Advanced Macroeconometrics – Assignment 4

Fynn Lore Tim Koenders Elisabeth Fidrmuc

June 29, 2023

Contents

The executable code that was used in compiling the assignment is available on GitHub at https://github.com/TimKoenders/Macrometrics4.

1

1.a $\lambda_1 = 0.1$ and $\lambda_2 = 0.5$

First we analyze at the case where standard hyperpriors are used (i.e. $\lambda_1 = 0.1$ and $\lambda_2 = 0.5$). Table 1 displays the autoregressive coefficients, Figure 1 shows impulse responses when using short run restrictions via Cholesky decomposition. Figure 3 portrays impulse response function when using sign restrictions and Figure 4 presents forecasts.

A shock in inflation can be interpreted as a (negative) supply shock. As supply becomes scarce, prices increase and so does inflation. Intuitively, it makes sense that as prices go up, demand and thus also output decline. This leads to lower revenues and so profit maximizing firms have to reduce costs and cut out labor. Unemployment increases. The Central Bank has to find a good balance in their monetary policy to control inflation whilst reducing unemployment. In the IRFs resulting from the Cholesky decomposition, the interest rate rises as the central bank tried to reduce inflation. In the IRFs resulting from sign restrictions however, the interest rate falls before gradually returning to its long run value, as the Central bank combats unemployment.

A shock in unemployment can be interpreted as a (negative) demand shock. Higher unemployment implies lower aggregate income and thus also lower aggregate demand. To fight unemployment, the central bank raises the interest rate, which remains persistently high for many periods. As the interest rate rises inflation decreases. Interestingly, for the IRFs resulting from sign restrictions, we can see that unemployment first increases but then decreases even below its long run value. It seems that the central bank lowers interest rate quite aggressively to lower unemployment which also leads to increases in inflation from period 5 onwards.

Finally, a shock in the treasury bill interest rate can be interpreted as a (positive) monetary policy shock. A higher interest rate implies lower inflation. As the interest rate falls back to its long run value, unemployment begins to rise.

According to the forecast, inflation is presumed to fall in the near future. Unemployment is expected to fall too, whilst the interest rate is predicted to remain relatively constant.

1.b $\lambda_1 = 1$ and $\lambda_2 = 5$

Table 2 displays the autoregressive coefficients, Figure 4 shows impulse responses when using short run restrictions via Cholesky decomposition. Figure 5 portrays impulse response function when using sign restrictions and Figure 6 presents forecasts.Compared to the initial hyperprior values, the coefficients seem to remain largely unaffected in magnitude, but the confidence intervals are broader in most cases. This implies that uncertainty surrounding the exact size of coefficients has increased.

A similar pattern can be observed for the impulse response function. Although the direction of movement is the same as before, it is looks as though the paths are more unstable. If we look at the effect of an inflation shock on unemployment for instance, we can see that the response variable first increases, then falls but then increases again, whereas before there was only an increase. However, the downturn is statistically insignificant. That is, there is more uncertainty surrounding the movement of the response variables.

The prediction seems to be largely unaffected by the change in hyperprior values.

1.c $\lambda_1 = 10$ and $\lambda_2 = 50$

Table 3 displays the autoregressive coefficients, Figure 7 shows impulse responses when using short run restrictions via Cholesky decomposition. Figure 8 portrays impulse response function when using sign restrictions and Figure 9 presents forecasts. The results for the autoregressive coefficients, the impulse response functions and forecasts are essentially the same as in the previous case.

1.d $\lambda_1 = 100$ and $\lambda_2 = 500$

Table 5 displays the autoregressive coefficients, Figure 10 shows impulse responses when using short run restrictions via Cholesky decomposition. Figure 11 portrays impulse response function when using sign restrictions and Figure 12 presents forecasts.The results are remarkably similar to the previous two cases. The confidence intervals surrounding the autoregressive parameters are wider, the paths of the impulse response functions are more unstable and the confidence intervals are broader. The forecast appears to be very similar but a deeper look reveals that the confidence interval is broader.

1.e Concluding Remarks

In conclusion we can say, that larger hyperprior values for λ_1 and λ_2 result in more unstable results and wider confidence intervals. Thus, the results are more uncertain. This makes sense, as the larger hyperpriors lead to a higher prior variance for the autoregressive parameter. The parameters can change more in the direction that the data pull them but this happens at the cost of higher uncertainty.

In our cases, it seems as though the choice of hyperprior λ_1 and λ_2 does not matter to a great extent. The results are very similar even as the hyperprior changes dramatically. In general, we would suggest using a prior that accommodates the individual circumstances. If one has no idea about the outcome of interest, larger hyperpriors are suitable. However, if the direction of outcome is relatively clear from the beginning and only the precise magnitudes are analyzed, a lower hyperprior is more appropriate.

In general, there are some possibilities to alleviate hyperparameter choices.

- Robustness analysis: Perform sensitivity analysis to assess the robustness of the results to different hyperprior choices. This involves evaluating how the posterior distributions and inferences change when different hyperprior distributions are used. By comparing the results across different hyperpriors, you can identify whether the conclusions are consistent or sensitive to the specific choices made. This is essentially, what we did in this task and we found out that the results are quite robust with only minor differences due to large changes in hyperpriors.
- Non-informative or weakly informative priors: Use non-informative or weakly informative priors for hyperparameters. Non-informative priors assign equal probabilities to all possible parameter values, while weakly informative priors have some limited prior knowledge incorporated. By using such priors, you can reduce the impact of specific hyperprior choices and allow the data to have a stronger influence on the posterior distributions.
- Empirical Bayes: In the empirical Bayes approach, you estimate the hyperparameters from the data itself. This involves using the observed data to inform the prior distribution of the hyperparameters. By incorporating data-driven estimates of hyperparameters, you can reduce the influence of subjective choices in specifying the hyperpriors. In this exercise, we use estimates obtained from OLS as the starting point of the analysis.
- Hierarchical models: Another approach is to use hierarchical models that explicitly model the uncertainty in the hyperparameters. In a hierarchical model, hyperparameters are treated as random variables with their own prior distributions. This allows the data to inform the hyperparameter estimates and also accounts for the uncertainty in the hyperprior choices.

	inf	une	tbi
inf_l1	1.187 [1.139, 1.242]	0.031 [0.002, 0.062]	0.078 [0.006, 0.153]
une_l1	-0.091 [-0.127, -0.056]	1.147 [1.093, 1.196]	-0.058 [-0.144, 0.027]
tbi_l1	0.012 [-0.004, 0.028]	0.005 [-0.012, 0.021]	0.946 [0.889, 0.999]
inf_l2	-0.112 [-0.166, -0.057]	-0.002 [-0.028, 0.024]	0.004 [-0.061, 0.067]
une_l2	0 [-0.03, 0.03]	-0.154 [-0.208, -0.1]	0.009 [-0.063, 0.078]
tbi_l2	-0.003 [-0.015, 0.009]	0.009 [-0.003, 0.021]	-0.062 [-0.111, -0.011]
inf_l3	-0.058 [-0.095, -0.023]	0 [-0.018, 0.017]	-0.012 [-0.056, 0.032]
une_l3	0.009 [-0.012, 0.029]	-0.075 [-0.109, -0.039]	0.015 [-0.033, 0.062]
tbi_l3	-0.003 [-0.011, 0.007]	0.007 [0, 0.015]	0.035 [0, 0.069]
inf_l4	-0.027 [-0.055, 0.001]	0.004 [-0.01, 0.017]	-0.004 [-0.038, 0.03]
une_l4	0.012 [-0.004, 0.028]	-0.011 [-0.037, 0.014]	0.025 [-0.012, 0.061]
tbi_l4	-0.002 [-0.008, 0.005]	0.004 [-0.002, 0.01]	0.003 [-0.026, 0.031]
Constant	0.421 [0.299, 0.547]	0.293 [0.172, 0.412]	0.258 [-0.021, 0.555]

