Multiple view Geometry: Exercise paper 2

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Exercise 1

a)

Translation matrix

$$T = \begin{bmatrix} I & T(t) \\ 0 & 1 \end{bmatrix}$$

b)

Rotation matrix

$$R = \begin{bmatrix} R(t) & 0 \\ 0 & 1 \end{bmatrix}$$

c)

Rotation followed by translation

$$R = \begin{bmatrix} R(t) & T(t) \\ 0 & 1 \end{bmatrix}$$

d)

Translation followed by rotation

$$R = \begin{bmatrix} R(t) & R(t)T(t) \\ 0 & 1 \end{bmatrix}$$

Exercise 2

$\Rightarrow$
Since we can choose any vector and to be x, we can choose it to me unit basis vector. Then we let $M = M_1 - M_2$ and we have $e_i^t M e_i = M_{ii} = 0$ for every i. For every $i \neq j$ we have $(e_i + e_j)^t M(e_i + e_j) = M_{ii} + M_{jj} + M_{ij} + M_{ji} = 0$, hence $M_{ij} = -Mji$ and matrix is is skew-symmetric
$\Leftarrow$
Using property of skew-symmetric matrices $M = -M^t$ and we have $x^t M x = (x^t M x)^t = -(x^t M x)$ Hence $x^t M x = 0$

Exercise 3

a)

$\hat{w}^3 = \hat{w}(ww^t - I) = (w \times w)w^t - \hat{w}I = \hat{w}$

b)

For the sake of my mental health I won't prove by induction but from first few terms it seems like formula for odd is $\hat{w}^{2n+1} = (-1)^n \hat{w}$ and for even $\hat{w}^{2n} = (-1)^{n+1}\hat{w}^2$

c)

For Rodrigues formula we let $v = \frac{w}{\|w\|}$ then we can write Taylor series for $e^{\hat{w}} = \sum_{n=0}^{+\infty} \frac{\hat{v}t^n}{n!} = I + \sum_{n=0}^{+\infty} \frac{t^{2n}}{(2n)!}\hat{v}^{2n} + \sum_{n=0}^{+\infty} \frac{t^{2n+1}}{(2n+1)!}\hat{v}^{2n+1}$ using results from part b and definition of cosine/sine Taylor series we get $I + \frac{\hat{w}^2}{\|w\|^2}(1 - cos(\|w\|) + \frac{\hat{w}}{\|w\|}(sin(\|w\|)$