

1. Given the AR(2) model $X_t = \frac{1}{2}X_{t-1} + \frac{1}{5}X_{t-2} + W_t$ where $W_t \sim \mathcal{N}(0, \sigma^2)$, determine whether the process is stationary.
2. Consider the MA(1) process $X_t = W_t + \theta W_{t-1}$, where $W_t \sim \mathcal{N}(0, \sigma^2)$. Show that the autocorrelation function (ACF) is zero for all lags greater than 1.
3. Consider the stationary ARMA(1,1) process $X_t = \phi X_{t-1} + W_t + \theta W_{t-1}$ where $W_t \sim \mathcal{N}(0, \sigma^2)$. Derive the MA representation of X_t .
4. Consider the following two ARMA processes:

$$X_t = \frac{1}{2}X_{t-1} - \frac{1}{4}W_{t-1} + W_t$$

and

$$Y_t = \frac{5}{6}Y_{t-1} - \frac{1}{6}Y_{t-2} - \frac{7}{12}W_{t-1} + \frac{1}{12}W_{t-2} + W_t$$

where $W_t \sim \mathcal{N}(0, \sigma^2)$. Show that X_t and Y_t are equivalent.

5. Rewrite the following models as ARMA processes:
 - (a) ARIMA(1,1,1)
 - (b) SARIMA(0,0,0)(1,1,1)_s