Project Presentation - AI1103

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Improving Error Probability Performance of Digital Communication Systems with Compact Nyquist Pulses

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Abstract

- Designing pulse shaping filters that satisfy the Nyquist condition for minimum intersymbol interference (ISI) is crucial to the performance of almost all digital transceiver systems.
- A method of improving the error probability performance of various Nyquist pulses, by multiplying them with a specific compactly supported function, is proposed.
- The resultant pulses are less sensitive to timing error and with smaller maximum distortion than the original pulses.

Prerequisites

Nyquist Pulses

The pulses which satisfy Nyquist ISI criterion which results in no intersymbol interference or ISI.
Let h(t) denote channel impulse response, then the condition for ISI free response is given by

$$h(nT_s) = \begin{cases} 1; & n = 0 \\ 0; & n \neq 0 \end{cases} \tag{1}$$

for all integers n and T_s is the symbol period.
Examples are sinc pulse, raised cosine pulses, etc.

Compactly Supported Functions

Functions with compact support on a topological space X are those whose closed support is a compact subset of X.
In other words, compactly supported function is zero outside of the compact set.

Prerequisites

Eye Diagrams

Eye diagrams are successful way of quickly and intuitively assessing the quality of a digital signal. It visualises the probability of signal being at each possible voltage across the symbol duration time.

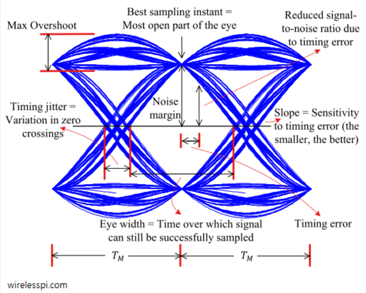

Figure: Typical Eye diagram with measurementss

PROBLEM

- The transceiver processes the received data on a sample by sample basis, resulting in timing error.
- The effect of the timing error is exacerbated by the intersymbol interference (ISI) phenomenon, which arises due to the bandlimited nature of the channel.
- Hence, we need to combat arising error probability due to combine effects of timing error and ISI phenomenon by designing pulse shaping filters that minimize the effects of ISI as well as noise.

Approaches

Consider a baseband pulse amplitude modulation (PAM) system given by

$$x(t) = \sum_{m=-\infty}^{\infty} d_m g(t - mT_{SYM}) + w(t) \tag{2}$$

$$h(nT_s) = \begin{cases} 1; & t = 0 \\ 0; & t = \pm 1, \pm 2, \cdots . \end{cases} \tag{3}$$

where T_{SYM} is the symbol duration, d_m is the transmitted symbols at the rate $\frac{1}{T_{SYM}}$, g(t) is the overall channel impulse response, and w(t) is the additive white Gaussian noise.

- These pulses are nyquist pulses. The most widely known ISI-free NP is the raised-cosine (rcos) pulse. Pulses like better than raised cosine(btrc)and fsech which have superior performance compared to rcos pulses have been proposed.
- To achieve low error probability without sacrificing bandwidth, linear combination of two pulses is proposed.

Different Approach

- Traditional method of pulse design targets the frequency response of the pulse.
- A different approach is to target the impulse response function without having to compromise much bandwidth.
- Here, the linear multiplication of existing nyquist pulses with a compactly supported function is proposed, which results in a compactly supported Nyquist pulse whose bandwidth can be controlled by a factor termed the scaling parameter.

Mathematical Definition

The expectation of ISI error probability P_e for a Nyquist pulse is given by

$$E[P_e] = \int P_e f_e \, d\epsilon \tag{4}$$

where f_e is probability density function of the time error ϵ.
Considering the case of binary antipodal signaling an additive white guassian noise(AGWN), P_e is evaluated as

$$P_e = \frac{1}{2} - \frac{2}{\pi} \sum_{m=1, m=\text{odd}}^{M} \frac{exp(-m^2\omega^2) sin(mg_0)}{2} . \prod_{k=N_1, k\neq 0}^{N_2} cos(m\omega g_k). \tag{5}$$

where

- M represents the number of coefficients considered in the approximate Fourier series of noise complementary distribution
- $\omega = \frac{2\pi}{T_f}$ where T_f is the period used in the series
- N_1 and N_2 represent the number of interfering symbols before and after the transmitted symbol
- $g_k = p_N(kT_{\text{SYM}} + \epsilon)$ is the sample version of g(t), where $p_N(t)$ is the pulse shape used and T_{SYM} is the symbol duration.

