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Introduction

In this project we tried to implement non-linear controller for pose stabilization. The benefits of using non-linear controller instead of linear are:

1. It stabilizes both position and robot orientation.
2. The resulting trajectory is much closer to what human steering would look like.

Some of the issues related to the usage of non-linear controller are the following:

1. It is harder to evaluate controller stability.
2. The implementation is more complex.
3. The resulting trajectory is likely to be non-smooth.
4. The chosed controller is only able to asymptotically converge. Due to the nature of the coordinate transformation, the control functions are not defined at $e = 0$.

Controller design

The chosen controller is design via Lyapunov techniques [1,2]. The main idea is to transform complex problem of designing a controller capable of both position and orientation stabilization in cartesian coordinates to a much simpler problem in polar coordinates.

The usual set of kinematic equation for unicycle robot are the following:

$\dot{x} = u \cos \phi$
$\dot{y} = u \sin \phi$, where u is linear speed, ω is angular speed, ϕ is orientation of the robot
$\dot{\phi} = \omega$

After transformation into polar coordinates, the system can be rewritten in the following way:

$\dot{e} = -u \cos(\theta - \phi)$
$\dot{\theta} = u \frac{\sin \alpha}{e}$, where e is the error distance and θ is its orientation in respect to goal's orientation
$\phi = \omega$

After substitution $\alpha = \theta - \phi$ (angle between the vehicle direction and distance vector e):

$$\dot{e} = -u \cos \alpha$$

$$\dot{\alpha} = -\omega + u \frac{\sin \alpha}{e}$$

$$\dot{\theta} = u \frac{\sin \alpha}{e}$$

The control lows are defined as follows:

$u = \gamma\, e \cos\alpha$

$\omega = k\,\alpha + \gamma \dfrac{\cos\alpha \sin\alpha}{\alpha}\ (\alpha + h\,\theta)$, where k, γ and h are tuning parameters

After obtaining control inputs, we can get actuator inputs by applying unicycle kinematics model.

Results

The controller hyperparameters were chosen to be the same as in [1]: $\gamma = 3, k = 6, h = 1$.

The designed controller was able to achieve origin point from any tested position. Example trajectories can be seen bellow.

Case 1: from $\begin{bmatrix} -1 \\ 1 \\ \frac{3\pi}{4} \end{bmatrix}$ to origin.

