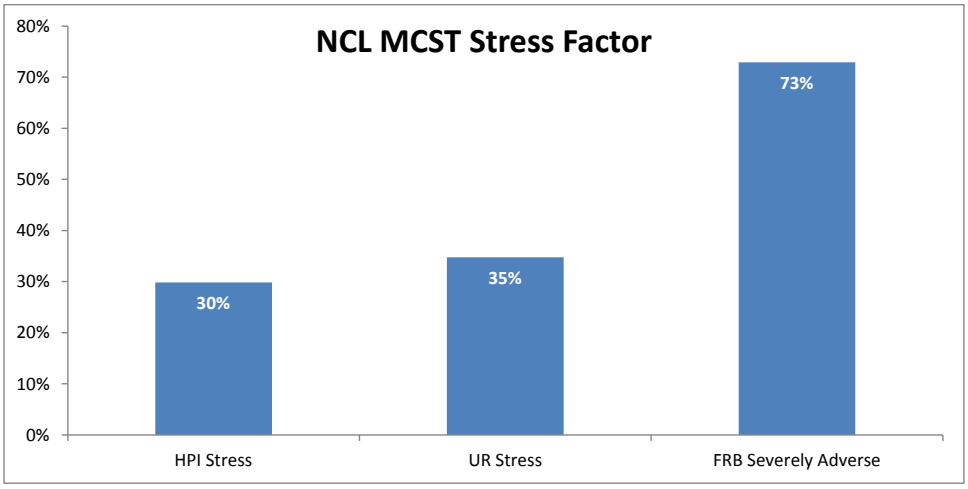
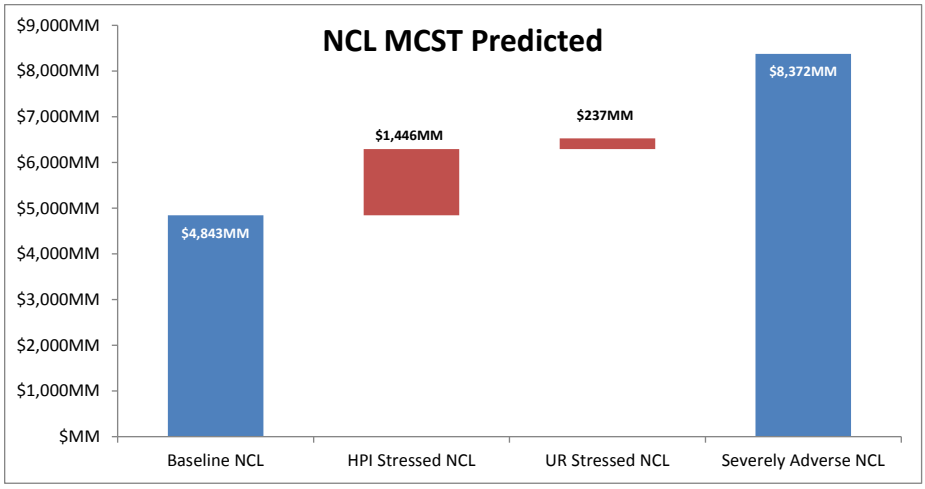



Figure 130: Stress contribution of macro variables for NCL (MCST scenario)




The sensitivity for NCL rate has also been evaluated. A univariate analysis is done for the same. As can be seen in the chart below, NCL is adequately sensitive to both unemployment rate and House Price Index. The sensitivity is relatively more for unemployment rate.

### 8.4.4 Sensitivity of model results to changes in model parameters

- Have sensitivity tests been performed at various levels of the model -- at the model component level (for e.g. PD, LGD, EAD), as well as final output (for e.g. NCL/ Balance)?

The objective of sensitivity testing is to determine how the model output (GCL/PD/EAD) is impacted by change in input factors during stress scenarios. Since stress scenarios are solely explained by macroeconomic factors, sensitivity testing has been done for these variables

only and not for credit quality variables. The results for the same have been shown in the previous sub-sections.

- What model parameters and values were tested?

  As explained above, sensitivity testing for model parameters is not applicable.

- What are the sensitivity test results?

  As explained above, sensitivity testing for model parameters is not applicable.

### 8.4.5 Sensitivity of model results to changes in the key assumptions

- Have sensitivity tests on assumptions been performed at various levels of the model - at the model component level (for e.g. PD, LGD, EAD), as well as at the level of model sub-components (such as vintage quality assumptions for PD in case of LookAhead models, etc.)?

