Multiple View Geometry: Exercise Paper 1

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Exercise 1

a)

Since number of vectors matches the dimension of $\mathbb{R}^3$ we can form a matrix if $det(A) \neq 0$ they are linearly independent

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

det(A) = 1 Hence the vectors are linearly independent, since they are linearly independent they span $\mathbb{R}^3$, and since they are linearly independent and span $\mathbb{R}^3$ they form basis in $\mathbb{R}^3$

b)

$$\begin{cases} 2a + 1b = 0 \\ 1a + 1b = 0, \quad a, b \in \mathbb{R} \\ 0a + 0b = 0 \end{cases} \iff \begin{cases} a = -b \\ 2a = -b, \quad a, b \in \mathbb{R} \end{cases}$$

So a = b = 0 and vectors are linearly independent, but since dim(span(v1,v2)) $\leq$ 2, while dim($\mathbb{R}^n$) = n

c)

The four vectors are linearly dependent, but we can form matrix where rows of matrix are linearly independent hence span $\mathbb{R}^3$, but since four vectors are not linearly independent they can not form basis in $\mathbb{R}^3$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Exercise 2

a)

By counterexample we have :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 4 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 8 \\ 9 & 12 & 15 \end{bmatrix}$$

The first two matrix are members of the set and since resulting matrix is not member of set it is not closed under multiplication hence not a group

b)

Since det(I) = 1, the set does not have identity element, hence not a group

c)

We can prove that it is a subgroup of GL if A and B are in G_3 and AB^{-1} in G_3. det(AB^{-1}) = det(A)det(B)$^{-1}$ = det(A) $\frac{1}{det(B)} \geq 0$

Exercise 3

We know that in $\mathbb{R}^3$ we have at most three linearly independent vectors, without loss of generality we have $v_1 = a_2v_2 + a_3v_3 + a_4v_4 + a_5v_5$, than we can write $\|v_1\|$ $= <v1, v1> = <v_1, a_2v_2 + a_3v_3 + a_4v_4 + a_5v_5> = a_2<v_1, v_2> + a_3<v_1, v_3> +$ $a_4<v_1, v_4> + a_5<v_1, v_5> = 0 + 0 + 0 + 0 = 0$ and that contradicts assumtion about non zero vectors

Exercise 4

SO(n) - special orthogonal group, O(n)- general orthogonal group, GL(n) - general linear group, SL(n)- special linear group, SE(n) - special euclidean group, E(n) - euclidean group, A(n) - affine group

Exercise 5

We have $\lambda_a < v_a, v_b >=< \lambda_a v_a, v_b >=< Av_a, v_b >=< v_a, A^t v_b >=< v_a, Av_b >=<$ $v_a, \lambda_b v_b >= \lambda_b < v_a, v_b >$ and since v_a and v_b are not orthogonal we have equal eigenvalues

Exercise 6

For a symmetric matrix A, it can be diagonalized as A= VDV^t, where V=$[v_1, \cdots, v_n]$ is the matrix of orthonormal eigenvectors and D=diag$(\lambda_1, \cdots, \lambda_n)$ is the diagonal matrix of eigenvalues. So we can decompose our expression as

$$(c^tV^t)VDV^t(Vc) = c^tDc = \sum_{i=1} c_i^2\lambda_i$$

The minimum is achieved when weight c_i put on smallest eigenvalue

Exercise 7

$\Rightarrow$
$A^tAx = A^t0 =$. Hence x $\in$ ker(A^tA)
$\Leftarrow$
$0 = x^tA^tAx =< Ax, Ax >$ Hence $Ax = 0$

Exercise 8

a)

A is $m \times n$, U is $m \times m$, S is $m \times n$ and V is $n \times n$

b)

Both matrix diagonalization but Eigenvalue decomposition is only applied to quadratic matrices

c)

The columns of V are eigenvectors of A^tA, the columns of U are eigenvalues AA^t

d)

The number of non-zero singular values is equal to the rank of A