

PROBABILITY AND RANDOM VARIABLES

Assignment 2

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Download latex-tikz codes from

https://github.com/VARSHITHAGANJI/AI1103_Probability_Assignment/blob/main/Assignment2.tex

PROBLEM

Gate EC Problem 9

Step 1. Flip a coin twice.

Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.

Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is Y is (upto two decimal places) ...

SOLUTION

Let flipping a coin twice be event H.

Sample space of event H = {HH, HT, TH, TT}

Let a random variable X; $X_1 = 1, X_2 = 2, X_3 = 3$

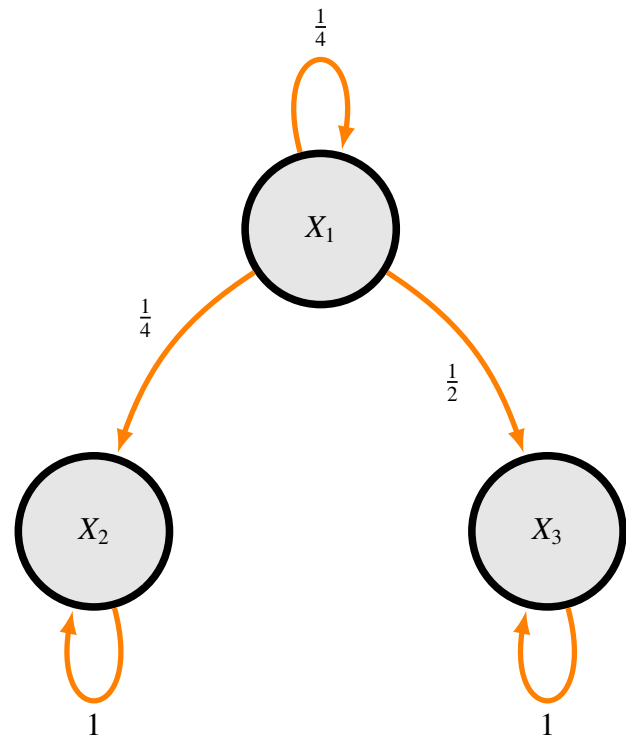
where X_1 represents outcome {TT}, X_2 represents getting outcome {TH} or output Y, X_3 represents getting output N.

The state transition matrix P is shown below :

$$\begin{array}{c|ccc} & X_1 & X_2 & X_3 \\ \hline X_1 & \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix} \\ X_2 & \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \\ X_3 & \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \end{array}$$

From the transition matrix, we have 1 transient state and 2 absorbing states.

Markov chain diagram



$$Q = \begin{bmatrix} \frac{1}{4} \end{bmatrix} \text{ and } R = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

$$N = (I - Q)^{-1} \quad (0.0.1)$$

$$= \left([1] - \begin{bmatrix} \frac{1}{4} \end{bmatrix} \right)^{-1} \quad (0.0.2)$$

$$= \begin{bmatrix} \frac{4}{3} \end{bmatrix} \quad (0.0.3)$$

We know that probability of being absorbed by state j after starting in state i is given by the $(i, j)^{th}$ entry of the matrix M, where $M = NR$.

$$M = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix}.$$

Hence the probability of being absorbed by state Y (1^{st} element of R) after starting with state X_1 (1^{st} element of Q) is $M_{1,1}$

$$\therefore \Pr(Y) = \frac{1}{3} = 0.33 \text{ (correct upto 2 decimal places)}$$