  The sensitivity tests to changes in key assumptions have been performed at final model output level (NCL).

- Have all the relevant business assumptions or inputs been tested? What key assumptions and values were tested?

  The business has tested the following key assumptions:

**Assumption #1: Constant quality of new origination**

Background

The US Branded Cards account level models assume that the distribution of FICO scores for new originations is consistent with the most recently observed distribution.

**Likelihood**

This assumption is likely to hold. Typically, in the stressed scenario, acquisition policies are tightened to control the origination FICO score mix

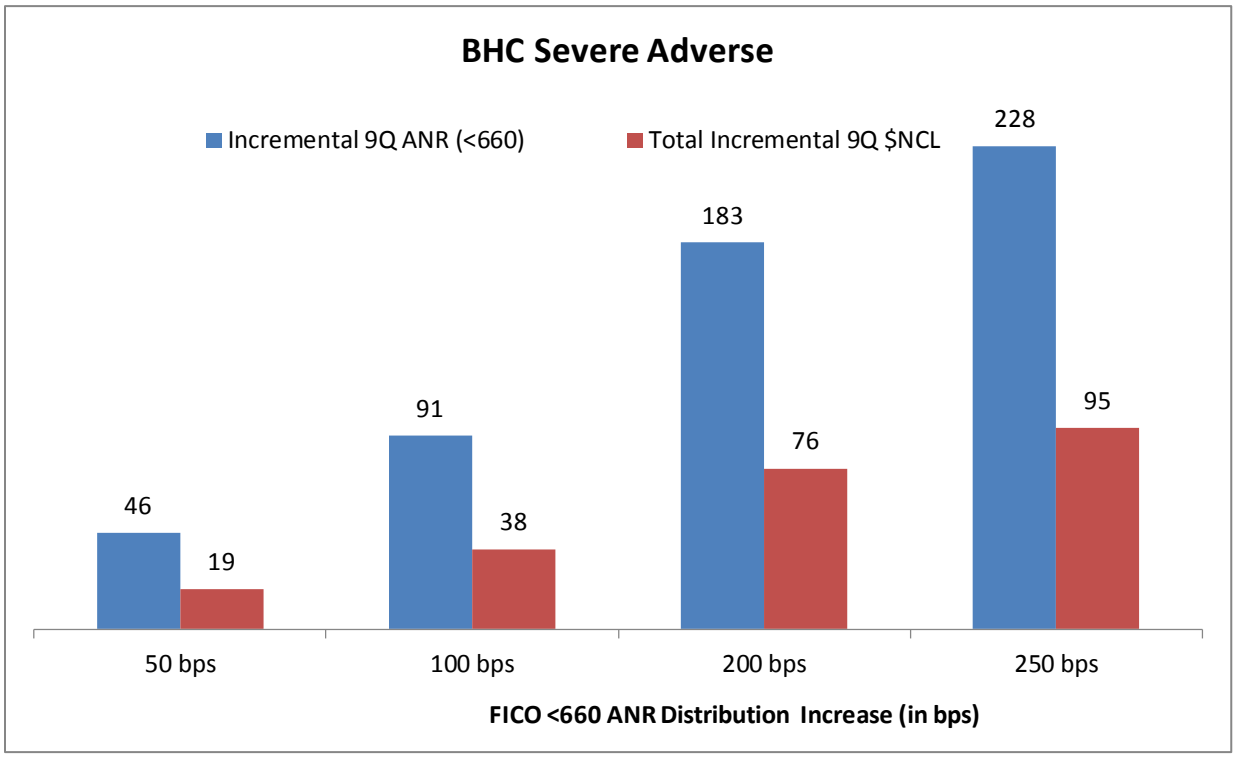
Since last recession, there have been put in place several governance mechanisms to take pro-active credit actions in anticipation of credit quality or performance deterioration. Specifically, instituted are Portfolio Stress Early Warning (PSEW) triggers with the objective to focus Risk Management attention on *external* economic impacts (e.g. Yield Curve) which will trip before internal monitoring triggers begin to trip. Additionally, there is in place well defined defined Risk

Appetite Framework (RAF) through which we track early vintage performance and concentration by risk score bands against pre-established guardrails and triggers

