# 1 Wiener filter

$\mathbf{v}_n * [h_0, h_1] = v(n) \cdot h_0 + v(n-1) \cdot h_1 = \mathbf{v}_n^t(n) \cdot \mathbf{h}$

We want to minimize: $\min_h E||\mathbf{w}_n - \mathbf{v}_n^t \cdot \mathbf{h}||^2$
in other words find the filter $\mathbf{h}$, so that the convolution with $\mathbf{v}_n$ will give $\mathbf{w}_n$
$E||\mathbf{w}_n - \mathbf{v}_n^t \cdot \mathbf{h}||^2 =$

$$\begin{aligned} E[\mathbf{h}^t \mathbf{w}_n^t \mathbf{w}_n \mathbf{h} + \mathbf{w}_n^2 - 2\mathbf{w}_n \mathbf{v}_n^t \mathbf{h}] = \\ \mathbf{h}^t E[\mathbf{v}_n^t \mathbf{v}_n^t]\mathbf{h} + E[\mathbf{w}_n^2] - 2E[\mathbf{w}_n \mathbf{v}_n^t]\mathbf{h} = \end{aligned} \tag{1}$$

However, $E[\mathbf{w}_n^2]$ does not depend on $\mathbf{h}$.

Then $\min_h E||\mathbf{w}_n - \mathbf{x}_n^t \cdot \mathbf{h}||^2$ is equal to:

$\min_h[\mathbf{h}^t r_{vv} \mathbf{h} - 2r_{wv}\mathbf{h}]$

We differentiate for $\mathbf{h}$ and we get:
$\frac{d}{d\mathbf{h}}[\mathbf{h}^t r_{vv}\mathbf{h} - 2r_{wv}\mathbf{h}] = 2\mathbf{h}^t \cdot r_{vv} - 2r_{wv}^t = 0 \Rightarrow \mathbf{h}^t = r_{wv}^t \cdot r_{vv}^{-1}$