PROBABILITIY AND RANDOM VARIABLES ASSIGNMENT 2

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Download latex codes from

https://github.com/VARSHITHAGANJI/AI1103_Probability_Assignment/blob/main/Assignment2.tex

QUESTION

Gate EC Problem 9
Step 1. Flip a coin twice.
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.
The probability that the output of the experiment is Y is (upto two decimal places) $\cdots$

SOLUTION

Given, a fair coin is tossed is tossed two times.
Let's define a Markov chain $\{X_n, n = 0, 1, 2, \dots\}$, where $X_n \in S = \{1, 2, 3\}$, such that

TABLE 1: States and their notations

Notation	State
$S = 1$	getting $\{TT\}$
$S = 2$	getting output Y
$S = 3$	getting output N

The state transition matrix for the Markov chain is

$$P = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.25 & 0.25 & 0.5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \tag{0.0.1}$$

Clearly, the state 1 are transient, while 2, 3 are absorbing. The standard form of a state transition matrix is

$$P = \begin{array}{c c} & \begin{array}{cc} A & N \end{array} \\ \begin{array}{c} A \\ N \end{array} & \begin{bmatrix} I & O \\ R & Q \end{bmatrix} \end{array} \tag{0.0.2}$$

where, Converting (0.0.1) to standard form, we get

TABLE 2: Notations and their meanings

Notation	Meaning
A	All absorbing states
N	All non-absorbing states
I	Identity matrix
O	Zero matrix
R, Q	Other submatices

$$P = \begin{array}{c c} & \begin{array}{ccc} 2 & 3 & 1 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 1 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.25 & 0.5 & 0.25 \end{bmatrix} \end{array} \tag{0.0.3}$$

From (0.0.2),

$$R = \begin{bmatrix} 0.25 & 0.5 \end{bmatrix}, Q = \begin{bmatrix} 0.25 \end{bmatrix} \tag{104.5}$$

The limiting matrix for absorbing Markov chain is

$$\bar{P} = \begin{bmatrix} I & O \\ FR & O \end{bmatrix} \tag{0.0.4}$$

where,

$$F = (I - Q)^{-1} \tag{0.0.5}$$

is called the fundamental matrix of P.

On solving, we get

$$\bar{P} = \begin{array}{c c} & \begin{array}{ccc} 2 & 3 & 1 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 1 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.33 & 0.17 & 0 \end{bmatrix} \end{array} \tag{0.0.6}$$

A element $\bar{p}_{ij}$ of $\bar{P}$ denotes the absorption probability in state j, starting from state i.
Then, the absorption probability in state 2 (i.e getting output Y) starting from state 1 is $\bar{p}_{12}$.

$$\therefore \bar{p}_{12} = 0.33 \, \text{(correct upto 2 decimal places)} \tag{0.0.7}$$

Markov chain diagram