



Horizontal Wind Model (HWM)

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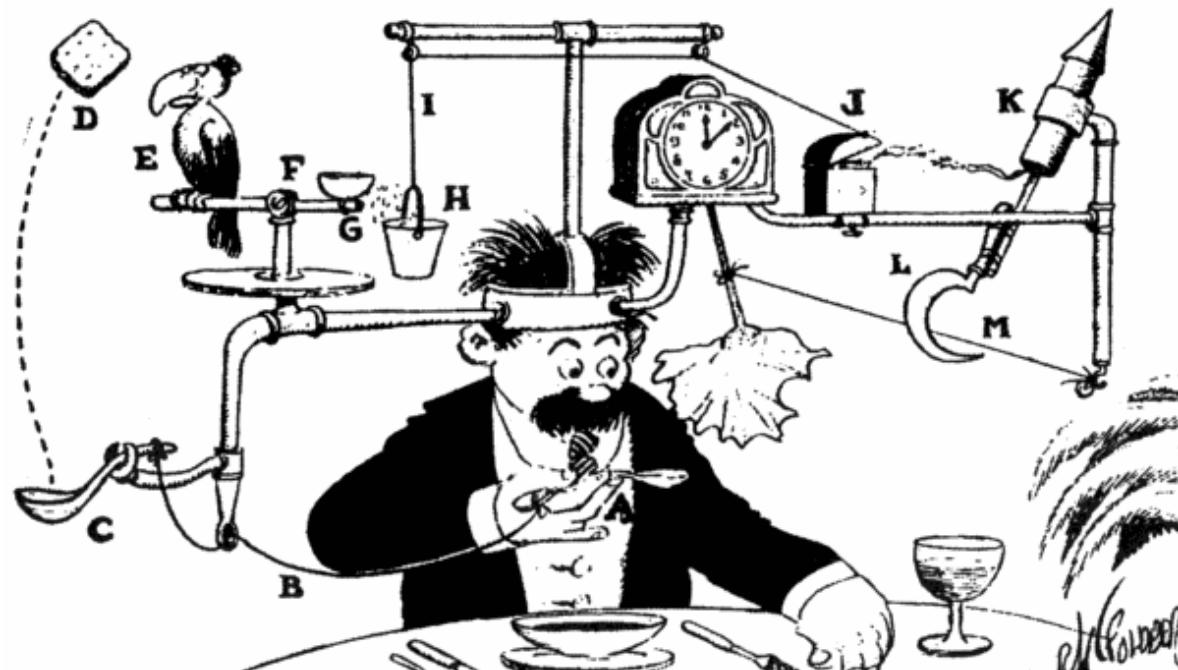
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Sometimes simple is preferable to complicated.

