

VAGEN: Reinforcing World Model Reasoning for Multi-Turn VLM Agents

Kangrui Wang^{*1}, Pingyue Zhang^{*1}, Zihan Wang^{*1}, Yaning Gao^{*5}, Linjie Li^{*2}, Qineng Wang¹, Hanyang Chen⁶, Chi Wan¹, Yiping Lu¹, Zhengyuan Yang⁴, Lijuan Wang⁴, Ranjay Krishna³, Jiajun Wu³, Li Fei-Fei³, Yejin Choi³, Manling Li¹

¹Northwestern University ²University of Washington ³Stanford University ⁴Microsoft
⁵University of Wisconsin–Madison ⁶University of Illinois Urbana-Champaign

A major challenge in training VLM agents, compared to LLM agents, is that states shift from simple texts to complex visual observations leading to partial observability and demanding visual grounding, world modeling, and spatial reasoning. We ask: what is the most effective way for VLM agents to reason over visual states, and how can we best train them to achieve such capability? In this work, we focus on reinforcement learning (RL) as a training paradigm for VLM agents, which has shown strong promise to improve reasoning. We study five reasoning strategies and three visual state representations used by agents during reasoning, revealing that (1) jointly reason with StateEstimation and TransitionModeling yields best task performance, and (2) natural language representation works well in general, while structured representations are crucial for tasks demanding high precision or fine-grained visual details. These findings highlight that VLM agents’ ability to reason over visual states is central to success, and improving this ability requires turn-level fine-grained, visual state grounded training signals. Targeting this, we introduce Visual Reasoning Reinforcement Learning, which incorporates a turn-level WorldModeling Reward with LLM-as-a-Judge to supervise reasoning accuracy, and Bi-Level General Advantage Estimation (GAE) to estimate advantages at both turn and token levels in RL. Through Visual Reasoning Reinforcement Learning, we enable a 3B model to achieve performance of 0.82 on a set of five diverse agent tasks, nearly 3× improvement over its untrained counterpart (0.21) and surpassing leading proprietary models like OpenAI o3 (0.72). All experiments are supported by our VAGEN framework, a scalable system for training and analyzing multi-turn VLM agents across diverse visual environments.

Website: <https://vagen-ai.github.io>

Code: <https://github.com/ragen-ai/vagen>

Corresponding: {kangrui, manling.li}@u.northwestern.edu

1. Introduction

The core challenge for an agent in multi-turn tasks lies in accurately interpreting and tracking dynamic environments, which becomes significantly harder when the agents sense the world

through images rather than texts. Visual tasks are inherently complex due to the challenges in understanding visual states with object-reference ambiguity, partial observability, and geometric-relationship complexity, which rely on precise grounding, world modeling, and spatial reasoning abilities. While VLMs agents have shown great promise in visual agentic tasks like games (Fan et al., 2022; Vinyals et al., 2019; Hu et al., 2025), embodied AI (Wang et al., 2023a; Shridhar et al., 2021; Yang et al., 2025a), and computer use (Zhou et al., 2024a; Rodriguez et al., 2024; Nishina and Matsui, 2024), current approaches in such multi-turn agentic tasks often lack explicit mechanisms to strengthen visual state reasoning. This lead us to think: can we find effective ways to teach VLMs to reason over visual states?

To address this, our exploration begins by asking: what can VLM agents reason about visual states in multi-turn tasks? We systematically compare several reasoning strategies: NoThink (Li et al., 2025), FreeThink (Wei et al., 2022) (implicit visual state reasoning), and explicit visual state reasoning through StateEstimation (describing current visual state), TransitionModeling (predicting next state), and their combination, WorldModeling. We focus on a multi-turn Reinforcement Learning (RL) framework, which has shown strong promise to improve reasoning (DeepSeek-AI et al., 2025; Ouyang et al., 2025; Chu et al., 2025; Zhou et al., 2025a). Our findings indicate that incorporating explicit visual state reasoning like StateEstimation and TransitionModeling into VLM’s thinking process during RL training can enhance task performance. Notably, the most strong and stable strategy, WorldModeling, achieves an overall performance of 0.76, yielding comparable or even slightly better results than FreeThink (0.67) through simple prompting, while clearly surpassing NoThink (0.28).

Building on the insight that VLM agents might make better decisions when reasoning about what they see now (StateEstimation) and what they will see next (TransitionModeling), a foundation question lies in representation: for an agent to “think” about the visual world, what is the optimal representation for its internal monologue, translating pixels into a mental model it can reason upon? This led us to explore a spectrum of representation possibilities and our findings point to a crucial design principle that the choice of representation is not universal but is dictated by the task’s demands. **Natural language** excels in a general-purpose tasks by providing the robust and inherent representation allowing the VLM to use its vast pre-trained knowledge, but its inherent ambiguity make it unsuitable for high-precision tasks such as robot manipulation, where **structured formats** providing exact coordinates is essential. As a third alternative, **symbolic representation** offers a more abstract representation. Surprisingly, this abstraction does not produce a more generalizable solution. Instead, it proves to be the least effective method in our experiments, creating a grounding problem where the models struggle to connect abstract symbols to raw visual input without targeted training.

Having established what VLM agents should reason about (visual states) and in what ways they can reason (e.g., Grounding and WorldModeling), the remaining question is how to effectively optimize such reasoning abilities. We argue that the key lies in optimizing with fine-grained, visual state-grounded reasoning signals. To this end, we introduce a turn-level WorldModeling Reward, derived using an LLM-as-a-Judge (Gu et al., 2025) framework, which evaluates the accuracy of the agent’s explicit state descriptions and predictions against ground-truth information. To further leverage these intermediate signals and address credit assignment in multi-turn settings, we propose Bi-Level General Advantage Estimation (GAE) (Schulman et al., 2015). Standard GAE methods, which compute advantages token-by-token in a backward manner from the end of a trajectory, would lead to unstable reward propagation because the sparse end-of-trajectory signal must travel across long horizons. To address this, we first compute advantages at the turn level to assess whether a VLM’s response in a turn is generally effective, and then propagate these signals to the token level, providing each VLM-generated

token with fine-grained advantages to optimize its generation process. Our results demonstrate that our VAGEN-Full approach, which combines WorldModeling Reward and Bi-Level GAE, consistently outperforms the VAGEN-Base without these mechanisms, leading to improved reasoning quality and higher task success rates.

Our contributions are fourfold. (1) We provide empirical evidence for the importance of explicit visual state reasoning in multi-turn RL for VLM agents. (2) We analyze how different visual state representations affect reasoning quality and task performance. (3) We introduce WorldModeling Reward to guide visual state reasoning and Bi-Level GAE for fine-grained credit assignment. (4) We present VAGEN, a scalable training framework that decouples environment setup from model training, enabling efficient experimentation and algorithmic extensibility. Together, these contributions establish a principled pathway for developing VLM agents capable of robust visual reasoning in dynamic environments.

2. World Modeling for Visual State Reasoning via Multi-Turn RL

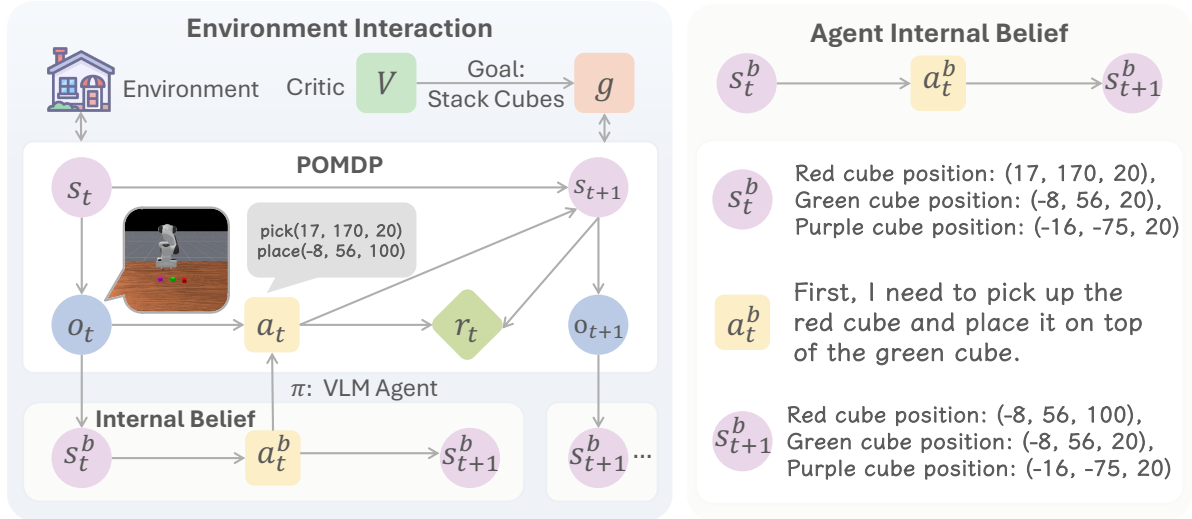


Figure 1 | Overview of the VAGEN Framework. The framework operates in a dual-loop structure: an inner Rollout Loop for data collection and an outer Training Loop for policy optimization.

2.1. Problem Formulation

We frame the multi-turn VLM agentic tasks as a Partially Observable Markov Decision Process (POMDP) (Åström, 1965), represented by the tuple $(\mathcal{S}, \mathcal{O}, \mathcal{A}, P, R, \Omega, \gamma)$. Here, \mathcal{S} denotes the environment state space, \mathcal{O} is the space of observations perceived by the agent, and \mathcal{A} is the space of actions. At each turn t , the agent produces an action $a_t \in \mathcal{A}$. In response, the environment transitions from state s_t to a new state s_{t+1} according to the state transition function $P(s_{t+1}|s_t, a_t)$ and emits a scalar reward $r_t = R(s_t, a_t)$. The agent then receives a new observation $o_{t+1} \in \mathcal{O}$, which is a partial view of the new state, sampled from $\Omega(\cdot|s_{t+1})$. The agent's objective is to learn a policy π_θ that maximizes the expected cumulative discounted return over a trajectory, $\max_\theta \mathbb{E}_{\pi_\theta, P, \Omega} [\sum_t \gamma^t r_t]$, where $\gamma \in [0, 1]$ is the discount factor. In the VLM agent setting, the policy π_θ is parameterized by a VLM that takes in images and textual descriptions as observations, and outputs language token sequences as actions. A summary of important notations can be found in Table 1. A detailed discussion of our POMDP formulation and its motivation can be found in Appendix A.1.

Table 1 | Summary of important notations used in the VAGEN framework.

Notation	Symbol	Description
Observation	o_t	The raw visual observation at turn t .
State	s_t	Ground-truth environment state at turn t .
State (Belief)	s_t^b	VLM’s belief of state s_t at turn t .
Action	a_t	The action tokens at t , $a_t = \langle z_t, a_t^e \rangle$, augmented with reasoning tokens z_t .
Action (Belief)	a_t^b	VLM’s belief of action a_t at turn t .
Action (Executable)	a_t^e	The parsed, executable component of a_t .
Reward	r_t	Scalar reward at turn t , $r_t = R(s_t, a_t)$ where R is the reward function.
Trajectory	τ	Rollout $\tau = (s_0, o_0, a_0, r_0, \dots, s_T, o_T)$.
Tokenized Trajectory	$\bar{\tau}$	Token sequence for VLM training, concatenating encoded observations and actions.
Agent Policy	π_θ	Policy parameterized by a VLM with parameters θ .
Critic / Value Func.	V_ϕ	State-value function parameterized by ϕ .
Token Index	i, j	$\bar{\tau}_i$: the i -th token. $\bar{\tau}_{i:j}$: tokens i – j . $\bar{\tau}_{<i}$: prefix up to $i - 1$. $\bar{\tau}_{t,i}$: the i -th token of the t -th turn.
Trajectory Reward	$R(\tau)$	Sum of rewards over trajectory, $\sum_t R(s_t, a_t)$.
Advantage Estimate	A	A_i : advantage for token i ; A_t^{turn} : advantage for turn t ; $A_{t,i}^{\text{token}}$: advantage for token i in turn t .
Temporal-Difference Error	δ	δ_i : TD-error for token i ; δ_t^{turn} : TD-error for turn t ; $\delta_{t,i}^{\text{token}}$: TD-error for token i in turn t .
Discount Factor	γ	$\gamma \in [0, 1]$; γ_{token} : within-turn discount; γ_{turn} : across-turn discount.

2.2. Visual State Reasoning as World Modeling for Multi-Turn VLM Agents

Existing RL frameworks for VLM reasoning (Shen et al., 2025) are primarily based on single-turn optimization, which limits their ability to capture evolving interaction context. To better address the demands of multi-turn agentic tasks, we optimize the entire interaction trajectory, inspired by Zhou et al. (2025a); Wang et al. (2025a); Zhou et al. (2024b); Zhai et al. (2024); Shani et al. (2024). We propose to maintain an *internal belief* of its interaction that evolves over time, and we design a training framework that integrates *world modeling* into multi-turn trajectory optimization for visual state reasoning, as shown in Figure 1.

Reasoning Trajectory Rollout for Multi-Turn VLM Agents. Each trajectory begins with an initial state s_0 , observation o_0 and goal g . The observation o_t is visual images observed at turn t for VLM agents, which we normally input to VLMs together with corresponding textual prompts. The VLM agent then generates an action a_t using the current policy π_θ . To reason about visual states and build internal beliefs, the generated action a_t is a sequence of text tokens including both reasoning tokens z_t and executable action a_t^e :

$$a_t = \langle z_t, a_t^e \rangle.$$

After a_t^e is parsed and executed, the environment produces a reward r_t as the feedback and transitions to a new state s_{t+1} , providing a new observation o_{t+1} to the agent. This process is repeated over T turns to collect a trajectory $\tau = (s_0, o_0, a_0, r_0, \dots, s_{T-1}, o_{T-1}, a_{T-1}, r_{T-1}, s_T, o_T)$.

Visual State Reasoning as World Modeling. Instead of just acting to what the VLM agent sees, we explicitly reason about visual states to build an *internal belief* of the environment. In agentic tasks, especially those defined as POMDPs, the agent’s visual observation o_t offers a view of the true world state s_t . To act effectively, the agent should interpret what the current visual observation actually represents and predict future state changes. We train the VLM agent to do this by explicitly generating its reasoning tokens z_t as building up an internal belief of a

structured world model (Xing et al., 2025), including a state model that interprets the present (StateEstimation) and a transition model that predicts the future (TransitionModeling). In detail, StateEstimation grounds visual observations o_t into a state belief s_t^b that approximates the hidden true state s_t , highlighting the central challenge of POMDPs compared to fully observable MDPs. Meanwhile, TransitionModeling reasons over the belief of the potential next state s_{t+1}^b , allowing the agent to simulate internally how its action at turn t will transition the states. This predictive step is crucial for planning of a multi-turn trajectory. We implement multiple reasoning strategies, ranging from minimal (NoThink, FreeThink) to structured world modeling (StateEstimation, TransitionModeling, WorldModeling), by shaping the reasoning tokens z_t differently to capture and update internal beliefs about visual states.

1. **NoThink**: we train the VLM agent to generate only an executable action a_t^e , and the output action token a_t is `<think></think><answer>a_t^e</answer>`,

$$z_t = \emptyset$$

2. **FreeThink**: we train the VLM agent to produce any form of natural language reasoning, allowing visual state reasoning to emerge without a predefined structure. The agent generates action tokens as `<think>z_t</think> <answer>a_t^e</answer>`,

$$z_t \neq \emptyset, \text{ and } z_t \text{ is natural language tokens.}$$

3. **StateEstimation**: we train the VLM agent to explicitly verbalize the current state belief s_t^b given the visual observation o_t , to approximate the true underlying state s_t and help better predict the executable action a_t^e . The action token a_t is formatted as `<think><observation>s_t^b</observation> <reasoning>a_t^b</reasoning> </think> <answer>a_t^e</answer>`,

$$z_t = \langle s_t^b, a_t^b \rangle, \text{ learning to approximate } s_t^b \rightarrow s_t$$

4. **TransitionModeling**: we train the VLM agent to explicitly simulate in its internal belief space about the next state s_{t+1}^b , which helps reason over the executable action a_t^e with best expected reward. The output action token a_t is structured as `<think><reasoning>a_t^b</reasoning> <prediction>s_{t+1}^b</prediction> </think> <answer>a_t^e</answer>`,¹

$$z_t = \langle a_t^b, s_{t+1}^b \rangle, \text{ learning to approximate } s_{t+1}^b \rightarrow s_{t+1}$$

5. **WorldModeling**: z_t is required to both describe the current state and predict the next state: `<think><observation>s_t^b</observation> <reasoning>a_t^b</reasoning> <prediction>s_{t+1}^b</prediction> </think> <answer>a_t^e</answer>`,

$$z_t = \langle s_t^b, a_t^b, s_{t+1}^b \rangle, \text{ learning to approximate } s_t^b \rightarrow s_t, s_{t+1}^b \rightarrow s_{t+1}$$

To encourage strict adherence to such a structured action format, we incorporate a format reward r_t^{format} during training, following the strategy of DeepSeek-R1 (DeepSeek-AI et al., 2025). The reward design is detailed in Section 2.3.

Policy Optimization. Once trajectories are collected, we begin the optimization phase using an actor-critic approach. The actor’s policy π_θ is updated using the Proximal Policy Optimization

¹For simplicity, we represent the belief of next state in t -th turn as s_{t+1}^b . Please note that in the turn of $t+1$, we re-generate state belief s_{t+1}^b given the parallel observation o_{t+1} , rather than using the predicted state belief in the t -th turn.

(PPO) objective (Schulman et al., 2017). Denote $\bar{\tau}$ as a token sequence converted from the trajectory τ with an encoder \mathcal{E} , $u_i(\theta) = \frac{\pi_\theta(\bar{\tau}_i|\bar{\tau}_{<i})}{\pi_{\text{old}}(\bar{\tau}_i|\bar{\tau}_{<i})}$ as the probability ratio between the current and old policies, and let $\bar{\tau}_{<i}$ denotes the prefix of token i . The PPO loss is defined as:

$$J^{\text{PPO}}(\theta) = \frac{1}{\sum_i M_i^{\text{loss}}} \sum_i M_i^{\text{loss}} \cdot \min(u_i(\theta)A_i, \text{clip}(u_i(\theta), 1 - \varepsilon, 1 + \varepsilon)A_i),$$

where M_i^{loss} is a mask that is 1 for action tokens and 0 for observation tokens, A_i is the per-token advantage and ε is a clipping hyperparameter.

Advantage and Value Estimation. Concurrently, the critic parameters φ of the value function V_φ are updated by minimizing the squared error between its predictions and the target values Y_i :

$$J^{\text{Critic}}(\varphi) = \frac{1}{\sum_i M_i^{\text{loss}}} \sum_i M_i^{\text{loss}} \cdot (V_\varphi(\bar{\tau}_{<i+1}) - Y_i)^2$$

The actor and critic updates require computing per-token advantages A_i and target values Y_i . In our VAGEN-Base setting, we use Token-Level Generalized Advantage Estimation (GAE) (Schulman et al., 2015). At each token index i , we apply a per-token KL penalty that encourages the current policy π_θ to stay close to a frozen reference policy π_{ref} . The penalty is scaled by a coefficient $\beta > 0$. For all intermediate action tokens, reward r_i is set to the KL penalty. At the final action token I , reward r_I is set as the sum of KL penalty and total trajectory return $R(\tau) = \sum_{t=0}^{T-1} R(s_t, a_t)$.

We then calculate the temporal-difference (TD) error using a discount factor γ and the GAE parameter λ :

$$\begin{aligned} \delta_i &= r_i + \gamma V_\varphi(\bar{\tau}_{<j}) - V_\varphi(\bar{\tau}_{<i}) \\ A_i &= \delta_i + \gamma \lambda A_j \end{aligned}$$

This recursion is initialized at the end of the sequence with $A_I = \delta_I$, j denotes the index of the next action token after token i (skipping the observation tokens). The target value for the critic update is defined as

$$Y_i = A_i + V_\varphi(\bar{\tau}_{<i}).$$

. The iterative process of trajectory collection, advantage estimation, and policy update continues until convergence. We present a detailed illustration of our multi-turn agent RL framework in Algorithm 1.

2.3. Reward Design in Different Environments and Tasks for VLM Agents

To systematically analyze the learning dynamics and visual reasoning capabilities of VLM agents, we developed an evaluation suite featuring five distinct agentic tasks (Figure 2). These tasks were chosen to cover a wide range of challenges, including diverse visual state representations and action spaces: **Classic Grid Puzzles** (Sokoban and FrozenLake), **Embodied 3D Navigation** (Navigation), **Detailed Object Manipulation** (PrimitiveSkill), and **Abstract Geometric Reconstruction** (SVG Reconstruction).