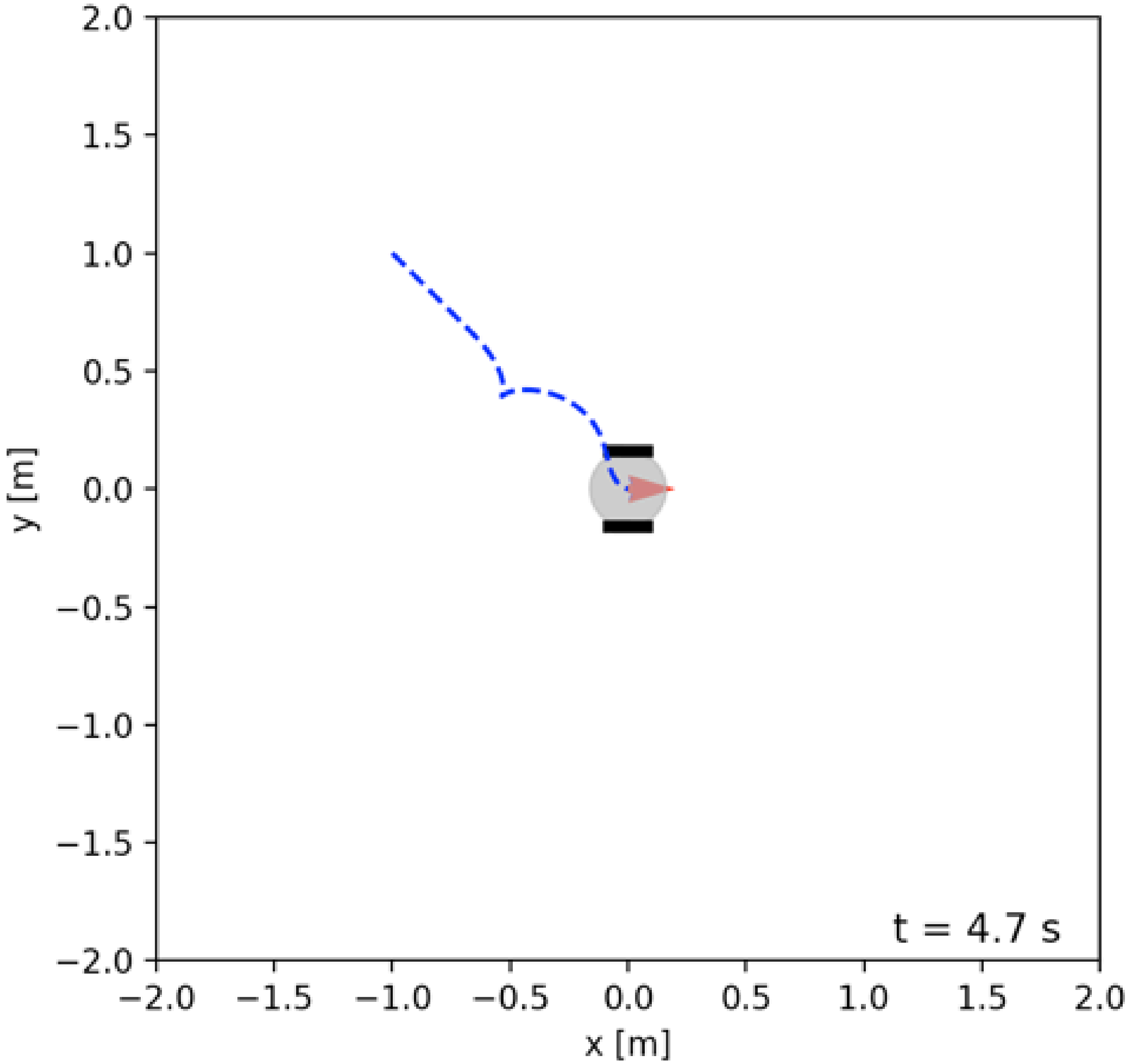

Figure 1. Path case 1.

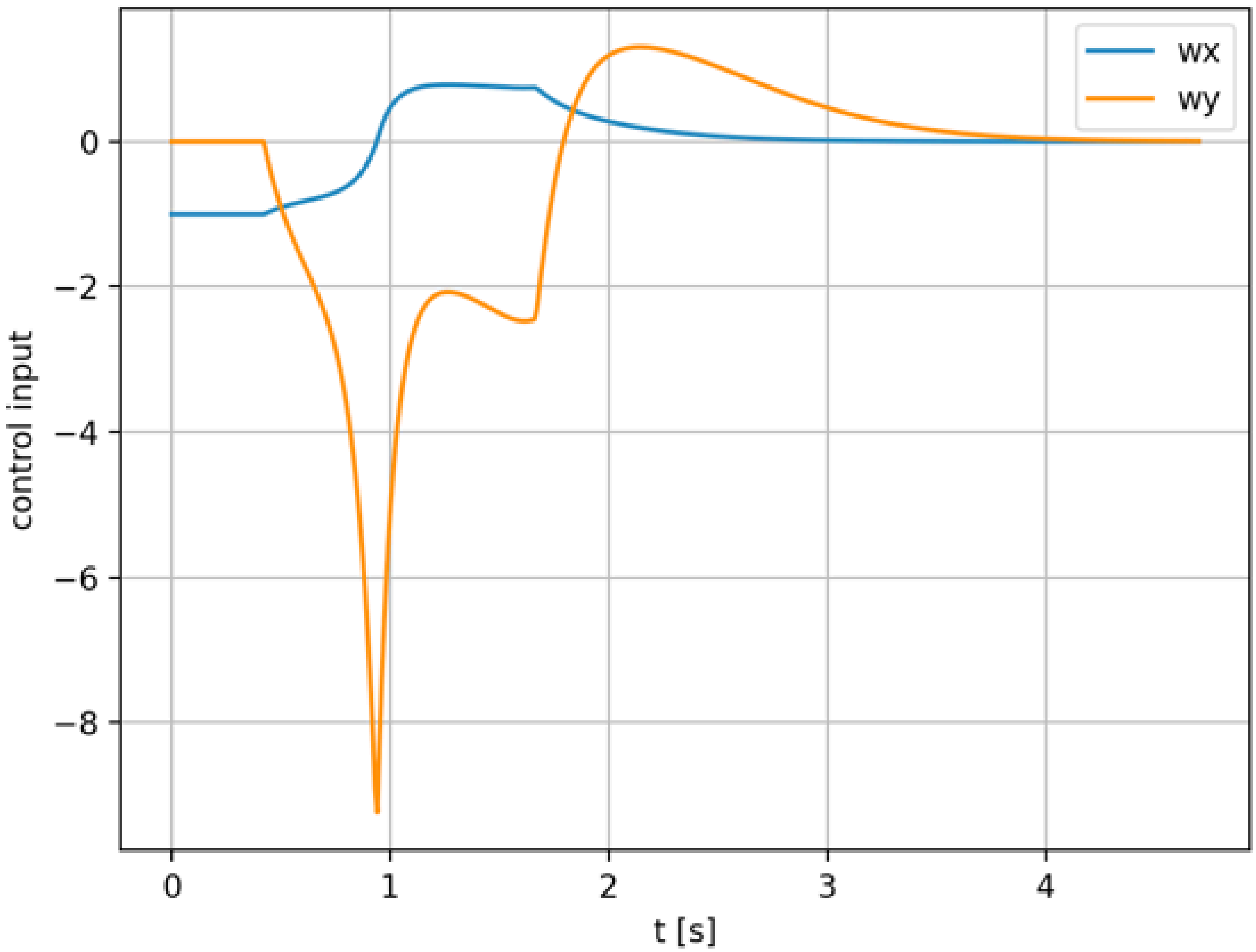

Figure 2. Control input case 1.