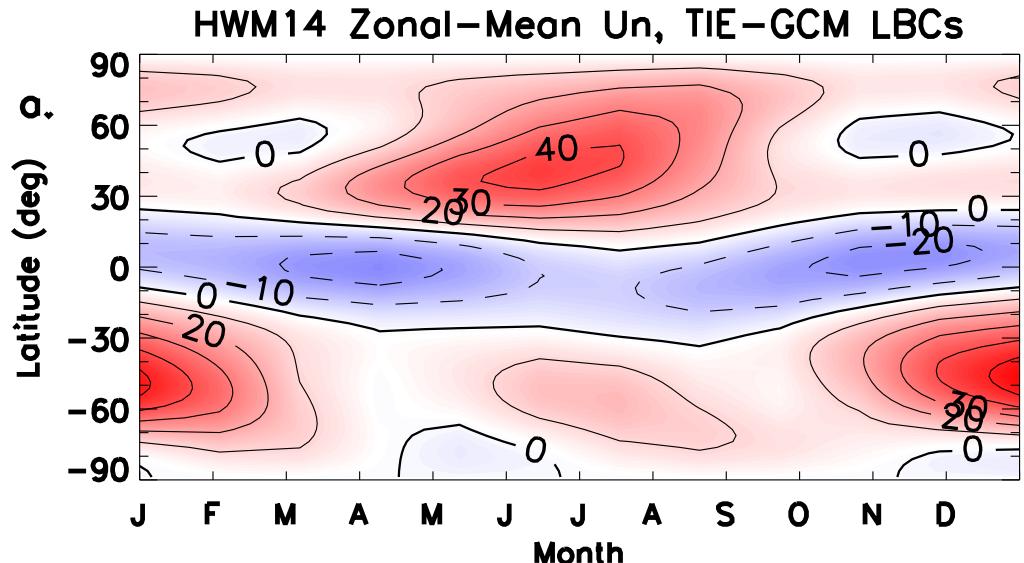


Motivation

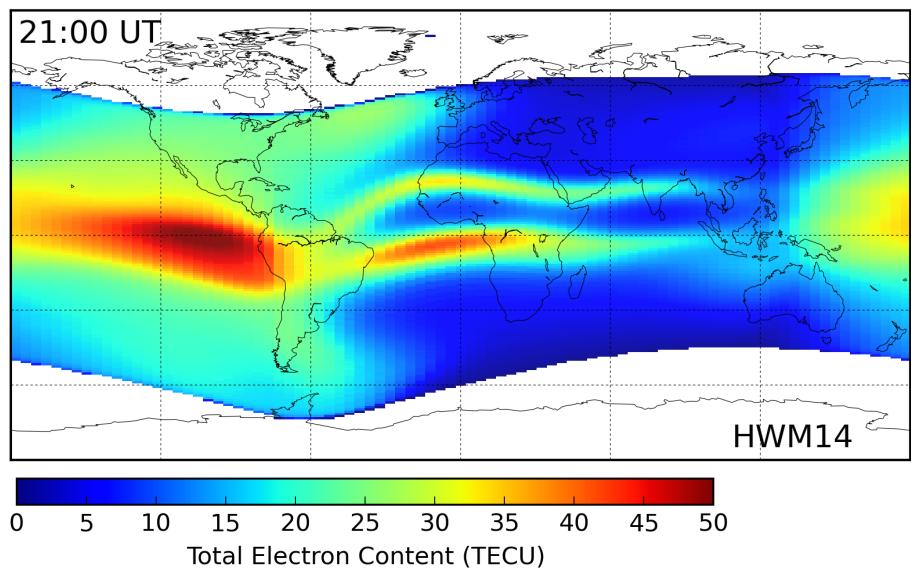
With the increasing complexity of coupled whole atmosphere models it is important to have an accurate observationally derived climatological specification of the atmosphere's wind fields.

HWM is able to provide a reasonable representation the variations of middle- and upper-atmosphere winds because they are predominantly driven by *in situ* solar heating under the periodic cyclical influence of the earth's rotation, tilt, and orbit around the sun.

When and where appropriate, HWM reduces the computational complexity of theoretical and applied calculations by avoiding the need to simultaneously compute the wind fields from first principles.