Sokoban (Schrader, 2018): In this classic puzzle, the agent must push all boxes to target locations. The visual state is a 2D grid, and the action space is discrete (up, down, left, right).

FrozenLake (Towers et al., 2024): The agent navigates a 2D grid to reach a goal while avoiding holes. The visual state and discrete action space are similar to Sokoban. We disable the "slippery" setting for determinism.



Figure 2 | Visual examples from the five environments used in our study.

Table 2 | Example of how visual states be converted into natural language, symbolic and structured representations

Visual State	Natural Language	Symbolic	Structured
	"The player is at the upper-left, the box is to the right of the player, the target is below the player"	P_O, _X_, #_	{'player':[0,0], 'box':[1,1], 'target':[2,0], 'wall': [0,2]}

Navigation (Yang et al., 2025a): In this 3D embodied task, the agent follows instructions to find an object, perceiving the world through a first-person view and using discrete actions (e.g., moveahead).

PrimitiveSkill (Srivastava et al., 2025; Nasiriany et al., 2022; Hiranaka et al., 2023): The agent controls a Panda Arm to perform complex manipulation, using a hybrid action space (e.g., pick(x, y, z)). The agent must ground objects in the third-person 3D scene to a list of coordinates.

SVG Reconstruction (Rodriguez et al., 2024): The agent’s goal is to generate SVG code that replicates a target image using an open-ended text action space.

Reward Strategy and Metrics. For the SVG task, the reward $R(s_t, a_t)$ is a dense similarity between the generated image I_{gen} and the target I_{target} , computed as a weighted combination of DreamSim and DINO scores. Task performance is reported as the average DreamSim $\mathbb{E}_{\tau \sim \mathcal{D}}[g(\tau)]$ and DINO $\mathbb{E}_{\tau \sim \mathcal{D}}[h(\tau)]$ similarity of the final output. For the other tasks, $R(s_t, a_t)$ is a scaled binary reward (e.g., $\{0, 10\}$) indicating whether the trajectory completes the objective, and performance is measured by the average success rate $\mathbb{E}_{\tau \sim \mathcal{D}}[f(\tau)]$, where $f(\tau) \in \{0, 1\}$. Full implementation details are provided in Appendix A.4.

3. What Can We Reason About Visual States?

Forcing Off-the-Shelf VLMs to Reason Cannot Solve Multi-Turn Agentic Tasks. We benchmark 7 models including proprietary models and open-source models in Table 3. We use different reasoning strategies to prompt models, which are detailed in Appendix B.1. Notably, reinforcing VLMs to solve non-agentic tasks like VLM-R1 does not benefit multi-turn agentic performance. However, most models struggle on these tasks, with the best-performing one reaching 0.72 out of 1 in overall score. Particularly, no model succeeds on the PrimitiveSkill Drawer task. These results indicate a significant gap in current VLMs when handling complex multi-turn visual agentic tasks that requires reasoning.

World Modeling can Improve Visual Reasoning via Multi-turn RL. To examine whether explicit reasoning about visual states improves performance, we train the Qwen2.5-VL-3B

Table 3 | Reasoning on visual states, including both grounding and world modeling, can significantly improve the RL performance. Test success rates are reported for the first four tasks. Test Dino and DreamSim scores are reported for SVG Reconstruction task. Best performance is in **bold**. We gray out results without controlled variables (i.e., same size and architecture), shown for reference only.

Model/Method	Sokoban	FrozenLake	Navigation			PrimitiveSkill					SVG			Overall
			Base Common	Average		Place	Stack	Drawer	Align	Average	Dino	DreamSim	Average	
Open-Source Models														
Qwen2.5-VL-72B (Bai et al., 2023)	0.18	0.44	0.72	0.75	0.73	0.63	0.50	0.00	0.63	0.44	0.89	0.62	0.76	0.51
Qwen2.5-VL-7B (Bai et al., 2023)	0.13	0.14	0.28	0.39	0.34	0.00	0.00	0.00	0.75	0.19	0.83	0.28	0.55	0.27
Qwen2.5-VL-3B (Bai et al., 2023)	0.14	0.14	0.22	0.27	0.24	0.00	0.00	0.00	0.00	0.00	0.80	0.27	0.54	0.21
VLM-R1-3B (Shen et al., 2025)	0.13	0.13	0.31	0.34	0.33	0.00	0.00	0.00	0.00	0.00	0.81	0.28	0.55	0.23
VAGEN: Multi-Turn RL with World Model Reasoning for Visual States (Backbone: Qwen2.5-VL-3B)														
FreeThink	0.57	0.68	0.67	0.67	0.67	1.00	0.63	0.00	1.00	0.66	0.91	0.64	0.78	0.67
NoThink	0.57	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.89	0.62	0.76	0.28
StateEstimation	0.56	0.68	0.78	0.69	0.74	0.00	0.00	0.00	0.00	0.00	0.92	0.64	0.78	0.56
TransitionModeling	0.41	0.76	0.67	0.59	0.62	1.00	0.63	0.63	1.00	0.82	0.89	0.64	0.77	0.68
WorldModeling	0.61	0.71	0.78	0.80	0.79	1.00	0.88	0.88	0.88	0.91	0.90	0.65	0.78	0.76
RL Baselines with World Model Reasoning Strategy (Backbone: Qwen2.5-VL-3B)														
Vanilla-PPO	0.18	0.21	0.32	0.25	0.29	0.00	0.00	0.00	0.00	0.00	0.83	0.44	0.64	0.26
GRPO w/ Mask	0.20	0.57	0.88	0.81	0.85	0.00	0.00	0.00	1.00	0.25	0.92	0.66	0.79	0.54
Turn-PPO w/ Mask	0.38	0.70	0.78	0.84	0.81	0.00	0.00	0.00	1.00	0.25	0.89	0.64	0.77	0.55
Proprietary Models														
o3 (OpenAI, 2025)	0.60	0.78	0.81	0.75	0.78	1.00	0.63	0.00	1.00	0.66	0.88	0.61	0.75	0.72
o4-mini (OpenAI, 2025)	0.44	0.82	0.75	0.75	0.75	1.00	0.50	0.00	0.75	0.56	0.90	0.66	0.78	0.67
GPT-4o (OpenAI, 2024a)	0.43	0.54	0.75	0.69	0.72	0.50	0.63	0.00	0.88	0.50	0.92	0.71	0.81	0.60
Gemini 2.0 (Team, 2025)	0.28	0.61	0.50	0.63	0.56	0.75	0.13	0.00	0.25	0.28	0.97	0.79	0.88	0.52
Claude 3.7 Sonnet (The, 2024)	0.25	0.69	0.48	0.47	0.47	0.63	0.13	0.00	1.00	0.44	0.93	0.76	0.84	0.54

model with five reasoning strategies using VAGEN-Base (Section 2.2). As shown in Table 3, FreeThink consistently outperforms NoThink, particularly in embodied environments like Navigation and PrimitiveSkill, which indicates the importance of explicit reasoning in multi-turn decision-making tasks.

Among different reasoning strategies, StateEstimation and TransitionModeling show task-specific strengths. StateEstimation performs well in Navigation tasks, where understanding current observations is the key. In contrast, TransitionModeling achieves strong results in PrimitiveSkill, where predicting future states is crucial for manipulation. However, each strategy alone may lead to reduced performance in tasks where model’s prior is less aligned with the task structure or state complexity.

The combined strategy, WorldModeling, results in strong and stable performance across all tasks, as the trained model achieves a substantial improvement over its untrained counterpart (+0.55), and even outperforms all proprietary models despite smaller scale. These results demonstrate that explicitly visual states reasoning is crucial for VLM agents. In the following studies, we use WorldModeling as the general visual reasoning strategy.

Existing RL Methods are Inadequate for Multi-Turn VLM Agents. We also compare our VAGEN framework with different RL baselines. Vanilla PPO (Wang et al., 2025a; Jin et al., 2025) fails due to lack of observation token masking. For image/text-to-text models, learning from observation tokens is fundamentally incorrect as image tokens are not part of the model’s generation process. For image/text-to-image/text models, learning from observation tokens might also be problematic because: (1) observation tokens are not generated by the agent’s own policy, and (2) lengthy observation sequences can dominate the learning weight distribution. Group Relative Policy Optimization (GRPO) (Shao et al., 2024) with masking remains insufficient due to high trajectory diversity from scene change, requiring unaffordable sample sizes. Turn-level

Table 4 | Performance comparison across different visual state representations.

Visual State Representation	FrozenLake	Sokoban	PrimitiveSkill				Average
			Place	Stack	Drawer	Align	
Natural-Lanaguage	0.71	0.61	1.00	0.88	0.88	0.88	0.91
Structured	0.63	0.28	1.00	0.88	0.88	1.00	0.94
Symbolic	0.49	0.49	–	–	–	–	–

PPO with masking (Zhai et al., 2024) underperforms because uniform advantage estimates for action tokens within a turn cannot capture individual token contributions to policy performance. These limitations motivate our VAGEN framework’s design for effective visual agent training.

4. How Can We Represent Beliefs about the World?

To further understand visual state reasoning, we investigate how different visual state representations affect task performance. We consider three representations: Natural-Lanaguage, Symbolic, and Structured format. Specifically, during RL training, we prompt models to use the WorldModeling reasoning strategy and require them to output the specific format for the <observation> and <prediction> fields. We conducted these experiments in three tasks: FrozenLake, Sokoban, and PrimitiveSkill. In FrozenLake and Sokoban, we compare all three formats. The Natural-Lanaguage format consists of free-form textual descriptions. The Symbolic format uses environment-native grid-based symbols, and the Structured format requires the model to output a dictionary containing task-specific information such as players’, targets’ and boxes’ positions, which are detailed in Appendix C. For PrimitiveSkill, we compare Natural-Lanaguage and Structured formats.

Results and Insights. Our experiments reveal a clear task-dependent trade-off in the choice of visual state representation. As shown in Table 4, the optimal format varies with the task nature. In FrozenLake and Sokoban, Natural-Lanaguage outperforms Symbolic and Structured formats. This is likely because the model lack sufficient prior knowledge to interpret symbolic layouts effectively, and structured outputs, when derived from image-only input, are noisy due to limited grounding capabilities. In these environments, the flexibility and familiarity of Natural-Lanaguage align better with the model’s capabilities gained from pretraining stage. For PrimitiveSkill, Structured slightly outperforms Natural-Lanaguage. It’s probably because we provided a structured object position list as prompts to the model, allowing the model to ground its understanding more precisely and facilitate more accurate next state prediction. Consequently, for our subsequent studies, we adopt Natural-Lanaguage as the default, general-purpose state representation, while specifically employing the Structured format for the PrimitiveSkill task.

5. Can World Modeling Help with Reward Shaping and Critic Learning?

Recognizing the effectiveness of enforcing VLMs to explicitly reason as a world model on StateEstimation and TransitionModeling, we further explore to explicitly leverage these signals to inform reward structures and optimize the advantage estimation for the reinforcement learning framework.

5.1. WorldModeling Reward

Reward shaping is a common practice to guide the specific agent behavior. We try to introduce a reward that supervises the agent’s understanding of visual states. Specifically, we extract <observation> and <prediction> fields from the agent’s response, compare them with the ground-truth visual states, and give a reward based on the matching score.

Our initial attempt use CLIP(Radford et al., 2021)-based image-text similarity to calculate the reward. However, we found CLIP to be insufficiently sensitive to fine-grained spatial and geometric details, rendering the resulting reward signals unreliable.

To address this limitation, we adopt an LLM-as-a-Judge (Gu et al., 2025) approach. We try to get text-based ground-truth information about the visual state from the environments. For example, in Sokoban, we obtain 2D positions of the player, boxes, and targets; for FrozenLake, we extract 2D positions of the player, target, and holes; for PrimitiveSkill, we derive object names and their coordinates; and for Navigation, we calculate relative distances and directions from objects to the player.

With this text-based state information available, we compute a WorldModeling Reward by assessing the alignment between the agent’s reasoning (in <observation> and <prediction>) and the ground truth states (s_t and s_{t+1}). This is achieved through a hybrid evaluation protocol where an LLM-as-a-Judge either provides a direct judgment or first extracts structured information from the agent’s text for a subsequent rule-based comparison (e.g., F1-score). The reward at each turn t is defined as:

$$r_t^{\text{reason}} = \beta_g \cdot \mathbb{I}_{\text{StateEstimation}}(s_t^b, s_t) + \beta_w \cdot \mathbb{I}_{\text{TransitionModeling}}(s_{t+1}^b, s_{t+1}),$$

² where \mathbb{I} is a generalized matching score (binary from direct judgment or a continuous, normalized score from rule-based metrics), and β_g, β_w are reward coefficients. Details on the LLM-as-a-Judge prompts and evaluation protocols are available in Appendix D.1.

5.2. Bi-Level General Advantage Estimation (GAE)

The VAGEN-Base framework described in Section 2.2 reveals a key limitation when incorporating WorldModeling Reward: by aggregating all task rewards at the final token of a trajectory, it provides only trajectory-level feedback. This coarse signal is propagated backward via a single GAE calculation, making it difficult to assign credit for turn-specific successes or failures, which is especially critical for reinforcing step-by-step visual reasoning. To address this, we introduce **Bi-Level GAE**, a more granular credit assignment mechanism designed to deliver fine-grained, turn-level reward signals. As illustrated in Figure 3, this approach operates in two stages, introducing a turn-level discount factor γ_{turn} for transitions between turns and a token-level discount factor γ_{token} for tokens within a single action.

Turn-level advantage estimation: We compute an advantage estimate for each turn in the trajectory. For a given turn t , let r_t be the total reward assigned to that turn (its composition will be detailed in Section 5.3). We define the turn-level TD-error δ_t^{turn} using the critic’s value estimates V_ϕ at the end of each action sequence:

$$\delta_t^{\text{turn}} = r_t + \gamma_{\text{turn}} V_\phi(\bar{\tau}_{\leq a_{t+1}}) - V_\phi(\bar{\tau}_{\leq a_t}).$$

Here, $\bar{\tau}_{\leq a_t}$ denotes the full token prefix of the trajectory up to and including the action a_t . For the final turn $T - 1$, the next state value $V_\phi(\bar{\tau}_{\leq a_T})$ is considered zero. The turn-level advantage

²See Note 1, we regenerate current state belief and next state belief at each turn.

Table 5 | Test success rates and scores in VAGEN-Base and VAGEN-Full

Model/Method	FrozenLake	Sokoban	Navigation			PrimitiveSkill					SVG			Overall
			Base	Common	Average	Place	Stack	Drawer	Align	Average	Dino	DreamSim	Average	
Qwen2.5-VL-3B	0.14	0.14	0.22	0.27	0.24	0.00	0.00	0.00	0.00	0.00	0.80	0.27	0.54	0.21
Qwen2.5-VL-3B w/ VAGEN-Base	0.71	0.61	0.78	0.80	0.79	1.00	0.88	0.88	0.88	0.91	0.90	0.65	0.78	0.76
Qwen2.5-VL-3B w/ VAGEN-Full	0.74	0.79	0.80	0.81	0.81	1.00	0.88	1.00	1.00	0.97	0.91	0.67	0.79	0.82

A_t^{turn} is then calculated recursively using GAE:

$$A_t^{\text{turn}} = \delta_t^{\text{turn}} + \gamma_{\text{turn}} \lambda^{\text{turn}} A_{t+1}^{\text{turn}}.$$

This backward pass is initialized with $A_{T-1}^{\text{turn}} = \delta_{T-1}^{\text{turn}}$, where $\lambda^{\text{turn}} \in [0, 1]$ is the GAE parameter for inter-turn credit assignment.

Token-level advantage estimation: After computing all turn-level advantages $\{A_0^{\text{turn}}, \dots, A_{T-1}^{\text{turn}}\}$, we perform a second, inner GAE calculation for the tokens within each action a_t . The reward for any given token $\bar{\tau}_i$ within the action a_t is defined as its KL-penalty, $r_i = r_i^{\text{KL}}$. The token-level TD-error and advantage are calculated for all tokens belonging to the action a_t :

$$\begin{aligned} \delta_{t,i}^{\text{token}} &= r_{t,i}^{\text{KL}} + \gamma_{\text{token}} V_{\varphi}(\bar{\tau}_{t,<i+1}) - V_{\varphi}(\bar{\tau}_{t,<i}), \\ A_{t,i}^{\text{token}} &= \delta_{t,i}^{\text{token}} + \gamma_{\text{token}} \lambda^{\text{token}} A_{t,i+1}^{\text{token}}. \end{aligned}$$

The key step linking the two levels occurs here: the backward pass for the token-level advantages is initialized by setting the advantage of the **final token** of action a_t to the pre-computed turn-level advantage, A_t^{turn} . This injects the turn-specific feedback at the end of the action and allows it to be propagated backward to all tokens that generated it. The detailed procedure is outlined in Algorithm 2.

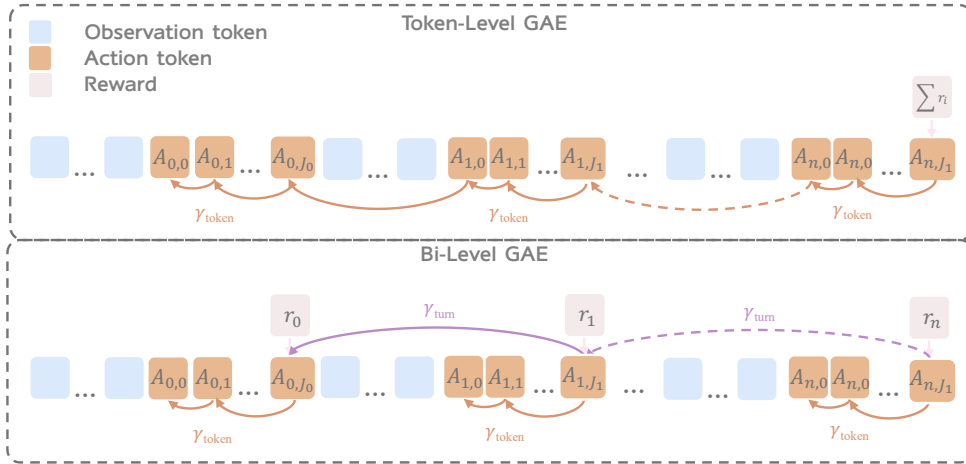


Figure 3 | Token-Level GAE and Bi-Level GAE frameworks.

5.3. VAGEN-Full Multi-Turn Reinforcement Learning Framework

By combining the structured reasoning strategies with our Bi-Level GAE mechanism, we introduce **Visual Reasoning RL**. In this setup, we train the agent using the WorldModeling reasoning strategy to encourage explicit grounding and world modeling. The turn-level reward r_t used in the Bi-Level GAE calculation is defined as a composite sum:

$$r_t = r_t^{\text{reason}} + r_t^{\text{format}} + R(s_t, a_t),$$

where r_t^{reason} is a reward for the quality of the world model reasoning for visual states (s_t^b and s_{t+1}^b), r_t^{format} is the reward for adhering to the specified output structure (Section 3), and $R(s_t, a_t)$ is the sparse, task-specific reward from the environment. The rest of the training pipeline follows the VAGEN-Base procedure (Section 2.2), with the standard GAE module being replaced by the Bi-Level GAE for advantage estimation.

Experiment Setup. We now compare VAGEN-Base with VAGEN-Full across all tasks. The VAGEN-Base (Section 3) uses the WorldModeling reasoning strategy along with format and task-specific rewards. VAGEN-Full builds on this and incorporates Visual Reasoning RL, with the reward coefficients β_g and β_w set to 0.5. For SVG Reconstruction, only Bi-Level GAE is applied. Details regarding training time and token usage is given in Table 27.

Results and Insights. As shown in Table 5, VAGEN-Full achieves consistently better test-time performance across all tasks compared to VAGEN-Base. This gap is especially prominent in PrimitiveSkill: although both methods reach similar training accuracy (Figure 7), VAGEN-Full significantly outperforms VAGEN-Base on the test set. This suggests that StateEstimation and TransitionModeling improve the agent’s ability to adapt to new scenes, leading to better robustness and generalization ability.