Compactly Supported Nyquist Pulses

Definition

Let U be a non empty set in $\Re^n$ for some positive integer n, and let $C^{\infty}_{com}(U)$ denote the space of compactly supported smooth functions on U.
If for some pulse p(t) the partial derivatives exist for all possible orders defined on its domain, then the product of p(t) and some function $p_C(t) \in C^{\infty}_{com}(U)$ such that

$$p_N(t) = p(t)p_C(t) \tag{6}$$

Frequency characteristics of $p_N(t)$

Lemma

Let z(t), v(t) and h(t) be some functions whose Fourier transforms are $Z(\omega)$, $V(\omega)$ and $H(\omega)$, respectively.
If z(t) = v(t)h(t), where $B_v = supp\{V(\omega)\}$ and $B_h = supp\{H(\omega)\}$ are the frequency supports (bandwidths) of v(t) and h(t) respectively, then it is true that $B_z = supp\{Z(\omega)\}$ is such that

$$B_z = B_v + B_h \tag{7}$$

Conclusion

If the bandwidth of $p_C(t)$ is much less than that of p(t), then that of $p_N(t)$ will approximately be equal to that of p(t), otherwise the bandwidth of $p_N(t)$ will always be greater than that of p(t).

Proposed compactly supported Function

The pulse is defined as the following

$$p_C(t) = (1 - x_1(t)) \sum_{k=0}^{\infty} - \left(\frac{0.5x_2(t)}{k!} \right)^k \qquad (8)$$

where $x_1(t) = (\frac{t}{\Gamma})^2, x_2(t) = (\frac{t}{\Gamma})^2$ and $\Gamma, (\Gamma > 0) \in \Re$ is a scaling parameter that defines the bandwidth of $p_C(t)$.

Reduced tail size is required to reduce peak to average power ratio.

The scaling parameter defines how the tails of$p_N(t)$ taper off.

For practical reasons, it is approximated as a continuous function expressed as

$$p_C(t) = (1 - x_1(t))\exp(-x_2(t)) \qquad (9)$$

Time Characteristics

As Γ decreases, the tails of $p_N(t)$ tend to zero more quickly resulting in less error probability.

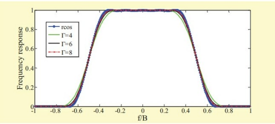

Figure: Time domain characteristics of $p_N(t)$ for rcos at $\alpha = 0.35$

Frequency Characteristics

Negligible null to null bandwidth price paid off for tapering off the pulse tails.

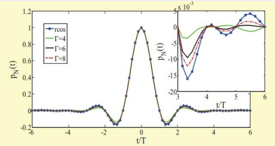

Figure: Frequency domain characteristics of $p_N(t)$ for rcos at $\alpha = 0.35$.

Noise margin and Timing jitter

The proposed pulse technique outperforms the respective conventional pulses with respect to noise margin.

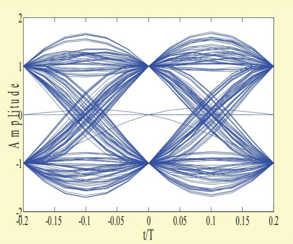

Figure: Eye diagram for rcos pulse at $\alpha = 0.35$

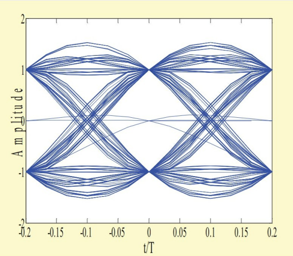

Figure: Eye diagram for $p_N(t)$ of rcos pulse at $\alpha = 0.35$ and $\Gamma = 8$

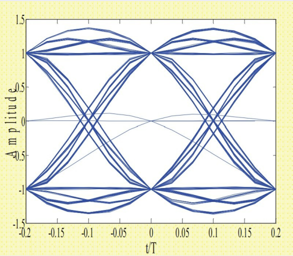

Figure: Eye diagram for $p_N(t)$ of btrc pulse at $\alpha = 0.35$ and $\Gamma = 4$

Optimal choice of Γ

- Depends on the application for which the pulse is used.
- To achieve a significantly lower error probability potential, the improved pulse technique with Γ value that tends to unity is optimal.
  The probability achieved is obtained at the expense of a slight bandwidth increase.
- For systems in which bandwidth resources is scarce, high values of Γ is better where there is no bandwidth compromise.

Summarizing

- A method of improving the performance of various conventional Nyquist pulses has been proposed.
- The improved pulse is the product of a conventional function with a compactly supported smooth function.
- Simulation results show that the proposed pulse technique outperforms the conventional pulses in terms of error probability in the presence of timing jitter and noise margin.