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A Structural Directed-Search Model of Students' Study Decisions and Program Choice

by

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The relevant model code is publicly available at https://github.com/VARFynn/University_Contributions/tree/main/01_Master/02_Paper/MMMOL

Abstract

This term paper develops a structural directed-search model of college choice in which students with heterogeneous abilities make portfolio application decisions to maximize expected future earnings. In the proposed framework, students face uncertainty about admission outcomes, application (search) costs, and must consider both the quality-ability match and program-specific admission cutoffs when constructing their application portfolios.

The proposed model captures the unique institutional setting of the Danish higher education system's GPA-based admission mechanism (kvote 1), where students can apply to up to eight programs and admission decisions are based solely on high school GPA. This term paper demonstrates that the combination of search costs and limitations on portfolio size leads to significant deviations from the socially optimal matching outcome in a frictionless baseline case. It further shows how the number of submitted application is affected by external (policy) parameters.

The framework allows for policy counterfactuals examining how changes to key parameters affect welfare and match efficiency. Specifically, it is analyzed how a change in student aid (i.e. SU) and application costs impacts sorting patterns and overall market congestion. This analysis is the first to explicitly model the mechanism through which student financial aid interacts with application behavior and, hence, endogenizes the application set including the number of included elements.

Contents

1 Introduction

Every year, tens of thousands of secondary school graduates face the decision of whether to pursue post-secondary education and, if so, which program to choose. This process involves two key dimensions of heterogeneity: differences in student abilities and variations in university program quality. The application decisions made by these prospective students will in the end affect both their professional and private trajectories. However, it is not uncommon to observe highly qualified graduates without university placements, while some programs struggle to fill their available seats. Furthermore, despite the opportunity to apply to multiple programs, many students choose to apply to only a single or a couple of programs.

Consequently, the question arises *how exactly students make their application decisions, whether this leads to an efficient outcome, and how the application process is affected by search frictions*. Understanding this process is helpful to understand the influence of policy interventions, e.g., raising student aid or application costs, on the labor market outcomes of the future. It is important to understand that better matching could not only affect individual wages, but also the overall output and tax revenues. Further, beyond the scope of this term paper, it can also lead to distributional effects. If the application process differs between students, this could be one further hindering factor for the social mobility of students. At the same time, a university education will also potentially affect who they will end up marrying (Almar et al., 2025) and it further affects how economies react to shocks (see implicitly Smeets et al., 2025).

Relevant Literature. This is of course not the first time in an economics paper that the students' application portfolio decision has been studied and subsequently modeled. Gale and Shapley (1962) were the first (prominent) example of a paper approaching the students' matching process (alongside marriage matching), though simplifying the application process. Since then, the model has been extended by numerous papers (as e.g. in Azevedo and Leshno, 2016) to account for more complex real-world settings and different institutional contexts, particularly in the realm of application clearing mechanisms.

In this context, it is important to decide between the assignment- or clearing mechanism and, thus, implicitly the model setup. In a decentralized context (e.g., Germany) it appears appropriate to rely on a probabilistic matching setup (á la Mortensen and Pissarides, 1994; Shimer and Smith, 2000), as students are confronted with an offer and acceptance deadline, not knowing if more offers arrive. However, in a centralized clearing, it becomes much more a problem of strategic search within a sub-set of programs, whereas the clearing-house decides the acceptance of the offers. Thus, a directed search matching (á la Moen, 1997; Acemoglu and Shimer, 1999; summarized in Wright et al., 2021) appears to be more reasonable in those cases.

Consequently, the question of how students form their application portfolios has also been addressed using structural estimation methods, which bring the theoretical models to actual admissions data. Chade et al. (2014) formalize the application process in a decentralized setting as a strategic portfolio decision under uncertainty, where students apply to multiple programs balancing the expected probability of acceptance against application costs and preferences. Epple et al. (2006) or Fu

(2014) model in a similar fashion the equilibrium between application behavior, admission policies, and enrollment decisions. Fu (2014) quantifies how application costs disproportionately deter low-income students, potentially distorting access and leading to inefficient matches. Likewise, Epple et al. (2006) show how tuition-setting and financial aid policies shape the stratification of students across institutions. However, basically all of these models assume a decentralized clearinghouse.

Within these models, there is typically some form of frictions, which are either exogenous or endogenous. These frictions are also visible in data. Most prominently, in the student aid literature the so called "under-matching" is considered the core driver of long-term inequality (see Hoxby and Avery, 2013; Hoxby and Turner, 2015, and implicitly Dynarski et al., 2021). Under-matching means that the so called high-achieving, low-income students do not apply to any selective colleges, even though those institutions would likely admit them with generous aid (Hoxby and Avery, 2013). The explanatory frictions for this under-matching are in the realm of information frictions e.g., not knowing about the existence of aid (Hoxby and Turner, 2015; Dynarski et al., 2021), but also search costs in form of actual costs of applying (Pallais, 2015), but associated also time costs (Knight and Schiff, 2022). The latter showed that a common application portal which lowered the application time led to higher application numbers. Yet, a correct identification of the overall associated costs remains ongoing research.

Institutional Setting. Denmark offers a unique institutional setting in the form of the GPA-based admission mechanism (kvote 1), where students can apply to up to eight programs and admission decisions are done by a clearing-house and (for the majority) solely on their high school GPA. Further, it has a unique student aid scheme, whereas all students receive a right for student support, while at the same time no tuition fees are charged. Thinking about the frictions, it also seems fair given Denmark's size to assume that all students know about all universities and their programs. This allows to think about the search costs more as cognitive search costs.

Value added. Hence, this term paper develops from scratch a simple choice model with two-sided heterogeneity and forward-looking students in a centralized setting, where students first make a discrete choice about whether to apply to a program and then select their preferred option from their application set, based on the expected net present value (NPV) of attending each program. The model assumes students are forward-looking and that there's a premium for a match of high-quality programs and high-ability students in form of future wage gains. It starts with a frictionless baseline case and then introduces search frictions by incorporating constraints on the number of applications, application costs, and potential congestion around the cutoff of programs in the admission process. The shift to a directed-search model fits well with Denmark's centralized clearinghouse system for university admissions as students have to ex-ante decide on their application set. The model is calibrated to Danish data[1] and used to run policy simulations examining shocks to the outside option, increases in application costs, and changes

[1] Transparency Note: Some data issues remain unresolved and will be discussed in the calibration and validation section. Given this is a term paper, I'll proceed under the assumption of accurate data in other sections.

to student financial aid. As the model endogenizes the application set, it allows to give deeper insights into the application process. However, this model is not run on the micro data and further estimated, but rather used on the calibrated parameters with a fixed-point algorithm.

For Denmark specifically, there does not exist much research on the application process and its efficiency. The closest paper is probably a recent working paper by Smeets et al. (2025), which develops a forward-looking dynamic model where students choose fields based on both expected earnings and non-pecuniary preferences. It assumes that forward-looking students choose fields based on idiosyncratic preferences, comparative advantages and the varying costs of switching occupations. Yet, it only partly focuses on the application process and does not consider search frictions.

Roadmap. This term paper is structured as follows. First, it starts with a Model Setup. In this model setup there is first an extensive introduction of the baseline case before search frictions in a directed search model are presented. There are no specifics on the institutional setting. However, these can e.g., be seen more thoroughly explained in Smeets et al. (2025). Second, there is a Calibration & Validation section, where the model is calibrated to Danish data. Third, there is a Results section, where the results are presented and some policy simulations are done. Finally, there is a Discussion & Conclusion section, where the results are discussed and the current limitations are thoroughly addressed.

2 Model Setup

In terms of model setup, this paper closely follows Galiani and Pantano's (2021) guide which breaks down the formulation of a strucutral model into twelve decision-making steps. It will also be presented once in the frictionless baseline case and once with introduced frictions in a directed-search-model framework (following Wright et al., 2021).[2]

2.1 Frictionless Baseline

In the frictionless case, students have full information in the game-theoretic sense. They know their type, the type distribution, and what others know (including that others know this, and so on). Thus, it does not matter if a student applies to all programs or the optimal one as the outcome will be the same. For simplicity, I assume that students apply to all sufficient programs. This can be modeled as follows and can be considered the/a societal optimum[3]:

[2]As this model started initially from a matching framework with two-sided heterogeneity, it might follow Shimer and Smith (2000) at some points very closely without explicitly mentioning it.