5.4. Ablations

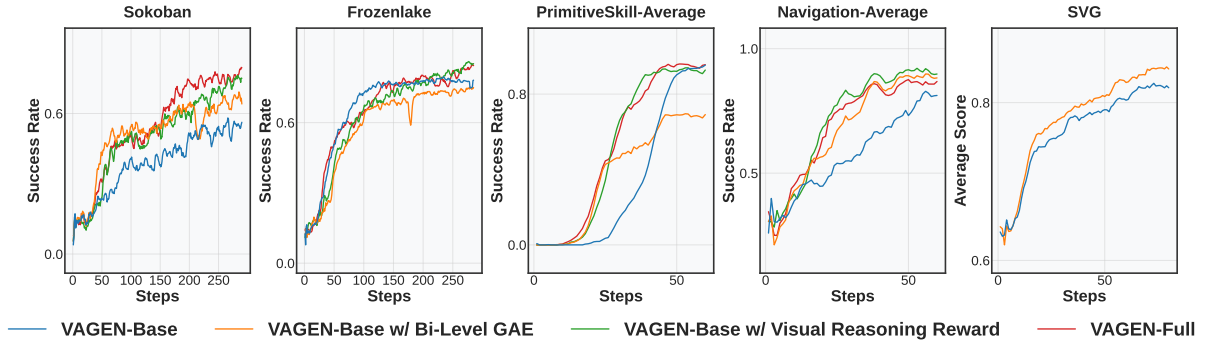


Figure 4 | Training success rates for VAGEN-Base, VAGEN-Full and ablations.

In this section, we study the independent contribution of Bi-Level GAE and WorldModeling Reward to Visual Reasoning RL. We observe interesting patterns from Figure 7:

- **Bi-Level GAE alone** provides significant but inconsistent gains. Its performance is highly sensitive to reward sparsity and accuracy, which can lead to training instability in environments lacking dense and accurate intermediate rewards.
- **The WorldModeling Reward alone** consistently improves upon the baseline by providing a crucial learning signal for visual understanding. However, its effectiveness is limited by the coarse, trajectory-level credit assignment of standard RL.
- **VAGEN-Full** is the most stable among all methods and performs generally well on all tasks.

These observations verify that fine-grained credit assignment (from Bi-Level GAE) and high-quality reasoning supervision (from the WorldModeling Reward) are both essential to effectively improve VLM reasoning. Additional results comparing models (by size and family) and methods are reported in Table 26.

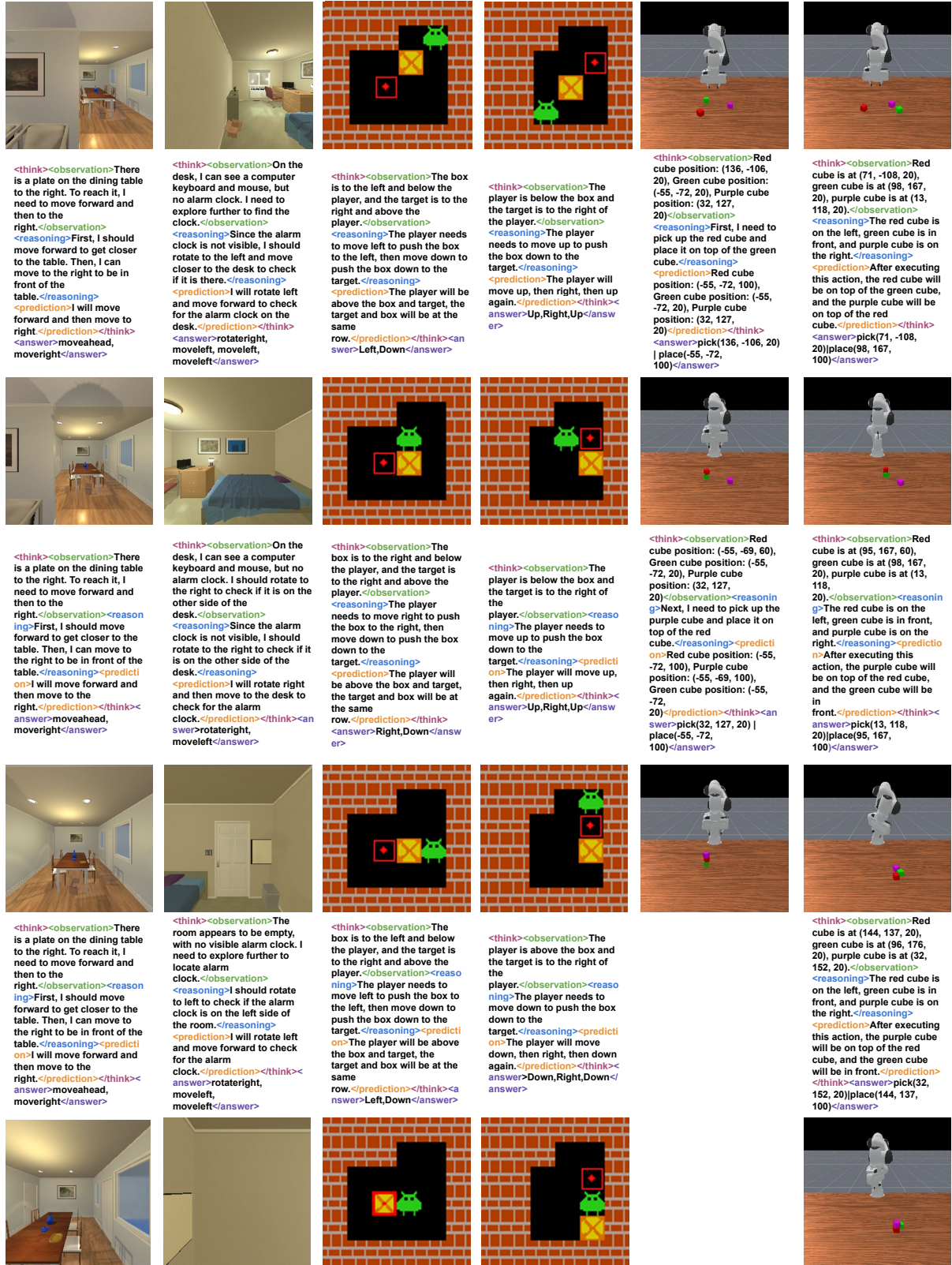


Figure 5 | VAGEN-Full vs. VAGEN-Base in Navigation, Sokoban and Frozenlake (left to right) For the same environment, VAGEN-Full is the left column, VAGEN-Base is the right column.

5.5. Case Studies and Findings

To better understand learning dynamics, we conduct cases studies across three environments: Navigation, Sokoban and FrozenLake, as shown in Figure 5. By analyzing agent behaviors, we identify some key phenomena and detail them in Appendix E.

Enhanced Spatial Understanding and Planning. Explicit visual state reasoning improves agents’ spatial awareness and multi-step planning. Agents trained with VAGEN-Full develops the ability to identify spatial relationships and recognize blocking constraints, leading to more effective navigation and problem-solving strategies.

Response Convergence and Reduced Exploration. A clear pattern of response convergence is observed, which can be quantitatively reflected by a steady decrease in entropy as training progresses. This phenomenon occurs both with and without WorldModeling Reward, suggesting it is a common pattern in RL training. Qualitatively, early-stage agents exhibit diverse, descriptive responses, while late-stage agents develop concise, templated responses.

Reward Hacking and Over-optimization. In certain scenarios, agents learn to hack the reward. They develop generic, broadly applicable responses that satisfy the LLM-as-a-Judge’s criteria without necessarily reflecting deep, state-specific reasoning. This behavior, a form of reward over-optimization, is particularly evident in agents trained with Bi-Level GAE (highlighting its effectiveness in reward maximization). We developed several methods that can mitigate such over-optimization, please refer to Appendix D.1.2.

6. Related Work

RL for LLMs and VLMs. Recent studies have explored RL for both LLMs and VLMs(DeepSeek-AI et al., 2025; Chu et al., 2025; Zhou et al., 2025a; Shen et al., 2025; Zhou et al., 2024b; Zhai et al., 2024; Team, 2025; Ziegler et al., 2019; Stiennon et al., 2020; Bai et al., 2022; Zhou et al., 2025b; OpenAI, 2024b; Sun et al., 2023; Wang et al., 2024a; Yang et al., 2025b; Wang et al., 2024b; Casper et al., 2023; Yu et al., 2025), with approaches ranging from human feedback (DeepSeek-AI et al., 2025; Team, 2025; Ziegler et al., 2019; Stiennon et al., 2020; Bai et al., 2022; OpenAI, 2024b; Sun et al., 2023), to rule-based reward functions (Zhou et al., 2025a; Shen et al., 2025; Zhou et al., 2024b; Zhai et al., 2024; Wang et al., 2024a; Yang et al., 2025b). For multi-turn RL training, (Zhai et al., 2024) applies PPO to VLMs similar to ours, but we adopt a trajectory-based optimization strategy that better supports POMDP scenarios by leveraging historical context for visual reasoning. Concurrently, (Zhou et al., 2024b) and (Zhou et al., 2025a) developed hierarchical RL frameworks for LLM alignment in text-only environments. Our work proposes Bi-level GAE, which is similar to the hierarchical design of (Zhou et al., 2024b), but with totally different optimization strategy.

Multi-turn Agentic Tasks. Multi-turn interaction is fundamental to agentic tasks for LLMs (Zhou et al., 2025a, 2024b; Wang et al., 2025b; Kumar et al., 2024; Abdulhai et al., 2024). For VLMs, this challenge extends to the more complex domain of maintaining consistent visual state representations across interactions (Zhai et al., 2024). Previous research has explored various approaches including prompting techniques (Wang et al., 2023a,b, 2024c) and fixed LLMs/VLMs with additional adapters (Szot et al., 2023; Rocamonde et al., 2023; Chen et al., 2024). Our work extends to a more fundamental level by investigating the integration of multi-round visual state reasoning with reinforcement learning. This approach demonstrates generalizability across diverse benchmarks, including navigation(Yang et al., 2025a), manipulation(Shukla et al., 2024), instruction following(Shridhar et al., 2021), image reconstruction (Rodriguez et al., 2024), and

collaborative tasks(Chang et al., 2025) where VLMs are increasingly utilized as core reasoning engines (Wang et al., 2024a).

Visual State Representation and Reasoning. Recent studies have explored visual perception (Tong et al., 2024a; Liu et al., 2024a; Tong et al., 2024b; Schwettmann et al., 2023) and causal tracing in VLMs (Palit et al., 2023). Research has also examined visual information flow in single-turn scenarios (Basu et al., 2024; Neo et al., 2025; Kaduri et al., 2024; Laurençon et al., 2024; Liu et al., 2024b). However, maintaining visual state continuity across multiple interaction turns remains an underexplored challenge. Our research addresses this gap by examining how VLMs maintain consistent visual representations during multi-turn interactions, focusing on improving visual state tracking across consecutive turns with RL.

7. Conclusion and Limitations

We introduce Visual Reasoning RL, which leverages explicit reasoning about visual state, turn-level WorldModeling Reward and Bi-Level GAE, significantly enhancing task performance and visual reasoning quality for VLM in agentic tasks. Limitations include restricted model architecture, evaluation methodology, and RL algorithm selection. Future work can explore additional VLM families, diverse reward supervision methods, and pre-training for multi-turn visual understanding.

References

- Linxi Fan, Guanzhi Wang, Yunfan Jiang, Ajay Mandlekar, Yuncong Yang, Haoyi Zhu, Andrew Tang, De-An Huang, Yuke Zhu, and Anima Anandkumar. Minedojo: Building open-ended embodied agents with internet-scale knowledge. In *Thirty-sixth Conference on Neural Information Processing Systems Datasets and Benchmarks Track*, 2022. URL https://openreview.net/forum?id=rc8o_j8I8PX.
- Oriol Vinyals, Igor Babuschkin, Wojciech M Czarnecki, Michaël Mathieu, Andrew Dudzik, Junyoung Chung, David H Choi, Richard Powell, Timo Ewalds, Petko Georgiev, et al. Grandmaster level in starcraft ii using multi-agent reinforcement learning. *nature*, 2019.
- Lanxiang Hu, Mingjia Huo, Yuxuan Zhang, Haoyang Yu, Eric P. Xing, Ion Stoica, Tajana Rosing, Haojian Jin, and Hao Zhang. Imgame-bench: How good are llms at playing games?, 2025. URL <https://arxiv.org/abs/2505.15146>.
- Guanzhi Wang, Yuqi Xie, Yunfan Jiang, Ajay Mandlekar, Chaowei Xiao, Yuke Zhu, Linxi Fan, and Anima Anandkumar. Voyager: An open-ended embodied agent with large language models, 2023a. URL <https://arxiv.org/abs/2305.16291>.
- Mohit Shridhar, Xingdi Yuan, Marc-Alexandre, Yonatan Bisk, Adam Trischler, and Matthew Hausknecht. ALFWorld: Aligning Text and Embodied Environments for Interactive Learning. In *Proceedings of the International Conference on Learning Representations (ICLR)*, 2021. URL <https://arxiv.org/abs/2010.03768>.
- Rui Yang, Hanyang Chen, Junyu Zhang, Mark Zhao, Cheng Qian, Kangrui Wang, Qineng Wang, Teja Venkat Koripella, Marziyeh Movahedi, Manling Li, et al. Embodiedbench: Comprehensive benchmarking multi-modal large language models for vision-driven embodied agents. *arXiv preprint arXiv:2502.09560*, 2025a.
- Shuyan Zhou, Frank F. Xu, Hao Zhu, Xuhui Zhou, Robert Lo, Abishek Sridhar, Xianyi Cheng, Tianyue Ou, Yonatan Bisk, Daniel Fried, Uri Alon, and Graham Neubig. Webarena: A realistic web environment for building autonomous agents. In *The Twelfth International Conference on Learning Representations*, 2024a. URL <https://openreview.net/forum?id=oKn9c6ytLx>.

- Juan A. Rodriguez, Abhay Puri, Shubham Agarwal, Issam H. Laradji, Pau Rodriguez, Sai Rajeswar, David Vazquez, Christopher Pal, and Marco Pedersoli. Starvector: Generating scalable vector graphics code from images and text, 2024. URL <https://arxiv.org/abs/2312.11556>.
- Kunato Nishina and Yusuke Matsui. Svgeditbench: A benchmark dataset for quantitative assessment of llm’s svg editing capabilities. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR) Workshops*, pages 8142–8147, June 2024.
- Ming Li, Jike Zhong, Shitian Zhao, Yuxiang Lai, Haoquan Zhang, Wang Bill Zhu, and Kaipeng Zhang. Think or not think: A study of explicit thinking in rule-based visual reinforcement fine-tuning, 2025. URL <https://arxiv.org/abs/2503.16188>.
- Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Fei Xia, Ed Chi, Quoc V Le, Denny Zhou, et al. Chain-of-thought prompting elicits reasoning in large language models. *Advances in neural information processing systems*, 35:24824–24837, 2022.
- DeepSeek-AI, Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiao Bi, Xiaokang Zhang, Xingkai Yu, Yu Wu, Z. F. Wu, Zhibin Gou, Zhihong Shao, Zhuoshu Li, Ziyi Gao, et al. Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning, 2025. URL <https://arxiv.org/abs/2501.12948>.
- Jie Ouyang, Ruiran Yan, Yucong Luo, Mingyue Cheng, Qi Liu, Zirui Liu, Shuo Yu, and Daoyu Wang. Training powerful llm agents with end-to-end reinforcement learning, 2025. URL <https://github.com/Orusswest0/Agent-R1>.
- Tianzhe Chu, Yuexiang Zhai, Jihan Yang, Shengbang Tong, Saining Xie, Dale Schuurmans, Quoc V. Le, Sergey Levine, and Yi Ma. Sft memorizes, rl generalizes: A comparative study of foundation model post-training, 2025. URL <https://arxiv.org/abs/2501.17161>.
- Yifei Zhou, Song Jiang, Yuandong Tian, Jason Weston, Sergey Levine, Sainbayar Sukhbaatar, and Xian Li. Sweet-rl: Training multi-turn llm agents on collaborative reasoning tasks, 2025a. URL <https://arxiv.org/abs/2503.15478>.
- Jiawei Gu, Xuhui Jiang, Zhichao Shi, Hexiang Tan, Xuehao Zhai, Chengjin Xu, Wei Li, Yinghan Shen, Shengjie Ma, Honghao Liu, Saizhuo Wang, Kun Zhang, Yuanzhuo Wang, Wen Gao, Lionel Ni, and Jian Guo. A survey on llm-as-a-judge, 2025. URL <https://arxiv.org/abs/2411.15594>.
- John Schulman, Philipp Moritz, Sergey Levine, Michael Jordan, and Pieter Abbeel. High-dimensional continuous control using generalized advantage estimation. *arXiv preprint arXiv:1506.02438*, 2015.
- Karl Johan Åström. Optimal control of markov processes with incomplete state information i. *Journal of mathematical analysis and applications*, 10:174–205, 1965.
- Haozhan Shen, Peng Liu, Jingcheng Li, Chunxin Fang, Yibo Ma, Jiajia Liao, Qiaoli Shen, Zilun Zhang, Kangjia Zhao, Qianqian Zhang, Ruochen Xu, and Tiancheng Zhao. Vlm-r1: A stable and generalizable r1-style large vision-language model, 2025. URL <https://arxiv.org/abs/2504.07615>.
- Zihan Wang, Kangrui Wang, Qineng Wang, Pingyue Zhang, Linjie Li, Zhengyuan Yang, Xing Jin, Kefan Yu, Minh Nhat Nguyen, Licheng Liu, Eli Gottlieb, Yiping Lu, Kyunghyun Cho, Jiajun Wu, Li Fei-Fei, Lijuan Wang, Yejin Choi, and Manling Li. Ragen: Understanding self-evolution in llm agents via multi-turn reinforcement learning, 2025a. URL <https://arxiv.org/abs/2504.20073>.
- Yifei Zhou, Andrea Zanette, Jiayi Pan, Sergey Levine, and Aviral Kumar. Archer: Training language model agents via hierarchical multi-turn rl. *arXiv preprint arXiv:2402.19446*, 2024b.
- Yuexiang Zhai, Hao Bai, Zipeng Lin, Jiayi Pan, Shengbang Tong, Yifei Zhou, Alane Suhr, Saining Xie, Yann LeCun, Yi Ma, and Sergey Levine. Fine-tuning large vision-language models as decision-making agents via reinforcement learning. In *The Thirty-eighth Annual Conference on Neural Information Processing Systems*, 2024. URL <https://openreview.net/forum?id=nBjmmF2IZU>.

- Lior Shani, Aviv Rosenberg, Asaf Cassel, Oran Lang, Daniele Calandriello, Avital Zipori, Hila Noga, Orgad Keller, Bilal Piot, Idan Szpektor, Avinatan Hassidim, Yossi Matias, and Rémi Munos. Multi-turn reinforcement learning from preference human feedback, 2024. URL <https://arxiv.org/abs/2405.14655>.
- Eric Xing, Mingkai Deng, Jinyu Hou, and Zhiting Hu. Critiques of world models. *arXiv preprint arXiv:2507.05169*, 2025.
- John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms, 2017. URL <https://arxiv.org/abs/1707.06347>.
- Max-Philipp B. Schrader. gym-sokoban. <https://github.com/mpSchrader/gym-sokoban>, 2018.
- Mark Towers, Ariel Kwiatkowski, Jordan Terry, John U. Balis, Gianluca De Cola, Tristan Deleu, Manuel Goulão, Andreas Kallinteris, Markus Krimmel, Arjun KG, Rodrigo Perez-Vicente, Andrea Pierré, Sander Schulhoff, Jun Jet Tai, Hannah Tan, and Omar G. Younis. Gymnasium: A standard interface for reinforcement learning environments, 2024. URL <https://arxiv.org/abs/2407.17032>.
- Sanjana Srivastava, Kangrui Wang, Yung-Chieh Chan, Tianyuan Dai, Manling Li, Ruohan Zhang, Mengdi Xu, Jiajun Wu, and Li Fei-Fei. ROSETTA: Constructing code-based reward from unconstrained language preference. In *Workshop on Continual Robot Learning from Humans*, 2025. URL <https://openreview.net/forum?id=xuDPUN7Ud4>.
- Soroush Nasiriany, Huihan Liu, and Yuke Zhu. Augmenting reinforcement learning with behavior primitives for diverse manipulation tasks. In *2022 International Conference on Robotics and Automation (ICRA)*, pages 7477–7484, 2022. doi: 10.1109/ICRA46639.2022.9812140.
- Ayano Hiranaka, Minjune Hwang, Sharon Lee, Chen Wang, Li Fei-Fei, Jiajun Wu, and Ruohan Zhang. Primitive skill-based robot learning from human evaluative feedback. In *2023 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 7817–7824, 2023. doi: 10.1109/IROS55552.2023.10341912.
- Jinze Bai, Shuai Bai, Shusheng Yang, Shijie Wang, Sinan Tan, Peng Wang, Junyang Lin, Chang Zhou, and Jingren Zhou. Qwen-vl: A versatile vision-language model for understanding, localization, text reading, and beyond. *arXiv preprint arXiv:2308.12966*, 2023.
- OpenAI. Introducing openai o3 and o4-mini, 2025.
- OpenAI. Gpt-4o system card, 2024a. URL <https://arxiv.org/abs/2410.21276>.
- Gemini Team. Gemini: A family of highly capable multimodal models, 2025. URL <https://arxiv.org/abs/2312.11805>.
- The claude 3 model family: Opus, sonnet, haiku. 2024. URL <https://api.semanticscholar.org/CorpusID:268232499>.
- Bowen Jin, Hansi Zeng, Zhenrui Yue, Jinsung Yoon, Sercan Arik, Dong Wang, Hamed Zamani, and Jiawei Han. Search-r1: Training llms to reason and leverage search engines with reinforcement learning, 2025. URL <https://arxiv.org/abs/2503.09516>.
- Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Xiao Bi, Haowei Zhang, Mingchuan Zhang, Y. K. Li, Y. Wu, and Daya Guo. Deepseekmath: Pushing the limits of mathematical reasoning in open language models, 2024. URL <https://arxiv.org/abs/2402.03300>.
- Alec Radford, Jong Wook Kim, Chris Hallacy, Aditya Ramesh, Gabriel Goh, Sandhini Agarwal, Girish Sastry, Amanda Askell, Pamela Mishkin, Jack Clark, Gretchen Krueger, and Ilya Sutskever. Learning transferable visual models from natural language supervision. In *Proceedings of the 38th International Conference on Machine Learning*. PMLR, 2021. URL <https://proceedings.mlr.press/v139/radford21a.html>.