Table 1: Autogressive coefficients $\lambda_1 = 0.1, \lambda_2 = 0.5$

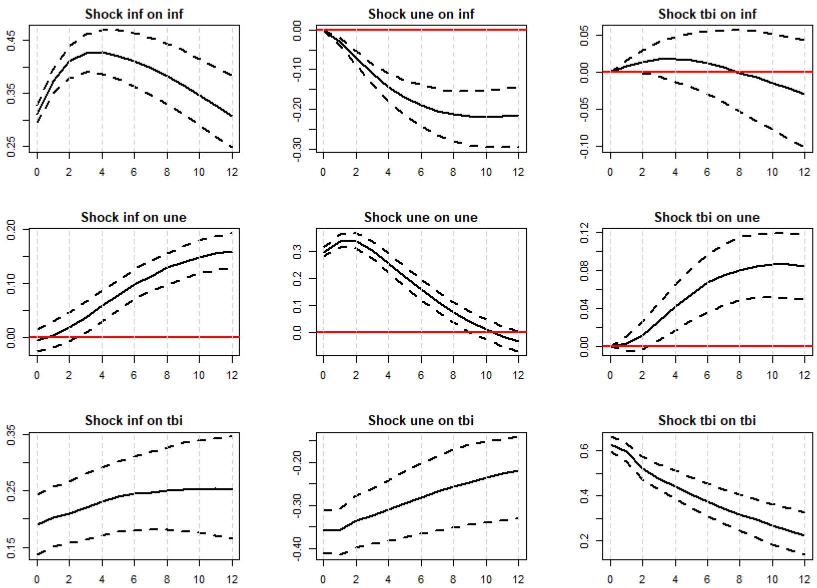

Figure 1: IRFs from Cholesky Decomposition,$\lambda_1 = 0.1, \lambda_2 = 0.5$

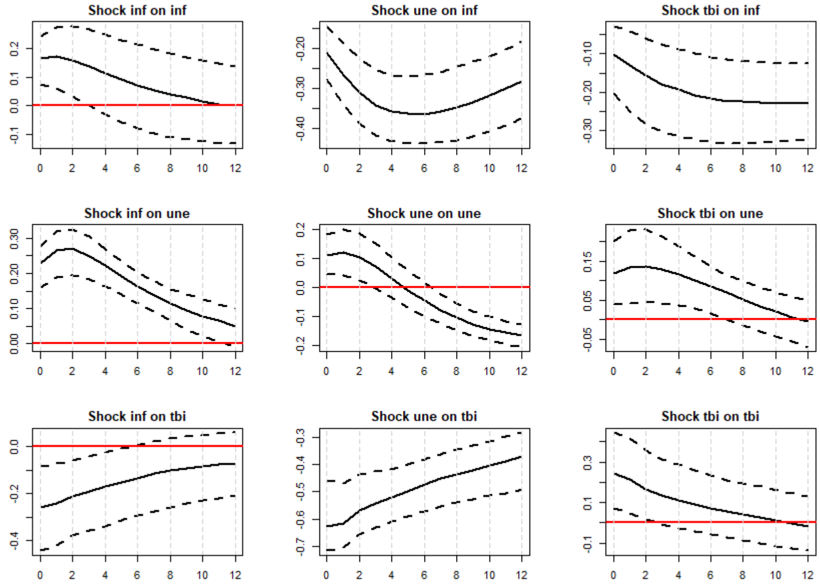

Figure 2: IRFs from Sign Restrictions, $\lambda_1 = 0.1, \lambda_2 = 0.5$

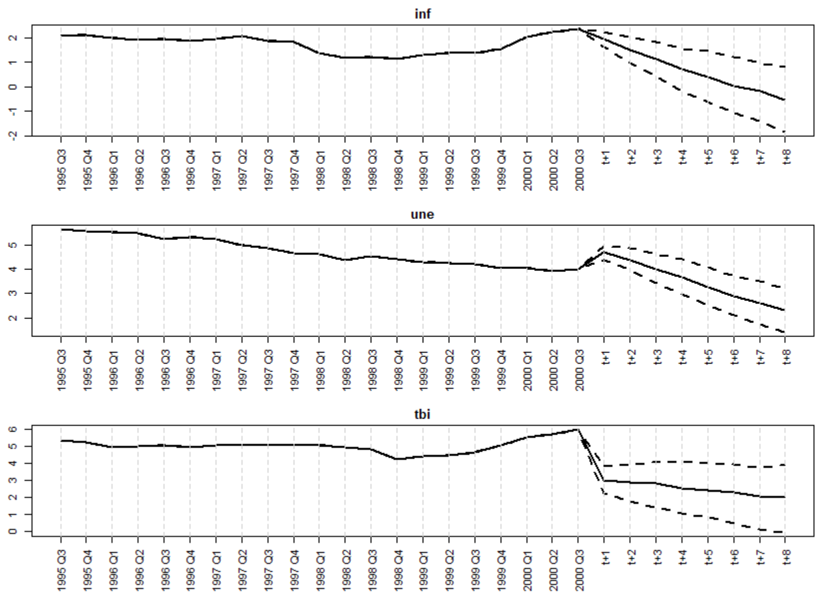

Figure 3: Predictions, $\lambda_1 = 0.1, \lambda_2 = 0.5$

	inf	une	tbi
inf_l1	1.488 [1.394, 1.584]	0.1 [0.01, 0.187]	0.158 [-0.063, 0.384]
une_l1	-0.209 [-0.322, -0.097]	1.5 [1.402, 1.599]	-0.655 [-0.897, -0.401]
tbi_l1	0.028 [-0.016, 0.069]	-0.021 [-0.065, 0.019]	1.07 [0.969, 1.172]
inf_l2	-0.459 [-0.629, -0.304]	-0.188 [-0.345, -0.034]	0.353 [-0.029, 0.751]
une_l2	0.191 [-0.003, 0.382]	-0.585 [-0.753, -0.423]	0.69 [0.246, 1.121]
tbi_l2	-0.08 [-0.14, -0.021]	0.056 [-0.003, 0.113]	-0.533 [-0.676, -0.39]
inf_l3	-0.074 [-0.228, 0.079]	0.063 [-0.09, 0.211]	-0.569 [-0.967, -0.17]
une_l3	-0.191 [-0.37, -0.006]	-0.128 [-0.277, 0.029]	-0.207 [-0.629, 0.244]
tbi_l3	0.069 [0.005, 0.136]	-0.048 [-0.103, 0.008]	0.496 [0.361, 0.629]
inf_l4	0.048 [-0.043, 0.136]	0.052 [-0.034, 0.139]	0.162 [-0.061, 0.379]
une_l4	0.157 [0.058, 0.258]	0.12 [0.034, 0.203]	0.18 [-0.054, 0.416]
tbi_l4	-0.02 [-0.065, 0.026]	0.04 [-0.002, 0.082]	-0.129 [-0.231, -0.031]
Constant	0.306 [0.171, 0.432]	0.296 [0.177, 0.418]	0.114 [-0.198, 0.417]