**Approach to performing the sensitivity and impact analyses**

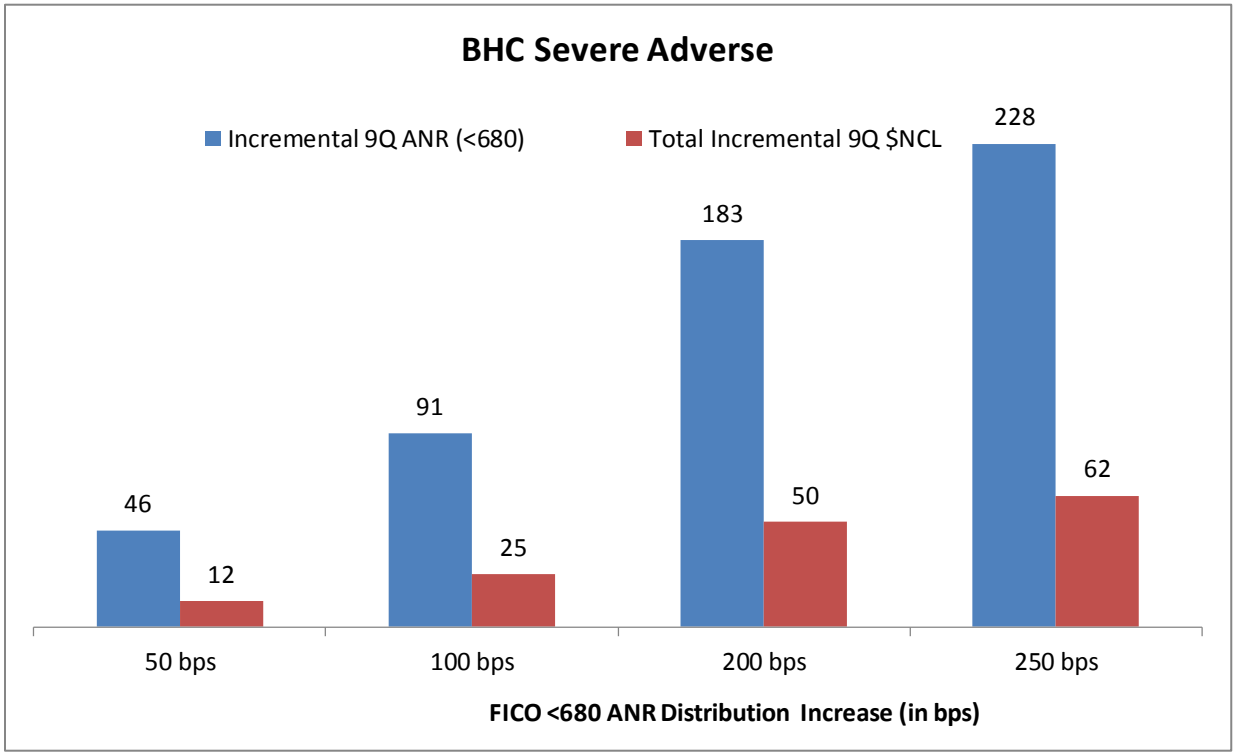
A NCL sensitivity analysis was conducted for potential changes in the originations FICO score distribution. For the purpose this analysis, the percentage of originations with FICO scores <660 was increased by 50 bp, 100 bp, 200 bp and 250 bp and the resulting increase in Average Net Receivable (ANR) and 9Q NCL was calculated under the BHC Stress (MCST severe adverse scenario).

Figure 131: Incremental NCL due to increase in ANR




Below is the same analysis for FICO <680. Incremental NCL (for 250bps increase in originations) is $62MM compared to $95MM for FICO< 660 as the 9Q $NCL% for FICO<680 is 53% lower than 9Q $NCL% for FICO<660

Figure 132: Incremental NCL due to increase in ANR




**Assumption #5:** No changes to regulatory environment

**Background**

Account level CCAR models for US Cards portfolios assume that there are no major changes to government regulation or regulatory policy interventions that could affect Citi's underwriting, payment requirements and collections practices.

**Likelihood**

This assumption is likely to hold. Typically, regulatory changes do not occur without warning and there is a waiting period before changes take effect. The waiting period would allow Citi to take early action to mitigate potential NCL impact.