[3]Obviously, this statement assumes a utilitarian social welfare function and should be taken cum grano salis.

Agents

Let $x \in [0,1]$ denote student ability with distribution function $L(x)$ of abilities with density $l(x)$. Let $y \in [0,1]$ denote college-program quality with distribution function $C(y)$ of program qualities with density $c(y)$. $\kappa(y) > 0$ denotes the exogenous seat capacity of program y.

Equilibrium

Let $A(x) \subseteq [0,1]$ then denote the sub-set of programs to which a type-x student applies, and let $NC(y) \in [0,1]$ denote the admission cut-off ability, which is endogenously determined and refers to the last admitted student. Let there further be a produced output in form of a match wage, $w(x,y)$, which results from the iteration of program quality and ability.

Then $f(x,y)$ can be considered the net present value (NPV) of attending program y for type x, and, hence, $W_0(x)$ can be considered the NPV of working immediately.

Finally, $M \subseteq [0,1]^2$ denotes the matching set. This matching set corresponds to the intersection of $A(x)$ and $NC(y)$. Consequently, this is referring to the set of all student-program pairs, where a student applied and would satisfy the admission condition, no matter if they will actually choose the program or not. It is further possible to created an Admission set, which captures the matching set in terms of actual admission.

In the frictionless baseline, the equilibrium is then a triple $\big(w, M, NC\big)$ s.t.

1. *Student optimality:* a type-x student applies to program y iff $f(x,y) \geq W_0(x)$ and $x \geq NC(y)$, then choosing the program with the highest NPV.

2. *Program optimality:* program y admits the top $\kappa(y)$ applicants, which pins down $NC(y)$;

As visible the programs are assumed to be rather simple in maximizing $f(x,y)$ given $A(x)$ and the constraint $\kappa(y)$

Planning Horizon and Forward-Looking Behavior

Let $T \in \mathbb{N}$ denote the finite lifetime horizon in years, and let $r > 0$ denote the annual discount rate. Let $\tau(y) \in \{1,\ldots,T\}$ be the length of program y. Let $c(y)$ denote the cost of attendance. The NPV of attending program y for ability type x is therefore in a general form given by

$$f(x,y) \equiv \sum_{t=\tau(y)+1}^{T} \frac{w(x,y)}{(1+r)^t} - \sum_{t=1}^{\tau(y)} \frac{c(y)}{(1+r)^t}. \tag{1}$$

However, as there are no program specific tuition fees in Denmark, but rather a general subsidy by the government in form of financial aid (i.e. SU), one can w.l.o.g. rewrite the NPV as

$$f(x,y) = \sum_{t=\tau(y)+1}^{T} \frac{w(x,y)}{(1+r)^t} - \sum_{t=1}^{\tau} \frac{c}{(1+r)^t}, \tag{2}$$

whereas $c = -SU$ is the student aid per year and τ is fixed for all programs. So, this implicitly assumes that we talk about a specific kind of programs, e.g. Bachelor programs. This can further be rewritten as

$$f(x,y) = w(x,y) \cdot \frac{(1+r)^{-\tau} - (1+r)^{-T}}{r} - c \cdot \frac{1-(1+r)^{-\tau}}{r} \tag{3}$$

In a similar fashion, the NPV of working immediately can then be expressed as

$$W_0(x) \equiv w_0(x) \sum_{t=1}^{T} \frac{1}{(1+r)^T}, \tag{4}$$

which, given it is a finite geometric series, can be rewritten as

$$W_0(x) = w_0(x) \frac{1-(1+r)^{-T}}{r} \tag{5}$$

Consequently, a forward-looking student chooses college over work iff $f(x,y) \geq W_0(x)$. This can be expressed substituting in the NPVs:

$$w_0(x) \frac{1-(1+r)^{-T}}{r} \leq w(x,y) \cdot \frac{(1+r)^{-\tau} - (1+r)^{-T}}{r} - c \cdot \frac{1-(1+r)^{-\tau}}{r} \tag{6}$$

Time Unit

As expressed above, the model is cast in discrete time with finite horizon T. One period is one year, which reflects the fact that student admissions are typically done once a year. The certain exemptions are disregarded for simplicity, as it should not affect the results.[4]

Choice Set

At the application stage a type-x student chooses between (i) working immediately or (ii) submitting applications to every program y that satisfies $f(x,y) \geq W_0(x)$. Each program y then ranks applicants by ability and admits the highest x's until its capacity $\kappa(y)$ is filled. Given the Danish clearing mechanism in combination with the lack of any frictions, a student will always end up at the highest possible program given the other students' types and accept that offer. This model does through the definition of the application set $A(x)$ not allow for a decline of an offer.

State Space

At the time of application, a student's individual state is $(x, W_0(x))$, while a program's state is $(y, \kappa(y))$.

[4] In the very specific case of Denmark, where students take several gap years, one could model this as a multi-period process, whereas a graduate has to has a couple of application periods at hand. Similarly one could also think about introducing the option for workers to switch into studying at a later point in time, which would be more in line with Humlum et al. (2017).

Objective Function

Given the decision, the student will maximize her NPV with the application decision captured in the set $A(x)$.

$$\max_{A(x)} NPV = \max_{A(x)}\{W_0(x), f(x,y)\}. \tag{7}$$

As the Universities are rather simple in this model, it is not necessary to define a specific objective function for them, as they will always admit the top $\kappa(y)$ applicants.

Observability

Both, students and the econometrician, observe ability $L(x)$, all program qualities $C(y)$, the capacities $\kappa(y)$, and the NPVs $f(x,y)$.

Observed Heterogeneity

Ability x is defined as GPA and, hence, oberserved. Program quality y is defined as the past combination of average starting wage and wage growth, thus, also observable.[5]

Unobserved Heterogeneity

The frictionless baseline abstracts from unobserved heterogeneity. Potential extensions may introduce idiosyncratic preference shocks or revealed hetereogenous preferences of previous choices making this a multi-heterogenous model.

Functional Form

Let the wage be $w(x,y) = \beta xy$ with $\beta > 0$.[6] Let the outside wage be $w_0(x) = (0.125)\alpha x$. with $\alpha > 0$. This allows to pin down the minimal acceptable program quality for a student with ability x as

$$\bar{y}(x) = \frac{w_0(x)\left(1-(1+r)^{-T}\right) + c\left(1-(1+r)^{-\tau}\right)}{\beta x\left((1+r)^{-\tau}-(1+r)^{-T}\right)} \tag{8}$$

Distributional Assumptions

Abilities are distributed $x \sim L(\cdot)$ with density $l(x)$. Program qualities are distributed $y \sim C(\cdot)$ with density $c(y)$.

[5]This obviously assumes that past wage growth implies future wage growth, which is not a stylized fact and expectations might differ. However, it is a good starting point for the analysis. One could think about drawing an expectation shock for individual students.

[6]This naturally creates positive assortative matching (PAM) by design, as it is a supermodular function. Currently, the exact form is lacking some empirical support, however r=0.41 from y and x in the data is visible.

2.2 Simple Comparative Statics – Baseline

This simple setup allows for simple comparative statics, which reflect the basic model features. Taking into account that $r, \alpha, \beta > 0$, and assuming that $0 < \tau < T$, the sign of the partial derivatives is as follows:

$$\frac{\partial \bar{y}}{\partial \beta} = -\frac{w_0(x)\left(1-(1+r)^{-T}\right) + c\left(1-(1+r)^{-\tau}\right)}{\beta^2 x\left((1+r)^{-\tau} - (1+r)^{-T}\right)} < 0 \tag{9}$$

$$\frac{\partial \bar{y}}{\partial c} = \frac{1-(1+r)^{-\tau}}{\beta x\left((1+r)^{-\tau} - (1+r)^{-T}\right)} > 0 \tag{10}$$

$$\frac{\partial \bar{y}}{\partial SU} = -\frac{1-(1+r)^{-\tau}}{\beta x\left((1+r)^{-\tau} - (1+r)^{-T}\right)} < 0 \tag{11}$$

First, a higher productivity parameter β reduces the minimal acceptable quality, as students can achieve better outcomes even at lower-quality programs. The student program is gaining relative advantage compared to the outside option. Consequently, a change in α exhibits the inverse effect. Second, increased costs of attendance make students more selective, as the gained benefit of programs decreases. Hence, greater student aid lowers the threshold for students to apply as it lowers the opportunity cost of attending University.