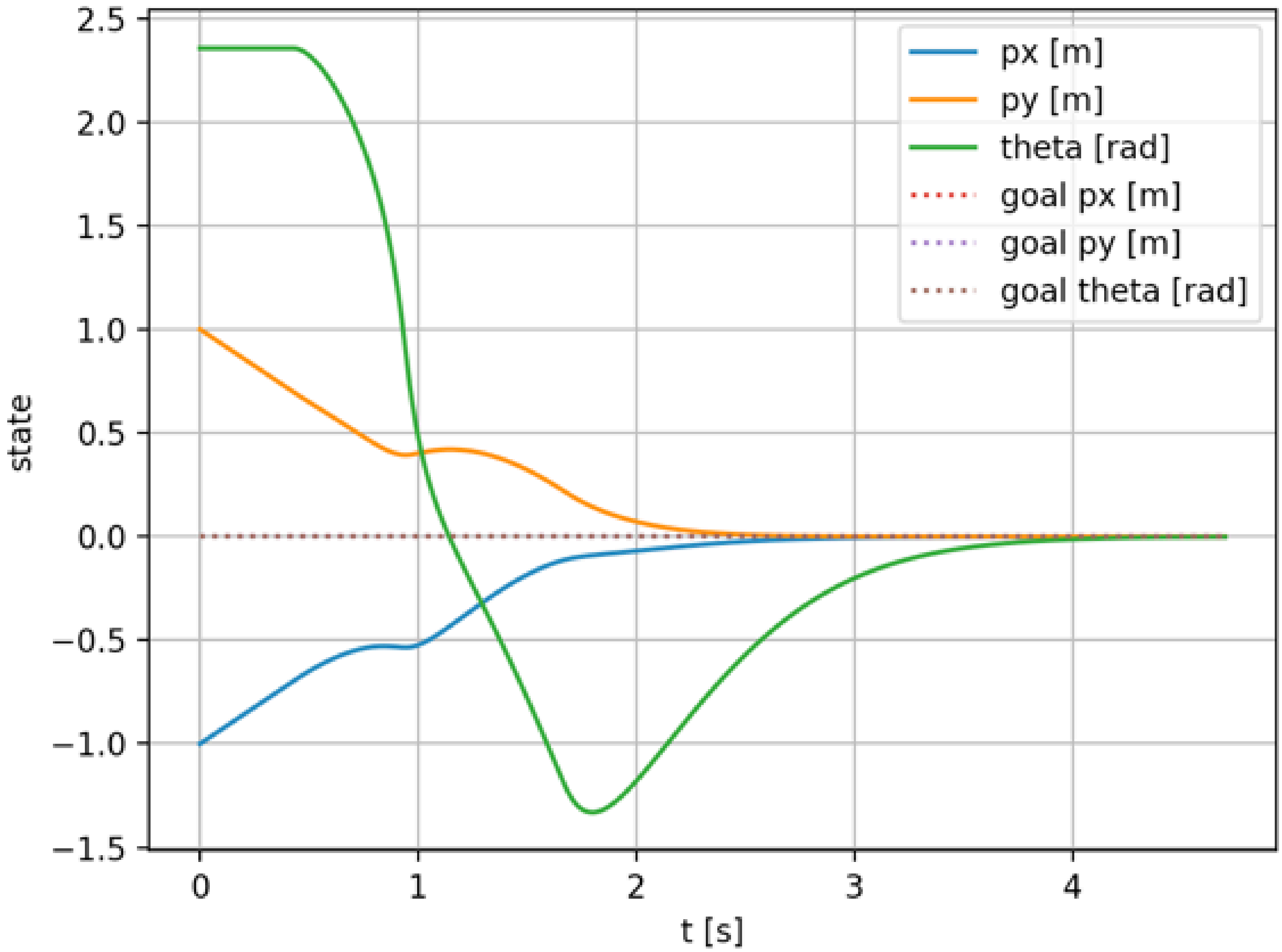

Figure 3. Position vs goal position case 1.

Case 2: from $\begin{bmatrix} -1.5 \\ -1 \\ -\pi \end{bmatrix}$ to origin.

