PROBABILITIY AND RANDOM VARIABLES ASSIGNMENT 4

GANJI VARSHITHA - AI20BTECH11009

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QUESTION
ISS Statistics Paper I Question 12

Two cannons A_1 and A_2 fire at the same target. Cannon A_1 fires on an average 9 projectiles in the time in which cannon A_2 fire 10 projectiles.But on an average 7 out of 10 projectiles from cannon A_1 and 6 out of 10 projectiles from cannon A_2 strike the target. If in the course of shooting, the target is struck by one projectile, then the probability that it is struck by projectile from cannon A_1 is

1) $\frac{20}{41}$

2) $\frac{21}{41}$

3) $\frac{6}{19}$

4) $\frac{63}{190}$

SOLUTION

Let's define a random variable X $\in$ $\{0, 1, 2\}$. We need to calculate the conditional probability

TABLE 1: Random variables

$X = 0$	event that the target is struck by a projectile
$X = 1$	event that cannon A_1 fires a projectile
$X = 2$	event that cannon A_2 fires a projectile

$\Pr(X = 1|X = 0)$.
By Bayes' Theorem, we get

$$\Pr(X = 1|X = 0) = \frac{\Pr(X = 0|X = 1)\Pr(X = 1)}{\sum_{i=1}^{2}\Pr(X = 0|X = i)\Pr(X = i)}$$

(0.0.1)

In (0.0.1),
$\Pr(X = 1|X = 0)$ represents the conditional probability of cannon A_1 striking the target.
Similarly, $\Pr(X = 2|X = 0)$ represents the conditional probability of cannon A_2 striking the target.
Given,

i	1	2
$\Pr(X = i)$	$\frac{9}{19}$	$\frac{10}{19}$

Also,

$$\Pr(X = 0|X = 1) = \frac{7}{10} \quad (0.0.2)$$

$$\Pr(X = 0|X = 2) = \frac{6}{10} \quad (0.0.3)$$

$$(0.0.4)$$

Substituting the values in (0.0.1), we get

$$\Pr(X = 1|X = 0) = \frac{\frac{7}{10}\frac{9}{19}}{\frac{7}{10}\frac{9}{19} + \frac{6}{10}\frac{10}{19}} \quad (0.0.5)$$

$$= \frac{63}{63 + 60} \quad (0.0.6)$$

$$= \frac{21}{41} \quad (0.0.7)$$

Therefore, option 2 is correct.