2.3 Directed-Search Model Extension

However, as this setup would not be able to explain the observed application behavior and lead to perfect sorting, it is necessary to introduce a search process. In this regard, an entry probability is introduced. It is assumed that students know about the programs, associated costs, and decided to search within a specific subset of programs based on the application behavior of the other applicants.

In order to extend the frictionless case, one can rearrange the objective function (equation 7) to include the outside option as college 0 (CO), which basically simplifies the objective function to

$$\max_{A(x)}\{f(x,y)\}. \tag{12}$$

Consequently, this C0 allows every student to join and has unlimited capacity ($\kappa = \infty$). At the application stage a type-x student then chooses a portfolio with $m(x)$ applications s.t.

$$A(x) \subseteq [0,1] \cup \{C0\} \qquad \text{and} \qquad m(x) \equiv |A(x) \setminus \{0\}| \leq a, \tag{13}$$

in order to still allow the maximum number of applications to universities (m(x)).

Let there now be an expected surplus $\tilde{s}(x,y)$

$$\tilde{s}(x,y) \equiv \pi(x,y)\, f(x,y), \tag{14}$$

where $\pi(x,y)$ is the probability of admission to program y for a student with ability x. Let there also be costs of applying $k(j)$ for each application $j \in A(x)$. Then the objective function can be rewritten as:

$$\max_{A(x)} \Big[\sum_{y\in A(x)} \tilde{s}(x,y)\Big] - \sum_{j=1}^{m(x)} k(j) \qquad \text{s.t.} \qquad m(x) \le a \tag{15}$$

Ordering the chosen programs by decreasing $\tilde{s}$[7], means for a student an application to program y_j is optimal as long as

$$\Delta_j(x) = \tilde{s}(x, y_j) - k(j) \ge 0, \quad \text{with} \quad \Delta_0(x) = \tilde{s}(x, 0) = W_0(x), \tag{16}$$

which is basically the same as saying that the marginal benefits of the application must be higher than the marginal costs for every submitted application. The introducton of the probability of admission $\pi(x, y)$ combined with the costs $k(j)$ is needed to reflect the observed application behavior of a few high likelihood programs, as otherwise every student would apply to all programs.

Given the public debate within Denmark of NC's being the main driver of the application behavior, it fair to assume that the probability of admission $\pi(x, y)$ can be defined as,

$$\pi(x,y) = \omega_y \Phi\left(\frac{x-\mu_y}{\sigma_y}\right) + (1-\omega_y)\pi_{\text{cong}}(x,y) \tag{17}$$

whereas the first term is the historical component and the second term is the congestion component, and $\omega_y \in [0, 1]$ determines the relative weight between the two and the congestion term[8]being

$$\pi_{\text{cong}}(x_i, y_j) = \begin{cases} 1.0 & \text{if } x_i > c_j \\ \frac{1}{n_{\text{NC}}} & \text{if } x_i = c_j \\ 0.0 & \text{if } x_i < c_j \end{cases} \tag{18}$$

The congestion component captures the competitive nature of admissions where programs rank applicants by ability and admit the highest-ranked students until capacity is filled. Even with full information, the cutoff creates uncertainty about the probability of admission, leading to a congestion component.[9] It is further important that

$$\pi(x, C0) = 1 \quad \text{and} \quad k(j) = 0 \tag{19}$$

holds in order to ensure that the NPV is always well-defined even if every selective program rejects the student.

However, this unconditional probabilities neglect the fact that the probability of admission is conditional on being accepted in the first choice. It is more appropriate

[7]Tiebreaker is random.

[8]Initially, the idea was to set the congestion term $\pi_{\text{cong}}(x, y) = \min\left\{1, \frac{\kappa(y)}{\int_x^1 I\{y\in A(z)\}\cdot l(z)\cdot dz}\right\}$. This implementation, did not yield a converging result.

[9]One could extend this to a $c_j + -\delta$, where δ is the uncertainty area around the cutoff. This would allow for a more flexible specification of the congestion component.

to consider the probability of admission to program y_j, where $y_i, y_j \in$ the before mentioned ordered list of potential applications ($\tilde{s}(x, y_{(1)}) \geq \tilde{s}(x, y_{(2)}) \geq \ldots \geq \tilde{s}(x, y_{(m)})$ and $i, j \in [1, m]$ with $j > i$, as follows

$$P(\text{attend } y_j) = \left[\prod_{i=1}^{j-1} (1 - \pi(x, y_i))\right] \cdot \pi(x, y_j). \tag{20}$$

Consequently, the objective function has to be rewritten as

$$\max_{A(x)} \left[\sum_{j=1}^{m(x)} \left(\prod_{i=1}^{j-1} (1 - \pi(x, y_i)) \cdot \pi(x, y_j) \cdot \tilde{s}(x, y_j)\right) - \sum_{j=1}^{m(x)} k(j)\right] \tag{21}$$

s.t. $m(x) \leq a$. This leads to the final decision rule for a student, which incorporates the same economic logic as before, but now also incorporates the conditional probability of admission to each program

$$\Delta_{(j)}(x) = \Big[\prod_{i=1}^{j-1} \big(1 - \pi(x, y_{(i)})\big)\Big]\, \tilde{s}\big(x, y_{(j)}\big) \;-\; k(j) \;\geq\; 0. \tag{22}$$

By design, $\tilde{s}(x, y_{(j)})$ is weakly decreasing in j, and $\pi(x, y_{(j)})$ is weakly increasing in j. Hence, the above decision rule is weakly decreasing in j, as flat (or increasing) k can never overturn the prior terms.

2.4 Simple Comparative Statics – Directed Search

By design of the model is the optimal application number $j^*(x)$ the largest j s.t. $\Delta_{(j)}(x) \geq 0$, which is the same as minimizing the number of applications j s.t. $\Delta_{(j)}(x) < 0$.

$$j^*(x) = \min\{j : \Delta_{(j)}(x) < 0\} - 1. \tag{23}$$

The corrected by minus one is needed in order to exclude C0, which will always be in such a minimization. Given the above mentioned weakly decreasing function of $\Delta_{(j)}(x)$, we know that a minimum is well-defined.

This setup features the following comparative statics,

$$\frac{\partial j^*(x)}{\partial f(x, y)} \geq 0 \Rightarrow \frac{\partial j^*(x)}{\partial \alpha} \leq 0 \quad \wedge \quad \frac{\partial j^*(x)}{\partial \beta} \geq 0, \tag{24}$$

$$\frac{\partial j^*(x)}{\partial SU} \geq 0, \tag{25}$$

$$\frac{\partial j^*(x)}{\partial \kappa(y)} \leq 0, \tag{26}$$

which shows that the number of applications is indeed endogenous. An increase in the expected surplus of a program might (in a partial equilibrium) lead to more applications, as the marginal benefit of an additional application is higher. On the other hand, an increase in the capacity of a program might lead to less applications, as the marginal benefit of an additional application is lower.

However, there are two important caveats to understand, which are at the same time the core model features. First, there is a damping factor in form of the congestion component in the probability of admission (for $\omega \in [0,1)$), which makes it overall a general equilbrium(-like) model. If $\kappa(y)$ converges to infinity, this factor loses weight which leads to partial and general equilibrium being the same. Yet, e.g. an increase in the expected surplus of all programs (which is basically an increase in SU) might lead to more applications in a partial equilibrium, but not as many in a general equilibrium, as the marginal benefit of an additional application is higher in the first round but factoring in the probability of admission in the second round. Second, there is setups in which the effect is always zero, e.g. when a is reached.