Table 2: $\lambda_1 = 1, \lambda_2 = 5$

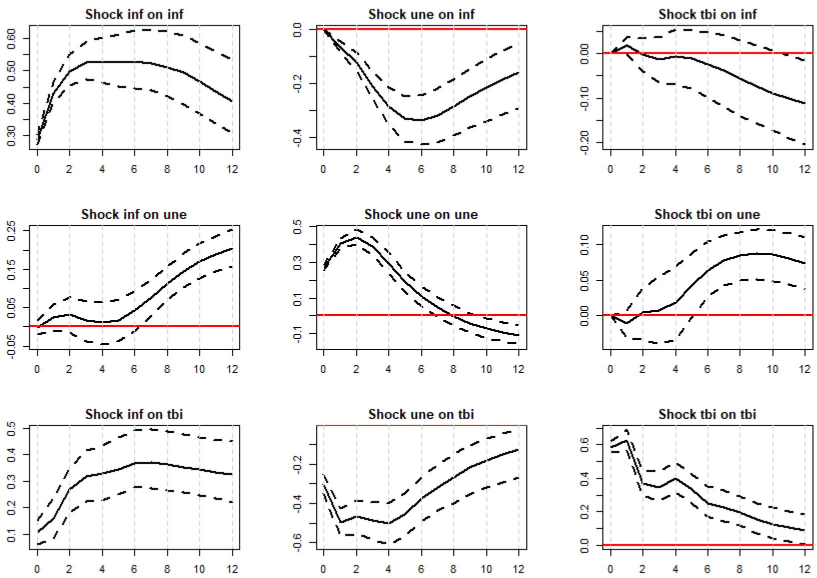

Figure 4: IRFs from Cholesky Decomposition, $\lambda_1 = 1, \lambda_2 = 5$

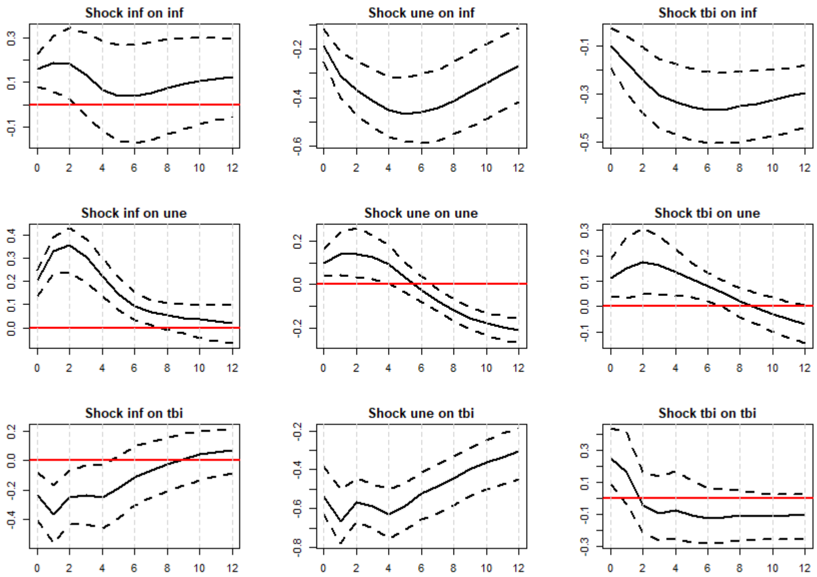

Figure 5: IRFs from Sign Restrictions, $\lambda_1 = 1, \lambda_2 = 5$

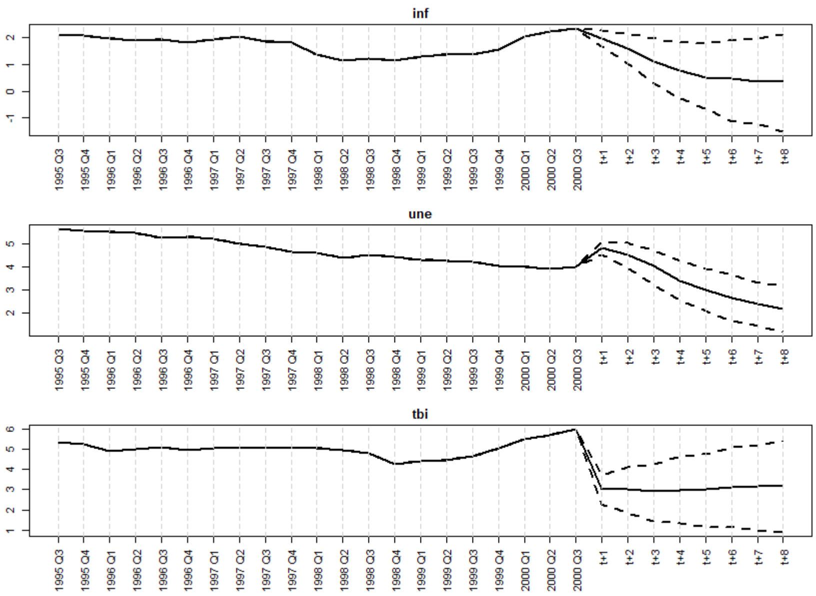

Figure 6: Predictions, $\lambda_1 = 1, \lambda_2 = 5$

	inf	une	tbi
inf_l1	1.501 [1.405, 1.603]	0.096 [0.007, 0.183]	0.17 [-0.056, 0.4]
une_l1	-0.207 [-0.324, -0.094]	1.515 [1.413, 1.615]	-0.647 [-0.889, -0.389]
tbi_l1	0.03 [-0.013, 0.075]	-0.026 [-0.069, 0.014]	1.1 [0.996, 1.208]
inf_l2	-0.484 [-0.665, -0.318]	-0.187 [-0.342, -0.031]	0.349 [-0.04, 0.763]
une_l2	0.193 [-0.008, 0.389]	-0.606 [-0.789, -0.431]	0.662 [0.194, 1.12]
tbi_l2	-0.084 [-0.147, -0.024]	0.067 [0.007, 0.123]	-0.601 [-0.754, -0.451]
inf_l3	-0.068 [-0.238, 0.094]	0.078 [-0.08, 0.224]	-0.619 [-1.031, -0.212]
une_l3	-0.191 [-0.377, -0.001]	-0.143 [-0.315, 0.044]	-0.125 [-0.584, 0.352]
tbi_l3	0.076 [0.011, 0.14]	-0.067 [-0.126, -0.009]	0.589 [0.439, 0.741]
inf_l4	0.055 [-0.042, 0.151]	0.043 [-0.044, 0.13]	0.202 [-0.019, 0.422]
une_l4	0.153 [0.052, 0.258]	0.141 [0.044, 0.231]	0.119 [-0.13, 0.364]
tbi_l4	-0.025 [-0.071, 0.023]	0.052 [0.007, 0.094]	-0.184 [-0.297, -0.079]
Constant	0.307 [0.175, 0.437]	0.295 [0.173, 0.422]	0.111 [-0.207, 0.414]