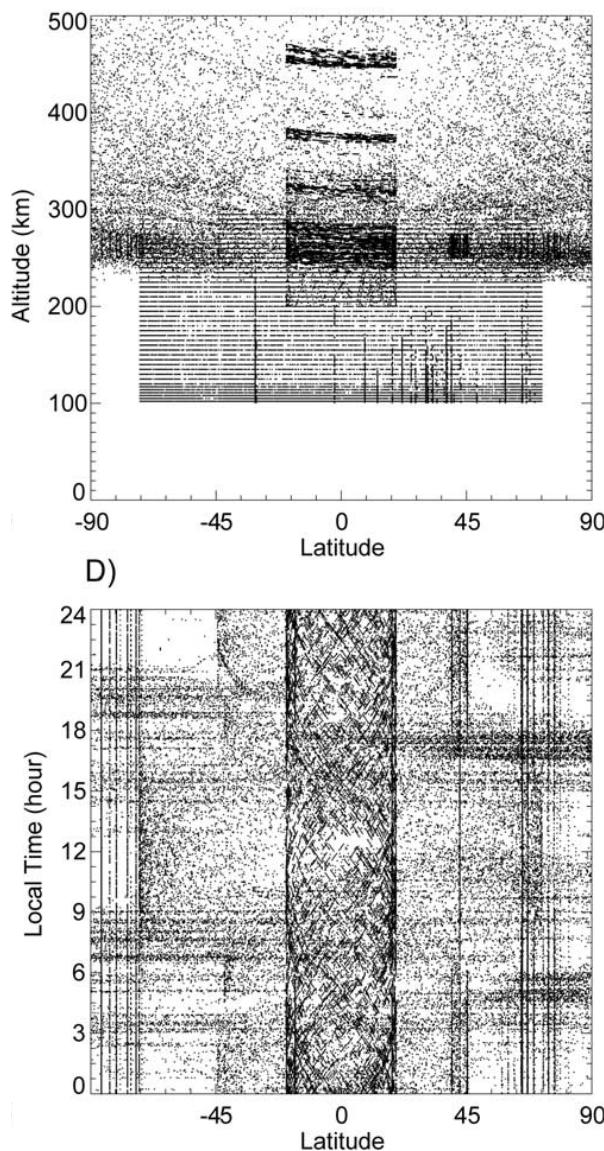
SAMI3 Total Electron Content



Observational Data

Instrument	Location	Height (km)	Years	Local Time	Days	Data Points	Reference
<i>Satellite</i>							
AE-E NATE ^a	±18.0°N	220–400	1975–1979	both	799	200,500	Spencer <i>et al.</i> [1973]
DE 2 WATS ^b	±89.0°N	200–600	1981–1983	both	536	391,500	Spencer <i>et al.</i> [1981]
DE 2 FPI ^c	±89.0°N	250	1981–1983	both	308	47,600	Hays <i>et al.</i> [1981]
UARS HRDI	±72.0°N	50–115	1993–1994	day	834	30,100,000	Hays <i>et al.</i> [1993]
UARS WINDII 5577 Å	±72.0°N	90–300	1991–1996	day	949	24,672,000	Shepherd <i>et al.</i> [1993]
UARS WINDII 6300 Å	±42.0°N	200–300	1991–1996	night	243	2,237,942	Shepherd <i>et al.</i> [1993]
<i>Sounding Rocket</i>							
Falling Sphere	8°S–60°N	8–98	1969–1991	both	1,186	96,205	Schmidlin <i>et al.</i> [1985]
Rocketsonde	38°S–77°N	2–90	1969–1991	both	5,082	843,000	Schmidlin <i>et al.</i> [1986]
TMA	31°S–70°N	59–277	1956–1998	both	276	92,792	Larsen [2002]
<i>Fabry-Perot Interferometer</i>							
Arecibo	18.4°N, 66.8°W	250	1980–1999	night	473	14,198	Burnside and Tepley [1989]
Arequipa	16.2°S, 71.4°W	250	1983–2001	night	1048	32,238	Meriwether <i>et al.</i> [1986]
Arrival Heights	77.8°S, 116.7°E	250	2002–2005	night	535	54,214	Hernandez <i>et al.</i> [1991]
Halley Bay	75.5°S, 26.6°W	250	1988–1998	night	799	82,614	Crickmore <i>et al.</i> [1991]
Millstone Hill	42.6°N, 71.5°W	250	1989–2002	night	1,770	68,333	Sipler <i>et al.</i> [1982]
Mount John	44.0°S, 170.4°E	89, 96, 250	1991–1996	night	560	2,660	Hernandez <i>et al.</i> [1991]
Søndrestrøm	67.0°N, 51.0°W	250	1984–2004	night	1,223	69,734	Killeen <i>et al.</i> [1995]
South Pole ^d	90.0°S	86, 250	1989–1999	night	1,091	163,044	Hernandez <i>et al.</i> [1991]
Svalbard ^e	78.2°N, 15.6°E	250	1980–1983	night	44	7,472	Smith and Sweeny [1980]
Thule	76.5°N, 68.4°W	250	1987–1989	night	172	21,500	Killeen <i>et al.</i> [1995]
Resolute Bay	74.7°N, 94.9°E	250	2003–2005	night	166	5,299	Wu <i>et al.</i> [2004]
Watson Lake	60.1°N, 128.6°W	250	1991–1992	night	135	28,000	Niciejewski <i>et al.</i> [1996]
<i>Incoherent Scatter Radar^f</i>							
Arecibo	18.3°N, 66.8°W	100–170	1974–1987	day	149	30,600	Harper [1977]
Chatanika	65.1°N, 147.4°W	90–130	1976–1982	day	97	38,721	Johnson <i>et al.</i> [1987]
European Incoherent Scatter	69.6°N, 19.2°E	100–120	1985–1987	day	29	2,900	Williams and Virdi [1989]
Millstone Hill	42.6°N, 71.5°W	120–400	1983–1987	both	142	23,536	Salah and Holt [1974]
Søndrestrøm	67.0°N, 50.9°W	150–400	1983–1987	both	146	19,600	Wickwar <i>et al.</i> [1984]
St. Santin ^f	44.6°N, 2.2°E	90–165	1973–1985	day	256	18,382	Amayenc [1974]
<i>Medium-Frequency Radar^g</i>							
Adelaide	34.5°S, 138.5°E	60–98	2001–2004	both	834	481,634	Vincent and Lesicar, 1991
Bribie Island	28.0°S, 153.0°W	60–98	1995	both	280	184,176	Reid [1987]
Davis	68.6°S, 78.0°E	50–100	2001–2004	both	730	526,160	Vincent and Lesicar [1991]
Poker Flat	65.1°N, 147.5°W	44–108	1979–1985	both	1857	2,746,684	Murayama <i>et al.</i> [2000]
Wakkani	45.4°N, 141.8°E	50–108	1998–2003	both	1538	1,874,672	Murayama <i>et al.</i> [2000]
Yamagawa	31.2°N, 130.6°E	60–98	1998–2003	both	1593	1,040,042	Murayama <i>et al.</i> [2000]
<i>Wind and Temperature Lidar</i>							
Fort Collins	40.6°N, 105.1°W	75–115	2002–2002	both	244	93,288	She <i>et al.</i> [2004]
<i>Numerical Weather Prediction Analysis^h</i>							
NOAA GFS Analysis	Global	0–35	2002–2007	both	1520	—	Kalnay <i>et al.</i> [1990]
NASA GEOS4 Analysis	Global	0–55	2002–2007	both	1520	—	Bloom <i>et al.</i> [2005]