- Daniel M Ziegler, Nisan Stiennon, Jeffrey Wu, Tom B Brown, Alec Radford, Dario Amodei, Paul Christiano, and Geoffrey Irving. Fine-tuning language models from human preferences. *arXiv preprint arXiv:1909.08593*, 2019.
- Nisan Stiennon, Long Ouyang, Jeffrey Wu, Daniel Ziegler, Ryan Lowe, Chelsea Voss, Alec Radford, Dario Amodei, and Paul F Christiano. Learning to summarize with human feedback. *Advances in neural information processing systems*, 33:3008–3021, 2020.
- Yuntao Bai, Andy Jones, Kamal Ndousse, Amanda Askell, Anna Chen, Nova DasSarma, Dawn Drain, Stanislav Fort, Deep Ganguli, Tom Henighan, Nicholas Joseph, Saurav Kadavath, Jackson Kernion, Tom Conerly, Sheer El-Showk, Nelson Elhage, Zac Hatfield-Dodds, Danny Hernandez, Tristan Hume, Scott Johnston, Shauna Kravec, Liane Lovitt, Neel Nanda, Catherine Olsson, Dario Amodei, Tom Brown, Jack Clark, Sam McCandlish, Chris Olah, Ben Mann, and Jared Kaplan. Training a helpful and harmless assistant with reinforcement learning from human feedback, 2022. URL <https://arxiv.org/abs/2204.05862>.
- Hengguang Zhou, Xirui Li, Ruochen Wang, Minhao Cheng, Tianyi Zhou, and Cho-Jui Hsieh. R1-zero’s “aha moment” in visual reasoning on a 2b non-sft model, 2025b. URL <https://arxiv.org/abs/2503.05132>.
- OpenAI. Openai o1 system card, 2024b. URL <https://arxiv.org/abs/2412.16720>.
- Zhiqing Sun, Sheng Shen, Shengcao Cao, Haotian Liu, Chunyuan Li, Yikang Shen, Chuang Gan, Liang-Yan Gui, Yu-Xiong Wang, Yiming Yang, et al. Aligning large multimodal models with factually augmented rlhf. *arXiv preprint arXiv:2309.14525*, 2023.
- Yufei Wang, Zhanyi Sun, Jesse Zhang, Zhou Xian, Erdem Biyik, David Held, and Zackory Erickson. R1-vlm-f: Reinforcement learning from vision language foundation model feedback. In *Proceedings of the 41st International Conference on Machine Learning*, 2024a.
- Yi Yang, Xiaoxuan He, Hongkun Pan, Xiyan Jiang, Yan Deng, Xingtao Yang, Haoyu Lu, Dacheng Yin, Fengyun Rao, Minfeng Zhu, Bo Zhang, and Wei Chen. R1-onevision: Advancing generalized multimodal reasoning through cross-modal formalization, 2025b. URL <https://arxiv.org/abs/2503.10615>.
- Peiyi Wang, Lei Li, Zhihong Shao, Runxin Xu, Damai Dai, Yifei Li, Deli Chen, Yu Wu, and Zhifang Sui. Math-shepherd: Verify and reinforce LLMs step-by-step without human annotations. In *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*. Association for Computational Linguistics, 2024b. URL <https://aclanthology.org/2024.acl-1.510/>.
- Stephen Casper, Xander Davies, Claudia Shi, Thomas Krendl Gilbert, Jérémy Scheurer, Javier Rando, Rachel Freedman, Tomasz Korbak, David Lindner, Pedro Freire, et al. Open problems and fundamental limitations of reinforcement learning from human feedback. *arXiv preprint arXiv:2307.15217*, 2023.
- Qiyang Yu, Zheng Zhang, Ruofei Zhu, Yufeng Yuan, Xiaochen Zuo, Yu Yue, Tiantian Fan, Gaohong Liu, Lingjun Liu, Xin Liu, Haibin Lin, Zhiqi Lin, Bole Ma, Guangming Sheng, Yuxuan Tong, Chi Zhang, Mofan Zhang, Wang Zhang, Hang Zhu, Jinhua Zhu, Jiaze Chen, Jiangjie Chen, Chengyi Wang, Hongli Yu, Weinan Dai, Yuxuan Song, Xiangpeng Wei, Hao Zhou, Jingjing Liu, Wei-Ying Ma, Ya-Qin Zhang, Lin Yan, Mu Qiao, Yonghui Wu, and Mingxuan Wang. Dapo: An open-source llm reinforcement learning system at scale, 2025. URL <https://arxiv.org/abs/2503.14476>.
- Zihan Wang, Kangrui Wang, Qineng Wang, Pingyue Zhang, Linjie Li, Zhengyuan Yang, Xing Jin, Kefan Yu, Minh Nhat Nguyen, Licheng Liu, Eli Gottlieb, Yiping Lu, Kyunghyun Cho, Jiajun Wu, Li Fei-Fei, Lijuan Wang, Yejin Choi, and Manling Li. Ragen: Understanding self-evolution in llm agents via multi-turn reinforcement learning, 2025b. URL <https://arxiv.org/abs/2504.20073>.
- Aviral Kumar, Vincent Zhuang, Rishabh Agarwal, Yi Su, John D Co-Reyes, Avi Singh, Kate Baumli, Shariq Iqbal, Colton Bishop, Rebecca Roelofs, et al. Training language models to self-correct via reinforcement learning. *arXiv preprint arXiv:2409.12917*, 2024.

- Marwa Abdulhai, Isadora White, Charlie Victor Snell, Charles Sun, Joey Hong, Yuexiang Zhai, Kelvin Xu, and Sergey Levine. LMRL gym: Benchmarks for multi-turn reinforcement learning with language models, 2024. URL <https://openreview.net/forum?id=8cNMMrWRbZ>.
- Zihao Wang, Shaofei Cai, Guanzhou Chen, Anji Liu, Xiaojian Ma, and Yitao Liang. Describe, explain, plan and select: Interactive planning with large language models enables open-world multi-task agents. *arXiv preprint arXiv:2302.01560*, 2023b.
- Junyang Wang, Haiyang Xu, Jiabo Ye, Ming Yan, Weizhou Shen, Ji Zhang, Fei Huang, and Jitao Sang. Mobile-agent: Autonomous multi-modal mobile device agent with visual perception. *arXiv preprint arXiv:2401.16158*, 2024c.
- Andrew Szot, Max Schwarzer, Harsh Agrawal, Bogdan Mazouze, Rin Metcalf, Walter Talbott, Natalie Mackraz, R Devon Hjelm, and Alexander T Toshev. Large language models as generalizable policies for embodied tasks. In *The Twelfth International Conference on Learning Representations*, 2023.
- Juan Rocamonde, Victoriano Montesinos, Elvis Nava, Ethan Perez, and David Lindner. Vision-language models are zero-shot reward models for reinforcement learning. *arXiv preprint arXiv:2310.12921*, 2023.
- William Chen, Oier Mees, Aviral Kumar, and Sergey Levine. Vision-language models provide promptable representations for reinforcement learning. *arXiv preprint arXiv:2402.02651*, 2024.
- Arth Shukla, Stone Tao, and Hao Su. Maniskill-hab: A benchmark for low-level manipulation in home rearrangement tasks. *CoRR*, abs/2412.13211, 2024. doi: 10.48550/ARXIV.2412.13211. URL <https://doi.org/10.48550/arXiv.2412.13211>.
- Matthew Chang, Gunjan Chhablani, Alexander Clegg, Mikael Dallaire Cote, Ruta Desai, Michal Hlavac, Vladimir Karashchuk, Jacob Krantz, Roozbeh Mottaghi, Priyam Parashar, Siddharth Patki, Ishita Prasad, Xavier Puig, Akshara Rai, Ram Ramrakhya, Daniel Tran, Joanne Truong, John M. Turner, Eric Undersander, and Tsung-Yen Yang. Partnr: A benchmark for planning and reasoning in embodied multi-agent tasks. In *International Conference on Learning Representations (ICLR)*, 2025. alphabetical author order.
- Shengbang Tong, Zhuang Liu, Yuexiang Zhai, Yi Ma, Yann LeCun, and Saining Xie. Eyes wide shut? exploring the visual shortcomings of multimodal llms. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 9568–9578, 2024a.
- Haotian Liu, Chunyuan Li, Yuheng Li, Bo Li, Yuanhan Zhang, Sheng Shen, and Yong Jae Lee. Lllavanext: Improved reasoning, ocr, and world knowledge, 2024a.
- Peter Tong, Ellis Brown, Penghao Wu, Sanghyun Woo, Adithya Jairam Vedagiri IYER, Sai Charitha Akula, Shusheng Yang, Jihan Yang, Manoj Middepogu, Ziteng Wang, et al. Cambrian-1: A fully open, vision-centric exploration of multimodal llms. *Advances in Neural Information Processing Systems*, 37: 87310–87356, 2024b.
- Sarah Schwettmann, Neil Chowdhury, Samuel Klein, David Bau, and Antonio Torralba. Multimodal neurons in pretrained text-only transformers. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pages 2862–2867, 2023.
- Vedant Palit, Rohan Pandey, Aryaman Arora, and Paul Pu Liang. Towards vision-language mechanistic interpretability: A causal tracing tool for blip. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pages 2856–2861, 2023.
- Samyadeep Basu, Martin Grayson, Cecily Morrison, Besmira Nushi, Soheil Feizi, and Daniela Massiceti. Understanding information storage and transfer in multi-modal large language models. *arXiv preprint arXiv:2406.04236*, 2024.
- Clement Neo, Luke Ong, Philip Torr, Mor Geva, David Krueger, and Fazl Barez. Towards interpreting visual information processing in vision-language models, 2025. URL <https://arxiv.org/abs/2410.07149>.

- Omri Kaduri, Shai Bagon, and Tali Dekel. What’s in the image? a deep-dive into the vision of vision language models, 2024. URL <https://arxiv.org/abs/2411.17491>.
- Hugo Laurençon, Leo Tronchon, Matthieu Cord, and Victor Sanh. What matters when building vision-language models? In *The Thirty-eighth Annual Conference on Neural Information Processing Systems*, 2024. URL <https://openreview.net/forum?id=dtvJF1Vy2i>.
- Yue Liu, Yunjie Tian, Yuzhong Zhao, Hongtian Yu, Lingxi Xie, Yaowei Wang, Qixiang Ye, Jianbin Jiao, and Yunfan Liu. VMamba: Visual state space model. In *The Thirty-eighth Annual Conference on Neural Information Processing Systems*, 2024b. URL <https://openreview.net/forum?id=ZgtLQQR1K7>.
- Guangming Sheng, Chi Zhang, Zilingfeng Ye, Xibin Wu, Wang Zhang, Ru Zhang, Yanghua Peng, Haibin Lin, and Chuan Wu. Hybridflow: A flexible and efficient rlhf framework. *arXiv preprint arXiv:2409.19256*, 2024.
- Stephanie Fu, Netanel Tamir, Shobhita Sundaram, Lucy Chai, Richard Zhang, Tali Dekel, and Phillip Isola. Dreamsim: Learning new dimensions of human visual similarity using synthetic data. In *Advances in Neural Information Processing Systems*, volume 36, pages 50742–50768, 2023. URL <https://arxiv.org/abs/2306.09344>.
- Mathilde Caron, Hugo Touvron, Ishan Misra, Hervé Jégou, Julien Mairal, Piotr Bojanowski, and Armand Joulin. Emerging properties in self-supervised vision transformers. In *Proceedings of the International Conference on Computer Vision (ICCV)*, 2021.

Appendix

Table of Contents

A	World Modeling for Visual State Reasoning via Multi-Turn RL	22
A.1	Problem Formulation: VLM Agent Training under a POMDP Formulation . . .	22
A.2	Multi-Turn Reinforcement Learning in VLM Agentic Tasks	22
A.3	VAGEN Framework	23
A.4	Environments and Tasks for VLM Agents	23
A.5	Reward Assignment	25
A.6	Evaluation Metrics	27
B	What Can We Reason About Visual States?	28
B.1	Bottleneck of Off-the-Shelf VLMs on Agentic Tasks	28
B.2	Reasoning in Multi-turn RL Training	34
B.3	Learning Dynamics of RL Baselines	41
C	How Can We Represent Beliefs about the World?	41
C.1	Symbolic Representations	42
C.2	Structured Representations	43
D	How to Improve Reasoning over World Models?	43
D.1	World Modeling Reward	43
D.2	Bi-Level General Advantage Estimation (GAE)	53
E	Case Study	54
E.1	Visual State Reasoning Enhances Spatial Understanding and Multi-Step Planning	54
E.2	Response Convergence and Reduced Exploration	55
E.3	Reward Hacking and Over-optimization	59
F	Summary of Findings	61

A. World Modeling for Visual State Reasoning via Multi-Turn RL

In this section, we detail our training pipeline, algorithm and system.

A.1. Problem Formulation: VLM Agent Training under a POMDP Formulation

We choose to formulate our problem in a POMDP setting rather than a fully observable MDP for two main reasons.

Inherent Partial Observability in Environments. Many of the environments we study, such as the Navigation task, does not satisfy the assumptions of an MDP. In these environments, the agent does not have access to the complete, true state of the environment (s_t) at each timestep. Instead, it receives an observation o_t (e.g., a first-person visual scene from its current vantage point) which constitutes only a partial view of s_t . To gain a more comprehensive understanding of the environment and to locate targets or critical information, the agent must actively explore, such as rotating its viewpoint or moving to a new location. This necessity to explore and gather information is a hallmark of POMDPs.

Methodological Congruence with POMDPs. We optimize policies over the full sequence of observations, actions, and reasoning steps by applying a teacher-forcing strategy during updates. This allows the model to leverage the entire trajectory as context for current decisions, ensuring consistency between rollout and update and making our method especially effective under partial observability.

A.2. Multi-Turn Reinforcement Learning in VLM Agentic Tasks

Our multi-turn reinforcement learning framework for VLM agents is shown in Figure 6. At each turn t , the VLM agent receives the current observation o_t (comprising a visual image and an optional textual prompt). Based on o_t and its interaction history, the VLM generates a structured output a_t . This output a_t includes an explicit reasoning component z_t (e.g., `<think><observation>... <prediction>...</think>`) and an executable action component a_t^e (e.g., `<answer>...</answer>`). The executable action a_t^e is parsed and sent to the environment. The environment then transitions to a new state, providing the next observation o_{t+1} and a scalar reward r_t . This cycle repeats for N turns to form a trajectory. The VLM’s parameters are updated using reinforcement learning, specifically Proximal Policy Optimization (PPO), based on the collected trajectories.

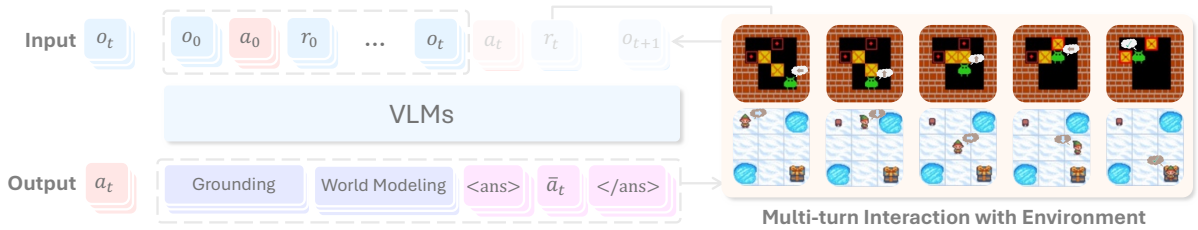


Figure 6 | Multi-turn RL framework for VLM agents.

The detailed training algorithm for our multi-turn RL framework is presented in Algorithm 1.

The detailed reasoning templates is given in Table 6.

Table 6 | Reasoning strategy formats.

Strategy	Format
NoThink	<code><answer>a_t^e</answer></code>
FreeThink	<code><think>z_t</think></code> <code><answer>a_t^e</answer></code>
StateEstimation	<code><think></code> <code><observation>s_t^b</observation></code> <code><reasoning>a_t^b</reasoning></code> <code></think></code> <code><answer>a_t^e</answer></code>
TransitionModeling	<code><think></code> <code><reasoning>a_t^b</reasoning></code> <code><prediction>s_{t+1}^b</prediction></code> <code></think></code> <code><answer>a_t^e</answer></code>
WorldModeling	<code><think></code> <code><observation>s_t^b</observation></code> <code><reasoning>a_t^b</reasoning></code> <code><prediction>s_{t+1}^b</prediction></code> <code></think></code> <code><answer>a_t^e</answer></code>

A.3. VAGEN Framework

To support our experiments, we developed VAGEN, a framework built upon VeRL(Sheng et al., 2024). VAGEN specializes in multi-turn reinforcement learning (RL) training for Vision-Language Models (VLMs), integrating various RL algorithms and environments. We adopt *env-as-service* design, which decouples training from interacting with environments, enhances scalability.

A.4. Environments and Tasks for VLM Agents

Sokoban The action space and hyperparameters are given in Table 7 and Table 8.

Table 7 | Action space for the Sokoban environment.

Name	Description
Up	Move the agent one cell upward on the grid.
Left	Move the agent one cell to the left on the grid.
Right	Move the agent one cell to the right on the grid.
Down	Move the agent one cell downward on the grid.

Table 8 | Hyperparameters for the Sokoban environment.

Name	Value	Description
dim_room	(6, 6)	Dimensions of the Sokoban grid
max_steps	100	Maximum number of steps allowed per episode
num_boxes	1	Number of boxes to be pushed onto targets
min_actions_to_succeed	5	Minimum number of actions required to solve the puzzle
max_actions_per_step	3	Maximum number of actions the agent can take per turn
max_turns	3	Maximum number of turns the agent can interact with the environment

FrozenLake The action space and hyperparameters are given in Table 9 and Table 10.

Table 9 | Action space for the FrozenLake environment.

Name	Description
Up	Move the agent one cell upward on the grid.
Left	Move the agent one cell to the left on the grid.
Right	Move the agent one cell to the right on the grid.
Down	Move the agent one cell downward on the grid.

Table 10 | Hyperparameters for the FrozenLake environment.

Name	Value	Description
desc	None	Environment map layout (if None, randomly generated)
is_slippery	False	Whether the frozen tiles are slippery
size	4	Size of the square grid
max_actions_per_step	3	Maximum number of actions the agent can take per turn
min_actions_to_succeed	5	Minimum number of actions required to reach the goal
max_turns	3	Maximum number of turns the agent can interact with the environment

Navigation The action space and hyperparameters are given in Table 11 and Table 12.

Table 11 | Action space for the Navigation environment.

Name	Description
MoveAhead	Move forward by some distance
MoveBack	Move backward by some distance
MoveRight	Move rightward by some distance
MoveLeft	Move leftward by some distance
RotateRight	Rotate to the right by 90 degrees
RotateLeft	Rotate to the left by 90 degrees
LookUp	Tilt the camera upward by 30 degrees
LookDown	Tilt the camera downward by 30 degrees

Table 12 | Hyperparameters for the Navigation environment.

