Problem Set 2
Linear Algebra

July 10, 2023

1. (i) Find vectors corresponding to points $(1,1)$ and $(3,4)$ two dimensional plane (say $\vec{v_1}$, $\vec{v_2}$).
(ii) Calculate the norm of both $\vec{v_1}$, $\vec{v_2}$ and inner product of $\vec{v_1}$, $\vec{v_2}$
(iii) Write the vector corresponding to point $(1,5)$ as linear combination of $\vec{v_1}$, $\vec{v_2}$.

2.
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 5 & 8 \end{bmatrix}$$
find A^T

3.
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 5 & 8 \end{bmatrix}$$
(i) Find REF(row echelon form) RREF(row reduced echelon form) of the matrix A and try to interpret row operations you have done as product of elementary matrices.
(ii)
$$\vec{v_1} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 2 \\ 5 \\ 5 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 3 \\ 6 \\ 8 \end{bmatrix}$$
Interpret whether these vectors are linearly independent or not.
(iii) Let us have some fun with elementary matrices,
consider E[1,2] matrix of order 3×3 calculate $E[1,2].E[1,2]$
Have you found anything intresting?
Try to interpret your observation as row operation done on second matrix

in above product?

4. Calculate the Row, column, null, and left null spaces of matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 5 & 8 \end{bmatrix}$$

Use the RREF already calculated for this matrix in previous question

5. The empty set is not a vector space. The empty set fails to satisfy only one of the requirements listed. Which one?

6. Let ∞ and $-\infty$ denote two distinct objects, neither of which is in R. Now define an addition and scalar multiplication on $R \cup \{\infty\} \cup \{-\infty\}$ as

$$t(\infty) = \begin{cases} -\infty, & \text{if t} < (0) \\ 0, & \text{if t} = 0 \\ \infty, & \text{if t} > 0 \end{cases} \tag{1}$$

and

$$t(-\infty) = \begin{cases} \infty, & \text{if t} < (0) \\ 0, & \text{if t} = 0 \\ -\infty, & \text{if t} > 0 \end{cases} \tag{2}$$

$$t + \infty = \infty + t = \infty, t + (-\infty) = (-\infty) + t = (-\infty)$$
$$\infty + \infty = \infty, (-\infty) + (-\infty) = (-\infty), \infty + (-\infty) = 0$$

Does $R \cup \{\infty\} \cup \{-\infty\}$ form a vector space over R