Problem set - 3

(1)

Expected Value of Question 1 = 0.8 * 100 = 80$

Scenario 1:

Person answer first Question correctly & proceeds to Question?.

Expected value of Qn 2 = 0.5 * 200
= 100$

Scenario 2:

Since quiz terminates if answering incorrectly the value is 0.

Total value

Scenario 1 ((0.8 * 100 + 0 * 200 = 80$)

+ Scenario 2 (0) = 80$

On comparision one can see that

E[Q1] = 80 & E[Q2] = 80

So to Maximize prize order doesn't Matter.

(2)

(a) P(No Power Cut in all 3 days)

$= (0.6)(0.6)(0.6)$

$= 0.216$

$\Rightarrow$ P(at least 1 Power Cut) $= 1 - P(\text{No Power Cut})$

$= 1 - 0.216$

$= 0.784$

(b) Marginal & Conditional Probabilities

P(Power Cut) $= P(\text{Power Cut}, \text{Sunny}) + P(\text{Power Cut}, \text{Rainy}) = 0.2 + 0.15 = 0.35$

P(No Power Cut) $= P(\text{No Power Cut}, \text{Sunny}) + P(\text{No Power Cut}, \text{Rainy}) = 0.6 + P$

$P(\text{Power Cut} \mid \text{Sunny}) = P(\text{Power Cut}, \text{Sunny}) / P(\text{Sunny})$

$= (0.2)/(0.2+0.6) = 0.25$

$P(\text{Power Cut} \mid \text{Rainy}) = P(\text{Power Cut}, \text{Rainy}) / P(\text{Rainy})$

$= 0.15/(0.15+P)$

$P(\text{No Power Cut} \mid \text{Sunny}) = P(\text{No Power Cut}, \text{Sunny}) / P(\text{Sunny})$

$= (0.6)/(0.2+0.6) = 0.75$

$P(\text{No Power Cut} \mid \text{Rainy}) = P(\text{No Power Cut}, \text{Rainy}) / P(\text{Rainy})$

$= P/(0.15+P)$

(3)

$P(x=k) = (e^{-\lambda} \lambda^k)/k!$ $\quad \lambda = np = (500)\left(\frac{1}{1000}\right) = 0.5$

$P(x=1) = (e^{-0.5} \cdot 0.5^1)/1!$

$= 0.3023$

(4) $P(X \le 1) = P(X=0) + P(X=1)$

$= 0.3033 + (e^{+0.5} * 0.5^0)/0!$

$= 0.3033 + 0.6065 = 0.9098$

$P(X \ge 2) = 1 - P(0 \le 1) = 1 - 0.9098$

$= 0.0902$

(4) U → Unguarded G → Guarded

~~P(0s~~

E(Points Scored) = E(Points Scored | G) * P(G) + E(Points Scored | UG) * P(UG)

$= 2 * 0.4 + 2 * 0.8 * (1-0.7)$ (0.7)

$= 0.56 + 0.48$

$= 1.04$

(5) The Probability density at any Point (x, y) within the triangle is 2.

Conditional Pdf is as:

$P(X|Y)$

$P(X|Y) = 2$ for $0 \le x \le y$

$P(X|Y) = 0$ for otherwise.

$P(Y|X=x)$:

$P(Y|X=x) = 2$ for $0 \le y \le x$

$P(Y|X=x) = 0$ for otherwise.

(6) $Z = (x-\mu)/\sigma = (45-34)/5$

$= 2.2$

$\phi(2.2) = 0.9861$

$P(\text{Temp} > 45^\circ C) = 1 - \phi(2.2) = 1 - 0.9861$

≈ 0.0139

$$\int_0^\infty A\left[e^{-x/5}\right] = 1$$

$$\Rightarrow A\left[-5e^{-\infty/5} - \left(-5e^{-(0/5)}\right)\right] = 1$$

$$= A\left[-5(0) - (-5*1)\right] = 1$$

$$A[5] = 1$$

$$A = 1/5$$

$$P(x > 10) = \int_{10}^\infty \frac{1}{5} e^{-x/5}\, dx$$

$$= \frac{1}{5}\left[-5(0) - (-5e^{-2})\right]$$

$$P(x > 10) = e^{-2}$$

$$P(x < 5) = \int_0^5 \frac{1}{5} e^{-x/5}\, dx$$

$$= \frac{1}{5}\left[-5 * e^{-1} + 5\right]$$

$$= e^{-1}$$

$$P(5 < x < 10) = \int_5^{10} f(x)\, dx$$

$$= \frac{1}{5}\left[5e^2 - 5e^{-1}\right]$$

$$P(5 < x < 10) = e^2 - e^{-1}$$