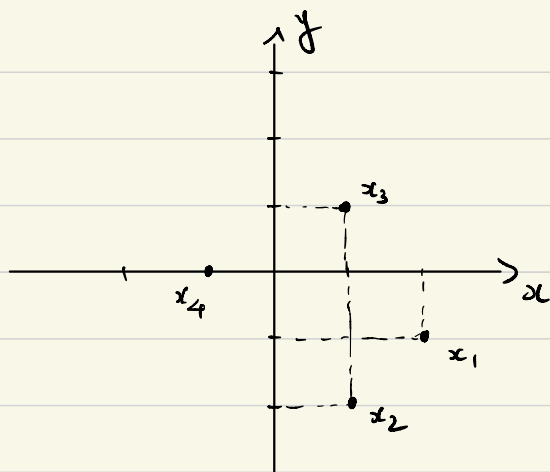




Exercise Round 1

Task 1

$$x^T l = 0$$



a) homogeneous coordinates:

$$x_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} ; x_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} ; x_4 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

b) Form 2 lines: l_1 through x_1, x_2 : $l_1 = x_1 \times x_2 = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$

l_2 through x_3, x_4 : $l_2 = x_3 \times x_4 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

c) x_5 is the intersection of l_1 and l_2

$$x_5 = l_1 \times l_2 = \begin{bmatrix} -7 \\ -4 \\ -1 \end{bmatrix} \rightarrow \text{in Cartesian Coordinate } x_5 = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

Task 2 a) + b)

Matrix representation

- translation : $T_1 = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$

2 DoF (degree of freedom)

- Euclidean transform : $T_2 = \begin{bmatrix} \cos \theta & -\sin \theta & t_x \\ \sin \theta & \cos \theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$ 3 DoF
(Rotation + translation)

- Similarity transform : $T_3 = \begin{bmatrix} s \cos \theta & -s \sin \theta & t_x \\ s \sin \theta & s \cos \theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$ 4 DoF

- Affine transform : $T_4 = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$ 5 DoF

- Projective transform : $T_5 = \begin{bmatrix} h_{11} & h_{21} & h_{31} \\ h_{12} & h_{22} & h_{32} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$ 8 DoF

c) The number of DoF in Projective transformation (8) is less than the number of elements in a 3×3 matrix because the last element in that matrix defines the ratios between other 8 elements. When we use the 9th value to normalize the elements, there are only 8 independent ratios left in the transformation matrix.