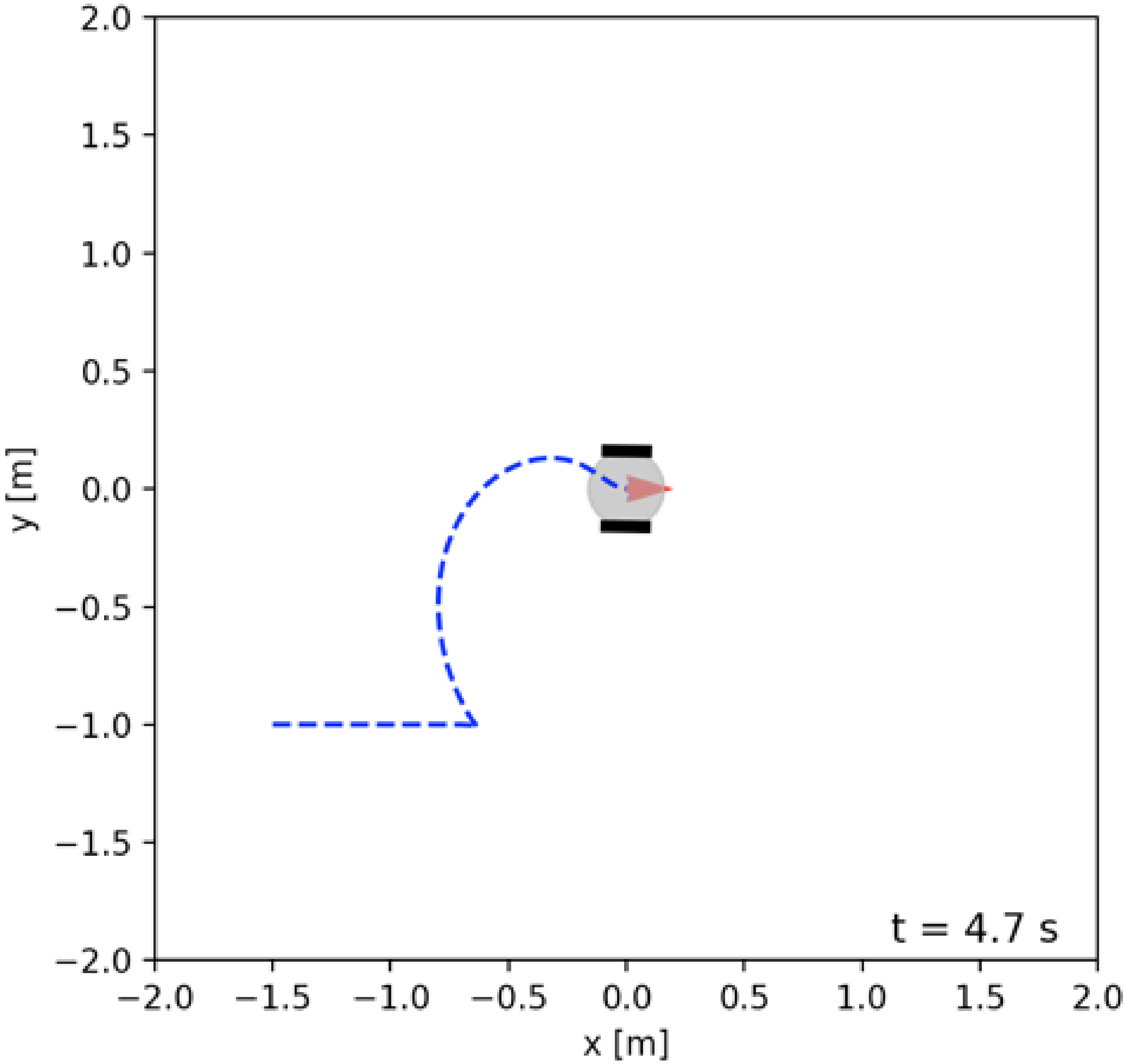

Figure 4. Path case 2.