3 Calibration & Validation

The calibration uses Danish register data (mainly KOT, UDGK)[10] and literature, with 2018 as the reference year. Parameters are categorized as exogenous, data-driven semi-exogenous, and data-based exogenous. Table 1 summarizes the parameters and their reasoning. Data-based exogenous parameters are theoretically set from data, though this is not yet well implemented.

Publicly available data for 2018 shows: 77,300 students completed gymnasial or vocational upper-secondary programs; 64,943 were admitted; 67,000 study seats existed; and 9,765 were rejected due to lack of spots (Danmarks Statistik, 2019; Uddannelses- og Forskningsministeriet, 2018). 81% of students got their first priority (Uddannelses- og Forskningsministeriet, 2018), while 58.8% did across cohorts (based on KOT). The average number of applications is 2.46, decreasing with GPA.[11] This data serves as the baseline and also as target moment when being calibrated.[12]

The student type distribution ($l(x)$) comes from the unweighted GPA distribution of 2018 graduates (UFGK; $f\colon [2,12] \to [0,1]$), exported in 0.01 bins (see Appendix A.2). Random draws within bins match exact student numbers. This approach, however, ignores that entry GPA depends on graduation year proximity and high school type (e.g. HHX).

The university type distribution ($c(y)$) uses expected average starting wage and wage growth, exported in 0.05 bins (see Appendix A.1). Random draws within bins are then used to match the exact program numbers. Due to UDD code and KOTNR aggregation mismatches[13], the distribution is then scaled to total program numbers. As more observed programs than actual ones exist, this suggests data loss in aggregation, visible when matching program numbers to application data. It may also be that old programs are included. Since reverse-matching does not (yet)

[10]For this course, I assume the data is known and refrain from deeper introduction.

[11]For first-time applicants, this is 1.94 (Hvidberg, 2023).

[12]So far, $\omega = 0.5$ and $k = 0.02$ yielded the best results. A further closer calibration is for the sake of writing this paper postponed.

[13]The problem and potential matching solutions for the KOTNR UDD code are also discussed in Hvidberg (2023); Smeets et al. (2025). Yet, on the precise program level, this is not yet accurately possible. So, this should be more understood as a proxy in form of the narrowest subject type definition.

work, this pre-matching-distribution, which was also mapped into $[0, 1]$, is asssumed to represent the true distribution nonetheless.

Table 1: Parameter calibration

Parameters	Values	Reasoning
Exogenous		
α	1.00	Mean earnings of non-college cohort.
β	1.20	College wage premium by GPA and program quality.
c	-0.15	There is no tuition fee, and SU amounts to approx. 15 % of the average gross-wage.
r	0.03	Literature values roughly $\in [0.01, 0.05]$.
T	54.00	Folkepension estimate for a 20 year old.
τ	3.00	Mean study length Bachelor program.
a	8.00	Maximum applications allowed by Danish law.
Data-driven Semi-exogenous (used to match the moments)		
ω	0.50	
k	0.02	
$k(j)$	0.00	Not yet used. There could be theoretical justification for setting this negative, which might be a good extension.
Data-based Exogenous		
$\kappa(y)$		*Source:* KOT – **Not identified (yet)**; hence, based on a random draw matching the overall average capacity.
$l(x)$		*Source:* UGDK – This is based in 0.01 bins of an unweighted grade average of the 2018 graduates, and then randomly drawn within a bin.
$c(y)$		*Source:* IDAN, IDAP, IND, UDDA, BEF, based on Almar et al. (2025) mapped into $\mathbb{R}^2$ – This is then taken in 0.05 bins, whereas the > 0.85 collapsed.
μ_y		*Source:* KOT – **Approximated** by correlation based on observed y; see note below.
σ_y	0.05	*Source:* KOT – **Not identified (yet)**; assumed to be 0.05 flat.

Currently, $k(y)$, μ_y, and σ_y cannot be assessed. Thus, $k(y)$ is a random draw from study places over program numbers, assuming no quality-spots relationship. If false, results are biased. The past threshold is a normal distribution draw with mean $\mu_y = y$ and standard deviation $\sigma_y = 0.05$. In practice, factors like cost of living may affect application demand, but this is not implemented.

Following Badescu et al. (2011), $\alpha = 1$ and $\beta = 1.2$, as EU-SILC data shows a 20% wage premium for tertiary degrees in Denmark.[14] SU accounts for 15% of average gross-wage, so $c = -0.15$.

With an average starting age when studying of around 20 years, $T = 54$ years until the expected pension age of 74 exists. This, however, ignores potential time inconsistencies as the discount factor $r = 0.03$ may be much more heterogeneous in practice (see Harrison et al., 2002 for Denmark).

4 Results & Counterfactuals

First, the difference between the frictionless and friction-induced search model will be described. Second, an increase in SU is simulated, before, third, an increase in student costs is.

4.1 General Results

Figure 1 displays the equilibrium matching set in the frictionless baseline model. The relationship between student ability and program quality is strictly increasing and exhibits positive assortative matching by design, resulting from the supermodular wage function. Since no frictions exist, all students apply to all sufficiently valuable programs in the Becker-like way, and matching is efficient in a utilitarian sense. The area itself displays where the students apply and which universities would accept them. However, the actual matching set after admissions is along the upper curve of the area.

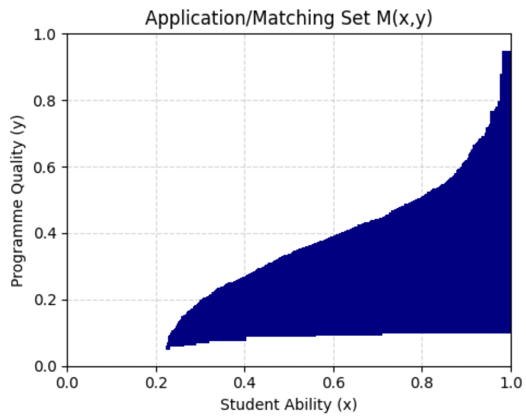

FIGURE 1: Matching set in the frictionless case

[14]Initially, w0(x) was 0.25 αx, implying this premium for the average type. However, this made university unattractive for many students due to a too-good outside option. Adjusting to 0.125 better fits the data, suggesting the estimated wage premium may be too high.

By contrast, figure 2 visualizes the observed application set in the directed-search model. This heatmap reveals clear deviations from frictionless optimal behavior: students with low to medium ability concentrate their applications around a narrow band of lower-quality programs, leading to congestion. While high-ability students still sort into top programs, they apply to fewer options than their low-quality peers. This is reflecting endogenous search behavior and the application constraints.