**Approach to performing the sensitivity and impact analyses**

NCL sensitivity analysis was conducted for potential changes in Collections practices that would restrict Citi as well as third party agencies to fewer Collections outbound calls.

For the purposes of analysis, the results from a prior test were leveraged, where call intensity was dropped from 4 times a day to 2 times a day. Data and roll impact was provided by the Collections team. Based on the test results, 0-WO bucket rate stress by ~35bps was observed. It was assumed that all of the stress in the 0-WO buckets flows in to GCL rate stress (i.e. 35bp increase in GCL rate). After netting down for adjustment and refunds and potential recoveries, NCL rate impact was estimated to be ~25bps.

The restriction on call intensity was assumed to begin from January 2017 and Baseline 9Q NCL impact was calculated to be ~$181MM. After applying stress to account for stress scenarios, 9Q NCL impact is estimated to be $314MM for MCST severely Adverse scenario.

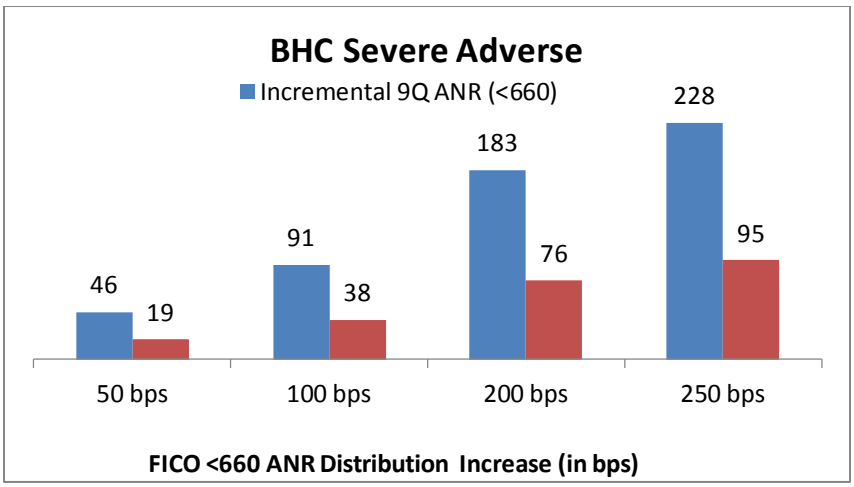
- What are the sensitivity test results?

**Assumption #1: Constant quality of new originations**

***Impact***

The resulting impact of sensitivity testing on the originations assumption is depicted below.

**Figure 133: Impact of increase in ANR On NCL**




| Portfolios | 9Q BHC Stress Loss Impact |
|---|---|
| Branded Cards | 0.1Bn |

As observed the impact on NCL with a 250 bps increase in ANR is 0.1Bn which is less than 1% of the total losses in the stress scenario. Hence the business is comfortable with the results since it has been observed that in the recent period the model over predicts by ~5%.

The detailed results are given in the attached sheet.

Sensitivity_analysis_n
ewaccounts_assumpti

**Assumption #5:** No changes to regulatory environment

***Impact***

| Portfolios | 9Q BHC Stress Loss Impact |
|---|---|
| Branded Cards | $0.3B |

The impact of regulatory changes is ~0.3Bn which is less than 1% of the total stress losses. Hence the business is comfortable with the results.

The detailed results are given in the attached sheet:

Assumptions
Testing_BC_regulatory

## 8.5 Benchmark Model Results and Other Triangulation Analyses

**Table 8.4.3.1: PD Back test CCAR 2018 and CCAR 2017 Account level Models**

| | CCAR 2018 Acct Level | | CCAR 2017 Acct Level | |
|---|---|---|---|---|
| | Recent | Stress | Recent | Stress |
| Q1 | 1.2% | 4.5% | -3.3% | 5.3% |
| Q2 | 0.3% | 3.3% | -3.5% | 3.8% |
| Q3 | 2.2% | -4.3% | -1.6% | -2.7% |
| Q4 | 4.9% | -2.9% | 0.7% | 1.2% |
| Q5 | 6.1% | -2.9% | 1.5% | -0.5% |
| Q6 | 6.5% | -2.1% | 3.0% | -1.6% |
| Q7 | 3.9% | -0.6% | 1.8% | 2.3% |
| Q8 | 6.9% | 1.8% | 0.1% | 4.5% |
| Q9 | 4.5% | 1.5% | 4.1% | 1.1% |
| 9 quarter | 3.89% | -0.27% | 0.08% | 1.39% |

