Robot End Effector Positioning for Cylinder Grasping

Technical Documentation

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1 Closest Approach to Cylinder with Robot End Effector

1.1 Problem Statement

Given a cylinder with centroid $\mathbf{C} = (x_0, y_0, z_0)$ and axis unit vector $\mathbf{A}$, we aim to position a robot end effector on a circle of radius r around the cylinder's centroid such that:

- The end effector is at the position on the circle closest to the origin
- The end effector's z-axis points toward the cylinder's centroid
- The end effector's orientation maintains constraints appropriate for grasping

1.2 Parametric Circle Construction

We construct a parametric circle of radius r around the cylinder centroid $\mathbf{C}$ as:

$$\mathbf{P}(\theta) = \mathbf{C} + r \cdot (\cos\theta \cdot \mathbf{u} + \sin\theta \cdot \mathbf{v}) \tag{1}$$

where $\mathbf{u}$ and $\mathbf{v}$ are orthonormal vectors perpendicular to the cylinder axis $\mathbf{A}$.

1.2.1 Constructing Orthonormal Basis

To find $\mathbf{u}$ and $\mathbf{v}$:

$$\mathbf{ref} = \begin{cases} (0,0,1) & \text{if } |\mathbf{A} \cdot (0,0,1)| \leq 0.9 \ (1,0,0) \\ \text{otherwise} \end{cases} \ \mathbf{u} \qquad = \frac{\mathbf{A} \times \mathbf{ref}}{|\mathbf{A} \times \mathbf{ref}|} \ \mathbf{v} = \frac{\mathbf{A} \times \mathbf{u}}{|\mathbf{A} \times \mathbf{u}|} \tag{2}$$

1.3 Finding Closest Point to Origin

To find the point on the circle closest to the origin, we minimize:

$$D(\theta) = |\mathbf{P}(\theta)|^2 = |\mathbf{C} + r(\cos\theta \cdot \mathbf{u} + \sin\theta \cdot \mathbf{v})|^2 \tag{3}$$

Expanding:

$$D(\theta) = |\mathbf{C}|^2 + 2r\mathbf{C} \cdot (\cos\theta \cdot \mathbf{u} + \sin\theta \cdot \mathbf{v}) + r^2 \tag{4}$$

Setting $A = -\mathbf{C} \cdot \mathbf{u}$ and $B = -\mathbf{C} \cdot \mathbf{v}$, the minimum occurs at:

$$\theta_{min} = \text{atan2}(B, A) \tag{5}$$

The closest point is then:

$$\mathbf{P}closest = \mathbf{C} + r \cdot (\cos\theta min \cdot \mathbf{u} + \sin\theta_{min} \cdot \mathbf{v}) \tag{6}$$

1.4 End Effector Orientation

The end effector orientation is defined by three orthonormal axes:

$$\mathbf{ee}z = \frac{\mathbf{C} - \mathbf{P}closest}{|\mathbf{C} - \mathbf{P}_{closest}|}\ \mathbf{ee}_y \qquad = \frac{\mathbf{A} \times \mathbf{ee}_z}{|\mathbf{A} \times \mathbf{ee}_z|}\ \mathbf{ee}x = \mathbf{ee}y \times \mathbf{ee}z \tag{7}$$

The rotation matrix $\mathbf{R}$ for the end effector is:

$$\mathbf{R} = \left[\mathbf{ee}x^\top\ \mathbf{ee}y^\top\ \mathbf{ee}z^\top\right]^\top \tag{8}$$

1.5 Robust IK Solution Finding

If the initial IK solution fails, we explore alternative points:

$$\mathbf{P}alt(\theta) = \mathbf{C} + r \cdot (\cos(\theta min + k\Delta\theta) \cdot \mathbf{u} + \sin(\theta min + k\Delta\theta) \cdot \mathbf{v}) \tag{9}$$

where $k \in 1, 2, ..., 11$ and $\Delta\theta = \frac{\pi}{6}$ (30 degrees). If angle variations fail, we try different radii:

$$\mathbf{P}alt(r_i) = \mathbf{C} + r_i \cdot (\cos\theta min \cdot \mathbf{u} + \sin\theta min \cdot \mathbf{v}) \tag{10}$$

where $r_i \in 0.3, 0.2, 0.35, 0.15, 0.4$. For each alternative position, the orientation calculation maintains the same constraints, ensuring the end effector's z-axis points toward the cylinder centroid.