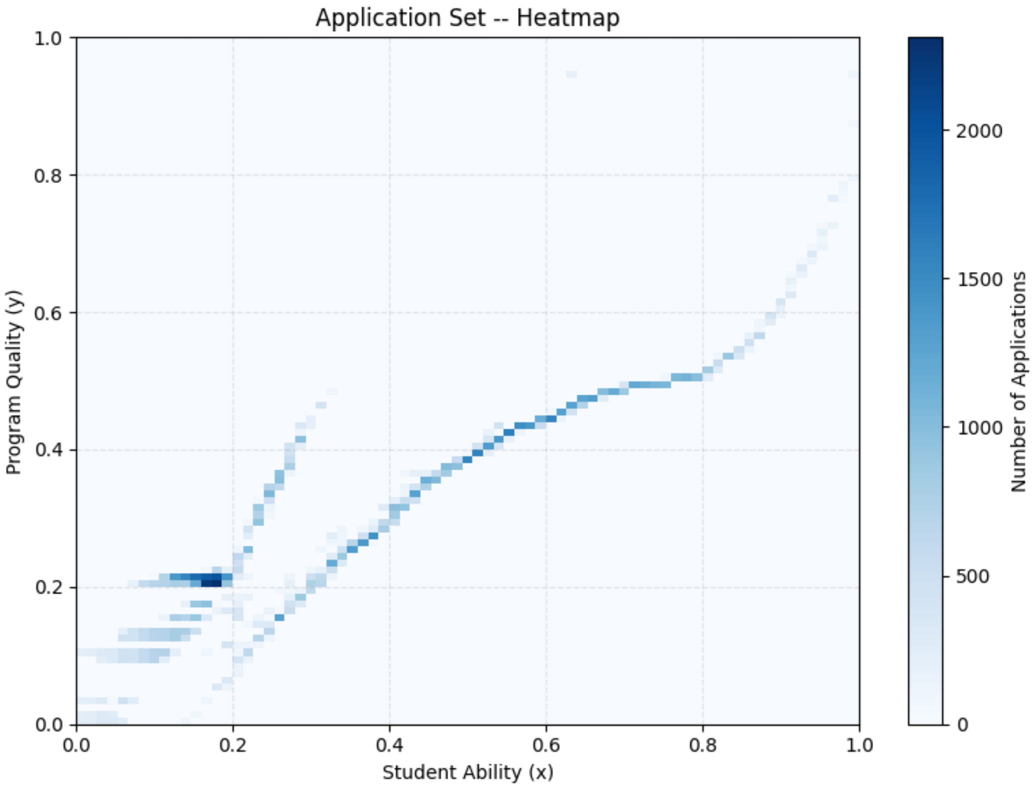

FIGURE 2: Application Set; Frictions

This also yields a different model outcomes. At the aggregate level, search frictions have sizeable effects on college participation and capacity utilization: While

TABLE 2: Aggregate Outcomes

Outcome	Frictionless	Directed Search
High School Graduates	77,296	77,296
Applications Submitted	-	148,480
Applications per Student		1.92
Students Admitted	66,373	51,515
Capacity Utilization	100%	77.6%
College Attendance Rate	85.9%	66.6%

the frictionless model fills all seats, the directed search model leads to substantial under-utilization of capacity and a drop of nearly 20 percentage points in college attendance.[15] This is by expectation about admission probabilities and the congestion. Especially, close to cutoff regions in tight subsets of the application set, the admission probabilities are low, which deter some students from applying at all. We can by the difference see that there is an efficiency loss induced by even modest search frictions.

Comparing, the data to the model simulation, figure 3, it one can see the distribution of applications per student (mean: 2.46) with simulated application numbers under the frictional model are still slightly higher than the model simulation (1.92). Importantly, the model captures the heterogeneity in the ability distribution. Similar to the actual data, low-ability students ($x < 0.4$) submit more applications on average (mean: 3.20). High-ability students ($x > 0.8$) apply to fewer programs (mean: 1.03), reflecting confidence in admission to top programs and diminishing returns from additional applications. This is also reflected in the data. However, the model is yet not perfectly calibrated, as especially the application numbers for the top students are under-predicted. Potentially a random preference or uncertainty shock could bring in some more congestion at the upper end of the ability distribution. This findings could also be further transfered to what one see's in (Hoxby and Avery, 2013), when one assumes different risk preferences induced by the financial situation.[16]

[15]Revealing that this model is yet not perfectly calibrated as it should be in the 90s.

[16]So far, all are basically assumed being risk-neutral.

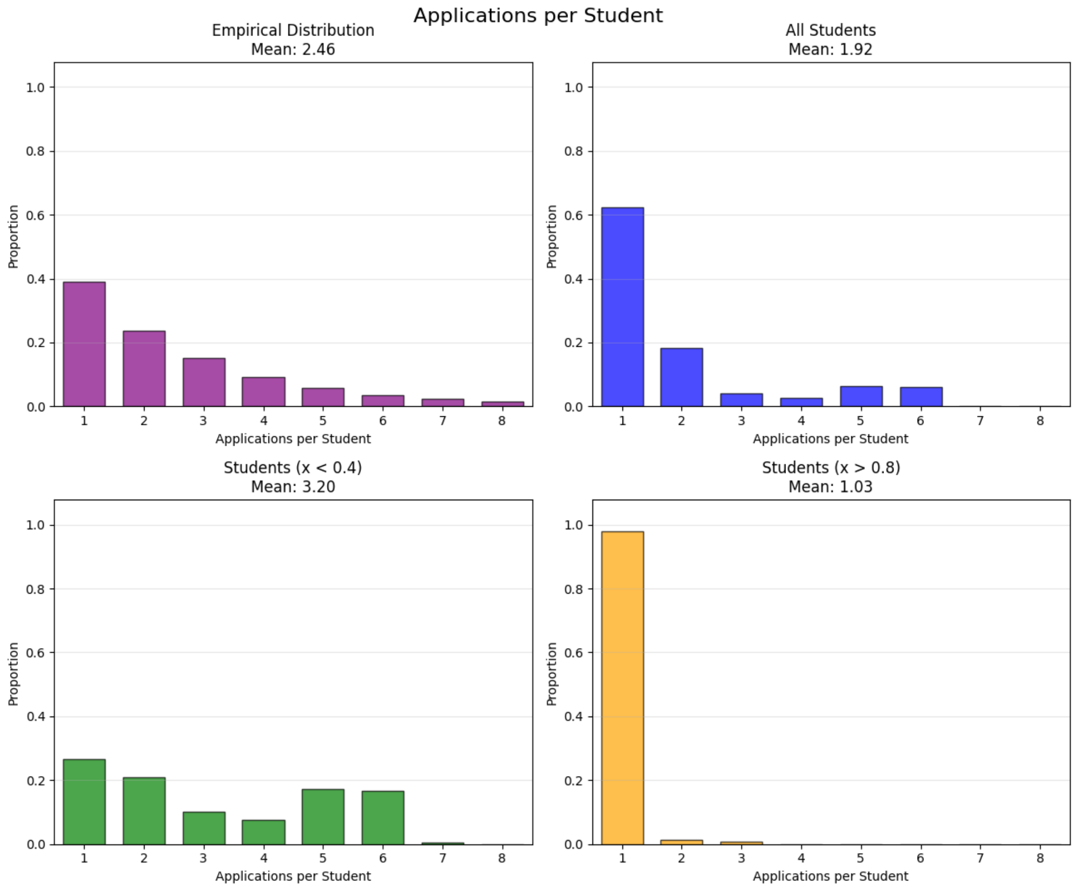

FIGURE 3: Application Numbers; Data vs. Model

Overall, one can see in figure 4 that the introduction of frictions leads to a lower average admitted student for some programs. A standard micro model would potentially predict this being a fixed effect, based on some standort parameters. This might of course be the case, but the model here suggests that it is also driven by strategic behavior, which might be under-estimated in a lot of other models. In the following policy simulations one will also see, that this factors are indeed influenced by external factors. It is also imporant to understand, that while it looks as if the upper end of the ability distribution is not affected, students submitting more applications e.g. due to being at the cutoff lead to negative trickle-down effects for the lower end of the ability distribution as it shifts some "application mass" to the left.

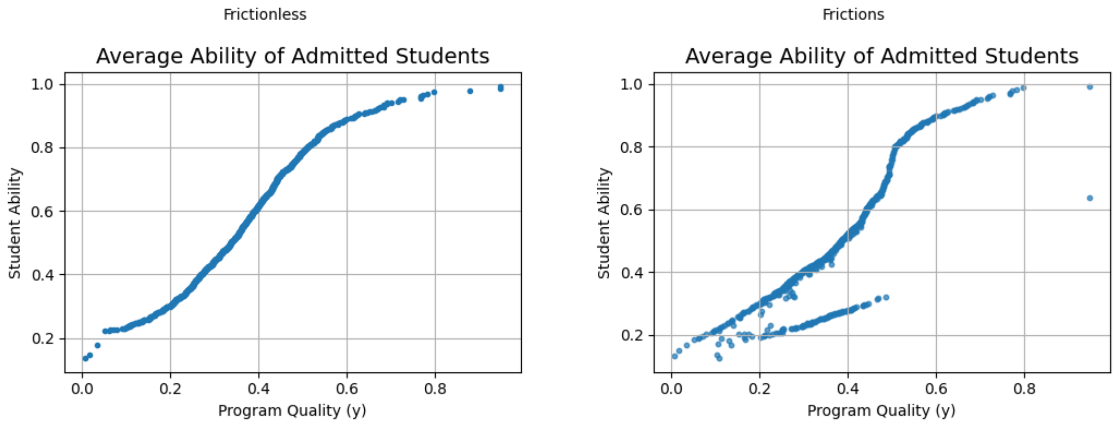

FIGURE 4: Average Admitted Student; Frictionless vs. Friction-based

4.2 Policy Simulation: Increase in SU // Outside Option Shock

To examine the impact of more generous student aid, the costs are lowered from -0.15 to -0.20. Intuitively, this lowers the opportunity cost of studying and increases the net present value (NPV) of university education across the board. This can mechanism wise be seen similar than a decrease in outside option wage.