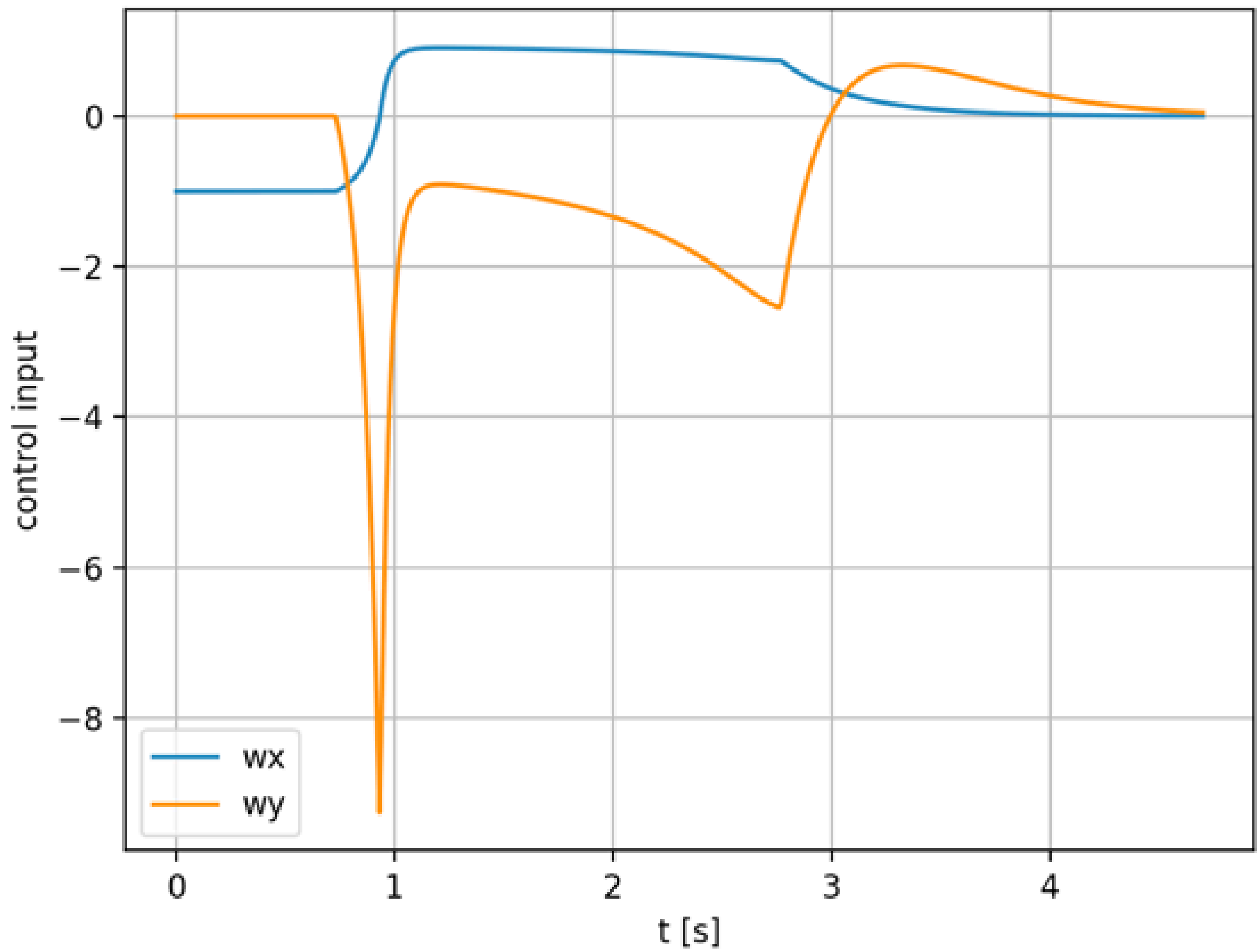

Figure 5. Control input case 2.