Table 8.4.3.1: EAD Back test CCAR 2018 and CCAR 2017 Account level Models

| | CCAR 2018 Acct Level | | CCAR 2017 Acct Level | |
|---|---|---|---|---|
| | Recent | Stress | Recent | Stress |
| Q1 | 2.2% | 0.0% | -4.2% | 2.0% |
| Q2 | 2.6% | -1.6% | -1.6% | 1.0% |
| Q3 | -1.4% | -1.3% | -0.8% | -6.0% |
| Q4 | -0.8% | -1.0% | 1.9% | -7.0% |
| Q5 | 0.7% | 1.6% | 4.3% | -6.0% |
| Q6 | 1.5% | 5.1% | 2.2% | -3.0% |
| Q7 | 1.4% | 2.6% | 5.5% | 0.0% |
| Q8 | 1.4% | -2.7% | 10.0% | 4.0% |
| Q9 | 3.1% | -7.0% | 10.1% | 0.0% |
| 9 quarter | 1.21% | -0.53% | 2.94% | -1.53% |

Table 8.4.3.1: GCL Back test CCAR 2018 and CCAR 2017 Account level Models

| | CCAR 2018 Acct Level | | CCAR 2017 Acct Level | |
|---|---|---|---|---|
| | Recent | Stress | Recent | Stress |
| Q1 | 2.9% | 0.7% | -6.5% | 1.2% |
| Q2 | -1.2% | -1.6% | -0.8% | 1.4% |
| Q3 | 0.5% | -2.6% | 0.7% | -4.8% |
| Q4 | 7.1% | 0.7% | 5.9% | 2.3% |
| Q5 | 11.0% | 1.5% | 9.5% | 0.3% |
| Q6 | 11.2% | 0.5% | 9.2% | -2.2% |
| Q7 | 13.1% | -4.7% | 10.8% | 0.5% |
| Q8 | 17.4% | -5.2% | 10.6% | 9.2% |
| Q9 | 16.0% | -9.4% | 13.0% | 0.9% |
| 9 quarter | 8.00% | -2.56% | 5.40% | 1.08% |

Table 8.4.3.1: Recovery Back test CCAR 2018 and CCAR 2017 Account level Models

| | CCAR 2018 Acct Level | | CCAR 2017 Acct Level | |
|---|---|---|---|---|
| | Recent | Stress | Recent | Stress |
| Q1 | 0.6% | -12.2% | 11.7% | -7.3% |
| Q2 | 2.9% | -9.4% | -0.4% | -4.0% |
| Q3 | 4.8% | -6.8% | -8.7% | -0.4% |
| Q4 | -4.8% | 9.5% | -6.9% | 10.8% |
| Q5 | 6.2% | 3.5% | 8.4% | 1.5% |
| Q6 | -4.5% | -0.9% | 5.3% | 2.8% |
| Q7 | -7.6% | -9.8% | -1.3% | 2.2% |
| Q8 | -8.8% | -2.3% | 5.0% | -5.9% |
| Q9 | -1.8% | 1.2% | 19.4% | -8.5% |
| 9 quarter | -1.20% | -3.09% | 3.24% | -1.41% |

The benchmark templates are attached below

| PD | EADF | GCL | RECOVERY | NCL |
|---|---|---|---|---|
| Benchmarking PD Template .xlsx | Benchmarking EADF Template .xlsx | Benchmarking GCL Template .xlsx | Benchmarking Recovery Template.xls | Benchmarking NCL Template.xlsx |

## 8.6 Nested Model Error

There is no nested error in the models. However, as NCL is a compound of the Account Level PD and EAD Models as well as the Recovery Rate Model there is the potential for Compound Model Error. However as shown in section 8.3 the NCL Back testing results are well within the MRM guidelines and thus the risk of compound model error does not impact the total results.

## 8.7 Other Performance Tests (if Applicable)

[Please describe any other performance tests conducted, including the types of tests, outcomes, and results]

# 9 References

[Please provide list of the references used to complete the MDD. Also attach documents if available]

All references have been mentioned as footnotes, wherever applicable.

# 10 Appendix

[Please enclose any documents or supporting exhibits referenced in the template exhibits.]

***Example:*** Scenario File, Vendor Supplier References, Grouping Results.