As the frictionless model already yields 100 %, the admissions will not be affected, while at the same time the matching set will be extended downwards as the minimum acceptable program quality increases.

In the directed-search setup is the change more nuanced. Figure 5 compares the average admitted student across programs under the baseline and increased SU scenarios. The overall structure of positive sorting remains, but with a slight flattening of the slope in lower-quality programs, indicating that more medium-ability students are now admitted to previously unattractive options.

This is also driven by the application behavior, which adjusts accordingly. Figure 6 shows a marginal increase in the average number of applications per student from 1.92 to 1.98, indicating that students broaden their application sets in response to a higher marginal benefit of studying and, thus, applying. This is particularly visible in the left tail of the ability distribution ($x < 0.4$), where the average number of applications increases to 3.40. This behavior is intuitive since students in this tail face greater uncertainty about their admission prospects. The higher search intensity increases their expected NPV. However, the increased search intensity also affects the matching set by raising admission probabilities for marginal programs, as students who might have otherwise not applied now choose to do so. At the aggregate, the increase in SU raises college attendance by approx. 3 percentage points.

Table 3: Policy Outcomes: Baseline vs. Increased SU

Outcome	Baseline ($\gamma = -0.15$)	Increased SU ($\gamma = -0.2$)
Applications per Graduate	1.92	1.98
Students Admitted	51,515	53,789
College Attendance Rate	66.6%	69.6%

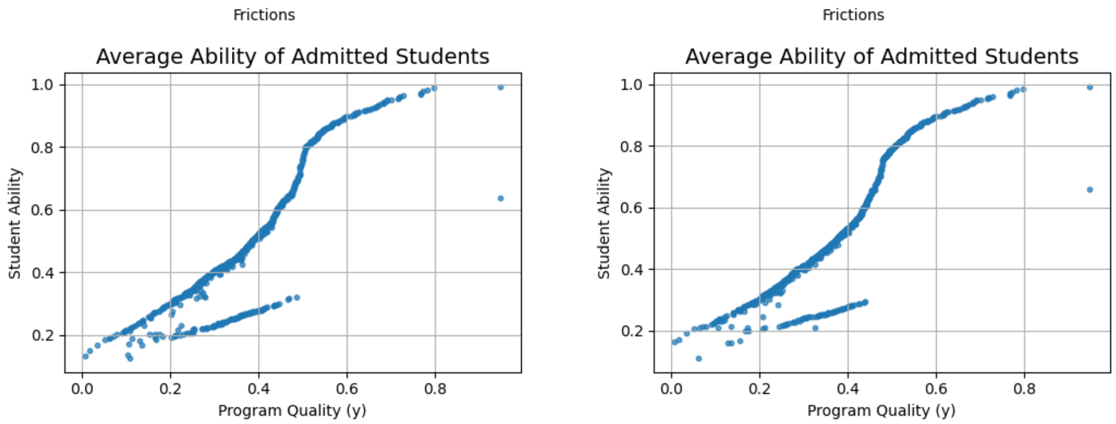

Figure 5: Average Admitted Student; Base vs. SU-Increased

4.3 Policy Simulation: Increase in Application Costs

Similarly, the application costs are increased from $k = 0.02$ to $k = 0.04$. As expected, this increases the marginal cost of expanding one's application portfolio and thus leads to more selective application behavior for some students.

Figure 8 confirms this effects. The mean number of applications per student drops from 1.92 (baseline) to 1.67, with the decline concentrated among low- and mid-ability students. The students with $x < 0.4$ now submit on average 2.40 applications (down from 3.20), while high-ability students ($x > 0.8$) are largely unaffected (mean: 1.02). This reflects a key intuition: when frictions rise, marginal applicants are the first to exit or reduce their participation.

However, this shift is also evident in the matching outcomes. As shown in figure 7, the average ability of admitted students remains broadly similar, but admission patterns in lower- and mid-quality programs become less tight.

Table 4: Policy Outcomes: Baseline vs. Increased Costs

Outcome	Baseline ($k = 0.02$)	Higher Cost ($k = 0.04$)
Applications per Graduate	1.92	1.67
Students Admitted	51,515	53,881
College Attendance Rate	66.6%	69.7%

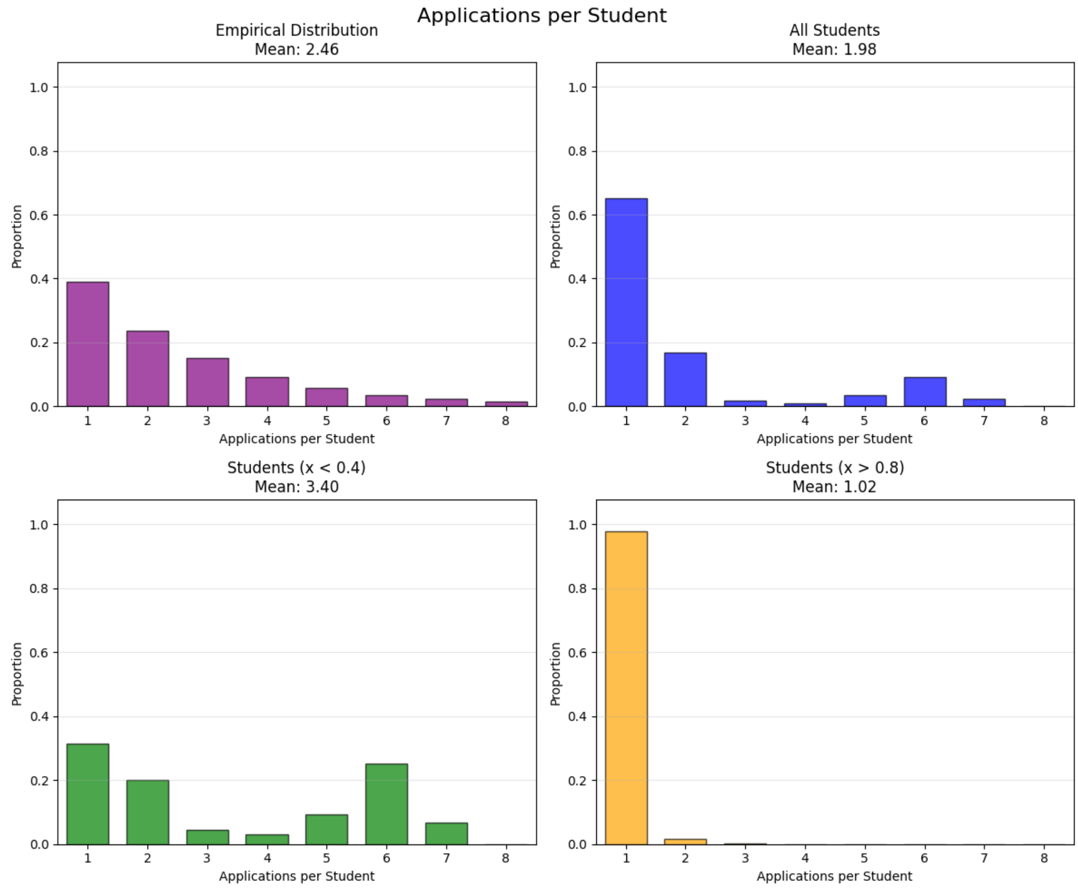

FIGURE 6: Application Numbers; After SU-Increase

However, interestingly, attendance rises on aggregate slightly due to reduced congestion and more decisive application choices, but the overall application intensity falls. This highlights that the trade-off between increases in benefit or costs and the market tightness. Once could now extend this trade-off for selectively targeting specific students. E.g. one could assume that student aid is only leading to higher benefits for the financly constraint students. Thus it might positively affect their marginal benefit of applying in the first round, yet it might in the second round lead to decreased admission probabilities.