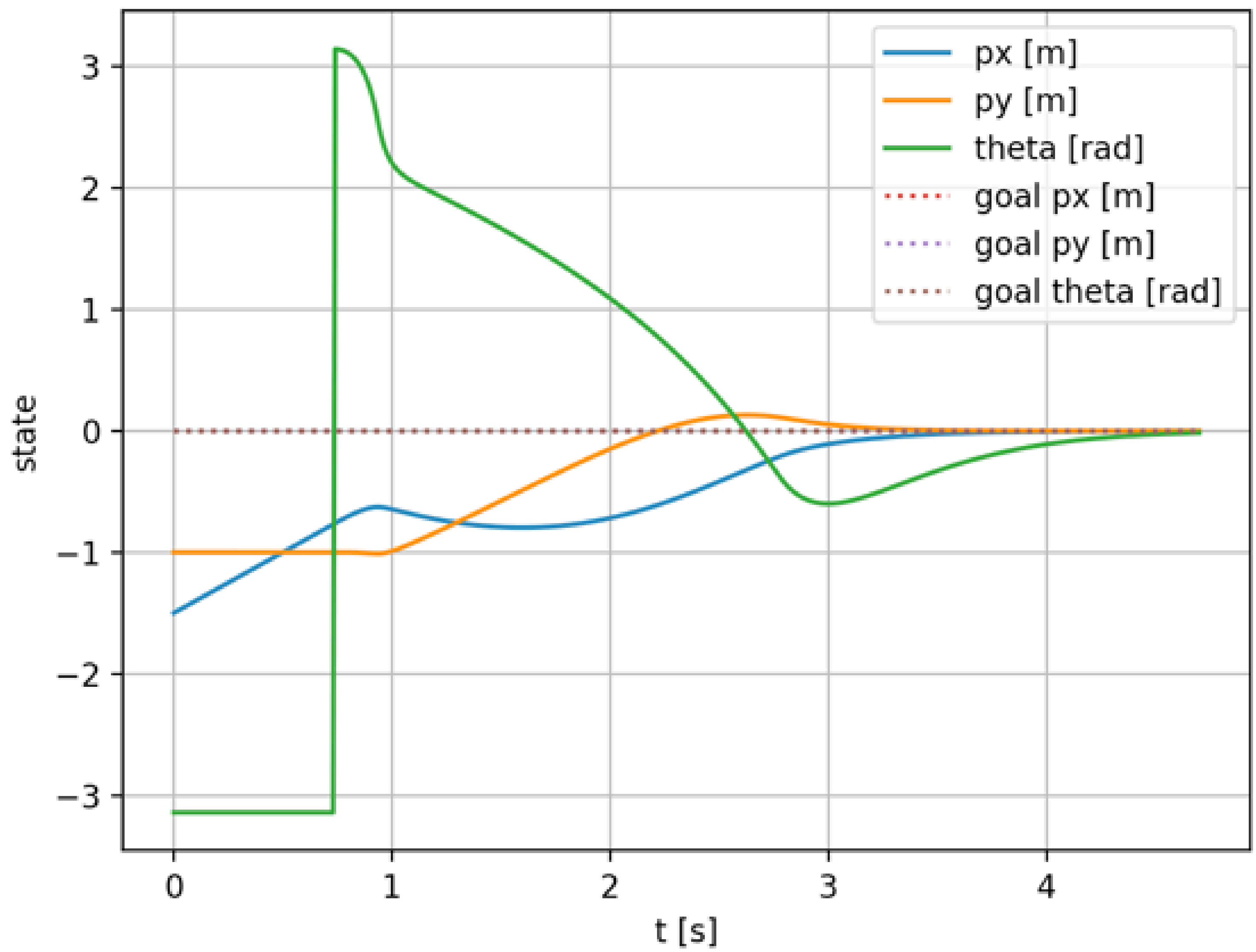

Figure 6. Robot state vs goal state case 2.

Case 3: from $\begin{bmatrix} 1.7 \\ 0.01 \\ \pi \end{bmatrix}$ to origin.