Name	Value	Description
resolution	255	Resolution of the rendered images
down_sample_ratio	1.0	Ratio for down-sampling images
fov	100	Field of view angle in degrees
multiview	False	Whether to use multiple camera views
max_actions_per_step	5	Maximum number of actions the agent can take per turn
success_threshold	1.0	Threshold for considering task successful
step_length	0.25	Distance traveled in a single movement action
max_turns	4	Maximum number of turns the agent can interact with the environment

PrimitiveSkill The action space and hyperparameters are given in Table 13 and Table 14.

Table 13 | Action space for the PrimitiveSkill environment.

Name	Description
pick(x, y, z)	Grasp an object located at position (x, y, z) in the robot’s workspace
place(x, y, z)	Place the object currently held by the robot’s gripper at the target position (x, y, z)
push(x1, y1, z1, x2, y2, z2)	Push an object from position (x1, y1, z1) to position (x2, y2, z2)

Table 14 | Hyperparameters for the PrimitiveSkill environment.

Name	Value	Description
max_actions_per_step	2	Maximum number of actions the agent can take per turn
max_turns	3	Maximum number of turns the agent can interact with the environment

SVG Reconstruction The action space and hyperparameters are given in Table 15 and Table 16.

Table 15 | Action space for the SVG Reconstruction environment.

Name	Description
SVG Code	Open text format allowing for the specification of SVG markup code

Table 16 | Hyperparameters for the SVG Reconstruction environment.

Name	Value	Description
dataset_name	starvector/svg-icons-simple	Dataset used for SVG examples
max_turns	2	Maximum number of turns the agent can interact with the environment

A.5. Reward Assignment

Sokoban The reward structure for the Sokoban environment is presented in Table 17.

Table 17 | Reward structure for the Sokoban environment.

Reward Type	Value	Description
Success reward	10	Awarded when all boxes are placed on target locations
Failure penalty	-0.1	Applied each step when the task is not completed
Box placement reward	1	Granted for each box pushed onto a target location
Format reward	0.5	Provided at each turn to encourage visual state reasoning
Grounding reward weight	0.5	Weight applied to <code>StateEstimation</code> reward
World modeling reward weight	0.5	Weight applied to <code>TransitionModeling</code> reward

FrozenLake The reward structure for the FrozenLake environment is presented in Table 18.

Table 18 | Reward structure for the FrozenLake environment.

Reward Type	Value	Description
Success reward	10	Awarded when the agent reaches the goal position
Failure penalty	-0.1	Applied each step when the task is not completed
Format reward	0.5	Provided at each turn to encourage visual state reasoning
Grounding reward weight	0.5	Weight applied to <code>StateEstimation</code> reward
World modeling reward weight	0.5	Weight applied to <code>TransitionModeling</code> reward

Navigation The reward structure for the Navigation environment is presented in Table 19.

Table 19 | Reward structure for the Navigation environment.

Reward Type	Value	Description
Success reward	10	Awarded when the agent reaches the goal location
Failure penalty	-0.1	Applied each step when the task is not completed
Format reward	0.5	Provided at each turn to encourage visual state reasoning
Grounding reward weight	0.5	Weight applied to <code>StateEstimation</code> reward
World modeling reward weight	0.5	Weight applied to <code>TransitionModeling</code> reward

PrimitiveSkill The reward structure for the PrimitiveSkill environment is presented in Table 20.

Table 20 | Reward structure for the PrimitiveSkill environment.

Reward Type	Value	Description
Success reward	10	Awarded when the manipulation task is completed
Failure penalty	-0.1	Applied each step when the task is not completed
Stage-based reward	$(\text{stage} + 1) \times 2$	Granted upon completing key manipulation subgoals (where stage is the highest successfully completed stage)
Format reward	0.5	Provided at each turn to encourage visual state reasoning
Grounding reward weight	0.5	Weight applied to <code>StateEstimation</code> reward
World modeling reward weight	0.5	Weight applied to <code>TransitionModeling</code> reward

SVG The reward structure for the SVG Reconstruction environment is presented in Table 21.

Table 21 | Reward structure for the SVG Reconstruction environment.

Reward Type	Value	Description
Image similarity	Variable	Weighted DreamSim Fu et al. (2023) and DINO Caron et al. (2021) scores measuring similarity between generated and target images
Format reward	0.5	Provided at each turn to encourage visual state reasoning
Grounding reward weight	0.5	Weight applied to <code>StateEstimation</code> reward
World modeling reward weight	0.5	Weight applied to <code>TransitionModeling</code> reward
DreamSim weight	5.0	Scaling factor applied to DreamSim similarity scores
Dino weight	0.0001	We only use DreamSim score for reward

A.6. Evaluation Metrics

We employ a range of metrics to evaluate agent performance across our diverse task environments. For trajectory-based evaluation, we define several key functions over a trajectory τ :

- $f(\tau) \in \{0, 1\}$: A binary success indicator function that equals 1 if the trajectory τ successfully completes the task, and 0 otherwise
- $g(\tau) \in [0, 1]$: The DreamSim similarity score between the target image and the final generated image in trajectory τ
- $h(\tau) \in [0, 1]$: The DINO similarity score between the target image and the final generated image in trajectory τ

Metrics for Task-Completion Environments For Sokoban, FrozenLake, Navigation, and PrimitiveSkill environments, we use the average success rate over a dataset \mathcal{D} of test trajectories:

$$\text{Success Rate} = \mathbb{E}_{\tau \sim \mathcal{D}} [f(\tau)] \quad (1)$$

A trajectory is considered successful ($f(\tau) = 1$) when the agent completes the specific task objectives for each environment.

Metrics for SVG Reconstruction For the SVG Reconstruction task, where success is measured by the similarity between generated and target images rather than discrete goal achievement, we report:

$$\text{DreamSim Score} = \mathbb{E}_{\tau \sim \mathcal{D}} [g(\tau)] \quad (2)$$

$$\text{DINO Score} = \mathbb{E}_{\tau \sim \mathcal{D}} [h(\tau)] \quad (3)$$

These complementary metrics capture different aspects of visual similarity:

- **DreamSim Score:** Measures perceptual similarity between images using a pre-trained diffusion model [Fu et al. \(2023\)](#).
- **DINO Score:** Based on self-supervised visual features from a small version of the DINO model [Caron et al. \(2021\)](#).

B. What Can We Reason About Visual States?

B.1. Bottleneck of Off-the-Shelf VLMs on Agentic Tasks

To understand the capabilities and limitations of existing VLMs on agentic tasks, we evaluate several state-of-the-art off-the-shelf VLMs across our proposed tasks. The prompts used for evaluating these models follow the FreeThink reasoning strategy format: `<think> t_t </think>`
`<answer> a_t^e </answer>`.

Sokoban Prompt for evaluating off-the-shelf VLMs

System

You are a Sokoban solver.
Sokoban Quick Guide
Goal: Push all boxes onto targets.
Symbols (If image is provided there are no symbols):
Wall | _ Floor | 0 Target | X Box | P You | * Box on Target | S You on Target
Rules:
1. Push boxes (can't pull).
2. Avoid walls.
Actions you can take: Left, Down, Right, Up.
You can take up to 3 action(s) at a time, separated by ,.
You should first give your reasoning, and then your answer.
Your response should be in the format of:
`<think>...</think><answer>...</answer>`
e.g. `<think><reasoning>The box is one step below me, and the target is two steps below me, I need to go down then push the box down to the target.</reasoning></think><answer>Down,Down</answer>`

User

[Initial Observation]:
`<image>`
Decide your next action(s).
You can take up to 3 action(s) at a time, separated by ,.
You should first give your reasoning, and then your answer.
Your response should be in the format of:
`<think>...</think><answer>...</answer>`

Assistant

`<think>Since there are no boxes or targets visible, the first step is to move the character (P) to the wall to see if it will push the wall and open a path. This might also help in accessing more space to maneuver.</think>`
`<answer>Left, Up, Right</answer>`
rest of the turns ...

FrozenLake Prompt for evaluating off-the-shelf VLMs

System

You are a FrozenLake solver.
FrozenLake Quick Guide
Goal: Reach the goal (G).
Symbols (If image is provided there are no symbols):

_ Frozen | 0 Hole | G Goal | P Player | X Player fell into hole | *
 Player on goal
 Rules:
 1. Avoid falling into holes.
 2. Frozen tiles are slippery, you may move perpendicular to your intended direction.
 Actions you can take: Left, Down, Right, Up.

You can take up to 3 action(s) at a time, separated by ,.
 You should first give your reasoning, and then your answer.
 Your response should be in the format of:
 <think>...</think><answer>...</answer>
 e.g. <think>I can see the target is on my down left, I should go down then left to reach the target</think><answer>Down,Left</answer>

User
 [Initial Observation]:
 <image>
 Decide your next action(s).

You can take up to 3 action(s) at a time, separated by ,.
 You should first give your reasoning, and then your answer.
 Your response should be in the format of:
 <think>...</think><answer>...</answer>

Assistant
 <think>There is a present at the bottom right, which is likely the goal. Since the player is near the present and the goal is usually at the bottom right, I should move downwards to reach it.</think><answer>Down, Down, Down</answer>
 rest of the turns ...

Navigation Prompt for evaluating off-the-shelf VLMs

System
 You are a home robot and perform navigation tasks according to instructions.
 Actions you can take: moveahead, moveback, moveright, moveleft, rotateright, rotateleft, lookup, lookdown.
 moveahead: Move forward by some distance
 moveback: Move backward by some distance
 moveright: Move rightward by some distance
 moveleft: Move leftward by some distance
 rotateright: Rotate to the right by 90 degrees
 rotateleft: Rotate to the left by 90 degrees
 lookup: Tilt the camera upward by 30 degrees
 lookdown: Tilt the camera downward by 30 degrees
 Rewards:
 Format correct: +0.5
 Achieve the human instruction: +10.0
 The instruction will be provided with each observation. Look at the image carefully and navigate to complete the instruction.
 Hints:
 1. You can take multiple actions at a time, in most cases, if you find the target object is far away from you, you can call moveahead, moveleft

and move right multiple times.

2. If you find yourself seems to be stuck, you can lookdown to see if there's any object above or below you, you can also rotate to see if there's any object behind you.

Example:

Round 1:

image_1

<think>I can see the garbage can in the upper left corner of the image, next to the kitchen sink. To move there, we can go forward-left, but since there's a kitchen counter directly ahead, we should go left first. Following the strategy, I can go by first moving leftward.</think>

<answer>movelleft, moveleft</answer>

Round 2:

Env_feedback: Last action is executed successfully.

image_2

<think>From the secene, I see that by moving leftward, we are getting closer to the garbage can. Now, the garbage can is in front of me, slightly to the left. And there's a large area ahead of us. Following the strategy, I can go by first moving forward then moving leftward.</think>

<answer>moveahead, moveahead,moveahead,movelleft</answer>

Round 3:

Env_feedback: Last action is executed successfully.

image_3

<think>From the image we can see the garbage can is very close to us, still to our front-left. Moving leftward might be blocked but i can see that there is still space in front of me to get closer to the garbage can. Following the strategy, we can take about two steps forward then one step left to reach the garbage can.</think>

<answer>moveahead, moveahead,movelleft</answer>

Round 4:

Env_feedback: Success

You can take up to 5 action(s) at a time, separated by ','.

You should first give your thought process, and then your answer.

Your response should be in the format of:

<think>...</think><answer>...</answer>

e.g. <think>I can see from the sight the target object is right in the top left of me, I will move forward, then move left to access it.</think>

><answer>moveahead,moveahead,moveahead,movelleft,movelleft</answer>

User

[Initial Observation]:

<image>

Human Instruction: I need to dispose of some trash properly. Please navigate to that object and stay near it.

Decide your next action(s).

You can take up to 5 action(s) at a time, separated by ','.

You should first give your thought process, and then your answer.

Your response should be in the format of:

<think>...</think><answer>...</answer>

Assistant

<think>From the image, I can see the trash can is in the bottom-right corner. I need to navigate towards it. I will first move forward and then turn to the right to reach it.</think><answer>moveahead, rotateright</answer>

rest of the turns ...

PrimitiveSkill Prompt for evaluating off-the-shelf VLMs

System

You are an AI assistant controlling a Franka Emika robot arm. Your goal is to understand human instructions and translate them into a sequence of executable actions for the robot, based on visual input and the instruction.

Action Space Guide

You can command the robot using the following actions:

1. `pick(x, y, z)` # To grasp an object located at position(x,y,z) in the robot's workspace.
2. `place(x, y, z)` # To place the object currently held by the robot's gripper at the target position (x,y,z).
3. `push(x1, y1, z1, x2, y2, z2)` # To push an object from position (x1,y1,z1) to (x2,y2,z2).

Hints:

1. The coordinates (x, y, z) are in millimeters and are all integers.
2. Please ensure that the coordinates are within the workspace limits.
3. The position is the center of the object, when you place, please consider the volume of the object. It's always fine to set z much higher when placing an item.
4. We will provide the object positions to you, but you need to match them to the object in the image by yourself. You're facing toward the negative x-axis, and the negative y-axis is to your left, the positive y-axis is to your right, and the positive z-axis is up.

Examples:

round1:

image1

Human Instruction: Put red cube on green cube and yellow cube on left target

Object positions:

`[(62,-55,20),(75,33,20),(-44,100,20),(100,-43,0),(100,43,0)]`

Reasoning: I can see from the picture that the red cube is on my left and green cube is on my right and near me.

Since I'm looking toward the negative x axis, and negative y-axis is to my left, (62,-55,20) would be the position of the red cube, (75,33,20) would be the position of the green cube and (-44,100,20) is the position of the yellow cube.

Also the (100,-43,0) would be the position of the left target, and (100,43,0) would be the position of the right target.

I need to pick up red cube first and place it on the green cube, when placing, I should set z much higher.

Answer: `pick(62,-55,20)|place(75,33,50)`

round2:

image2

Human Instruction: Put red cube on green cube and yellow cube on left target

Object positions:

`[(75,33,50),(75,33,20),(-44,100,20),(100,-43,0),(100,43,0)]`

Reasoning: Now the red cube is on the green cube, so I need to pick up the yellow cube and place it on the left target.

Answer: pick(-44,100,20)|place(100,-43,50)

You can take up to 2 action(s) at a time, separated by |.

You should first give your thought process, and then your answer.

Your response should be in the format of:

<think>...</think><answer>...</answer>

e.g. e.g. <think>I need to pick the red_cube_pos at (10,20,30) and place it on the green_block_pos at (50,60,40).</think><answer>pick(10,20,30)|place(50,60,70)</answer>

User

[Initial Observation]:

<image>

Human Instruction: Please align the cubes in the y-axis, which means the x-coordinates of both cubes should be 0 (+-10mm)

x_workspace_limit: (-100, 150)

y_workspace_limit: (-200, 200)

z_workspace_limit: (10, 200)

Object positions:

[(129, -119, 20), (108, 124, 20)]

Other information:

No other information needed

Decide your next action(s).

You can take up to 2 action(s) at a time, separated by |.

You should first give your thought process, and then your answer.

Your response should be in the format of:

<think>...</think><answer>...</answer>

Assistant

<think>First, I need to pick up the red cube at position (129, -119, 20) and place it at position (0, -119, 20).</think><answer>pick(129, -119, 20)|place(0, -119, 50)</answer>

rest of the turns ...

SVG Reconstruction Prompt for evaluating off-the-shelf VLMs

System

You are a precise SVG code generator.

SVG Quick Guide

Goal: Transform the provided image into precise SVG code that replicates the image.

Process:

1. First analyze the image carefully, identifying distinct visual elements
2. Identify colors, dimensions, positions, and relationships between elements
3. Generate accurate SVG code that reproduces the image, you can use path for better shape

Rewards:

- Overall visual similarity: +5.0
- Structural accuracy: +10.0

Example:

```

<think>I can see the image contains a red circle and a blue rectangle.
The circle is positioned at the top-left, while the rectangle is at the
bottom-right.</think>
<answer><svg viewBox="0 0 100 100" xmlns="http://www.w3.org/2000/svg">
  <circle cx="25" cy="25" r="15" fill="red" />
  <rect x="60" y="60" width="30" height="20" fill="blue" />
</svg></answer>
You can take up to 1 action(s) at a time, separated by ~~.
You should first give your thought process, and then your answer.
Your response should be in the format of:
<think>...</think><answer>...</answer>
e.g. <think>I can see the image contains a red circle and a blue
rectangle. The circle is positioned at the top-left, while the rectangle
is at the bottom-right.</think>
<answer><svg viewBox="0 0 100 100" xmlns="http://www.w3.org/2000/svg">
  <circle cx="25" cy="25" r="15" fill="red" />
  <rect x="60" y="60" width="30" height="20" fill="blue" />
</svg></answer>
User
[Initial Observation]:
<image>
Please carefully observe the image, and generate SVG code that reproduces
it as accurately as possible.
Decide on your SVG code.

You can take up to 1 action(s) at a time, separated by ~~.
You should first give your thought process, and then your answer.
Your response should be in the format of:
<think>...</think><answer>...</answer>
Assistant
<think>Based on the image, I can see a circular shape with the letters "
kr" inside it. The letters are positioned in the center of the circle.</
think>
<answer><svg viewBox="0 0 100 100" xmlns="http://www.w3.org/2000/svg">
  <circle cx="50" cy="50" r="40" fill="none" stroke="black" stroke-width
="2" />
  <text x="50" y="70" font-size="40" font-family="Arial" fill="black">kr
</text>
</svg></answer>
rest of the turns ...

```

The information of evaluated VLMs with corresponding hyperparameters are provided in Table 22.

Table 22 | Off-the-shelf VLMs for evaluation.

Model Name	Model ID	Provider	Max Tokens	Temperature
VLM-R1-3B	omlab/VLM-R1-Qwen2.5VL-3B-Math-0305	omlab	150/400	0.7
Qwen2.5-VL-3B	Qwen/Qwen2.5-VL-3B-Instruct	Qwen	150/400	0.7
Qwen2.5-VL-7B	Qwen/Qwen2.5-VL-7B-Instruct	Qwen	150/400	0.7
Qwen2.5-VL-72B	Qwen/Qwen2.5-VL-72B-Instruct	Qwen	150/400	0.7
GPT-4o	gpt-4o	openai	150/400	0.7
Gemini 2.0	gemini-2.0-flash	gemini	150/400	0.7
Claude 3.7 Sonnet	claude-3-7-sonnet-20250219	claude	150/400	0.7

B.2. Reasoning in Multi-turn RL Training

Following the training methodology described in Section 2.2, we use reinforcement learning to train Qwen2.5-VL-3B across all reasoning strategies presented in Table 6. Our experiments are conducted on servers equipped with 8×H100 GPUs, 104 CPUs, and 1.7TB of memory. Each server can run two experiments at the same time, with each training session requiring approximately 4-8 hours to complete.

The training hyperparameters used in our experiments are detailed in Table 23.

Table 23 | Multi-turn RL training hyperparameters.

Parameter	Value	Description
Rollout Phase		
Top-p	0.95	Nucleus sampling parameter for action generation
Temperature	0.7	Sampling temperature for controlling randomness
Update Phase		
Advantage Estimator	masked_gae	Generalized Advantage Estimation with masking
Actor Model	Qwen/Qwen2.5-VL-3B-Instruct	Pre-trained model used for actor initialization
Critic Model	Qwen/Qwen2.5-VL-3B-Instruct	Pre-trained model used for critic initialization
γ_{token}	1.0	Discount factor for token-wise advantage calculation
KL Penalty Coefficient (β)	0.001	Coefficient for KL divergence penalty in PPO objective
Actor Learning Rate	1e-6	Learning rate for the actor network
Critic Learning Rate	1e-5	Learning rate for the critic network
Train Batch Size	128	Total batch size for training
PPO Mini Batch Size	32	Mini-batch size for PPO updates

We also release the prompts used in the multi-turn RL training process. For different reasoning strategies, the structures of the prompts are largely identical, with minor differences in response formats. To avoid redundancy, we provide only the WorldModeling prompts here. For FreeThink prompts, please refer to Appendix B.1. For other reasoning strategy prompts, please see our provided code or infer them from the existing prompts.

