

LINEAR SYSTEMS AND SIGNAL PROCESSING

GATE ASSIGNMENT 1

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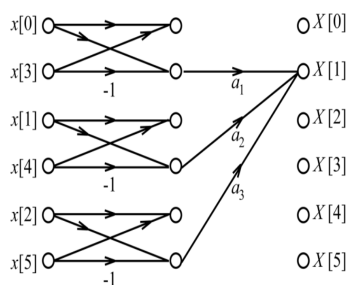
Download latex codes from

https://github.com/VARSHITHAGANJI/EE3900_GATE_ASSIGNMENTS/blob/main/GATE_ASSIGNMENT1/GATE_ASSIGNMENT1.tex

QUESTION

GATE EC-2019 Question 28

Consider a six-point decimation-in-time Fast Fourier Transform (FFT) algorithm, for which the signal-flow graph corresponding to $X[1]$ is shown in the figure. Let $W_6 = \exp(-j\frac{2\pi}{6})$. In the figure, what should be the values of the coefficients a_1, a_2, a_3 in terms of W_6 so that $X[1]$ is obtained correctly?



- 1) $a_1 = -1, a_2 = W_6, a_3 = W_6^2$
- 2) $a_1 = 1, a_2 = W_6^2, a_3 = W_6$
- 3) $a_1 = 1, a_2 = W_6, a_3 = W_6^2$
- 4) $a_1 = -1, a_2 = W_6^2, a_3 = W_6$

SOLUTION

Considering six-point decimation-in-time(DIT) FFT, we have

$$X(k) = \sum_{n=0}^5 x(n) W_6^{kn} \quad (0.0.1)$$

where twiddler factor $W_6 = \exp(-j\frac{2\pi}{6})$. For obtaining $X[1]$, put $k=1$ in (0.0.1)

$$X(1) = x(0) W_6^0 + x(1) W_6^1 + x(2) W_6^2 + x(3) W_6^3 + x(4) W_6^4 + x(5) W_6^5 \quad (0.0.2)$$

By symmetric property of twiddler factor

$$W_N^{k+\frac{N}{2}} = -W_N^k, \text{ where } N=6.$$

Putting $k = 0, 1, 2$, we get

$$W_6^3 = -W_6^0 \quad (0.0.3)$$

$$W_6^4 = -W_6^1 \quad (0.0.4)$$

$$W_6^5 = -W_6^2 \quad (0.0.5)$$

respectively.

Substituting (0.0.3), (0.0.4), (0.0.5) in (0.0.1), we get,

$$X(1) = (x(0) - x(3)) + (x(1) - x(4)) W_6^1 + (x(2) - x(5)) W_6^2 \quad (0.0.6)$$

From figure, we have

$$X(1) = (x(0) - x(3)) a_1 + (x(1) - x(4)) a_2 + (x(2) - x(5)) a_3$$

Comparing with (0.0.6), we get

$$a_1 = 1, a_2 = W_6, a_3 = W_6^2$$

\therefore Option 3 is correct.