Problem set-2

1

(i) $\vec{v_1} = (1,1) - (0,0) = (1,1)$

$\vec{v_2} = (3,4) - (0,0) = (3,4)$

(ii) Norm $=$ sqrt$(1^2 + 1^2) = \sqrt{2}$

sqrt$(3^2 + 4^2) = 5$

Inner Product $= \vec{v_1} \cdot \vec{v_2} = 1.3 + 1.4 = 7$

(iii) $(1,5) = a\vec{v_1} + b\vec{v_2}$

$1 = a(1) + (b)(3)$

$5 = (a)(1) + b(4)$

$\Rightarrow a = 2 \quad b = 1/2$

$(1,5) = 2\vec{v_1} + 1/2\,\vec{v_2} = 2(1,1) + 1/2\,(3,4)$

2

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 5 & 8 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 5 & 5 \\ 3 & 6 & 8 \end{bmatrix}$$

3 1

1. Swap ~~R~~ R_1 & R_2 $\rightarrow E_1$

$$\begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 1 & 5 & 8 \end{bmatrix}$$

2. Subtract R_1 multiplied $1/4$ from R_2 $\rightarrow E_2$

$$\begin{bmatrix} 4 & 5 & 6 \\ 0 & -3/4 & -3/4 \\ 1 & 5 & 8 \end{bmatrix}$$

3. " " " from R_3 $\rightarrow E_3$

$$\begin{bmatrix} 4 & 5 & 6 \\ 0 & -3/4 & -3/4 \\ 0 & 3/4 & 7/4 \end{bmatrix}$$

4. Multiply R_2 by $-4/3$ $\rightarrow E_4$

$$\begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 1 \\ 0 & 3/4 & 7/4 \end{bmatrix}$$

5. Subtract R_2 multiplied by $3/4$ from R_3 $\rightarrow E_5$

$$\begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

6. Multiply R_3 by $1/2$ $\rightarrow E_6$

$$\begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

7. Subtract R_3 from R_2 $\rightarrow E_7$

$$\begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

8. Subtract ~~~~ $6R_3$ from R_1 $\rightarrow E_8$

$$\begin{bmatrix} 4 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

9. Subtract $5R_2$ from R_1 $\rightarrow E_9$

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{REF}$$

10. Divide R_1 by 4 $\rightarrow E_{10}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

↓

RREF

REF $= E_9 \cdot E_8 \ldots E_1$

RREF $= E_6 \cdot E_5 \cdot E_4 \ldots E_1$

(4)

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 5 & 8 \end{bmatrix} \rightarrow [\vec{v_1}, \vec{v_2}, \vec{v_3}]$

Here. Rank → 3

Since Rank is same as number of vectors they are linearly independent..

REF(A) =

3

(iii)

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & & \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$

$F[1,2] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Result is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The interesting obs here is Product is Identity Matrix which implies that by applying the row operation of swapping the first & second rows twice results in no change to matrix

(4)

lets Consider the same RREF Obtained in the above question

$RREF(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(1) Row Space:

Row Space is spanned by Rows of RREF

RowSpace(A) = Span{[1,0,0] [0,1,0] [0,0,1]}

Column Space:

Column space is spanned by the original Matrix.

Span{[1 4 1] [2 5 5] [3 6 8]}

Null space:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 1 & 5 & 3 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Nullspace (A) = Span$\{[-1 2, 1]\}$ $\{$

Left Null space.

It consists of all vectors which when multiplied by transpose of Matrix result in zero vector.

$$A^T 10 = \left[\begin{array}{ccc|c} 1 & 4 & 1 & 0 \\ 2 & 5 & 5 & 0 \\ 3 & 6 & 8 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Similar to Nullspace Left Nullspace = Span$\{[1 2, -1]\}$

(5) Empty set fails to satisfy the requirement of having a zero vector. A vector space must have a zero vector. Since Empty space doesn't have any elements it cannot have a zero vector & thus it does not satisfy this requirement.

(6) No the set $R \cup \{\infty\} \cup \{-\infty\}$ does not form a vector space over R

Because

(1) Closure Under Addition is failed to satisfy by the set as in a vector space sum must belong to the vector space itself which is not the case

② Closure under Scalar Multiplication:

The scalar multiplication defined for (∞) & $(-\infty)$ violates closure under scalar multiplication. For example $(-1) * \infty = -\infty$ & $(-1)(-\infty) = \infty$

③ Existence of zero vector:

A VS must have a 0 vector. Hence zero vector is not defined.

④ Existence of Additive Inverses:

For every vector v there must exist an additive inverse $-v$ such that $v + (-v) = 0$

But here there is no defined additive inverse for ∞.