1. Consider the stochastic process $X_t = t + W_t$, where W_t is an iid white noise process with $Var(W_t) = \sigma^2$.

   (a) Compute the auto-covariance function of X_t. Based on this, what would the theoretical ACF be if we tried to apply the standard formula? Comment on whether this is valid.

   (b) Figure 1 represents the ACF generated from a realization of X_t with 1000 values. Explain why it looks different from your result above.

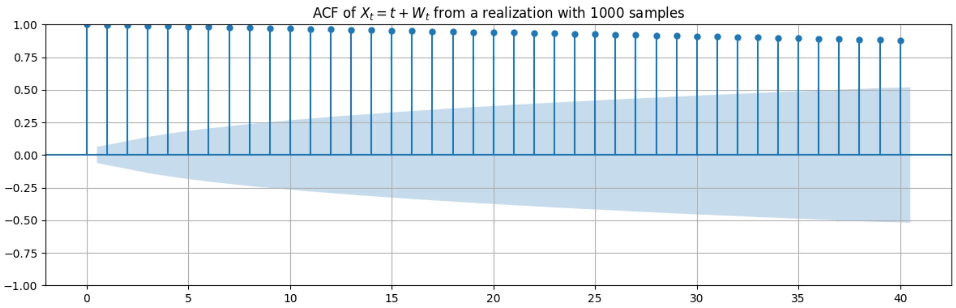

Figure 1: ACF from a realization of X_t with 1000 values

2. For each of the provided stochastic processes, compute its mean, variance, and auto-covariance $cov(X_s, X_t)$, and determine whether it is stationary. In the following, consider that W_t is an iid white noise process with $Var(W_t) = \sigma^2$.

   (a) $X_t = 5 + W_t$.

   (b) $X_t = t \cdot W_t$.

   (c) $X_t = 2W_t + W_{t-1}$.

   (d) $X_t = (-1)^t W_t$

3. Make each of the following stochastic processes stationary without using decomposition, and review any induced artificial dependencies. In the following, consider that W_t is an iid white noise process with $Var(W_t) = \sigma^2$.

   (a) $X_t = t^2 + W_t$

   (b) $X_t = \cos\left(\frac{2\pi t}{12}\right) + W_t$

   (c) $X_t = t + \cos\left(\frac{2\pi t}{6}\right) + W_t$

   (d) $X_t = t^2 + \sin\left(\frac{2\pi t}{4}\right) + W_t$