Table 3: Autogressive coefficients $\lambda_1 = 10, \lambda_2 = 50$

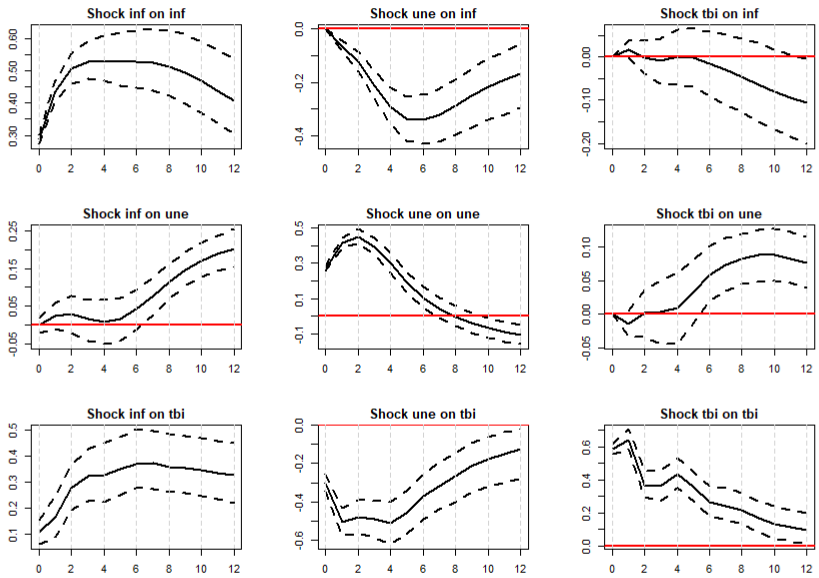

Figure 7: IRFs from Cholesky Decomposition, $\lambda_1 = 10, \lambda_2 = 50$

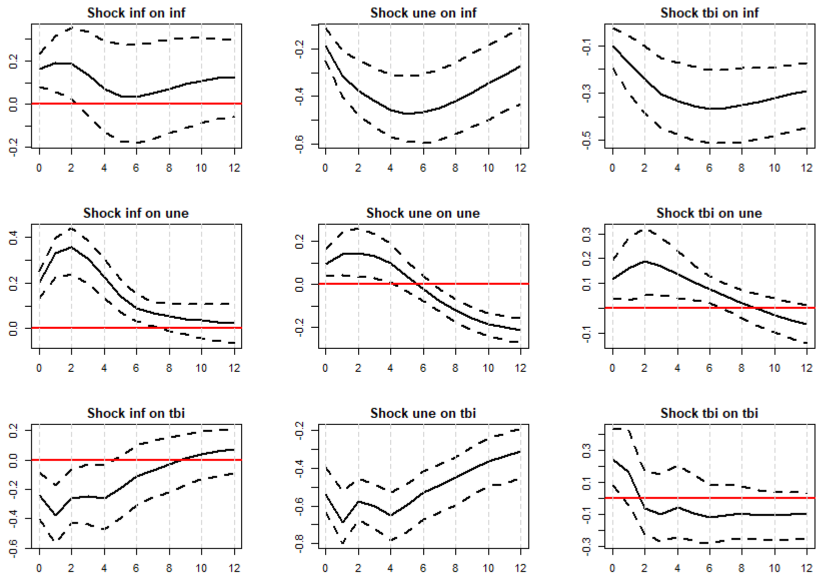

Figure 8: IRFs from Sign Restrictions, $\lambda_1 = 10, \lambda_2 = 50$

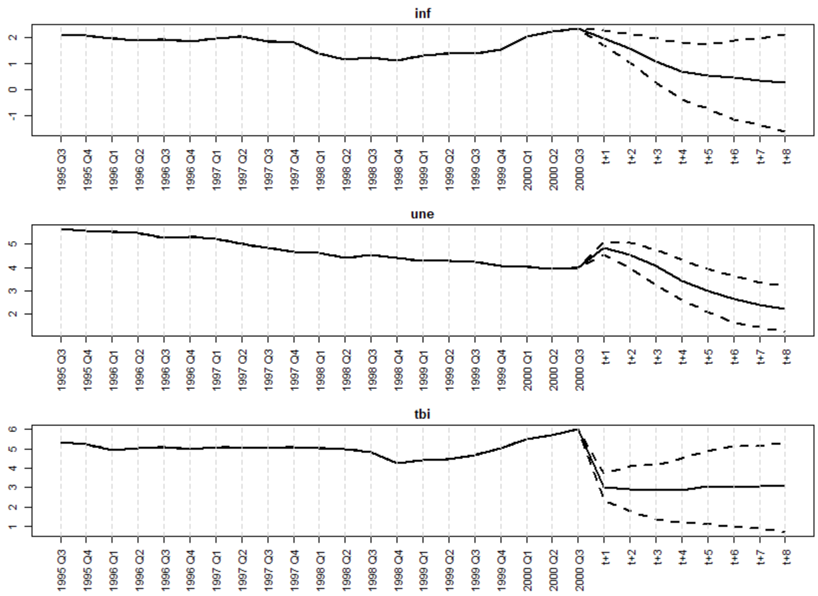

Figure 9: Predictions, $\lambda_1 = 10, \lambda_2 = 50$

	inf	une	tbi
inf_l1	1.501 [1.405, 1.604]	0.096 [0.007, 0.183]	0.17 [-0.056, 0.4]
une_l1	-0.207 [-0.324, -0.094]	1.515 [1.413, 1.615]	-0.647 [-0.889, -0.389]
tbi_l1	0.03 [-0.013, 0.075]	-0.026 [-0.069, 0.014]	1.101 [0.997, 1.209]
inf_l2	-0.485 [-0.665, -0.318]	-0.187 [-0.342, -0.031]	0.349 [-0.04, 0.763]
une_l2	0.193 [-0.008, 0.389]	-0.606 [-0.79, -0.431]	0.662 [0.194, 1.121]
tbi_l2	-0.084 [-0.147, -0.024]	0.067 [0.007, 0.123]	-0.602 [-0.755, -0.452]
inf_l3	-0.068 [-0.238, 0.094]	0.078 [-0.08, 0.224]	-0.619 [-1.031, -0.213]
une_l3	-0.191 [-0.377, -0.001]	-0.144 [-0.316, 0.044]	-0.123 [-0.583, 0.353]
tbi_l3	0.076 [0.011, 0.14]	-0.067 [-0.126, -0.009]	0.59 [0.44, 0.742]
inf_l4	0.055 [-0.042, 0.151]	0.043 [-0.045, 0.13]	0.202 [-0.019, 0.423]
une_l4	0.153 [0.051, 0.258]	0.141 [0.045, 0.232]	0.118 [-0.131, 0.363]
tbi_l4	-0.025 [-0.071, 0.022]	0.052 [0.007, 0.094]	-0.185 [-0.298, -0.08]
Constant	0.307 [0.175, 0.437]	0.295 [0.173, 0.422]	0.111 [-0.207, 0.414]

