PROBABILITY AND RANDOM VARIABLES
Assignment 2

GANJI VARSHITHA - AI20BTECH11009

Download latex-tikz codes from

https://github.com/VARSHITHAGANJI/AI1103_Probability_Assignment/blob/main/Assignment2.tex

Problem

Gate EC Problem 9
Step 1. Flip a coin twice.
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.
The probability that the output of the experiment is Y is (upto two decimal places) $\cdots$

Solution

Let flipping a coin twice be event H.
Sample space of event H = $\{HH, HT, TH, TT\}$
Probability of getting $\{TH\}$ as an outcome of event H is same as getting output Y which is given by

$$\Pr(TH) = \frac{1}{4}$$

Let the event of flipping a coin twice again be A.
We know,

$$\Pr(A) = \Pr(TT) = \frac{1}{4}$$

Let a random variable X be the number of times flipping the coin twice to get the output Y in the last trial.
X $\in \{1, 2, 3, 4, \cdots \infty\}$
Suppose the outcome $\{TH\}$ or Y occurs in k^{th} trial.

$$\Pr(X = k) = \Pr(TH)\left(\Pr(A)^{k-1}\right)$$

We know output Y occurs in 1^{st} trial or 2^{nd} trial or 3^{rd} trial and so on till ∞
This gives us

$$Y = \bigcup_{i=1}^{\infty} (X = i)$$

$$\begin{aligned}
\Pr(Y) &= \sum_{i=1}^{\infty} \Pr(X = i) \\
&= \Pr(TH) \sum_{i=1}^{\infty} \Pr(A)^{i-1} \\
&= \Pr(TH) \sum_{i=0}^{\infty} \Pr(A)^{i} \\
&= \Pr(TH) \frac{1}{1 - \Pr(A)} \\
&= \frac{1}{4}\left(\frac{1}{1 - \frac{1}{4}}\right) \\
&= \frac{1}{3} \qquad (0.0.1)
\end{aligned}$$

$\therefore \Pr(Y) = 0.33$ (Round off to two decimal places)