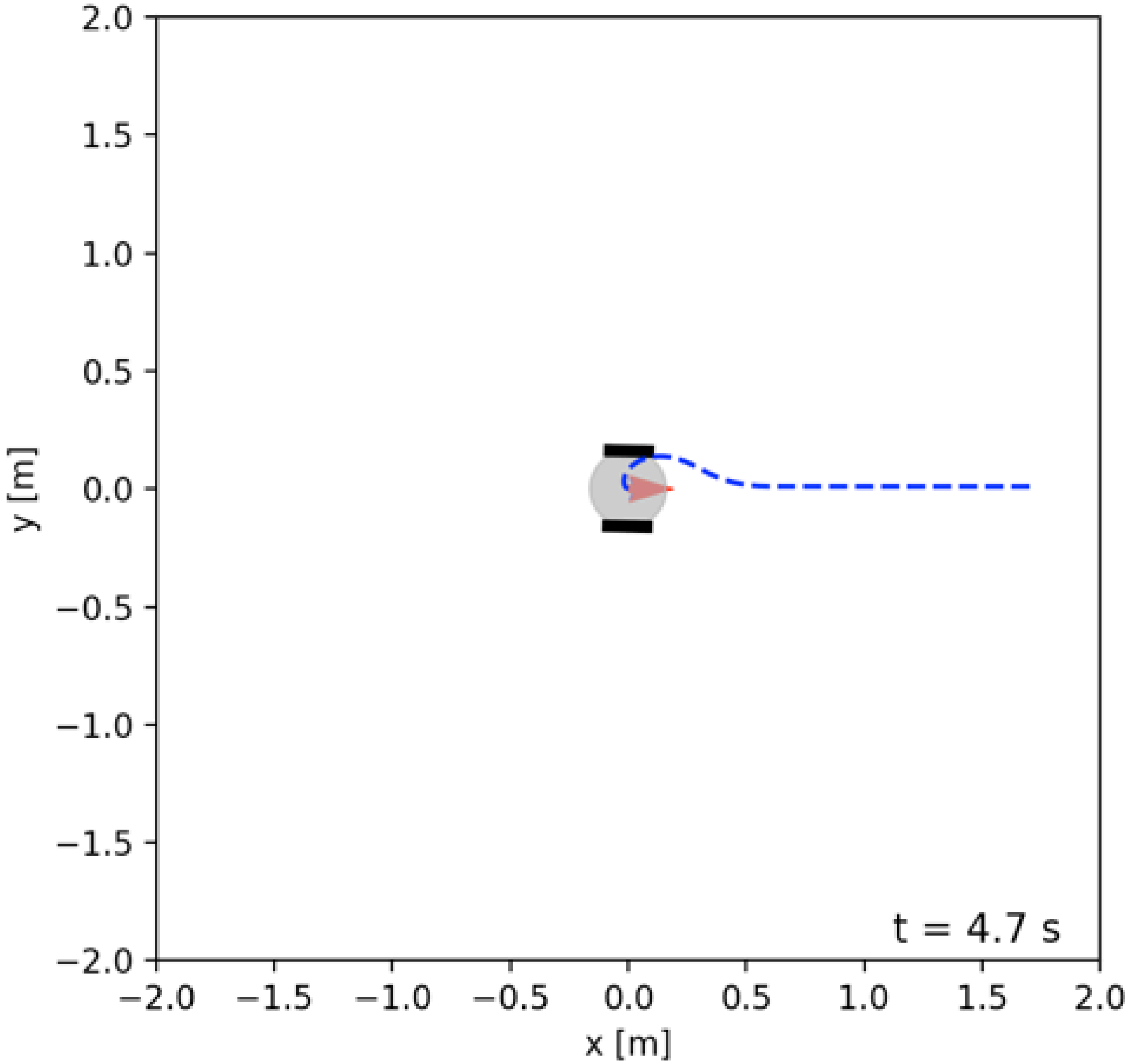

Figure 7. Path case 3.

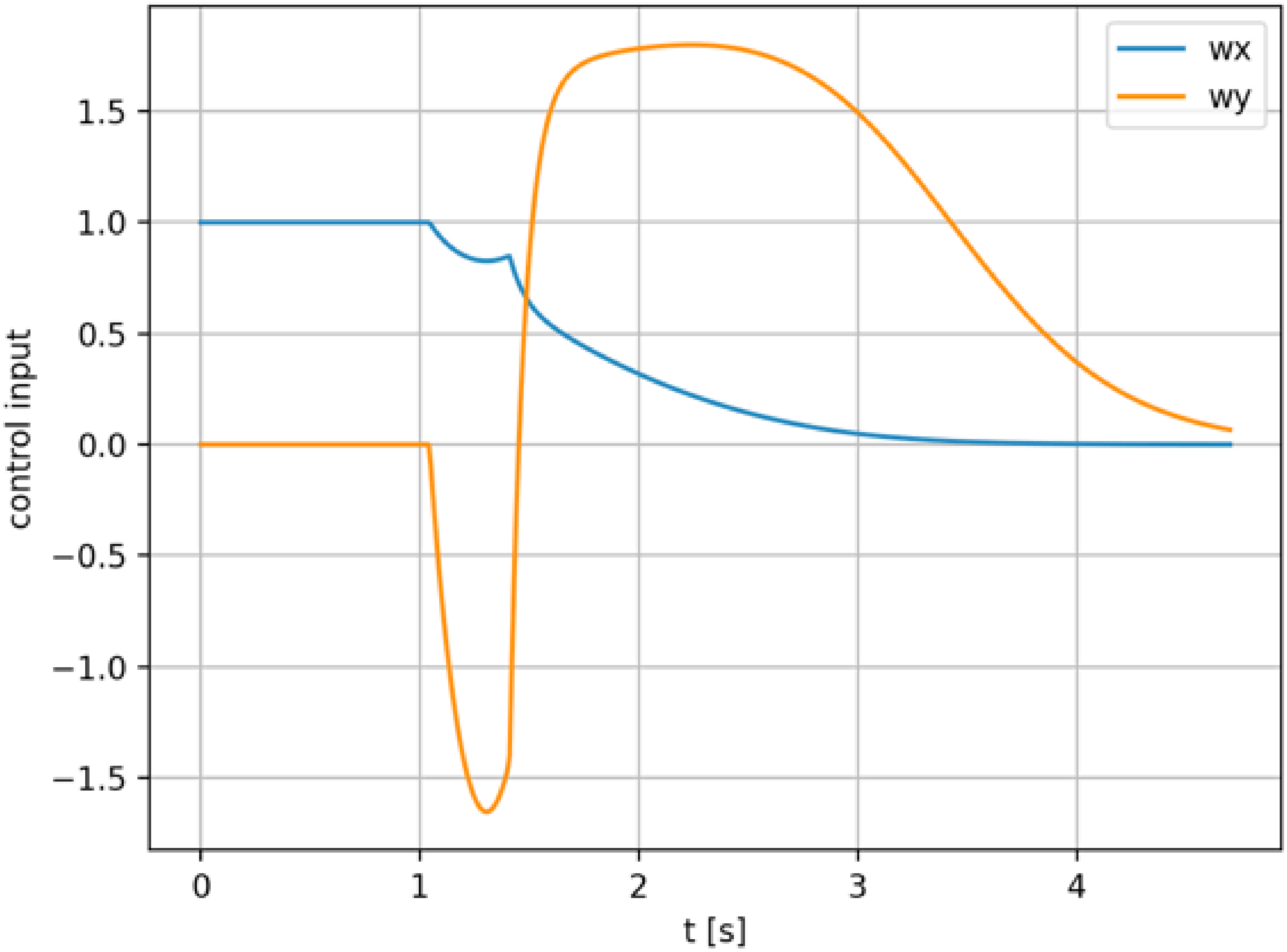

Figure 8. Control input case 3.