Table 4: $\lambda_1 = 100, \lambda_2 = 500$

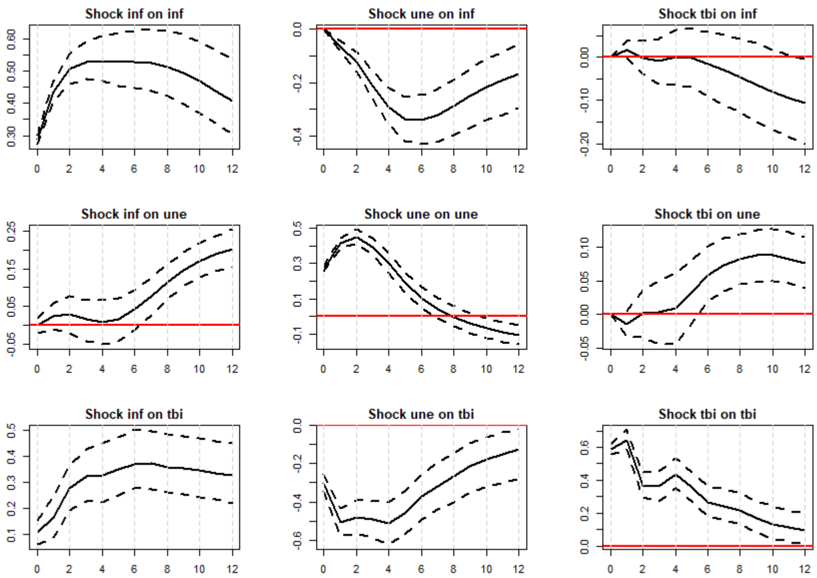

Figure 10: IRFs from Cholesky Decomposition, $\lambda_1 = 100, \lambda_2 = 500$

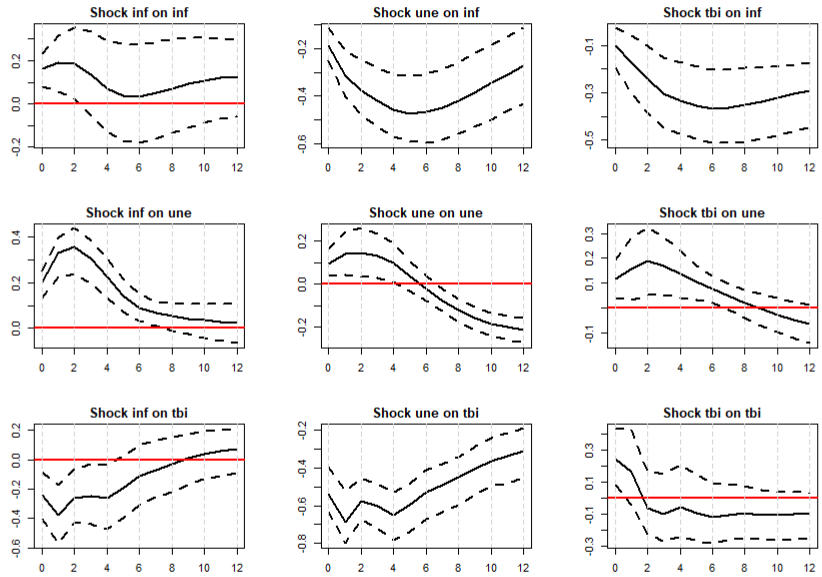

Figure 11: IRFs from Sign Restrictions, $\lambda_1 = 100, \lambda_2 = 500$

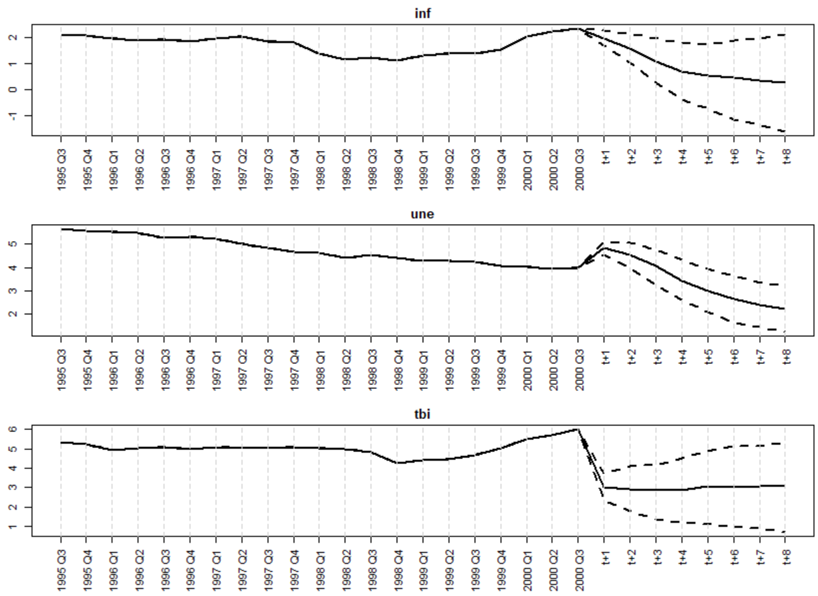

Figure 12: Prediction, $\lambda_1 = 100, \lambda_2 = 500$

2

2.a Bayesian VAR

We estimated the VAR model proposed by Kilian (2009) in a bayesian fashion and obtained its structural form through a recursive ordering of variables. To address the issue of aggressive shrinkage associated with the original Minnesota Prior, we adjusted both λ_1 and λ_2 by a factor of 3. This adjustment was made as higher lambda values induce less shrinkage.

2.b Figure 2 and 3 of Killian (2009)

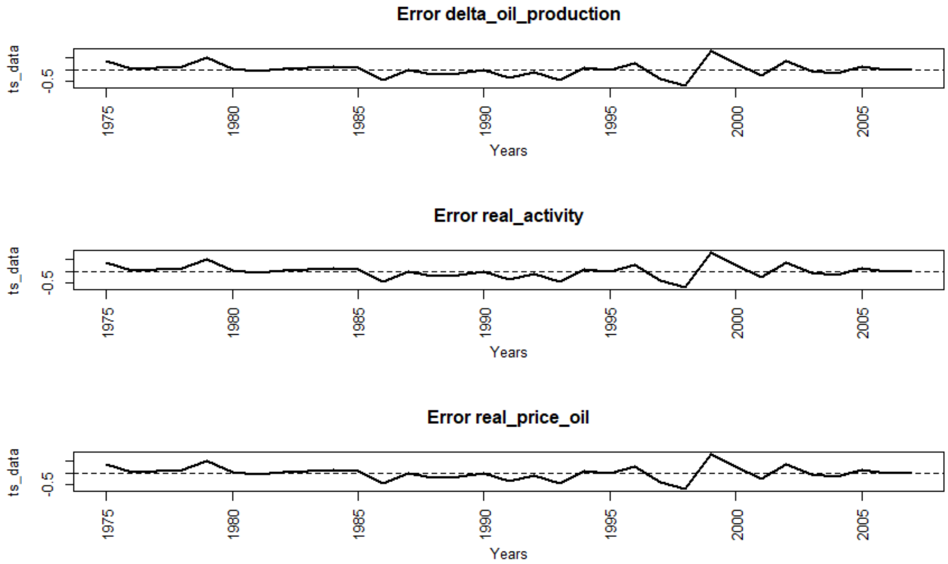

Figure 13: Figure 2, Kilian (2009)

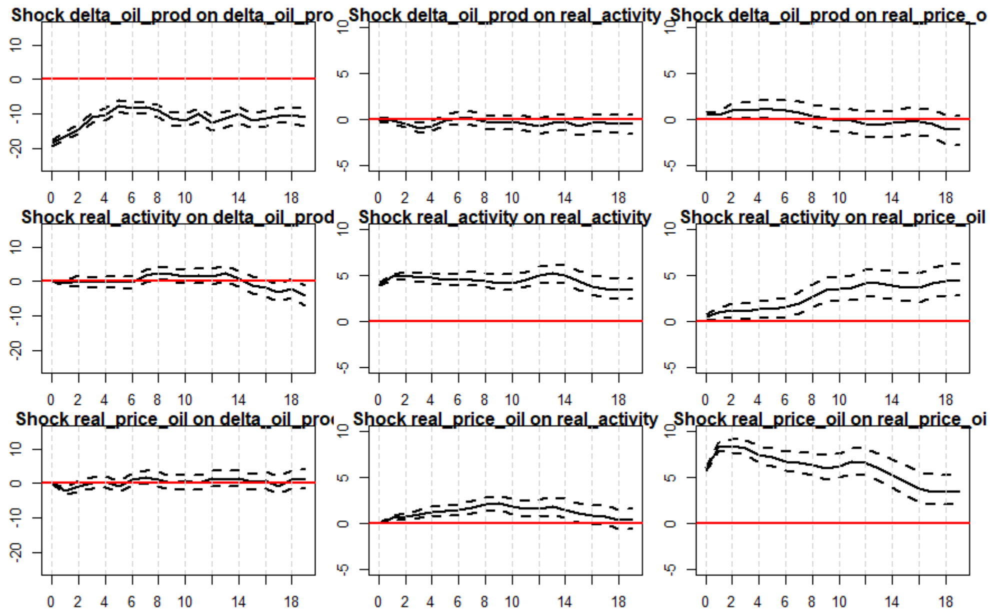

Figure 14: Figure 3, Kilian (2009)

2.c Sign Restrictions

From an econometric perspective, it is crucial to set sign restrictions in a way that avoids linear dependence among the columns, ensuring identification. Additionally, from an economic standpoint, these restrictions should align with meaningful economic relationships. Following the specification by Kilian Murphy (2012), we implemented the following sign restrictions:

An oil supply shock is expected to reduce oil production, decrease real activity, and increase the real price of oil. An unexpected shock to aggregate demand is anticipated to increase all three variables: oil production, real activity, and the real price of oil. An oil-specific demand shock is predicted to raise oil production and the real price of oil while reducing real activity. To estimate the model and identify the structural form, we utilized the Bayesian VAR framework, incorporating the aforementioned sign restrictions. To achieve less aggressive shrinkage, we adjusted the values of λ_1 and λ_2 as done previously.