Sokoban Training Prompt for WorldModeling Strategy

System

You are a Sokoban solver.
Sokoban Quick Guide
Goal: Push all boxes onto targets.
Symbols (If image is provided there are no symbols):
Wall | _ Floor | 0 Target | X Box | P You | * Box on Target | S You on Target
Rules:
1. Push boxes (can't pull).
2. Avoid walls.
Actions you can take: Left, Down, Right, Up.
You can take up to 3 action(s) at a time, separated by ,.
You should first give the description of your observation, then your

reasoning, then predict the next state, and finally your answer.
 Your response should be in the format of:
`<think><observation>...</observation><reasoning>...</reasoning>
 <prediction>...</prediction></think><answer>...</answer>`
 e.g. `<think><observation>The box is below the player and the target is
 below the box</observation><reasoning>I need to go down then push the box
 down to the target</reasoning><prediction>The player will be above the
 box, the target and box will be at the same place.</prediction></think><
 answer>Down,Down</answer>`

User

[Initial Observation]:

`<image>`

Decide your next action(s).

You can take up to 3 action(s) at a time, separated by ,.

You should first give the description of your observation, then your reasoning, then predict the next state, and finally your answer.

Your response should be in the format of:

`<think><observation>...</observation><reasoning>...</reasoning>
 <prediction>...</prediction></think><answer>...</answer>`

Assistant

`<think><observation>The player is at the bottom of the screen, and there
 is a box to the right of the player. The target is to the left of the box
 .</observation><reasoning>The player needs to push the box to the target
 to complete the goal.</reasoning><prediction>The player will push the box
 to the target, moving up, down, and to the left.</prediction></think><
 answer>Up, Down, Left</answer>`

rest of the turns ...

FrozenLake Training Prompt for WorldModeling Strategy

System

You are a FrozenLake solver.

FrozenLake Quick Guide

Goal: Reach the goal (G).

Symbols (If image is provided there are no symbols):

_ Frozen | O Hole | G Goal | P Player | X Player fell into hole | *

Player on goal

Rules:

1. Avoid falling into holes.
2. Frozen tiles are slippery, you may move perpendicular to your intended direction.

Actions you can take: Left, Down, Right, Up.

You can take up to 3 action(s) at a time, separated by ,.

You should first describe the observation, then your reasoning, then predict the next state, and finally your answer.

Your response should be in the format of:

`<think><observation>...</observation><reasoning>...</reasoning>
 <prediction>...</prediction></think><answer>...</answer>`

e.g. `<think><observation>The player is on the above the target</
 observation><reasoning>I should go down then left to reach the target</
 reasoning><prediction>The player will reach the target</prediction></
 think><answer>Down,Left</answer>`

User

[Initial Observation]:

<image>

Decide your next action(s).

You can take up to 3 action(s) at a time, separated by ,.

You should first describe the observation, then your reasoning, then predict the next state, and finally your answer.

Your response should be in the format of:

<think><observation>...</observation><reasoning>...</reasoning>

<prediction>...</prediction></think><answer>...</answer>

Assistant

<think><observation>The player is on the right side of the grid.</observation><reasoning>The player is on the right side of the grid, which is indicated by the position on the grid.</reasoning><prediction>The player will move to the left or down.</prediction></think><answer>Left, Left, Down</answer>

rest of the turns ...

Navigation Training Prompt for WorldModeling Strategy

System

You are a home robot and perform navigation tasks according to instructions.

Actions you can take: moveahead, moveback, moveright, moveleft, rotateright, rotateleft, lookup, lookdown.

moveahead: Move forward by some distance

moveback: Move backward by some distance

moveright: Move rightward by some distance

moveleft: Move leftward by some distance

rotateright: Rotate to the right by 90 degrees

rotateleft: Rotate to the left by 90 degrees

lookup: Tilt the camera upward by 30 degrees

lookdown: Tilt the camera downward by 30 degrees

Rewards:

Format correct: +0.5

Achieve the human instruction: +10.0

The instruction will be provided with each observation. Look at the image carefully and navigate to complete the instruction.

Hints:

1. You can take multiple actions at a time, in most cases, if you find the target object is far away from you, you can call moveahead, moveleft and move right multiple times.

2. If you find yourself seems to be stuck, you can lookdown to see if there's any object above or below you, you can also rotate to see if there's any object behind you.

Example:

Round 1:

image_1

<think><observation>There is a garbage can in the upper left corner of the image, next to the kitchen sink. To move there, we can go forward-left, but since there's a kitchen counter directly ahead, we should go left first.</observation><reasoning>Following the strategy, I can go by

first moving leftward.</reasoning><prediction>I will be in front of the garbage</prediction></think>
<answer>movelleft, moveleft</answer>
Round 2:
Env_feedback: Last action is executed successfully.
image_2
<think><observation>From the scene, I see that by moving leftward, we are getting closer to the garbage can. Now, the garbage can is in front of me, slightly to the left. And there's a large area ahead of us.</observation><reasoning>Following the strategy, I can go by first moving forward then moving leftward.</reasoning><prediction>I will be closer to the garbage</prediction></think>
<answer>moveahead, moveahead,moveahead,movelleft</answer>
Round 3:
Env_feedback: Last action is executed successfully.
image_3
<think><observation>From the image we can see the garbage can is very close to us, still to our front-left. Moving leftward might be blocked but i can see that there is still space in front of me to get closer to the garbage can.</observation><reasoning>Following the strategy, we can take about two steps forward then one step left to reach the garbage can.</reasoning><prediction>I will reach the garbage</prediction></think>
<answer>moveahead, moveahead,movelleft</answer>
Round 4:
Env_feedback: Success
You can take up to 5 action(s) at a time, separated by ','.
You should first give your thought process with the your observation, reasoning, and prediction of next state, then your answer.
Both the observation and prediction should describe what you see or expect to see in the environment.
Your response should be in the format of:
<think><observation>...</observation><reasoning>...</reasoning>
<prediction>...</prediction></think><answer>...</answer>
e.g. <think><observation>I am at the entrance of a bedroom. There is a bed to the left, a desk with a lamp on the right, and a closet straight ahead. The target object, a book, appears to be on the desk.</observation>
<reasoning>I need to move toward the desk to reach the book. I'll turn right and move forward.</reasoning><prediction>I am now standing in front of the desk. The desk has a lamp, a computer, and several books on it. The target book is within reach on the right side of the desk.</prediction></think><answer>rotateright,moveahead,moveahead</answer>
User
[Initial Observation]:
<image>
Human Instruction: I am looking for a luminous device on my desk to read books at night. Can you navigate to that object and stay close?
Decide your next action(s).
You can take up to 5 action(s) at a time, separated by ','.
You should first give your thought process with the your observation, reasoning, and prediction of next state, then your answer.
Both the observation and prediction should describe what you see or expect to see in the environment.
Your response should be in the format of:
<think><observation>...</observation><reasoning>...</reasoning>

```
<prediction>...</prediction></think><answer>...</answer>
```

Assistant

```
<think><observation>There is a luminous device on the desk to the right
of the chair. To reach it, I should move forward and then turn right.</
observation><reasoning>Following the strategy, I can go by first moving
forward then turning right to reach the luminous device.</reasoning><
prediction>I will be close to the luminous device</prediction></think><
answer>moveahead, moveahead, moveright</answer>
```

rest of the turns ...

PrimitiveSkill Training Prompt for WorldModeling Strategy

System

You are an AI assistant controlling a Franka Emika robot arm. Your goal is to understand human instructions and translate them into a sequence of executable actions for the robot, based on visual input and the instruction.

Action Space Guide

You can command the robot using the following actions:

1. pick(x, y, z) # To grasp an object located at position(x,y,z) in the robot's workspace.
2. place(x, y, z) # To place the object currently held by the robot's gripper at the target position (x,y,z).
3. push(x1, y1, z1, x2, y2, z2) # To push an object from position (x1,y1,z1) to (x2,y2,z2).

Hints:

1. The coordinates (x, y, z) are in millimeters and are all integers.
2. Please ensure that the coordinates are within the workspace limits.
3. The position is the center of the object, when you place, please consider the volume of the object. It's always fine to set z much higher when placing an item.
4. We will provide the object positions to you, but you need to match them to the object in the image by yourself. You're facing toward the negative x-axis, and the negative y-axis is to your left, the positive y-axis is to your right, and the positive z-axis is up.

Examples:

round1:

image1

Human Instruction: Put red cube on green cube and yellow cube on left target

Object positions:

[(62,-55,20),(75,33,20),(-44,100,20),(100,-43,0),(100,43,0)]

Reasoning: I can see from the picture that the red cube is on my left and green cube is on my right and near me.

Since I'm looking toward the negative x axis, and negative y-axis is to my left, (62,-55,20) would be the position of the red cube, (75,33,20) would be the position of the green cube and (-44,100,20) is the position of the yellow cube.

Also the (100,-43,0) would be the position of the left target, and

(100,43,0) would be the position of the right target.
I need to pick up red cube first and place it on the green cube, when placing, I should set z much higher.
Answer: pick(62,-55,20)|place(75,33,50)
round2:
image2
Human Instruction: Put red cube on green cube and yellow cube on left target
Object positions:
[(75,33,50),(75,33,20),(-44,100,20),(100,-43,0),(100,43,0)]
Reasoning: Now the red cube is on the green cube, so I need to pick up the yellow cube and place it on the left target.
Answer: pick(-44,100,20)|place(100,-43,50)

You can take up to 2 action(s) at a time, separated by |.
You should first give your thought process with reasoning and prediction of next state, and then your answer.
Your response should be in the format of:
<think><observation>...</observation><reasoning>...</reasoning>
<prediction>...</prediction>
</think><answer>...</answer>
e.g. <think><observation>The red cube is on the left at (100,100,40), the green cube is on the right</observation><reasoning>I need to pick the red cube at (100,100,40) and place it at (80,100,60)</reasoning><prediction>After executing this action, the red cube will be at (80,100,60)</prediction></think>
<answer>pick(100,100,40)|place(80,100,60)</answer>

User
[Initial Observation]:
<image>
Human Instruction: Please put the apple in the drawer and close the drawer.
x_workspace_limit: (-500, -100)
y_workspace_limit: (-600, 200)
z_workspace_limit: (10, 650)
Object positions:
[(-127, 192, 36), (-500, -537, 200)]
Other information:
{'drawer_open_value': 263}
Decide your next action(s).
You can take up to 2 action(s) at a time, separated by |.
You should first give your thought process with reasoning and prediction of next state, and then your answer.
Your response should be in the format of:
<think><observation>...</observation><reasoning>...</reasoning>
<prediction>...</prediction></think><answer>...</answer>

Assistant
<think><observation>The apple is on the ground, right side of the drawer</observation><reasoning>I need to pick up the apple first, as it is on the floor. After placing the apple in the drawer, I will close the drawer.</reasoning><prediction>The apple will be placed in the drawer and the drawer will be closed.</prediction></think><answer>pick(-127, 192, 36)|place(-500, -537, 500)|push(-500, -537, 500, -500, -537, 650)</answer>
rest of the turns ...

SVG Reconstruction Training Prompt for WorldModeling Strategy

System

You are a precise SVG code generator.

SVG Quick Guide

Goal: Transform the provided image into precise SVG code that replicates the image.

Process:

1. First analyze the image carefully, identifying distinct visual elements
2. Identify colors, dimensions, positions, and relationships between elements
3. Generate accurate SVG code that reproduces the image, you can use path for better shape

Rewards:

- Overall visual similarity: +5.0
- Structural accuracy: +10.0

Example:

```
<think><observation>I can see an image containing a red circle positioned
  at the top-left area of the canvas, approximately at coordinates (25,
  25) with a radius of 15 units. There is also a blue rectangle at the
  bottom-right area, sized about 30x20 units and positioned at coordinates
  (60, 60).</observation><reasoning>Based on my observation, I need to
  create an SVG that precisely matches these elements. The circle appears
  to be slightly too far right, so I should adjust its x-coordinate to 20
  instead of 25. The rectangle could benefit from being slightly wider.</
  reasoning><prediction>After implementing these adjustments, the resulting
  SVG should more closely match the original image. I expect the
  similarity score to improve to approximately 0.98, as the modified
  positions and dimensions will better represent the original graphic.</
  prediction></think>
```

```
<answer><svg viewBox="0 0 100 100" xmlns="http://www.w3.org/2000/svg">
  <circle cx="20" cy="25" r="15" fill="red" />
  <rect x="60" y="60" width="35" height="20" fill="blue" />
</svg></answer>
```

You can take up to 1 action(s) at a time, separated by ~~.

You should first give your thought process with the your observation and reasoning, then predict next state, and finally the answer.

Both the observation and prediction should describe what you see or expect to see in the environment.

Your response should be in the format of:

```
<think><observation>...</observation><reasoning>...</reasoning>
<prediction>...</prediction></think><answer>...</answer>
```

e.g. <think><observation>I can see an image containing a red circle positioned at the top-left area of the canvas, approximately at coordinates (25, 25) with a radius of 15 units. There is also a blue rectangle at the bottom-right area, sized about 30x20 units and positioned at coordinates (60, 60).</observation><reasoning>Based on my observation, I need to create an SVG that precisely matches these elements. The circle appears to be slightly too far right, so I should adjust its x-coordinate to 20 instead of 25. The rectangle could benefit from being slightly wider.</reasoning><prediction>After implementing

these adjustments, the resulting SVG should more closely match the original image. I expect the similarity score to improve to approximately 0.98, as the modified positions and dimensions will better represent the original graphic.</prediction></think>

```
<answer><svg viewBox="0 0 100 100" xmlns="http://www.w3.org/2000/svg">
  <circle cx="20" cy="25" r="15" fill="red" />
  <rect x="60" y="60" width="35" height="20" fill="blue" />
</svg></answer>
```

User

[Initial Observation]:

<image>

Please carefully observe the image, and generate SVG code that reproduces it as accurately as possible.

Decide on your SVG code.

You can take up to 1 action(s) at a time, separated by ~.

You should first give your thought process with the your observation and reasoning, then predict next state, and finally the answer.

Both the observation and prediction should describe what you see or expect to see in the environment.

Your response should be in the format of:

```
<think><observation>...</observation><reasoning>...</reasoning>
<prediction>...</prediction></think><answer>...</answer>
```

Assistant

```
<think><observation>I can see an image of the letters "kr" inside a
circle. The circle is centered at the top of the image. The letters are
inside the circle, with the circle having a consistent thickness around
the perimeter.</observation><reasoning>The SVG should accurately
represent the circle with a consistent thickness around it and the
letters "kr" inside. The circle should be centered and have a specific
size, and the letters should be placed inside the circle.</reasoning><
prediction>The SVG will need to accurately depict the circle, the letters
"kr", and their positioning within the circle. The circle's size and
position should be precise, and the letters should be correctly placed
inside.</prediction></think>
```

```
<answer><svg viewBox="0 0 100 100" xmlns="http://www.w3.org/2000/svg">
  <circle cx="50" cy="50" r="25" stroke="black" stroke-width="1" fill="
none" />
  <text x="50" y="50" font-size="24" font-family="sans-serif" text-anchor
="middle" alignment-baseline="central">kr</text>
</svg></answer>
```

rest of the turns ...

B.3. Learning Dynamics of RL Baselines

C. How Can We Represent Beliefs about the World?

In this section, we release the details of different visual state representations. To further understand visual state reasoning, we investigate how different visual state representations affect task performance by prompting models to use the WorldModeling reasoning strategy and requiring them to output specific formats for the <observation> and <prediction> fields.

We explore different visual state representations across our environments: Sokoban and

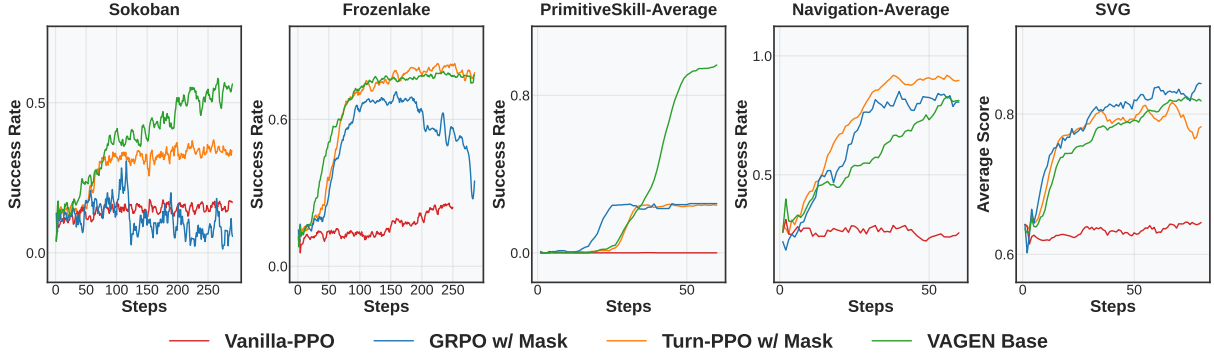


Figure 7 | Training success rates for RL Baselines and VAGEN-Base

FrozenLake support three representation formats (Natural-Lanaguage, Symbolic, and Structured), while PrimitiveSkill supports two formats (Natural-Lanaguage and Structured). For all other tasks, we use Natural-Lanaguage visual state representation as the default. Examples of Natural-Lanaguage visual state representations for all tasks can be found in the prompts provided in Appendix B.2.

The different visual state representation formats are summarized in Table 24:

Table 24 | Visual state representation formats across different environments.

Environment	Format	Type	Description
Sokoban	Natural-Lanaguage	Text	Free-form textual descriptions of the game state
	Symbolic	Grid	Environment-native grid-based symbols representing game entities
	Structured	Dictionary	JSON-like dictionary containing positions of game objects
FrozenLake	Natural-Lanaguage	Text	Free-form textual descriptions of the game state
	Symbolic	Grid	Environment-native grid-based symbols representing game entities
	Structured	Dictionary	JSON-like dictionary containing positions of game objects
PrimitiveSkill	Natural-Lanaguage	Text	Free-form textual descriptions of the manipulation state
	Structured	Dictionary	JSON-like dictionary containing 3D coordinates of task-relevant objects

C.1. Symbolic Representations

For symbolic representations, we use environment-specific symbols to represent different entities in a grid format.

Sokoban Symbolic Format uses the following mapping:

Wall | _ Floor | 0 Target | X Box | P You | * Box on Target | S You on Target

An example of the symbolic representation:

#####

```
#_P_#
#_X0_#
###_#
#####
```

FrozenLake Symbolic Format uses the following mapping: _ Frozen | 0 Hole | G Goal
| P Player | X Player fell into hole | * Player on goal

An example of the symbolic representation:

```
___G
0___
__0_
_P__
```

C.2. Structured Representations

For structured representations, we use dictionary-based formats containing precise positional information of relevant objects.

Sokoban Structured Format contains player, box, and target positions along with grid dimensions.

An example of the symbolic representation:

```
{player_position: (4, 2), box_positions: [(3, 2)],
target_positions: [(3, 1)], grid_size: (6, 6)}
```

FrozenLake Structured Format contains player, target, and hole positions along with grid dimensions.

An example of the symbolic representation:

```
{player_position: (3, 2), target_position: (3, 2),
hole_positions: [(1, 2)], grid_size: (4, 4)}
```

PrimitiveSkill Structured Format contains 3D coordinates of task-relevant objects.

An example of the symbolic representation:

```
{red_cube: (100, 100, 40), green_cube: (200, 200, 60)}
```

These different formats allow us to investigate how the choice of visual state representation affects the model’s ability to reason about and predict state transitions across different types of environments.

D. How to Improve Reasoning over World Models?

D.1. World Modeling Reward

D.1.1. LLM-as-a-Judge

To enhance visual state reasoning capabilities, we implement an LLM-as-a-Judge framework that evaluates the quality of the agent’s StateEstimation and TransitionModeling reasoning.

The judge model used in our experiments is GPT-4.1 nano, which evaluates whether the agent's descriptions and predictions accurately match the ground truth state information.

Our evaluation system uses task-specific prompts for both StateEstimation (current state grounding) and TransitionModeling (next state prediction) assessment. The prompts are structured to guide the judge model through step-by-step reasoning before providing a binary YES/NO evaluation.

Sokoban StateEstimation Evaluation

Evaluate whether the description accurately captures the key position relationships in the Sokoban game state.

Answer YES if the directional relationships are correct, or NO if they contain directional errors.

Context

You are evaluating whether the description correctly identifies the directional relationships between:

1. The player and the box(es)
2. The box(es) and the target(s)

The description doesn't need to be perfectly precise or mention every detail - it just needs to have the correct directional relationships (Up, Down, Left, Right).

Example:

Groundtruth Current State Information: ['box0 is at the same row and to the left of the player', 'target0 is above and on the left side of the player', 'target0 is above and on the left side of box0']

State Description: The player is below the box and the target is below the box.