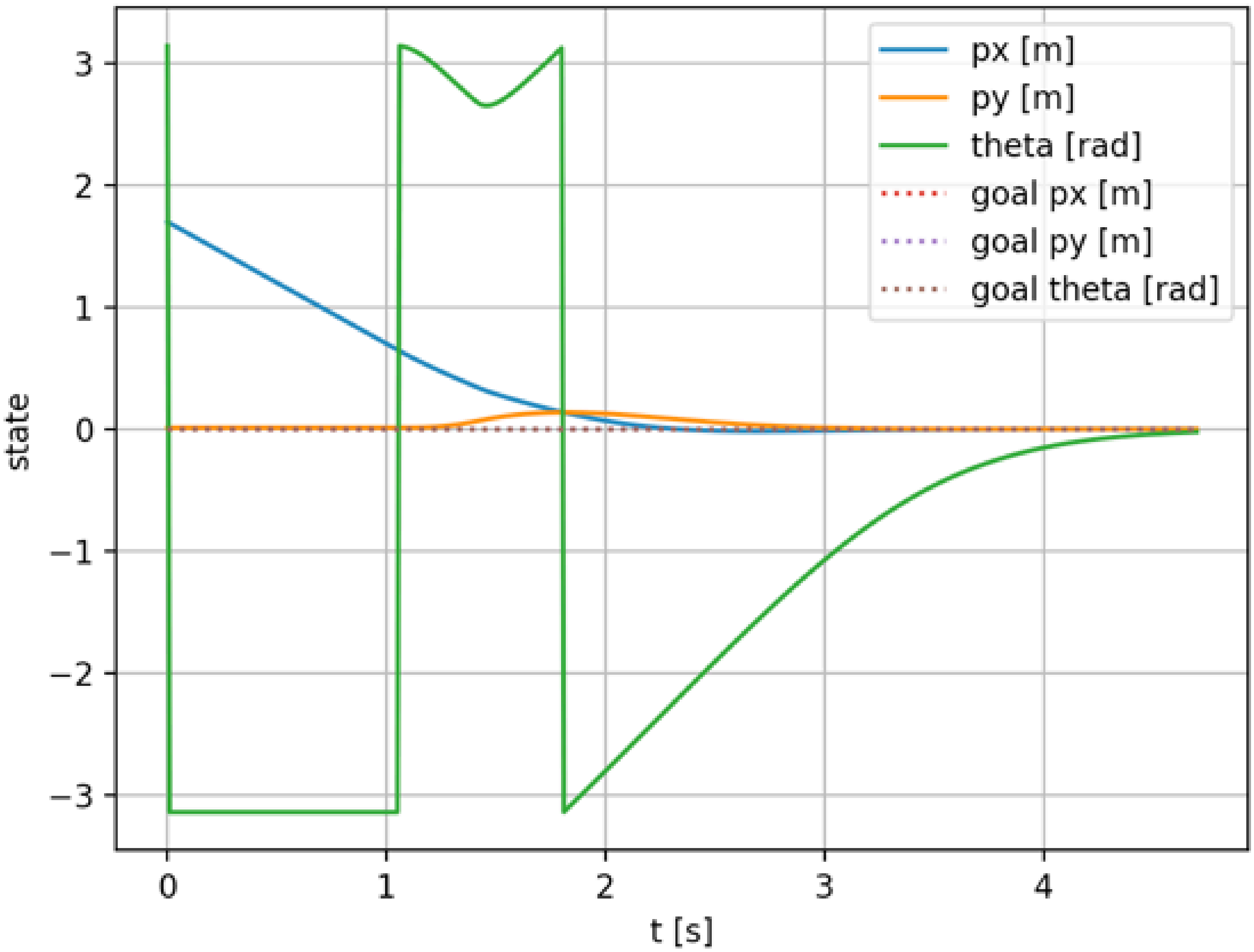

Figure 9. Robot state vs goal state case 3.

Conclusion

Non-linear controllers are powerful tools and allow much more freedom in robots' controlls, but the price for that is higher complexity both in implementation and math involved in rigorous proofs for their convergence.

References

[1] M. Aicardi, G. Casalino, A. Bicchi and A. Balestrino, "Closed loop steering of unicycle like vehicles via Lyapunov techniques," in IEEE Robotics & Automation Magazine, vol. 2, no. 1, pp. 27-35, March 1995, doi: 10.1109/100.388294.

[2] https://stanfordasl.github.io/aa274a/pdfs/notes/lecture4.pdf