The sign restrictions can be written more compactly,

$$e_t = \begin{pmatrix} e_t^{\Delta\text{prod}} \\ e_t^{\text{rea}} \\ e_t^{\text{rpo}} \end{pmatrix} = \begin{pmatrix} - & + & + \\ - & + & - \\ + & + & + \end{pmatrix} \begin{pmatrix} e_t^{\text{oil supply shock}} \\ e_t^{\text{aggregate demand shock}} \\ e_t^{\text{oil market specific demand shock}} \end{pmatrix}$$

Imposing these restrictions we can identify and estimate the model, and retrieve the following impulse response functions.

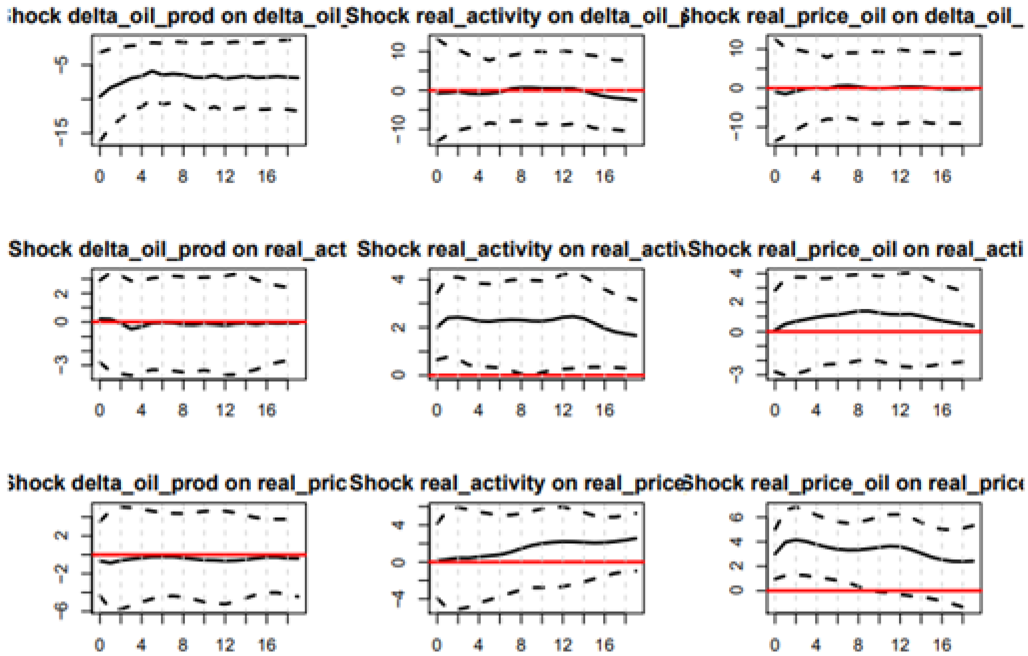

Figure 15: IRF's with sign-restrictions

We can identify the following shortcomings of using short-run restrictions,

Loss of Information: By imposing short restrictions, you are effectively constraining the relationships between variables in the model. This can lead to a loss of information and oversimplification of the true underlying dynamics. If the imposed restrictions do not accurately reflect the actual relationships among variables, the model may produce biased or misleading results.

Possible misrepresentation: Imposing stringent restrictions in the immediate term can result in a misrepresentation of the fundamental economic relationships at play. The imposed restrictions may not accurately reflect the true dynamics of the variables, impacting the reliability of estimated parameters and subsequent analysis.

Model-specific constraints: Short-term restrictions are typically tailored to the specific model being estimated. If economic theory or prior knowledge suggests alternative limitations, incorporating them into the short-term identification framework may not be straightforward.

We can identify the following shortcomings of using sign restrictions,

Uncertainty: Sign-based constraints solely determine the impulse response function (IRF) in terms of direction, leaving the magnitude undetermined. Consequently, multiple plausible solutions for the IRFs that satisfy the sign restrictions can exist.

Restrictions needed: It relaxes the exclusion restrictions but at the cost of restricting other parameters what were unrestricted using short-term restrictions.

Non-uniqueness of identification: In certain cases, sign-based restrictions may fail to uniquely determine the parameters of interest. This particularly holds true when the variables exhibit high correlation or when the data provide limited information regarding the dynamics of the variables.

2.d Oil market shocks and unemployment

We first test whether the unemployment rate series is stationary. The test yielded the following results:

Augmented Dickey-Fuller Test Dickey-Fuller = -3.9764, Lag order = 7, p-value = 0.01035 alternative hypothesis: stationary

These results suggest that based on the Augmented Dickey-Fuller test, there is evidence to support the hypothesis that the series is stationary. Hence, we include the CPI variable in levels.

We then replicate figure 3 of the paper with the extra unemployment rate shock using both short-term and sign-restrictions.

For the short-run restrictions, we use Cholesky decomposition and place the variables in decreasing order of exogeneity. The lagging nature of unemployment in relation to the business cycle reinforces the argument for a recursive ordering. This is because unemployment does not show immediate effects in the impulse response functions. By incorporating unemployment into the model while imposing short-run restrictions, we observe the following patterns of change. In response to an oil supply shock, unemployment declines. However, when confronted with an aggregate demand shock, unemployment initially decreases but subsequently rises again. In the event of an oil-specific demand shock, unemployment rises. Conversely, when there is a shock to unemployment itself, we observe a decline in oil production followed by an increase. Additionally, both real economic activity and the real price of oil decrease during such circumstances.

We impose the following sign restrictions to recover the structural form:

$$e_t = \begin{pmatrix} e_t^{\Delta\text{prod}} \\ e_t^{\text{rea}} \\ e_t^{\text{rpo}} \\ e_t^{\text{UNRATE}} \end{pmatrix} = \begin{pmatrix} - & + & + & + \\ - & + & - & - \\ + & + & + & - \\ + & - & - & + \end{pmatrix} \begin{pmatrix} e_t^{\text{oil supply shock}} \\ e_t^{\text{aggregate demand shock}} \\ e_t^{\text{oil market specific demand shock}} \\ e_t^{\text{Unemployment shock}} \end{pmatrix}$$

