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PROBABILITY AND RANDOM VARIABLES Assignment 2

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Download latex-tikz codes from

https://github.com/VARSHITHAGANJI/
AI1103_Probability_Assignment/blob/main/
Assignment2.tex

PROBLEM

Gate EC Problem 9

Step 1. Flip a coin twice.

Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.

Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is Y is (upto two decimal places) \cdots

Solution

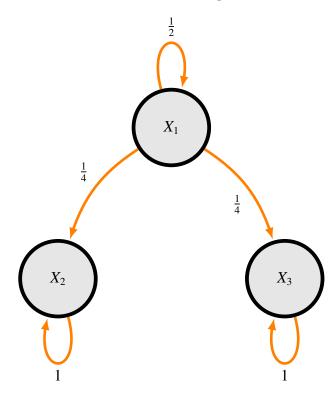
Let flipping a coin twice be event H. Sample space of event $H = \{HH, HT, TH, TT\}$ Let a random variable X; $X_1 = 1, X_2 = 2, X_3 = 3$ where X_1 represents outcome $\{TT\}$, X_2 represents getting outcome $\{TH\}$ or output Y, X_3 represents getting output N.

The state transition matrix P is shown below: X_1 X_2 X_3

$$\begin{array}{cccc}
X_1 & X_2 & X_3 \\
X_2 & \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{array}$$

From the transition matrix, we have 1 transient state and 2 absorbing states.

Markov chain diagram



Q =
$$\begin{bmatrix} \frac{1}{4} \end{bmatrix}$$
 and R = $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \end{bmatrix}$

$$N = (I - Q)^{-1} \tag{0.0.1}$$

$$= \left(\begin{bmatrix} 1 \end{bmatrix} - \left[\frac{1}{4} \right] \right)^{-1} \tag{0.0.2}$$

$$= \left[\frac{4}{3}\right] \tag{0.0.3}$$

We know that probability of being absorbed by state j after starting in state i is given by the $(i, j)^{th}$ entry of the matrix M, where M = NR.

$$\mathbf{M} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix}.$$

Hence the probability of being absorbed by state Y (1st element of R) after starting with state X_1 (1st element of Q) is $M_{1,1}$

 \therefore Pr $(Y) = \frac{1}{3} = 0.33$ (correct upto 2 decimal places)