PROBABILITY AND RANDOM VARIABLES
Assignment 2

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PROBLEM

Gate EC Problem 9
Step 1. Flip a coin twice.
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.
The probability that the output of the experiment is Y is (upto two decimal places) $\cdots$

SOLUTION

Let flipping a coin twice be event H.
Sample space of event H = $\{HH, HT, TH, TT\}$
Probability of getting $\{TH\}$ as an outcome of event H is same as getting output Y which is given by

$$\Pr(TH) = \frac{1}{4}$$

Let the event of flipping a coin twice again be A.
We know,

$$\Pr(A) = \Pr(TT) = \frac{1}{4}$$

Let a random variable X be the number of times flipping the coin twice to get the output Y in the last trial.
$X \in \{1, 2, 3, 4, \cdots \infty \}$
Suppose the outcome $\{TH\}$ or Y occurs in k^{th} trial.

$$\Pr(X = k) = \Pr(TH)(\Pr(A)^{k-1})$$

We know output Y occurs in 1st trial or 2nd trial or 3rd trial and so on till ∞
This gives us

$$Y = \bigcup_{i=1}^{\infty} (X = i)$$

$$\Pr(Y) = \sum_{i=1}^{\infty} \Pr(X = i) \tag{0.0.1}$$

$$= \Pr(TH) \sum_{i=1}^{\infty} \Pr(A)^{i-1} \tag{0.0.2}$$

$$= \Pr(TH) \sum_{i=0}^{\infty} \Pr(A)^{i} \tag{0.0.3}$$

$$= \Pr(TH) \frac{1}{1 - \Pr(A)} \tag{0.0.4}$$

$$= \frac{1}{4} \left(\frac{1}{1 - \frac{1}{4}} \right) \tag{0.0.5}$$

$$= \frac{1}{3} \tag{0.0.6}$$

$\therefore \Pr(Y) = 0.33$ (Round off to two decimal places)