Table 1. HWM07 Observational Database Summary



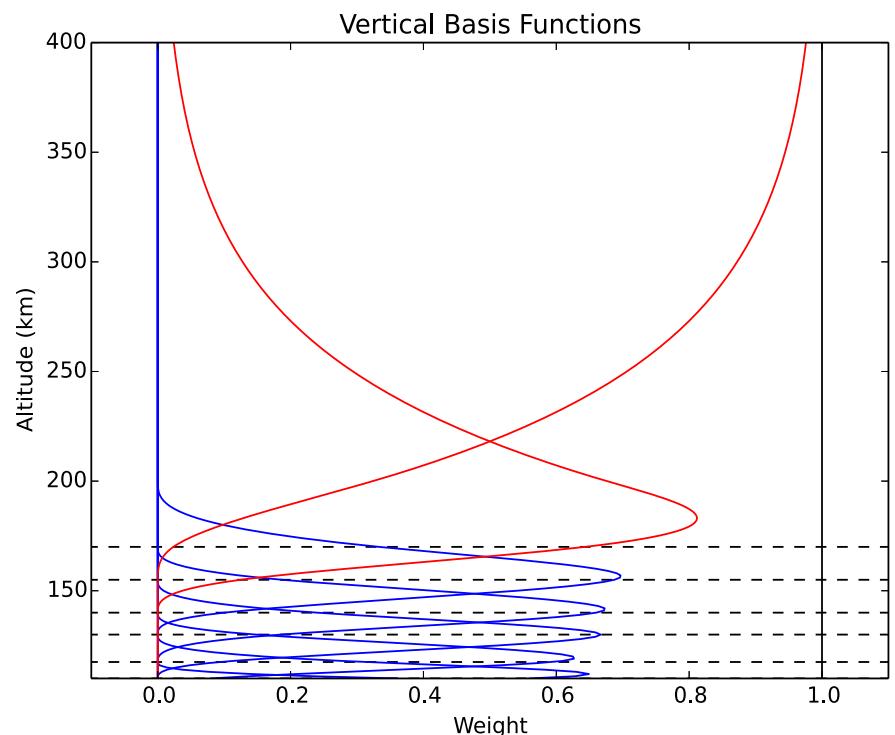
Mathematical Formulation

$$u(\tau, \delta, \theta, \phi) = \sum_{n=0}^N \sum_{s=0}^S \Psi_1(\tau, \theta, s, n) + \sum_{s=0}^S \sum_{l=1}^L \sum_{n=0}^N \Psi_2(\tau, \delta, \theta, s, l, n) + \sum_{s=0}^S \sum_{m=1}^M \sum_{n=0}^N \Psi_3(\tau, \phi, \theta, s, m, n)$$

$$\Psi_1(\tau, \theta, s, n) = -C_r^{s,n} \cdot \sin(n\theta) \cdot \cos(s\tau) + C_i^{s,n} \cdot \sin(n\theta) \cdot \sin(s\tau)$$

$$\begin{aligned} \Psi_2(s, l, n, \tau, \delta, \theta) = & C_{a_r}^{s,l,n} \cdot V_n^l(\theta) \cdot \cos(l\delta) \cdot \cos(s\tau) + \\ & C_{a_i}^{s,l,n} \cdot V_n^l(\theta) \cdot \sin(l\delta) \cdot \cos(s\tau) + \\ & B_{a_r}^{s,l,n} \cdot W_n^l(\theta) \cdot \cos(l\delta) \cdot \cos(s\tau) + \\ & B_{a_i}^{s,l,n} \cdot W_n^l(\theta) \cdot \sin(l\delta) \cdot \cos(s\tau) + \\ & C_{b_r}^{s,l,n} \cdot V_n^l(\theta) \cdot \cos(l\delta) \cdot \sin(s\tau) + \\ & C_{b_i}^{s,l,n} \cdot V_n^l(\theta) \cdot \sin(l\delta) \cdot \sin(s\tau) + \\ & B_{b_r}^{s,l,n} \cdot W_n^l(\theta) \cdot \cos(l\delta) \cdot \sin(s\tau) + \\ & B_{b_i}^{s,l,n} \cdot W_n^l(\theta) \cdot \sin(l\delta) \cdot \sin(s\tau), \end{aligned}$$

$$U(\tau, \delta, \theta, \phi, z) = \sum_j \beta_j(z) u_j(\tau, \delta, \theta, \phi)$$



$$V_n^l(\theta) = \frac{1}{\sqrt{n(n+1)}} \frac{d}{d\theta} P_n^l(\theta)$$

$$W_n^l(\theta) = \frac{1}{\sqrt{n(n+1)}} \frac{m}{\cos(\theta)} P_n^l(\theta)$$

$$u: \{C_r, C_i, B_r, B_i\} \leftrightarrow -v: \{B_r, B_i, -C_r, -C_i\}$$

$$w_{\text{los}} = u \sin \varphi + v \cos \varphi$$

Parameter Estimation

7×10^6 rows
(observations)

