PROBABILITIY AND RANDOM VARIABLES ASSIGNMENT 4

GANJI VARSHITHA - AI20BTECH11009

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QUESTION

ISS Statistics Paper I Question 12
Two cannons A_1 and A_2 fire at the same target. Cannon A_1 fires on an average 9 projectiles in the time in which cannon A_2 fire 10 projectiles.But on an average 7 out of 10 projectiles from cannon A_1 and 6 out of 10 projectiles from cannon A_2 strike the target. If in the course of shooting, the target is struck by one projectile, then the probability that it is struck by projectile from cannon A_1 is

1) $\frac{20}{41}$

2) $\frac{21}{41}$

3) $\frac{6}{19}$

4) $\frac{63}{190}$

SOLUTION

TABLE 1: Listing of events

S	event that the target is struck by a projectile
A_1	event that cannon A_1 fires a projectile
A_2	event that cannon A_2 fires a projectile

We need to calculate the conditional probability $\Pr(A_1|S)$.
By Bayes' Theorem, we get

$$\Pr(A_1|S) = \frac{\Pr(S|A_1)\Pr(A_1)}{\Pr(S|A_1)\Pr(A_1) + \Pr(S|A_2)\Pr(A_2)} \tag{0.0.1}$$

In (0.0.1),
$\Pr(S|A_i)$ represents the conditional probability of cannon A_i striking the target.
Given,

E	A_1	A_2
$\Pr(E)$	$\frac{9}{19}$	$\frac{10}{19}$

Also,

$$\Pr(S|A_1) = \frac{7}{10} \tag{0.0.2}$$

$$\Pr(S|A_2) = \frac{6}{10} \tag{0.0.3}$$

$$\tag{0.0.4}$$

Substituting the values in (0.0.1), we get

$$\Pr(A_1|S) = \frac{\frac{7}{10}\frac{9}{19}}{\frac{7}{10}\frac{9}{19} + \frac{6}{10}\frac{10}{19}} \tag{0.0.5}$$

$$= \frac{63}{63 + 60} \tag{0.0.6}$$

$$= \frac{21}{41} \tag{0.0.7}$$

Therefore, option 2 is correct.