Then we obtain the following impulse response functions,

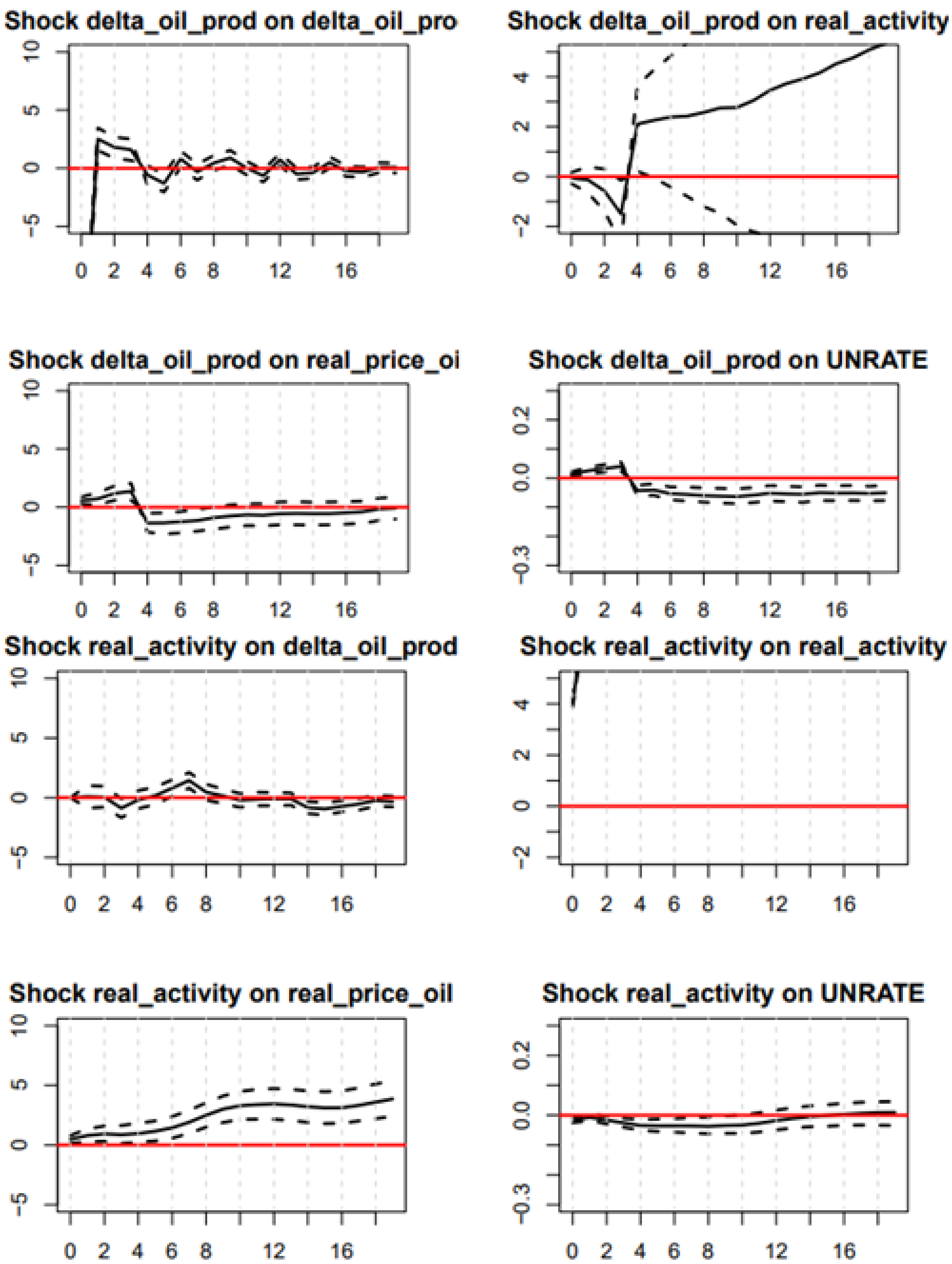

Figure 16: IRF's with short-run restrictions

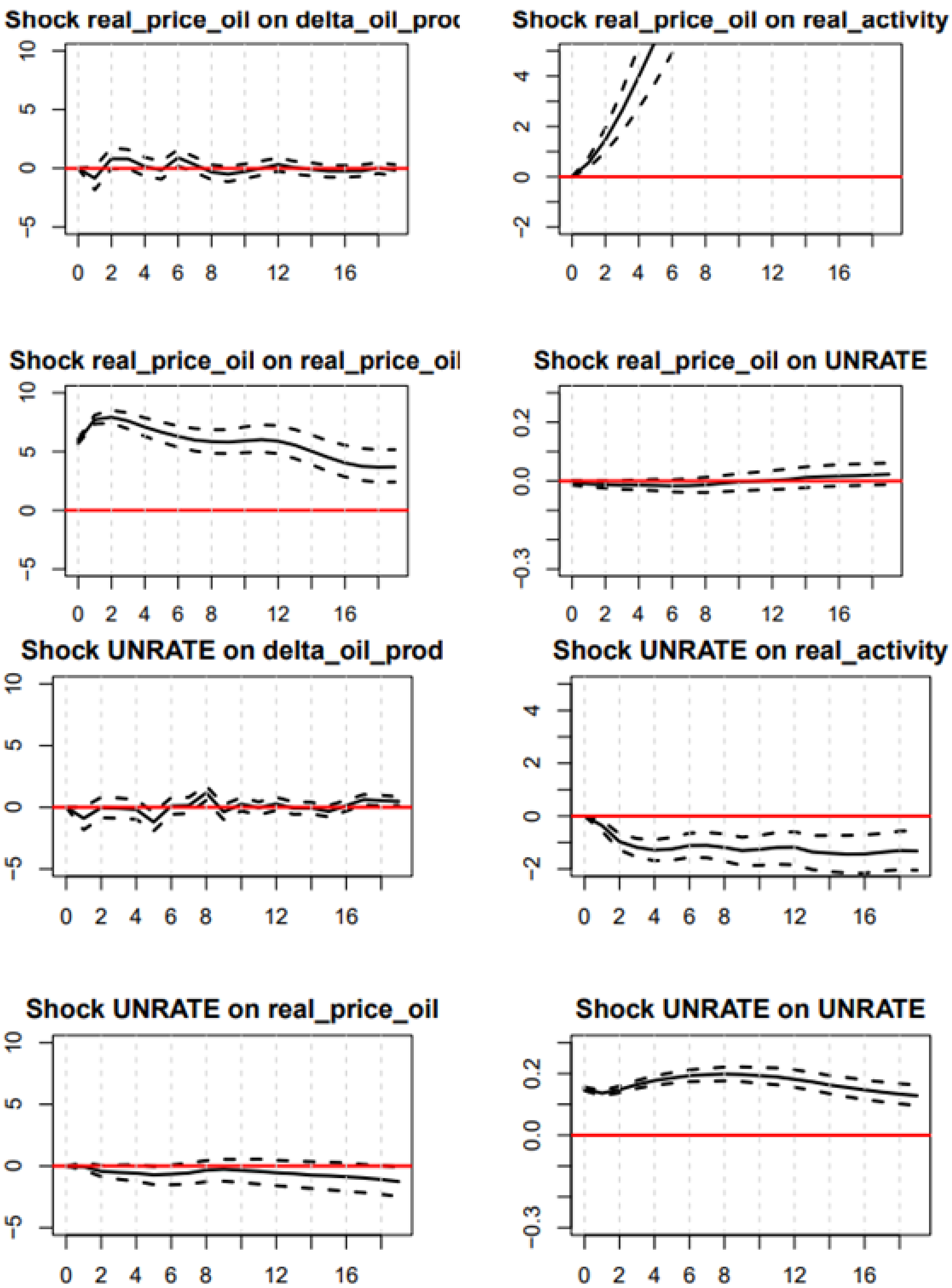

Figure 17: IRF's with short-run restrictions

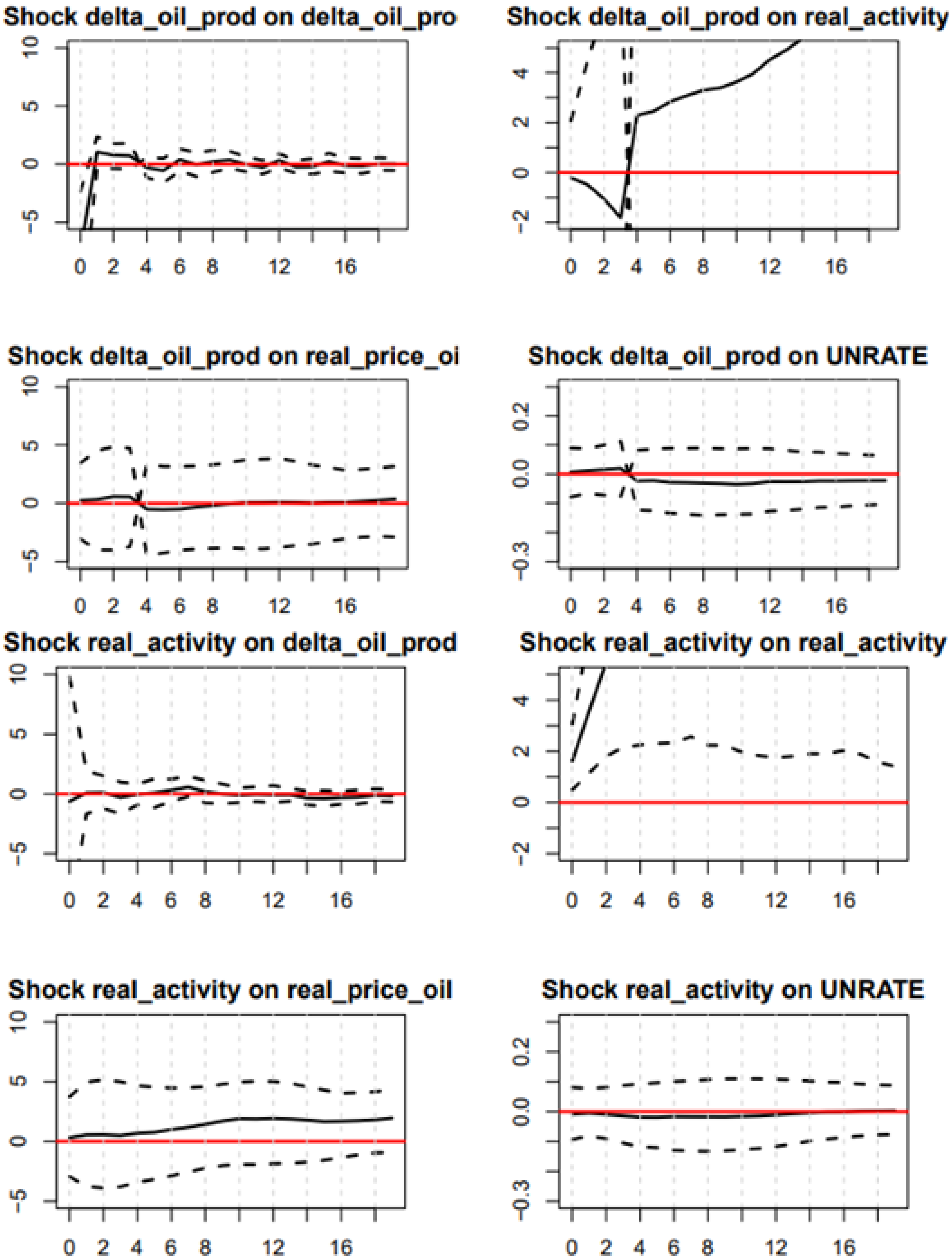

Figure 18: IRF's with sign-restrictions

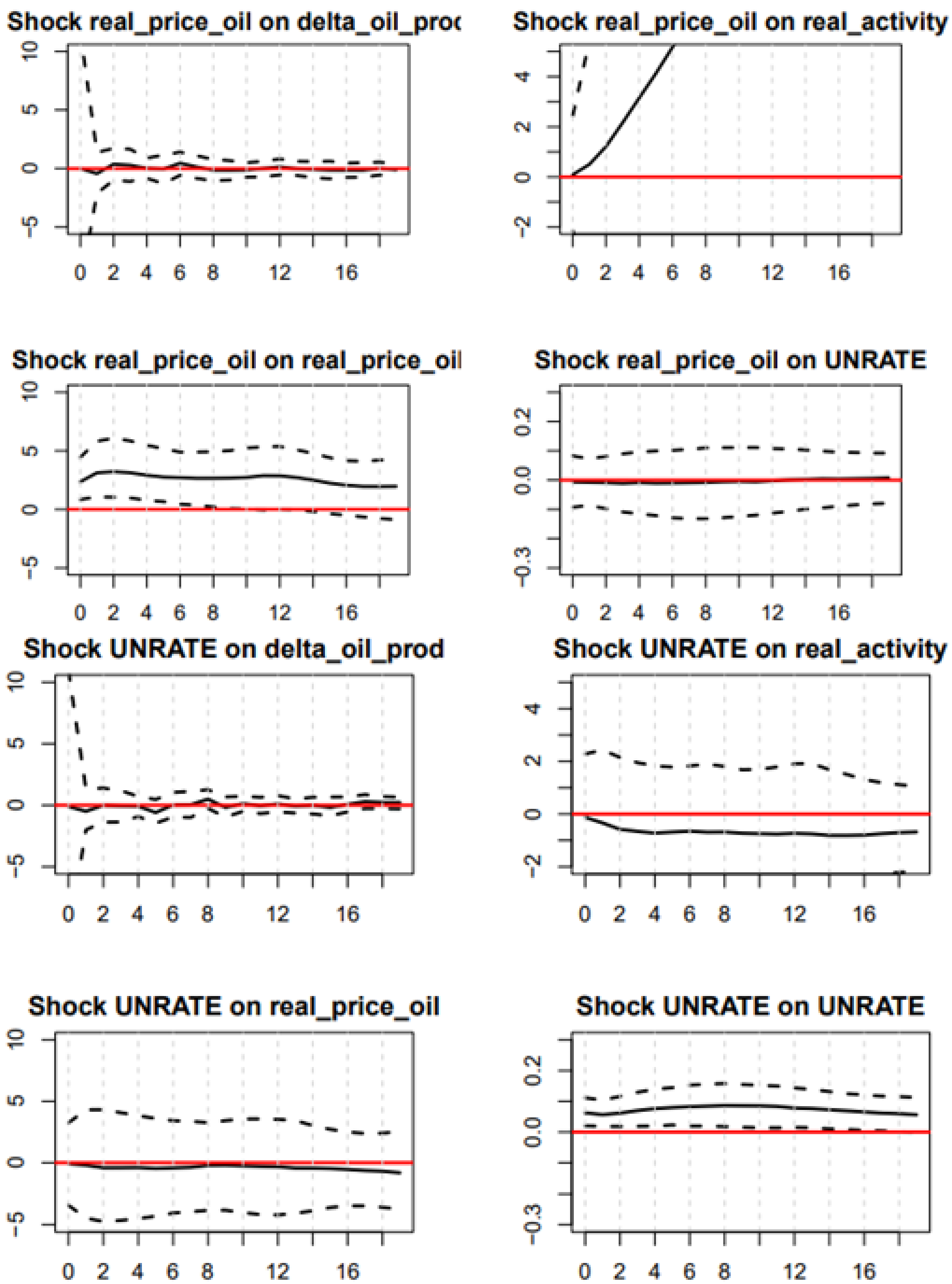

Figure 19: IRF's with sign-restrictions

Short-run restrictions and the incorporation of unemployment into the model reveal the following trends. In the presence of an oil supply shock, we first observe an increase in the unemployment rate but then we observe a persistent negative effect. If there is a shock specific to oil demand, it leads to a rise in unemployment. Conversely, when there is a positive shock to unemployment, there is no clear pattern to the growth in oil production. Additionally, after a positive shock the unemployment, both real economic activity and the real price of oil witness a decrease.

Implementing sign restrictions in the model gives different responses. In the case of an oil supply, demand or price shock shock, no noticeable changes occur to the unemployment rate. In the scenario of a shock to unemployment rate, minimal changes can be observed in the other variables. Most significantly we observe a decline in aggregate demand.