24,000 columns
(parameters)

$$\begin{matrix}
 \beta_0\Psi_0 & \beta_1\Psi_1 & \beta_2\Psi_2 & \beta_3\Psi_3 \\
 \beta_1\Psi_1 & \beta_2\Psi_2 & \beta_3\Psi_3 & \beta_4\Psi_4 \\
 \beta_2\Psi_2 & \beta_3\Psi_3 & \beta_4\Psi_4 & \beta_5\Psi_5 \\
 & \ddots & & \\
 & \beta_{j+1}\Psi_{j+1} & \beta_{j+1}\Psi_{j+1} & \beta_{j+1}\Psi_{j+1} & \beta_{j+1}\Psi_{j+1} \\
 & \ddots & & \\
 & \beta_{24}\Psi_{24} & \beta_{25}\Psi_{25} & \beta_{26}\Psi_{26} & \beta_{27}\Psi_{27} \\
 & \beta_{25}\Psi_{25} & \beta_{26}\Psi_{26} & \beta_{27}\Psi_{27} & \beta_{28}\Psi_{28} \\
 & & \beta_{26}\Psi_{26} & \beta_{27}\Psi_{27} & \beta_{28}\Psi_{28} & \beta_{29}\Psi_{29}
 \end{matrix}$$

$$\mathbf{d} = \mathbf{Gm}$$

$$\begin{matrix}
 \mathbf{m}_0 \\
 \mathbf{m}_1 \\
 \mathbf{m}_2 \\
 \mathbf{m}_3 \\
 \mathbf{m}_4 \\
 \vdots \\
 \mathbf{m}_j \\
 \vdots \\
 \mathbf{m}_{26} \\
 \mathbf{m}_{27} \\
 \mathbf{m}_{28} \\
 \mathbf{m}_{29}
 \end{matrix}
 \times
 \begin{matrix}
 \mathbf{d}_0 \\
 \vdots \\
 \mathbf{d}_{22} \\
 \mathbf{d}_{23} \\
 \mathbf{d}_{24} \\
 \vdots \\
 \mathbf{d}_{25} \\
 \mathbf{d}_{26}
 \end{matrix}
 = \begin{matrix}
 \mathbf{d}_0 \\
 \vdots \\
 \mathbf{d}_{22} \\
 \mathbf{d}_{23} \\
 \mathbf{d}_{24} \\
 \vdots \\
 \mathbf{d}_{25} \\
 \mathbf{d}_{26}
 \end{matrix}$$

Iterative linear least-squares optimal estimation procedure (e.g. Rodgers *et al.*, 2000])

$$\begin{aligned}
 \mathbf{m}_{n+1} &= \mathbf{m}_n + [\mathbf{G}^T \mathbf{S}_\epsilon^{-1} \mathbf{G} + \mathbf{S}_n^{-1}]^{-1} \mathbf{G}^T \mathbf{S}_\epsilon [\mathbf{d} - \mathbf{Gm}_n] \\
 \mathbf{S}_{n+1} &= [\mathbf{G}^T \mathbf{S}_\epsilon^{-1} \mathbf{G} + \mathbf{S}_n^{-1}]^{-1}
 \end{aligned}$$

Convergence after approximately ten iterations with 2×10^6 observation each

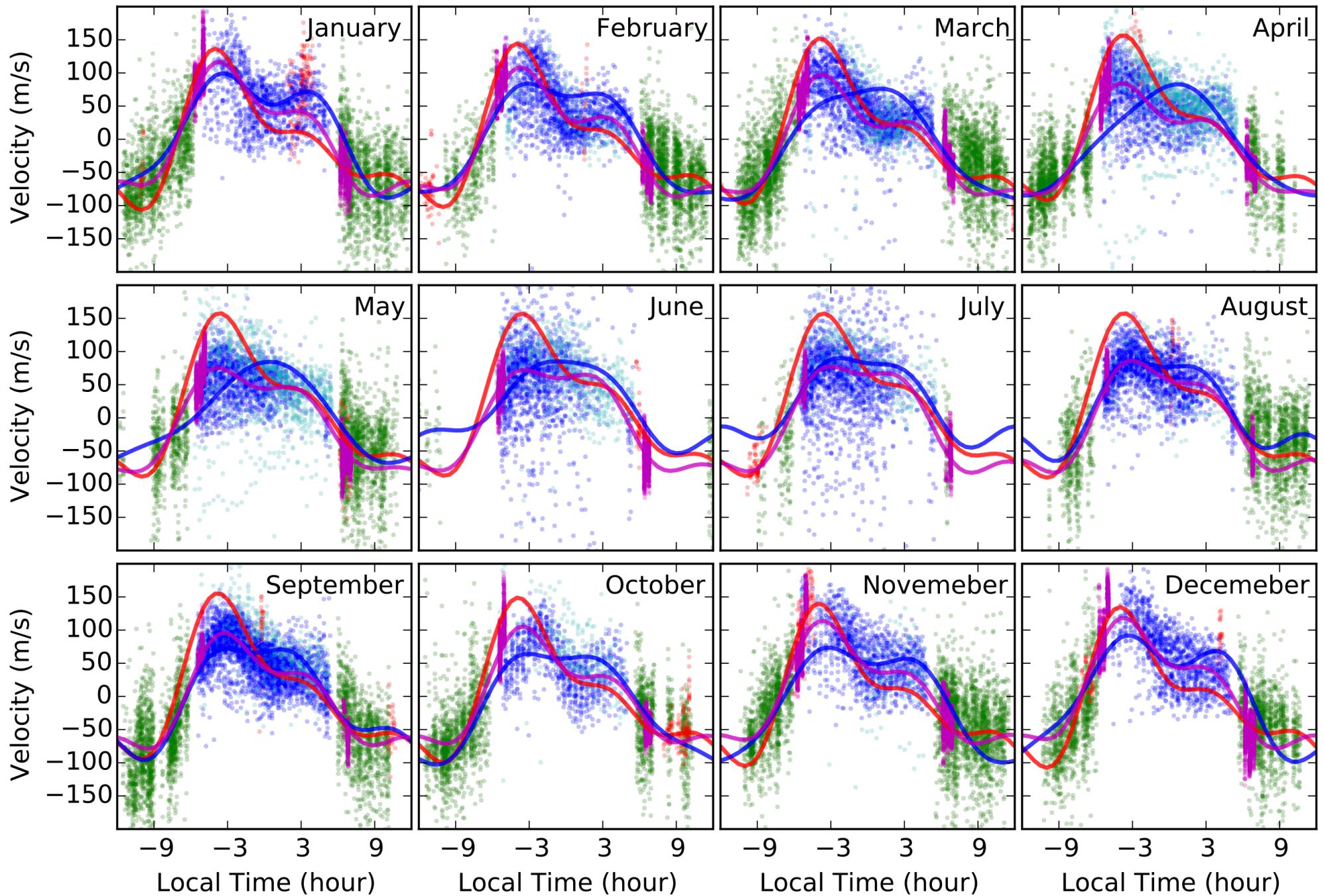
\mathbf{S}_n Parameter Covariance
 $\mathbf{S}_\epsilon = \text{diag}\|1/\sigma_i^2\|$
 ~ 10 to 60 m/s
 $\sigma_i = 37.5$ m/s

