1. Suppose you have a sinusoidal signal with an actual frequency of $f = 15$ Hz. If the signal is sampled at a rate of $\Delta = 20$ Hz, what frequency would be observed in the sampled data. Explain the phenomena.

2. Compute the discrete Fourier transform of the realization $\boldsymbol{x} = \{2, 1, 0, 1\}$ and then compute the inverse discrete Fourier transform to retrieve the original data.

3. Evaluate and represent graphically

$$S = \sum_{j=0}^{8} e^{2\pi i \frac{j}{10}}$$

4. Consider a sinusoid $x_t = R\cos(2\pi(ft+\phi))$. Determine the effect of changing the time origin and scale $u = \frac{t-a}{b}$ on the amplitude, phase, and frequency.

5. In the lecture, we saw that fitting a sinusoid $x_t = R\cos(2\pi(ft + \phi))$ with known frequency f to a time series $\{x_0, \ldots, x_{n-1}\}$ could be achieved by minimizing the sum of squared residuals.

   Derive the solution of this optimization problem, assuming that the number of observations n is an integer multiple k of the period $P = 1/f$, i.e., $n = kP$.