CFI, MT'ly

Session-I

Problem Set

① To find the probablity that a random chosen player is an all-rounder we need to find the the number of players who fit this criterion and divide it by the total Number Of players.

Since there are 12 players can bat & 12 player who can bowl, it is possible that all of them are all rounders. However there might be some players who can only bat or only bowl among these 12.

Extreme Case:

When 12 player who can bat can also bowl & 12 player who can bowl can also bat then No of allrounders = 12

The Probability = 12/20

= 0.6

But this is Extreme Case

Another Case:

None of the players who can bat can bowl & none of these players who can bowl can bat.

Here No of Allrounders = 0

Therefore No of allrounders can be btw [0, 12]

∴ Probability will be btw

[0, 0.6]

(2) Total Area $= \pi r^2 = 256\pi \text{ cm}^2$

Red region $= \pi(2^2) = 4\pi \text{ cm}^2$

Blue region $= \pi(4^2 - 2^2) = 12\pi \text{ cm}^2$

Green region $= \pi(8^2 - 4^2) = 48\pi \text{ cm}^2$

$\therefore$ Black region = Total − Red − Blue − Green

$= 256\pi - 4\pi - 12\pi - 40\pi$

$= 192\pi \text{ cm}^2$

$\therefore$ Probability of Event = Black / Total

$= 192\pi / 256\pi$

$= 0.75$

(3)

Choice 1 → Yashwanth wins if there is atleast 1 head

Choice 2 → Rushi wins if there is at least one head

Probability of Choice 1 $= 0.5 \times 0.5 = 0.25$ → Probability of tails in both toss

$\therefore$ Probability of Yashwant wining in choice 1 is $= 1 - 0.25$

$= 0.75$

~~Choice~~ 2 →

Probability of choice 2:

Using the compliment method

the possibility of getting at least one head is $1 - 0.25 = 0.75$

Therefore the probability of Rushi wining in Choice 2 is also 0.75.

we can see that both probabilities are same here Yashwant choice of elimination is incorrect

The mistake here is, assuming that probability of both choices would be different. However since the complement of an event & the event itself have complementary probabilities, the probabilities of both choices are same.

(2)

$2^4 = 16$

~~50~~ (2×25)

Total No of Possible Paths = ~~2~~ ~~27~~ ~~2~~ ~~2~~ 5

Possible Paths = 3

Probability = $3/16 = 0.1875$

(5)

Total defective cycle = $(20 \times 3) + (30 \times 4) + (50 \times 2)$

$= 0.6 + 1.2 + 1 = 2.8\%$

using Bayes Theorem

Probability = $(3\% \times 20\%) / 2.8\%$

$= 0.006 / 0.028$

$= 0.2143$

or

21.43%

(6) There can be ~~ea~~ 2 Scenario's

1 → Both Proper Machines are checked first. In second check the technician will check both damaged Machines. This will lead to correct identification of Damaged machine.

2 → One Proper and One Damaged in 1st check. In second check the remaining machine. Here it will not be identified correctly.

Probability of each scenario

(1) Chk 1: ~~Pro~~ Checking Proper Machine first $= \left(\frac{2}{4}\right)\left(\frac{1}{3}\right)$
$= \frac{1}{6}$

Chk 2: Checking Damage machine
$= \frac{1}{1}$

∴ Probability $= \frac{1}{6} * \frac{1}{1} = \frac{1}{6}$

(2)

Probability of checking 1D & 1P machine in first check $= \left(\frac{2}{4}\right)\left(\frac{2}{3}\right) = \frac{4}{12}$

In second check probability of checking rem machine & not identifying correctly $= \left(\cancel{\frac{2}{2}}\right)$
$\left(\frac{4}{12}\right)\left(\frac{1}{1}\right)$
$= \frac{4}{12}$

Total Probability $= \frac{1}{6} + \frac{4}{12} = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}$

i.e. Probability is 50% or 0.5.

(7)

Probability that bag containing 5 B & 1 NB

$= P(5B) \times P(1ND) = \left(\frac{10}{15}\right)\left(\frac{1}{6}\right)$

$= \frac{1}{9}$

$P(6B) = \frac{1}{6}$

P(Two B) = P(Bag Cont 5B & 1NB) + P(Bag Cont 6B)

$= \frac{1}{9} + \frac{1}{6} = \frac{5}{18}$

$P(\text{At least 5 Ball} \mid 2B) = (3)\left(\frac{1}{18} + \frac{1}{2}\right)$

$= \frac{1}{3} / \frac{5}{6}$

$\boxed{= \frac{2}{5}}$

(8)

(a) Prob of Giri City Attendance =

$(0.9 \times 1) + (0.1)(0.3)$

$= 0.93$

(b) A → Not present B → Girish Gior Atty

$\rightarrow P(A \mid B) = (P(B \mid A) \times P(A))(P(B))$

$= (0.3)(0.1) / (0.93)$

$= 0.0323$

(10)

M → Suffering from Measles

F → " " " flu

R → Having Rash

$P(M|R) = \left(P(R|M) * P(M) \right) / P(R)$

$P(R) = P(R|M) * P(M) + P(R|F) * P(F)$

$= 0.95 * 0.8 + 0.08 * 0.2$

$= 0.776$

$P(M|R) = \left(P(R|M) * P(M) \right) / P(R)$

$= (0.95 * 0.8) / 0.776$

≈ 0.9794

(11) Scenario 1: Surya Arrives & waits for 2C

Scenario 2: 2C arrives first & waits for Surya.

Scenario 3: Both Surya & 2C arrive at the same time.

Prob (Scenario 1) $= \left(\frac{1}{2}\right)\left(\frac{10}{60}\right) = \frac{1}{12}$

Prob (Scenario 2) $= \left(\frac{1}{2}\right)\left(\frac{10}{60}\right) = \frac{1}{12}$

Prob (Scenario 3) $\simeq 0$

Probability of Meeting $= \frac{1}{12} + \frac{1}{12}$

$= \frac{1}{6}$

(12) A: Aircraft is present

D: Radar detects an AC

$P(D/A) = 0.99$ $\quad P(D/\text{Not}A) = 0.10$ $\quad P(A) = 0.05$

$P(D) = P(D/A) * P(A) + P(D/\text{Not}) * P(\text{Not}A)$

$P(D/\text{Not}A) = 1 - P(\text{Not}D/\text{Not}A)$

$P(\text{Not}D/\text{Not}A) = 1 - 0.10 = 0.90$

$P(\text{Not}A) = 1 - P(A) = 1 - 0.05 = 0.95$

$P(D) = (0.99)(0.05) + (0.10)(0.95)$

$= 0.1445$

Using Bayes Theorem

$P(A/D) = (P(D/A) * P(A)) / P(D)$

$= (0.99)(0.05) / (0.1445)$

$= 0.0495 / 0.1445$

≈ 0.3425

(9) Total Number of people = $m+n$

Using Catalan Number

No of arrangement can be Catalan $(m+n)$ $-$ Catalan (n)

Prob = $(\text{Catalan}(m+n) - \text{Catalan}(n)) / \text{Catalan}(m+n)$

$= (12m^2 + 12mn - 7m - n + 1) / (16m^2 + 16mn +$