Statistical Performance Measures

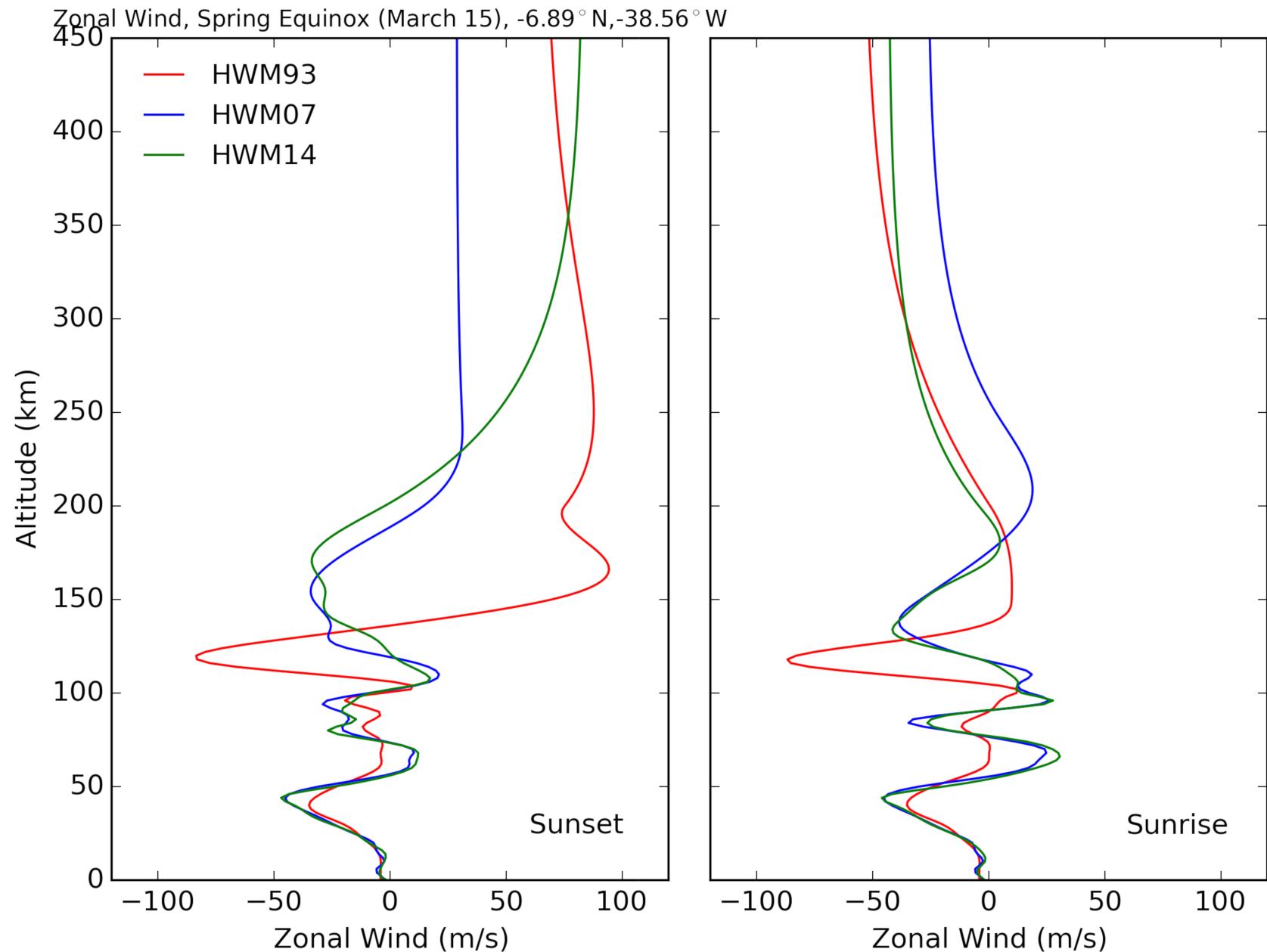
Additional statistics provided in Drob *et al.*, (2014)