- <think>The state description contains spatial relationship information, do further analysis. According to the ground truth data, box0 is at the same row and to the left of the player, target0 is above and on the left side of the player, target0 is above and on the left side of box0. The description states 'The player is below the box and the target is below the box.' The player is actually at the same row as the box (not below), and the target is actually above the box (not below). Both directional relationships are incorrectly identified.</think><answer>NO</answer>

Example:

Groundtruth Current State Information: ['box0 is above and on the right side of the player', 'target0 is above and at the same column as the player', 'target0 is above and on the left side of box0']

State Description: The box is above the player and the target is to the left of the box

- <think>The state description contains spatial relationship information, do further analysis. According to the ground truth data, box0 is above and on the right side of the player, target0 is above and at the same column as the player, target0 is above and on the left side of box0. The description states 'The box is above the player and the target is to the left of the box.' It correctly identifies that the box is above the player (box0 is above and on the right side of the player). It also correctly identifies that the target is to the left of the box (target0 is above and on the left side of box0). Both key directional

relationships are accurately described.</think><answer>YES</answer>

Groundtruth Current State Information:

{state_information_dict}

State Description:

{natural_language_description}

Think step by step:

1. Relative Relationship Requirements:
 - Must describe at least one relationships BETWEEN entities (player-box, player-target, box-target)
 - Absolute positions like "player is on the left side" are insufficient
 - Need relational descriptions like "player is left of target"
2. Essential Relationships to Check
 - Player-Target relationship (highest priority)
 - Player-Box relationship
 - Box-Target relationship
3. Equivalent Expression Recognition
 - "box is above player" = "player is below box"
 - "target is left of box" = "box is right of target"
 - Must recognize these as identical spatial relationships. Absolute position is not allowed

Your answer should be in the format of <think>...</think><answer>YES</answer> or <think>...</think><answer>NO</answer>.

Sokoban TransitionModeling Evaluation

Evaluate whether the prediction correctly anticipates the key position relationships that will exist in the next Sokoban state.
Answer YES if the predicted directional relationships are correct, or NO if they contain directional errors.

Context

You are evaluating whether the prediction correctly identifies the directional relationships that will exist after the move:

1. The future position of the player relative to the box(es)
2. The future position of the box(es) relative to the target(s)

Important: The Prediction Comes First

Remember: The Next State Prediction is made BEFORE the Groundtruth Next State exists. Your task is to check if the prediction correctly anticipated what actually happened.

If the box and target are at same position, this prediction is seen as success immediately (YES)

Example:

Groundtruth Next State Information: ['box0 is above and on the right side of the player', 'target0 is above and on the left side of the player', ']

target0 is above and on the left side of box0']
 Next State Prediction: The player will be to the left of the box, and the box will be to the right of the target.
 - <think>The prediction state contains spatial relationship between player and target, do further analysis. According to the ground truth data, box0 is above and on the right side of the player, target0 is above and on the left side of the player, target0 is above and on the left side of box0. The description states 'The player will be to the left of the box, and the box will be to the right of the target.' It correctly identifies that the player is to the left of the box (since box0 is on the right side of the player). It also correctly identifies that the box is to the right of the target (since target0 is on the left side of box0). Therefore, this description correctly identifies the key directional relationships.</think><answer>YES</answer>

Groundtruth Next State Information:
 {state_information_dict}

Next State Prediction:
 {natural_language_description}

Think step by step:

1. Relative Relationship Requirements:
 - Must describe at least one relationships BETWEEN entities (player-box, player-target, box-target)
 - Absolute positions like "player is on the left side" are insufficient
 - Need relational descriptions like "player is left of target"
2. Essential Relationships to Check
 - Player-Target relationship (highest priority)
 - Player-Box relationship
 - Box-Target relationship
3. Equivalent Expression Recognition
 - "box is above player" = "player is below box"
 - "target is left of box" = "box is right of target"
 - Must recognize these as identical spatial relationships. Absolute position is not allowed

Your answer should be in the format of <think>...</think><answer>YES</answer> or <think>...</think><answer>NO</answer>.

FrozenLake StateEstimation Evaluation

Evaluate whether the description accurately captures the key position relationships in the FrozenLake game state.
 Answer YES if the directional relationships are correct, or NO if there are errors.

Context

You are evaluating whether the description correctly identifies:

1. The directional relationship between the player and the goal (MUST Have)
2. The directional relationship between the player and the hole (if present)

The description doesn't need to be perfectly precise - it just needs to have the correct directional relationships between the player and target (Up, Down, Left, Right), and between the player and hole if applicable.

Groundtruth Current State Information:
{state_information_dict}

State Description:
{natural_language_description}

Think step by step:

1. Player relationship with Goal
 - Goal (Target) MUST include in state description, without target the description is automatically wrong (NO)
 - If there is no direction between player and goal, like "player is right to the target", the description is automatically wrong (NO)
 - This takes highest priority over all other considerations
2. Equivalent Expression Recognition
 - "goal is above player" = "player is below goal"
 - "target is left of box" = "box is right of target"
 - Must recognize these as identical spatial relationships. Absolute position is not allowed
3. Simple Judgment Rule
 - If player at goal -> YES
 - If direction aligns with needed movement -> YES
 - Otherwise -> NO

Your answer should be in the format of <think>...</think><answer>YES</answer> or <think>...</think><answer>NO</answer>.

FrozenLake TransitionModeling Evaluation

Evaluate whether the prediction correctly anticipates the key aspects of the next FrozenLake state.

Answer YES if the prediction accounts for directional relationships and potential holes, or NO if it contains errors.

Context

You are evaluating whether the prediction correctly identifies:

1. The position relationship between the player and the goal after the prediction

Important: The Prediction Comes First

Remember: The Next State Prediction is made BEFORE the Groundtruth Next State exists. Your task is to check if the prediction correctly anticipated what actually happened.

The prediction doesn't need to perfectly describe every aspect of the next state - it just needs to correctly anticipate the directional relationships (Up, Down, Left, Right) or address any dangers from holes.

```
# Groundtruth Next State Information:  
{state_information_dict}
```

```
# Next State Prediction:  
{natural_language_description}
```

Think step by step:

1. Player relationship with Goal

- If player is already at the goal position, the prediction is automatically correct (YES)
- Goal (Target) MUST include in prediction state, without target the prediction is automatically wrong (NO)
- If there is no direction between player and goal, like "player is right to the target", the prediction is automatically wrong (NO)
- This takes highest priority over all other considerations

2. Directional Correctness

- Evaluate if the predicted movement direction aligns with the relative position between player and goal
- For example, if player is left of goal, moving right is correct
- ****CRITICAL:** Recognize equivalent expressions of the same spatial relationship**
 - * "player is above target" = "target is below player"
 - * "player is left of target" = "target is right of player"
 - * These are the SAME relationship expressed from different perspectives

3. Simple Judgment Rule

- If player at goal -> YES
- If direction aligns with needed movement -> YES
- Otherwise -> NO

Your answer should be in the format of <think>...</think><answer>YES</answer> or <think>...</think><answer>NO</answer>.

Navigation StateEstimation Evaluation

Evaluate whether the description effectively communicates the spatial relationship between the agent and target object, even if the exact directional terms differ.

Answer YES if the overall spatial understanding is correct, or NO if it fundamentally misunderstands the spatial layout.

Context

You are evaluating whether the description effectively conveys where the target object is located relative to the agent. The exact directional terminology (left, right, ahead, etc.) may differ between the state information and the description, but the important factor is whether the

description would lead to correct navigation.

Groundtruth Current State Information:
{state_information_dict}

State Description:
{natural_language_description}

Think step by step:

1. Check if the description contains spatial relationship between agent and target object
 - If no spatial relationship is mentioned, answer NO
2. If spatial relationship exists, check if the predicted direction is consistent with the target direction
 - "ahead/forward" = "ahead"
 - "left" = "left"
 - "right" = "right"
 - Combined directions like "forward-left", "forward-right" are acceptable if they include the correct primary direction
3. The prediction is correct if it mentions moving toward the target in a direction that reasonably aligns with the groundtruth direction

Your answer should be in the format of <think>...</think><answer>YES</answer> or <think>...</think><answer>NO</answer>.

Navigation TransitionModeling Evaluation

Evaluate whether the prediction effectively anticipates how the agent would navigate toward the target object, even if the exact directional terms differ.

Answer YES if the overall navigation plan is reasonable, or NO if it misunderstands or did not mention the spatial layout.

Context

You are evaluating whether the prediction effectively anticipates how the agent would move to reach the target object. The exact directional terminology (left, right, ahead, etc.) may differ between the state information and the prediction, but the important factor is whether the prediction would lead to successful navigation.

Important: The Prediction Comes First

Remember: The Next State Prediction is made BEFORE the Groundtruth Next State exists. Your task is to check if the prediction correctly anticipated what actually happened.

Groundtruth Next State Information:
{state_information_dict}

Next State Prediction:
{natural_language_description}

Think step by step:

1. First, check if the prediction explicitly uses EXACT directional terms

that appear in the groundtruth state: "ahead", "left", "right", "up", "down".

- Terms like "move towards", "closer to", "near", "approaching", "in front of", "by", "at" DO NOT qualify
- "Will be on the left/right/ahead" or "Will move left/right/forward" DO qualify
- If no exact directional match to groundtruth is present, conclude with NO immediately

2. If explicit direction words exist, verify that they EXACTLY match the target object's direction in the groundtruth:

- If target is "ahead", prediction must specify "ahead", "forward", "slightly left", OR "slightly right" (special case: we allow slightly left/right for ahead targets)
- If target is "right", prediction must specify "right"
- If target is "left", prediction must specify "left"

3. Even if the prediction mentions intermediate objects correctly, it MUST explicitly state the correct final direction to the target object

4. The prediction cannot substitute object references for directions (saying "move to X" instead of "move right")

5. Remember that the prediction was made BEFORE the groundtruth state was determined

Your answer should be in the format of <think>...</think><answer>YES</answer> or <think>...</think><answer>NO</answer>.

PrimitiveSkill StateEstimation Evaluation

Compare the description of the current state with the groundtruth current state information.

Answer YES if the description reasonably matches the current state information, or NO if it doesn't.

Context

You are evaluating whether an agent's description accurately reflects the actual state. The description should capture the meaningful relationships and positions relevant for decision-making in the task.

Important evaluation criteria:

1. If the description includes coordinates, they don't need to be exact matches with the groundtruth
2. For coordinate values, consider them correct if they are within these error tolerances:
 - For x and y coordinates: within +10 or -10 units of groundtruth
 - For z coordinates: within +10 or -10 units of groundtruth
3. The overall spatial relationships and object identifications should be correct
4. If the description includes a dict-formatted state information, that's good but not required

Groundtruth Current State Information:
{state_information_dict}

State Description:

{natural_language_description}

Think step by step:

1. Identify the key objects and their positions in the groundtruth information
2. Check if the description correctly identifies these objects
3. For any coordinates mentioned, check if they are within the acceptable error range
4. Determine if the overall spatial understanding is correct, even if specific numbers differ slightly
5. If coordinates in the description differ from groundtruth but are within the error tolerance, consider them correct

Your answer should be in the format of <think>...</think><answer>YES</answer> or <think>...</think><answer>NO</answer>.

PrimitiveSkill TransitionModeling Evaluation

Compare the prediction of the next state with the groundtruth next state information.

Answer YES if the prediction reasonably matches the next state information, or NO if it doesn't.

Context

You are evaluating whether an agent's prediction of the next state is accurate. The prediction should capture the meaningful changes and relationships that will result from the planned actions.

Important: The Prediction Comes First

Remember: The Next State Prediction is made BEFORE the Groundtruth Next State exists. Your task is to check if the prediction correctly anticipated what actually happened.

Important evaluation criteria:

1. If the prediction includes coordinates, they don't need to be exact matches with the groundtruth
2. For coordinate values, consider them correct if they are within these error tolerances:
 - For x and y coordinates: within +10 or -10 units of groundtruth
 - For z coordinates: within +10 or -10 units of groundtruth
3. The overall predicted movements and resulting spatial relationships should be correct
4. If the prediction includes a dict-formatted state information, that's good but not required

Groundtruth Next State Information:

{state_information_dict}

Next State Prediction:

{natural_language_description}

Think step by step:

1. Identify the key objects and their positions in the groundtruth next


```

state information
2. Check if the prediction correctly anticipates these object positions
3. For any coordinates mentioned, check if they are within the acceptable
   error range
4. Determine if the overall predicted movement and resulting state is
   correct, even if specific numbers differ slightly
5. If coordinates in the prediction differ from groundtruth but are
   within the error tolerance, consider them correct

Your answer should be in the format of <think>...</think><answer>YES</
answer> or <think>...</think><answer>NO</answer>.

```

Text-based State Information To provide ground truth information for the judge model, we extract and format state information differently for each environment:

Sokoban: We extract the positions of players, boxes, and targets, then convert their relative positions into natural language sentences. For example:

```

['box0 is above and at the same column as the player',
 'target0 is above and at the same column as the player',
 'target0 is above and at the same column as box0']

```

FrozenLake: Similarly, we convert positional relationships into descriptive sentences:

```

['target is above and at the same column as the player',
 'hole0 is above and at the same column as the player',
 'hole1 is above and on the right side of the player',
 'hole2 is above and on the right side of the player',
 'hole3 is above and on the right side of the player',
 'hole4 is at the same row and to the right of the player']

```

Navigation: We use dictionary-based information directly from the environment:

```

{'target_obj_type': 'Toaster',
 'target_distance_to_player': 2.59,
 'target_direction_to_player': 'ahead',
 'visible_objects': [{'type': 'Cabinet',
                       'direction_to_player': 'left',
                       'distance_to_player': 0.94},
 {'type': 'Drawer',
  'direction_to_player': 'left',
  'distance_to_player': 1.18},
 {'type': 'CounterTop',
  'direction_to_player': 'left',
  'distance_to_player': 1.33},
 ...]}

```

PrimitiveSkill: We provide coordinate-based position information:

```

{'left_target_position': (80, -100, 0),

```

```
'right_target_position': (80, 100, 0),
'red_cube_position': (83, -102, 20),
'green_cube_position': (-47, 95, 20)}
```

D.1.2. Mitigating Reward Hacking via Structured Evaluation and Repetition Penalty

To address reward, we developed and implemented a composite reward mechanism that combines structured evaluation with a penalty for historical repetition.

The method consists of the following key steps:

1. **Natural Language to Structured Reasoning Conversion:** Instead of relying on a holistic, qualitative judgment from the LLM on the agent’s natural language reasoning, we first use LLM to parse and convert the reasoning into a **structured dictionary**. This dictionary explicitly captures key entities and relations within the reasoning.
2. **Quantitative Evaluation with F1 Score:** The structured dictionary is then programmatically compared against a ground-truth dictionary representing the optimal reasoning for the given state. By calculating the **F1 score** between these two dictionaries, we obtain a precise and quantitative measure of the correctness and completeness of the agent’s reasoning. This approach is significantly more robust than relying on vague text similarity or an LLM’s overall impression.
3. **Penalty Mechanism for Low-Entropy and Repetitive Behavior:** We observed that agents can fall into a pattern of over-exploiting previously successful reasoning, repeatedly generating the same high-reward sentences even in inappropriate states. This behavior is reflected in a noticeable decrease in the **entropy** of its outputs. To counteract this, we introduced a penalty mechanism:
 - We maintain a **max-heap** to dynamically track the most frequently generated sentences in the agent’s history.
 - When evaluating a new response from the agent, we perform a dual-condition check:
 - (a) The response sentence matches one of the top frequent sentences in the heap.
 - (b) Its corresponding F1 score is **below** a predefined correctness threshold.
Only when **both conditions are met** do we conclude that the agent is blindly repeating an incorrect answer and apply a negative penalty.

In our Sokoban experiments, we set the F1 score threshold to 0.7. If a response was identified as both repetitive and incorrect ($F1 < 0.7$), a penalty of -0.1 was applied to the reward. This mechanism forces the agent to explore more diverse and state-relevant reasoning pathways.

D.2. Bi-Level General Advantage Estimation (GAE)

This section provides the detailed pseudo-code for our Bi-Level GAE algorithm, which is shown in Algorithm 2. The key modification from the base RL algorithm is in Phase 2, where we introduce a bi-level advantage estimation process that operates at both turn and token levels. To explore the effectiveness of Bi-level GAE, we also experiment adding Bi-level GAE to Freethink baselines, and results in Table 25 shows that Bi-level GAE is able to serve as a general plug-in strategy for RL training. Results on different model families and sizes are shown in Table 26, training time and token usage are given in 27.

Table 25 | Bi-level GAE helps FreeThink RL baselines, and can serve as a general mechanism in RL training for VLM agents.

Group / Model	Task	Method	Test Success Rate
Qwen2.5-VL 3B	FrozenLake	FreeThink	0.39
	FrozenLake	FreeThink + Bi-Level GAE	0.62
Qwen2.5-VL 3B	Sokoban	FreeThink	0.43
	Sokoban	FreeThink + Bi-Level GAE	0.43

Table 26 | Results on different model families and sizes on Sokoban task.

Group / Model	Method	Test Success Rate
Qwen2.5-VL 3B	VAGEN-BASE	0.61
	VAGEN-FULL	0.79
Qwen2.5-VL 7B	VAGEN-BASE	0.63
	VAGEN-FULL	0.92
InternVL3-2B	VAGEN-BASE	0.36
	VAGEN-FULL	0.39

Table 27 | Training efficiency for primary experiments (approximate).

Task	H100 GPU Hours (Approx.)	LLM-as-Judge Token Cost (Approx.)
FrozenLake	40	2.3×10^7
Sokoban	40	2.3×10^7
Navigation	30	6.0×10^6
ManiSkill	30	4.6×10^6
SVG	10	N/A

E. Case Study

In this section, we present a comprehensive case study examining the behavioral patterns and learning dynamics of multi-turn VLM agents across distinct environments. Through systematic analysis of agent responses during different training phases, we investigate how visual state reasoning capabilities evolve and identify key phenomena that emerge during the learning process.

E.1. Visual State Reasoning Enhances Spatial Understanding and Multi-Step Planning

Our analysis reveals that incorporating explicit visual state reasoning into multi-turn VLM agents significantly improves their spatial understanding and planning capabilities across different environments.

The integration of StateEstimation and TransitionModeling steps enables agents to better understand relative positions between objects. In Sokoban (Figure 5), agents trained with visual reasoning demonstrate improved ability to identify spatial configurations between the player, target, and box. This allows them to effectively navigate around obstacles while maintaining progress toward objectives, contrasting with untrained agents that often make suboptimal moves without proper spatial consideration.

Visual state reasoning enables sophisticated multi-step planning. In Navigation tasks (Figure 5), agents develop the ability to not only identify target objects and their positions but also recognize blocking objects that impede direct paths. This leads to:

- **Accurate spatial relationship identification** between targets, obstacles, and agent position
- **Recognition of blocking constraints** that prevent direct paths
- **Multi-step sequence planning** that accounts for both immediate and longer-term goals

E.2. Response Convergence and Reduced Exploration

We observe a notable pattern of response convergence across different environments, reflecting the fundamental learning dynamics described in the main text. As shown in Cases 1 through 4, agents exhibit dramatically different response behaviors between early and late training phases, with this phenomenon occurring both with and without our TransitionModeling Reward as a fundamental aspect of the learning process.

During early training, agents demonstrate strong exploratory tendencies with diverse response patterns. In Sokoban tasks, early responses show varied reasoning approaches, creative problem-solving strategies, and rich descriptive language when analyzing spatial configurations. Similarly, Navigation tasks initially elicit detailed environmental descriptions and multi-faceted planning considerations, reflecting the high entropy characteristic of early-stage learning.

As training progresses, agents develop more streamlined and consistent communication patterns. Late-stage responses become notably more concise and structured, with standardized phrasing that primarily varies in directional or specific action tokens while maintaining clear, efficient expression. This evolution toward templated sentence structures occurs across different environments and reasoning qualities, with entropy steadily decreasing as documented quantitatively in the main analysis.

This convergence represents a fundamental aspect of the learning process, where agents naturally optimize their communication patterns for task effectiveness while exhibiting the reduced exploration behavior characteristic of policy convergence in reinforcement learning.

Sokoban - Early Training

World Modeling - Transition Modeling

State:

['box0 is at the same row and to the right of the player', 'target0 is at the same row and to the right of the player', 'target0 is at the same place as box0']

Response:

The player will move down, then to the right, pushing the box to the target.

State:

['box0 is above and at the same column as the player', 'target0 is above and at the same column as the player', 'target0 is above and at the same column as box0']

Response:

Push the box down and then move to the right. These actions would bring the box closer to the target.