Overall, these results suggest that even small frictions be time-based, or cognitive can shape application portfolios and matching outcomes. It has also been shown, that at first negative looking effects, like an increase in application costs, can have positive effects on aggregate outcomes, when they reduce the overall market tightness, leading to more efficient application behavior. Yet, the same might hold for the expected costs or benefits, which might be shaped by the public debate.

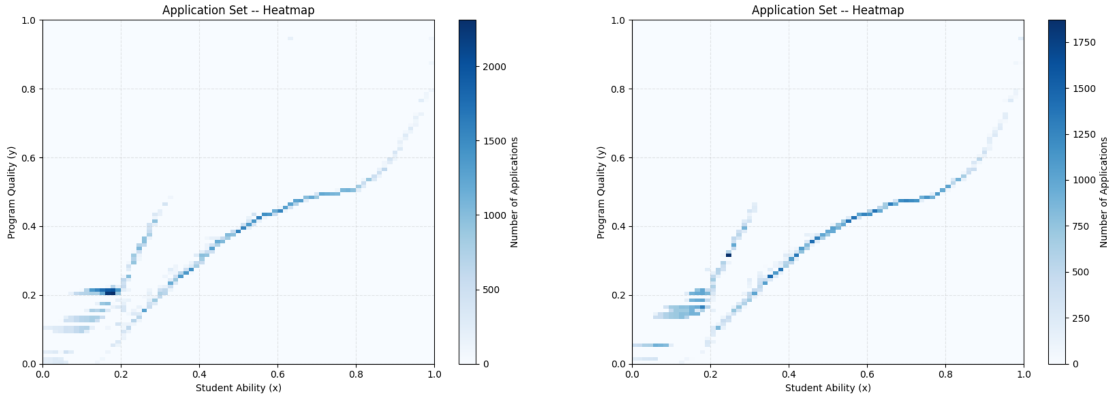

FIGURE 7: Average Admitted Student; After Costs-Increase

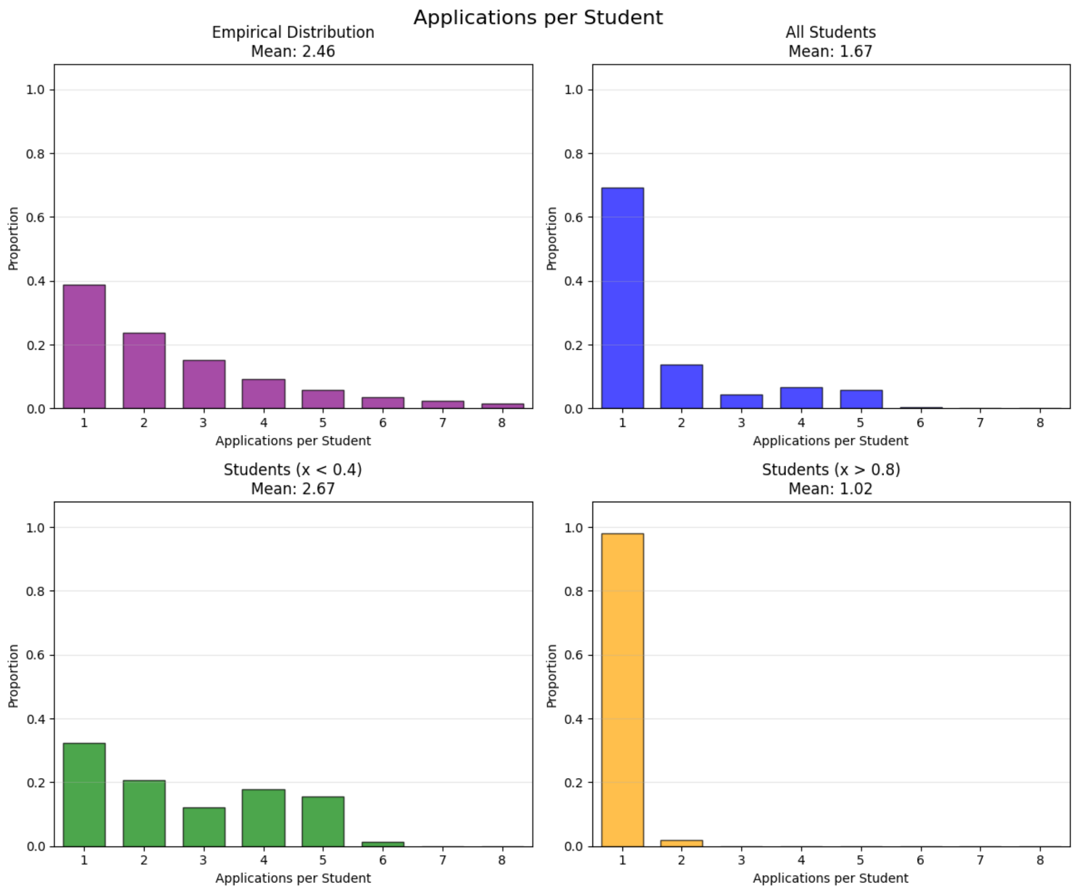

FIGURE 8: Application Numbers; After Costs-Increase

5 Discussion and Conclusion

“Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful.” (Box and Draper, 1987, p. 424) While this model is currently far from being “useful” in a predictive sense and does not reflect real-world behavior in full complexity, it still offers a structured lens to think through application dynamics under frictions. The results illustrate how even modest constraints on application behavior in form of congestion and probabilities can lead to inefficient matches, under-utilization of capacity, and a drop in attendance that matter for both individual welfare and aggregate efficiency. It is also needed to explain the observed patterns in the data.

That said, the model comes with notable limitations. It abstracts from frictions on the labor market, assumes a stylized universal wage function, and treats universities as simple maximizers of ability without strategic behavior or multi-dimensional admission criteria. It also way too often assumes a parametric form, which is yet to be shown empirically. Student preferences are homogeneous and forward-looking, ignoring non-monetary factors, local constraints and potential further heterogeneity. This could be included with a preference type shock, like its e.g. done in Smeets et al. (2025). Peer effects, dropout risks, the option of declining offers and evolving student abilities are also omitted. Finally, it assumes perfect information and does not model how information constraints might differ by background, which is a key factor in access inequality. However, this could in theory be modeled.

These simplifications restrict the model’s realism, but they also isolates the mechanisms. It is also a reasonable assumption that this case might allow to abstract from information and search frictions, because the first might neglectable in Denmark. Notably, the endogenous application set challenges assumptions in some reduced-form analyses. F.x. papers estimating the effect of student aid without accounting for how such aid may shift portfolio decisions and thereby affect others might then be in danger of violating SUTVA.

Overall, the model is a first step toward understanding how student-college matching in a setup with a central clearninghouse works. While abstract, it can help structure future empirical work and policy thinking around improving match quality and educational access. The next step is to bring the model onto the yearly-micro-data level and set up a proper estimation.

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A Mapping Students and Program Types

A.1 Mapping Programs Types

Let x_i denote the average wage and y_i the wage growth for program i, with $i = 1, \dots, n$ calculated as in Almar et al. (2025). Define the min-max normalized variables:

$$\tilde{x}_i = \frac{x_i - x^{\min}}{x^{\max} - x^{\min}}, \quad \tilde{y}_i = \frac{y_i - y^{\min}}{y^{\max} - y^{\min}}$$

where $x^{\min} = \min_j x_j$, $x^{\max} = \max_j x_j$, and analogously for y.