	N	β_{obs}	β_{93}	β_{07}	β_{14}	σ_{obs}	σ_{93}^{rms}	σ_{07}^{rms}	σ_{14}^{rms}
Line-of-Sight, Fabry-Perot Interferometer									
Arrival Heights	138690	-14.56	-13.19	-0.59	-6.74	103.55	101.06	68.60	69.07
Resolute Bay	17377	0.41	-0.83	2.17	0.52	107.91	65.74	71.30	67.98
Arecibo	8051	11.62	-2.59	-9.25	-5.58	34.09	33.97	35.54	30.84
Millstone Hill	7503	-20.05	-9.76	-12.50	-9.92	59.95	48.42	47.71	45.79
Søndrestrøme	3730	2.97	-15.97	-27.72	-8.82	110.57	87.24	93.62	81.23
Cross-Track, Satellite									
AE-E NATE	57428	7.74	7.76	5.31	1.81	68.73	56.58	55.40	51.40
GOCE	573672	95.79	19.58	40.46	6.81	48.83	50.41	60.60	36.05
Zonal, Fabry-Perot Interferometer									
Arecibo	79108	36.77	12.65	-8.04	-6.19	52.66	53.07	47.59	44.71
Arequipa	99198	84.48	-25.81	-3.84	0.74	71.66	78.52	72.13	68.83
Halley Bay	91245	14.85	15.51	-5.13	-9.07	93.81	93.29	66.44	65.35
Jicamarca	1901	48.33	-43.89	-23.19	-7.74	59.29	79.79	64.31	59.54
Millstone Hill	68175	22.12	28.37	-9.74	-9.62	70.32	68.29	63.56	63.21
Mount John	1949	35.17	-1.38	-8.55	1.70	70.81	49.92	56.79	54.97
Movil	24227	63.54	-26.99	-9.16	4.39	55.55	60.72	56.49	48.35
Psigah	12610	35.01	15.84	-27.40	-13.64	57.67	51.25	56.36	51.32
Renoir	12483	57.23	-34.74	-6.05	-0.71	44.94	63.27	46.09	40.32
Søndrestrøme	10442	-15.31	20.62	-7.31	-3.94	102.68	114.75	97.96	104.09
Svalbard	1353	-40.74	8.83	-3.57	-9.32	129.08	140.66	125.08	129.76
Thule	15643	2.78	27.58	-1.95	-7.45	150.12	133.50	118.98	118.30
Watson Lake	4979	-20.23	52.09	13.47	5.69	67.85	86.19	68.89	64.96
Poker Flat	450925	-14.35	-4.16	0.49	-9.09	70.35	65.00	72.56	60.30
Zonal, Satellite/Rocket									
DE2 WATS	7233	-19.95	1.00	-6.92	-6.52	125.73	88.99	89.17	94.83
TMA	2774	9.40	-2.02	6.37	-5.82	53.60	57.35	34.79	33.21
UARS WINDII 557.7 nm	415238	-39.77	-8.32	-4.37	-5.19	72.22	72.03	59.50	59.31

Statistical Performance Measures



Observed Morphology



Programming Tips

Official HWM14 source <http://onlinelibrary.wiley.com/doi/10.1002/2014EA000089/full>
supplemental information: [ess224-sup-0002-supinfo.tgz](#)

Please read the ‘README file’ and the Hedin *et al.*, (1994), Drob *et al.*, (2007), and Drob *et al.* (2014) papers.

```
subroutine hwm14(iyd,sec,alt,glat,glon,stl,f107a,f107,ap,w)
implicit none
integer(4),intent(in) :: iyd      ! year and day as yyddd
real(4),intent(in)   :: sec      ! ut(sec)
real(4),intent(in)   :: alt      ! altitude(km)
real(4),intent(in)   :: glat     ! geodetic latitude(deg)
real(4),intent(in)   :: glon     ! geodetic longitude(deg)
real(4),intent(in)   :: stl      ! not used !!!
real(4),intent(in)   :: f107a    ! not used !!!
real(4),intent(in)   :: f107     ! not used !!!
real(4),intent(in)   :: ap(2)    ! ap(1) not used !!!, ap(2) current 3hr
real(4),intent(out)  :: w(2)     ! w(1) +northward, w(2) +eastward, (m/s)
```