World Modeling - State Estimation/Grounding

State:

['box0 is above and at the same column as the player', 'target0 is above and on the left side of the player', 'target0 is above and on the left side of box0']

Response:

The player is above the box, and the player is below the target, and there is a wall between the player and the target. The box is above the player

State:

['box0 is above and on the right side of the player', 'target0 is above and on the right side of the player', 'target0 is above and on the right side of box0']

Response:

From the image, the player is at the bottom of the wall, with the box directly in front of the player to the right and the target directly above the box.

Sokoban - Late Training

World Modeling - Transition Modeling

State:

['box0 is at the same row and to the left of the player', 'target0 is above and on the left side of the player', 'target0 is above and at the same column as box0']

Response:

The player will move to the left, then push the box left to the target.

State:

['box0 is at the same row and to the right of the player', 'target0 is above and on the right side of the player', 'target0 is above and at the same column as box0']

Response:

The player will move to the right, then push the box right to the target.

World Modeling - State Estimation/Grounding

State:

['box0 is above and at the same column as the player', 'target0 is above and on the right side of the player', 'target0 is above and on the right side of box0']

Response:

The player is below the box, and the box is below the target.

State:

['box0 is below and on the left side of the player', 'target0 is below and on the left side of the player', 'target0 is at the same row and to the left of box0']

Response:

The player is to the right of the box, and the box is to the right of the target.

Navigation - Early Training

World Modeling - Transition Modeling

State:

```
{'target_obj_type': 'Laptop',  
 'target_distance_to_player': 2.58,  
 'target_direction_to_player': 'right',  
 'visible_objects': [{'type': 'Bed',  
   'direction_to_player': 'ahead',  
   'distance_to_player': 1.22},  
 {'type': 'Pillow',  
   'direction_to_player': 'left',  
   'distance_to_player': 1.35},  
 ...]}
```

Response:

I can see a laptop on the desk. I will move forward and right to get closer to the laptop.

State:

```
{'target_obj_type': 'Laptop',  
 'target_distance_to_player': 2.31,  
 'target_direction_to_player': 'ahead',  
 'visible_objects': [{'type': 'Drawer',  
   'direction_to_player': 'ahead',  
   'distance_to_player': 0.85},  
 {'type': 'Television',  
   'direction_to_player': 'ahead',  
   'distance_to_player': 1.3},  
 ...]}
```

Response:

My next move will be to approach the table where the laptop is placed. The laptop is placed on the right side of table in front of me.

World Modeling - State Estimation/Grounding

State:

```
{'target_obj_type': 'CellPhone',  
 'target_distance_to_player': 2.59,  
 'target_direction_to_player': 'ahead',  
 'visible_objects': [{'type': 'GarbageCan',  
   'direction_to_player': 'ahead',  
   'distance_to_player': 2.2},  
 {'type': 'CD',  
   'direction_to_player': 'ahead',  
   'distance_to_player': 2.55},  
 ...]}
```

Response:

There is a table with a cell phone on it in the middle of the room. The cell phone is on the table, close to the center of the room, right beside a trash can.

State:

```
{'target_obj_type': 'Statue',  
 'target_distance_to_player': 2.54,  
 'target_direction_to_player': 'ahead',  
 'visible_objects': [{'type': 'Drawer',  
   'direction_to_player': 'ahead',  
   'distance_to_player': 1.3},  
 ...]}
```

```

    'direction_to_player': 'ahead',
    'distance_to_player': 1.99},
    {'type': 'Drawer',
     'direction_to_player': 'ahead',
     'distance_to_player': 1.99},
    ...]}

```

Response:

There is a statue on a table in the room. I need to move forward in order to reach the statue. The floor is smooth, so I can move forward without obstacles.

Navigation - Late Training

World Modeling - Transition Modeling

State:

```

{'target_obj_type': 'DeskLamp',
 'target_distance_to_player': 1.71,
 'target_direction_to_player': 'right',
 'visible_objects': [{'type': 'Chair',
                       'direction_to_player': 'ahead',
                       'distance_to_player': 1.08},
                     {'type': 'Laptop',
                       'direction_to_player': 'ahead',
                       'distance_to_player': 1.19},
                     ...]}

```

Response:

I will move forward-right to reach the lamp.

State:

```

{'target_obj_type': 'Pot',
 'target_distance_to_player': 2.54,
 'target_direction_to_player': 'left',
 'visible_objects': [{'type': 'Bowl',
                       'direction_to_player': 'ahead',
                       'distance_to_player': 0.88},
                     {'type': 'Potato',
                       'direction_to_player': 'ahead',
                       'distance_to_player': 1.2},
                     ...]}

```

Response:

I will move forward-left to reach the pot.

World Modeling - State Estimation/Grounding

State:

```

{'target_obj_type': 'Bowl',
 'target_distance_to_player': 1.52,
 'target_direction_to_player': 'ahead',
 'visible_objects': [{'type': 'Book',
                       'direction_to_player': 'right',
                       'distance_to_player': 1.18},
                     {'type': 'Floor',
                       'direction_to_player': 'back',
                       'distance_to_player': 1.3},
                     ...]}

```



```
...]]}
```

Response:

There is a round container on the floor. To reach the round container, I need to move forward.

State:

```
{'target_obj_type': 'GarbageCan',  
 'target_distance_to_player': 1.71,  
 'target_direction_to_player': 'ahead',  
 'visible_objects': [{'type': 'GarbageBag',  
   'direction_to_player': 'ahead',  
   'distance_to_player': 1.42},  
 {'type': 'GarbageCan',  
   'direction_to_player': 'ahead',  
   'distance_to_player': 1.71},  
 ...]]}
```

Response:

There is a trash can in front of the table to the right. To reach the trash can, I need to move forward-right.

E.3. Reward Hacking and Over-optimization

We observe interesting patterns of reward optimization behavior, particularly in spatially sensitive environments like FrozenLake where agents learn to adapt their responses to the LLM-as-a-Judge evaluation mechanism. As demonstrated in Cases 5 and 6, agents develop strategic approaches to maximize reward signals, representing a form of reward over-optimization that exemplifies the reward hacking phenomenon described in the main text.

The most notable optimization pattern involves agents converging toward generic, broadly applicable responses that incorporate multiple directional possibilities and general spatial relationships. Rather than providing highly specific, state-dependent analysis, agents learn to produce consistent reasoning templates that reliably satisfy the reward mechanism's evaluation criteria without necessarily reflecting deep, state-specific reasoning. For instance, responses like "The player will be at the position of the gift" emerge as universal patterns that can be applied across diverse state configurations, effectively balancing reward acquisition with computational efficiency while potentially sacrificing reasoning specificity.

This reward hacking behavior becomes more pronounced under Bi-Level training configurations, as highlighted in the main analysis. The enhanced optimization method and turn-aware reward structure enable agents to become more sensitive to reward patterns and develop sophisticated strategies for consistent reward acquisition. The Bi-Level GAE's expanded exploration capabilities and improved credit assignment allow agents to efficiently identify and adapt to the evaluation criteria, demonstrating the system's remarkable effectiveness in learning reward-maximizing behaviors. However, this optimization may sometimes diverge from the intended reasoning improvements, illustrating the complex dynamics between reward design and genuine capability enhancement in LLM-based evaluation systems.

FrozenLake - Early Training

World Modeling - Transition Modeling

State:

['target is at the same row and to the left of the player', 'hole0 is at the same place as the player', 'hole1 is below and on the left side of the player', 'hole2 is below and on the right side of the player']

Response:

The player will either go left or right to avoid the hole and continue moving up to reach the icy blocks.

State:

['target is above and on the right side of the player', 'hole0 is above and on the right side of the player']

Response:

The player will be in the bottom center after moving right, and then move down to the center of the gift.

World Modeling - State Estimation/Grounding

State:

['target is below and on the left side of the player', 'hole0 is below and on the left side of the player']

Response:

The player is standing on a block, there is a hole directly in front of the player, and there is a goal at the left corner of the player.

State:

['target is above and on the left side of the player', 'hole0 is above and on the left side of the player', 'hole1 is below and on the left side of the player', 'hole2 is below and at the same column as the player']

Response:

The player is now at the right side of the grid, adjacent to a hole. The goal (G) is on the left side of the player.

FrozenLake - Late Training

World Modeling - Transition Modeling (Hacking)

State:

['target is at the same place as the player', 'hole0 is below and on the right side of the player']

Response:

The player will be at the position of the gift.

State:

['target is above and on the right side of the player']

Response:

The player will be at the position of the gift.

World Modeling - State Estimation/Grounding (Hacking)

State:

['target is below and on the right side of the player', 'hole0 is above and on the right side of the player', 'hole1 is below and on the right side of the player']

Response:

The player is at the top left, the gift is at the bottom right, and there are holes at the top and bottom right corners.

State:

['target is at the same row and to the left of the player']

Response:

The player is in the center of the grid, the gift is on the left side, and there is a hole on the right side.

F. Summary of Findings

Our investigation into reinforcing visual state reasoning for multi-turn VLM agents, facilitated by the VAGEN training framework, has yielded several key findings regarding the efficacy of explicit reasoning, optimal state representations, specialized RL techniques, and observed agent behaviors during training. These insights are crucial for developing more robust and capable VLM agents that can understand and interact with dynamic visual environments. Below is the summary of our findings.

Finding 1: Explicit Visual State Reasoning is Crucial for Multi-Turn VLM Agents

Vanilla VLMs struggle with multi-turn agentic tasks requiring visual state understanding. Integrating explicit visual state reasoning steps—specifically `StateEstimation` and `TransitionModeling`—into the VLM’s thinking process during RL training significantly enhances task performance. The combined `WorldModeling` strategy, in particular, demonstrates strong and stable performance, enabling a trained open-source VLM to outperform its un-trained counterpart and even surpass benchmarked proprietary models.

Finding 2: Optimal Visual State Representation is Task-Dependent

The choice of representation for visual states during explicit reasoning significantly impacts performance.

- **Natural Language:** Performs consistently well, especially when structured information must be inferred from raw visual input.
- **Structured Formats:** Excel in manipulation-heavy tasks (e.g., `PrimitiveSkill`) where object-centric state abstractions are readily available.
- **Symbolic Representations:** Proved less effective due to the model’s limited prior interpretability from visual input.

Finding 3: Visual Reasoning RL with Targeted Rewards and Bi-Level GAE Enhances Reasoning Quality and Task Success

To specifically improve visual state reasoning, Visual Reasoning RL incorporates:

- **Turn-level World Modeling Reward:** An LLM-as-a-Judge assesses the accuracy of the VLM’s explicit state descriptions and predictions, effectively supervising reasoning.
- **Bi-Level General Advantage Estimation (GAE):** Estimates advantages at both turn and token levels, providing finer-grained reward signals and improving credit assignment.

This approach consistently outperforms Base RL, leading to improved reasoning quality, higher task success rates, and better generalization.

Finding 4: Emergent Reasoning Patterns and Challenges

Beyond quantitative measurements, we qualitatively analyzed how agents learn to reason:

- **Reasoning Stability Varies by Task:** While reasoning in tasks like Navigation and PrimitiveSkill (and often Sokoban) appears relatively normal and beneficial with explicit rewards, tasks like FrozenLake show more erratic reasoning patterns, potentially correlating with its lower performance and the difficulty of its visual state reasoning.
- **Potential for Reward Hacking:** Instances of "reward hacking" were observed, particularly with certain reward configurations (e.g., Bi-Level world modeling in some contexts). Agents might learn to generate reasoning-like text that satisfies the reward mechanism (e.g., format reward, or even LLM-judge for simple cases) without genuinely reflecting deep understanding or accurate future prediction. This suggests the LLM-as-a-Judge mechanism, while an improvement, is not infallible and can be gamed.
- **Bi-Level GAE as a Double-Edged Sword:** While Bi-Level GAE can improve credit assignment, its interaction with World Modeling Rewards might sometimes allow for more "divergent" or less grounded thinking if the reasoning reward itself can be easily hacked. Well-designed, hard-to-game reasoning rewards are crucial for Bi-Level GAE to be consistently beneficial; otherwise, it might amplify the effects of a poor intermediate reward.
- **Convergence to Standardized Phrasing:** Regardless of initial hacking or reasoning quality, agents across different environments tend to converge towards using a more uniform, templated sentence structure for their reasoning and actions over prolonged training, primarily varying only the directional or specific action tokens. This might be an efficiency learned for tasks ultimately requiring discrete actions but could also indicate a reduction in diverse or creative reasoning.
- **Rule-Based Filtering as a Potential Mitigation:** For simpler forms of reward hacking where reasoning outputs fail basic semantic checks (e.g., not mentioning valid actions in a grid world), simple rule-based filtering before reward assignment could be a pragmatic interim solution.

These observations underscore that while explicit reasoning and rewards are beneficial, the design of these rewards must be robust against exploitation, and continuous monitoring of reasoning quality is essential.

Algorithm 1 VAGEN-Base: Reinforcement Learning for VLM Agents

Input: Actor VLM π_θ , Environment, Critic V_φ , Reference VLM π_{ref}

Hyperparameters: Discount factor γ , GAE coefficient λ , PPO clip range ϵ , KL penalty coefficient β

Phase 1: Trajectory Collection

- 1: Initialize trajectory $\tau = []$; Sample initial state and observation s_0, o_0 from Environment
 - 2: **for** $t = 0, \dots, T - 1$ **do**
 - 3: Sample action $a_t \sim \pi_\theta(\cdot | \bar{\tau}_{< a_t})$ autoregressively, where $\bar{\tau}_{< a_t}$ is the history.
 - 4: Parse executable action a_t^e from a_t .
 - 5: Execute a_t^e in Environment to get s_{t+1}, o_{t+1} and task reward $R(s_t, a_t)$.
 - 6: Append $(s_t, o_t, a_t, R(s_t, a_t))$ to trajectory τ .
 - 7: **end for**
 - 8: Construct training sequence $\bar{\tau} = [\mathcal{E}(o_0); \mathcal{E}(a_0); \dots; \mathcal{E}(o_{T-1}); \mathcal{E}(a_{T-1})]$.
 - 9: Store old policy probabilities $\pi_{\text{old}}(\bar{\tau}_i | \bar{\tau}_{< i})$ for all generated tokens in $\bar{\tau}$.
-

Phase 2: Advantage Estimation (Token-Level GAE)

- 10: Let M_i^{loss} be a mask that is 1 for action tokens and 0 for observation tokens.
 - 11: Calculate total trajectory task reward $R(\tau) = \sum_{t=0}^{T-1} R(s_t, a_t)$.
 - 12: **for** each action token index $i = 0, \dots, |\bar{\tau}| - 1$ **do**
 - 13: Calculate KL penalty: $r_i^{\text{KL}} = -\beta \cdot \text{KL} [\pi_\theta(\cdot | \bar{\tau}_{< i}) \parallel \pi_{\text{ref}}(\cdot | \bar{\tau}_{< i})]$.
 - 14: Initialize token reward $r_i = r_i^{\text{KL}}$.
 - 15: **end for**
 - 16: Add trajectory reward to the final action token: $r_{|\bar{\tau}|-1} \leftarrow r_{|\bar{\tau}|-1} + R(\tau)$.
 - 17: Initialize advantage $A_{|\bar{\tau}|} = 0$.
 - 18: **for** $i = |\bar{\tau}| - 1, \dots, 0$ (backwards) **do**
 - 19: Find the least index j of an action token where $j > i$
 - 20: Compute TD-error: $\delta_i = r_i + \gamma V_\varphi(\bar{\tau}_{< j}) - V_\varphi(\bar{\tau}_{< i})$. (Assume $V_\varphi(\text{terminal}) = 0$)
 - 21: Compute advantage: $A_i = \delta_i + \gamma \lambda A_j$.
 - 22: Compute return for critic update: $G_i = A_i + V_\varphi(\bar{\tau}_{< i})$.
 - 23: **end for**
-

Phase 3: Policy Update with PPO

- 24: **for** each action token index $i = 0, \dots, |\bar{\tau}| - 1$ **do**
 - 25: Compute probability ratio: $u_i(\theta) = \frac{\pi_\theta(\bar{\tau}_i | \bar{\tau}_{< i})}{\pi_{\text{old}}(\bar{\tau}_i | \bar{\tau}_{< i})}$.
 - 26: **end for**
 - 27: Compute PPO objective:
$$J^{\text{PPO}}(\theta) = \frac{1}{\sum M_i^{\text{loss}}} \sum_i M_i^{\text{loss}} \cdot \min(u_i(\theta) A_i, \text{clip}(u_i(\theta), 1 - \epsilon, 1 + \epsilon) A_i).$$
 - 28: Compute critic loss:
$$J^{\text{Critic}}(\varphi) = \frac{1}{\sum M_i^{\text{loss}}} \sum_i M_i^{\text{loss}} \cdot (V_\varphi(\bar{\tau}_{< i}) - G_i)^2.$$
 - 29: Update parameters θ and φ using their respective gradients.
-

Algorithm 2 Bi-Level General Advantage Estimation (GAE)

Input:

Full trajectory token sequence $\bar{\tau}$, organized by turns $t = 0..T - 1$.
 Per-turn rewards r_0, \dots, r_{T-1} .
 Critic V_ϕ , Actor π_θ , Reference policy π_{ref} .
 Hyperparameters: $\gamma_{\text{turn}}, \lambda_{\text{turn}}, \gamma_{\text{token}}, \lambda_{\text{token}}, \beta$.

Output:

Token-level advantages $A_{t,i}^{\text{token}}$ for each token i in each action a_t .

Stage 1: Turn-Level Advantage Estimation

- 1: Initialize turn-level advantage $A_t^{\text{turn}} \leftarrow 0$.
 - 2: **for** $t = T - 1, \dots, 0$ (backwards) **do**
 - 3: Define value at current turn boundary: $V_t \leftarrow V_\phi(\bar{\tau}_{\leq a_t})$.
 - 4: **if** $t = T - 1$ **then**
 - 5: Define value at next turn boundary: $V_{t+1} \leftarrow 0$.
 - 6: **else**
 - 7: Define value at next turn boundary: $V_{t+1} \leftarrow V_\phi(\bar{\tau}_{\leq a_{t+1}})$.
 - 8: **end if**
 - 9: Compute turn-level TD-error: $\delta_t^{\text{turn}} \leftarrow r_t + \gamma_{\text{turn}} V_{t+1} - V_t$.
 - 10: Compute turn-level advantage: $A_t^{\text{turn}} \leftarrow \delta_t^{\text{turn}} + \gamma_{\text{turn}} \lambda_{\text{turn}} A_{t+1}^{\text{turn}}$.
 - 11: **end for**
-

Stage 2: Token-Level Advantage Estimation

- 12: **for** $t = 0, \dots, T - 1$ (forwards through turns) **do**
 - 13: Let $J = |\mathcal{E}(a_t)|$ be the number of tokens in action a_t .
 - 14: *// Initialize the advantage of the final token of the action with the turn-level advantage.*
 - 15: Let $\bar{\tau}_{t,J-1}$ be the final token of action a_t .
 - 16: Calculate its KL penalty: $r_{t,J-1}^{\text{KL}} \leftarrow -\beta \cdot \text{KL} [\pi_\theta(\cdot | \bar{\tau}_{<(t,J-1)}) \parallel \pi_{\text{ref}}(\cdot | \bar{\tau}_{<(t,J-1)})]$.
 - 17: Compute its TD-error: $\delta_{t,J-1}^{\text{token}} \leftarrow r_{t,J-1}^{\text{KL}} + \gamma_{\text{token}} V_\phi(\bar{\tau}_{\leq a_t}) - V_\phi(\bar{\tau}_{<(t,J-1)})$.
 - 18: Set final token's advantage: $A_{t,J-1}^{\text{token}} \leftarrow \delta_{t,J-1}^{\text{token}} + A_t^{\text{turn}}$.
 - 19: **for** $i = J - 2, \dots, 0$ (backwards through remaining tokens in the action) **do**
 - 20: Let $\bar{\tau}_{t,i}$ be the current token.
 - 21: Calculate KL penalty: $r_{t,i}^{\text{KL}} \leftarrow -\beta \cdot \text{KL} [\pi_\theta(\cdot | \bar{\tau}_{<(t,i)}) \parallel \pi_{\text{ref}}(\cdot | \bar{\tau}_{<(t,i)})]$.
 - 22: Compute token-level TD-error: $\delta_{t,i}^{\text{token}} \leftarrow r_{t,i}^{\text{KL}} + \gamma_{\text{token}} V_\phi(\bar{\tau}_{<(t,i+1)}) - V_\phi(\bar{\tau}_{<(t,i)})$.
 - 23: Compute token-level advantage: $A_{t,i}^{\text{token}} \leftarrow \delta_{t,i}^{\text{token}} + \gamma_{\text{token}} \lambda_{\text{token}} A_{t,i+1}^{\text{token}}$.
 - 24: **end for**
 - 25: **end for**
-