I then define a linear mapping function $f : \mathbb{R}^2 \to [0,1]$, which combines the normalized variables with weight $\alpha \in [0,1]$,

$$s_i = f(x_i, y_i) = \alpha \cdot \tilde{x}_i + (1 - \alpha) \cdot \tilde{y}_i,$$

where i for simplicity now set $\alpha = 0.5$. To ensure the final score lies in $[0,1]$, I again apply a second min-max normalization to s_i:

$$z_i = \frac{s_i - s^{\min}}{s^{\max} - s^{\min}}, \quad \text{where } s^{\min} = \min_j s_j, \; s^{\max} = \max_j s_j$$

The resulting value $z_i \in [0,1]$ provides a composite indicator that reflects both wage level and growth and is in this term-paper used as proxy for the quality of a program.

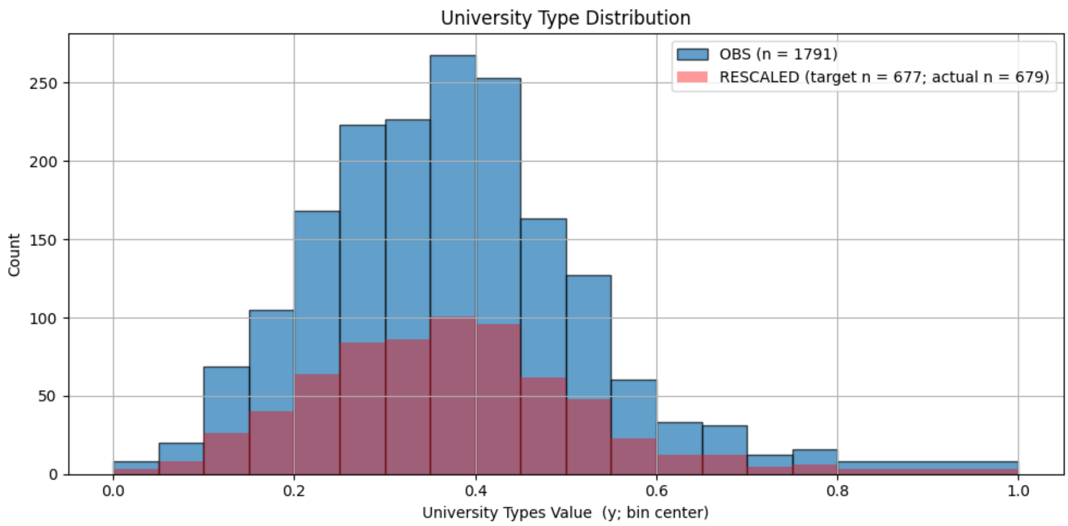

Figure 9: University Type Distribution

NOTE: This is based on all observed program combinations over all years. The mapping back to KOTNR was not fully possible due to aggregation mismatches of KOTNR; it is thus rescaled to match the actual program numbers.

A.2 Mapping Students Types

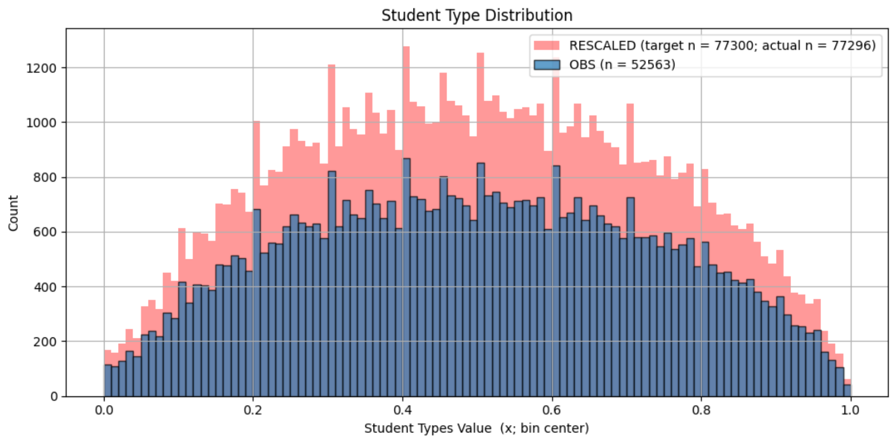

FIGURE 10: Student Type Distribution

NOTE: This is based on the 2018 graduates in UDGK and rescaled to match the full application pool. Hence, it is assumed that there is no selection into being an unobserved-graduate (e.g. foreigners).

B Further Figures

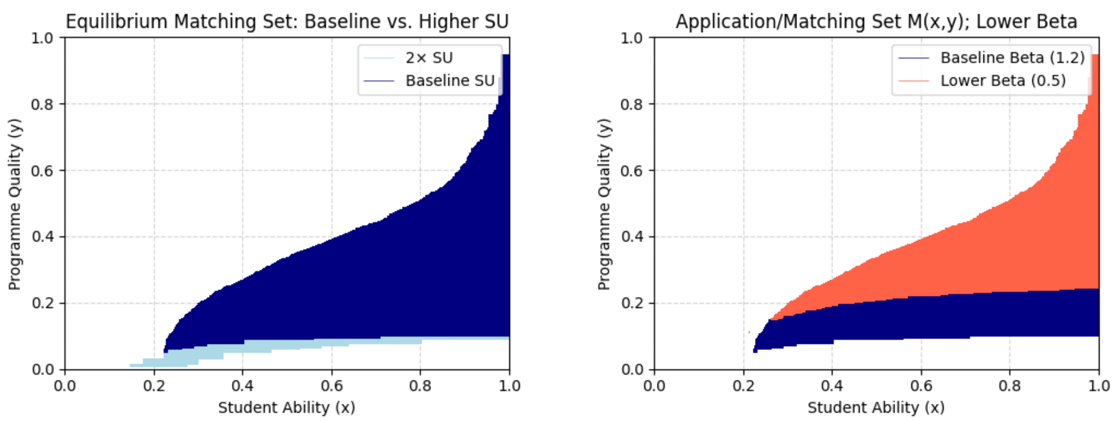

FIGURE 11: Matching set in the frictionless case

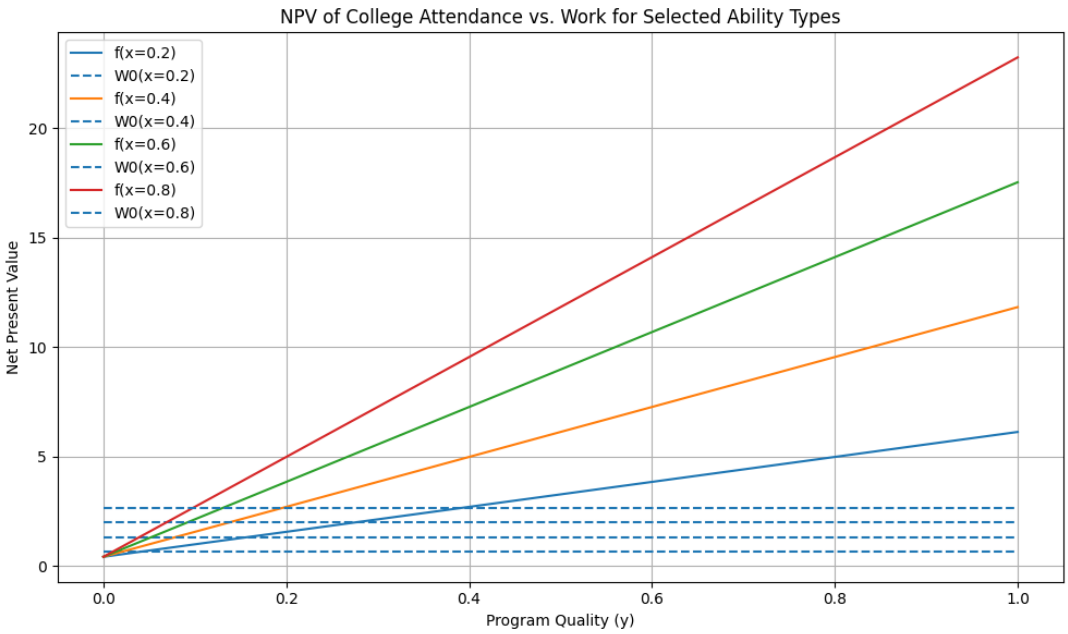

FIGURE 12: NPV Curves by Ability