HWM14 compiles with Python Numpy’s f2py script without any code modifications.

```
%f2py -c -m hwm hwm14.f90 -fcompiler=gnu95

import hwm
[u,v] = hwm.hwm14(iyd,sec,alt,glat,glon,stl,f107a,f107,ap=[-1,-1])
```

Programming Tips

FORTRAN90

```
subroutine generic()

    use qwm,only content, wavefactor, tidefactor ! <= switches and factors
    implicit none

    ! some code ..

    ap(1:2) = -1                      ! Disturbance Wind Model off

    ! some code ..

    content(2:3) = .false. ! Stationary waves and tides off (zonal mean only)

    ! some code ..

    content(1) = .false.      ! Zonal means off
    content(3) = .true.       ! Only the migrating tides now

    ! some code ..

    tidefactor(3) = 0.0          ! Turn off the terdiurnal tide
    tidefactor(2) = 2.0          ! Scale the migrating semidiurnal tide by x2

    ! some code ..

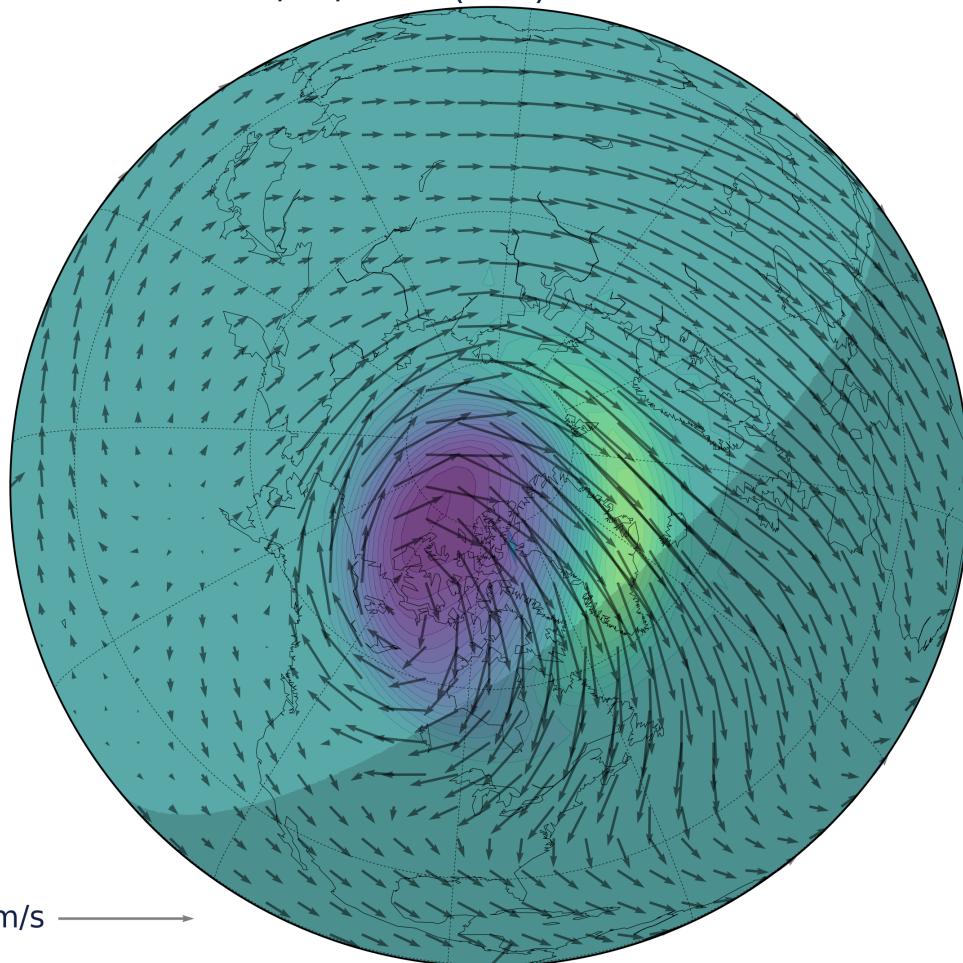
end subroutine generic
```

Python Module (via Numpy's f2py script)

```
hwm.qwm.content[1:3] = -1    # 0 = .false. -1 = .true.    Python to Fortran
hwm.qwm.tidefactor[0] = 1.5 # Scale the diurnal tide    python[n] = fortran(n+1)
```

Interactive Demo

06/21/2016 (173) 03:00 UTC



HWM14

353 m/s →

Wiemer05

Day	172
Altitude	275
UT (hour)	3
Ap	21

Φ (deg)	-90
B_T (nT)	5
ρ (#/cm ³)	9
V (km/s)	468