PROBABILITY AND RANDOM VARIABLES Assignment 2

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Download latex-tikz codes from

https://github.com/VARSHITHAGANJI/AI1103_Probability_Assignment/blob/main/Assignment2.tex

Problem

Gate EC Problem 9

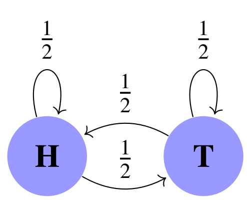
Step 1. Flip a coin twice.
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.
The probability that the output of the experiment is Y is (upto two decimal places) $\cdots$

Solution

Let flipping a coin twice be event H.
Sample space of event H = $\{HH, HT, TH, TT\}$
Let a random variable X; $X_1 = 1, X_2 = 2, X_3 = 3$ where X_1 represents outcome $\{TT\}$, X_2 represents getting outcome $\{TH\}$ or output Y, X_3 represents getting output N.
The state transition matrix P is shown below :

$$\begin{array}{cc} & \begin{array}{ccc} X_1 & X_2 & X_3 \end{array} \\ \begin{array}{c} X_1 \\ X_2 \\ X_3 \end{array} & \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Markov chain diagram



From the transition matrix, we have 1 transient state and 2 absorbing states.
Q = $\begin{bmatrix} \frac{1}{4} \end{bmatrix}$ and R = $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \end{bmatrix}$

$$N = (I - Q)^{-1} \tag{0.0.1}$$

$$= \left([1] - \begin{bmatrix} \frac{1}{4} \end{bmatrix} \right)^{-1} \tag{0.0.2}$$

$$= \begin{bmatrix} \frac{4}{3} \end{bmatrix} \tag{0.0.3}$$

We know that probability of being absorbed by state j after starting in state i is given by the $(i, j)^{th}$ entry of the matrix M, where M = NR.
M = $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix}$.
Hence the probability of being absorbed by state Y (1^{st}element of R) after starting with state X_1 (1^{st}element of Q) is $M_{1,1}$

$\therefore \Pr(Y) = \frac{1}{3} = 0.33$ (correct upto 2 decimal places)