LINEAR SYSTEMS AND SIGNAL PROCESSING
GATE ASSIGNMENT 3

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QUESTION

GATE EC-2005 Question 31

In what range should Re(s) remain so that the Laplace transform of the function $e^{(a+2)t+5}$ exists?

(a) Re(s) > a+2
(b) Re(s) > a+7
(c) Re(s) < 2
(d) Re(s) > a+5

SOLUTION

Lemma 0.1 (Laplace transform formula).

Time Function $f(t) = \mathcal{L}^{-1}\{F(s)\}$	Laplace transform of f(t) $F(s) = \mathcal{L}\{f(t)\}$	ROC
$e^{\alpha t}$	$\frac{1}{s-\alpha}$	Re(s) > α

Given function is given by

$$f(t) = e^{(a+2)t+5} \qquad (0.0.1)$$
$$= e^5 e^{(a+2)t} \qquad (0.0.2)$$

Let $g(t) = e^{((a+2)t)}$ which is in the form of $e^{\alpha t}$ where $\alpha = a + 2$.

Let the Laplace transform of $f(t)$ and $g(t)$ be $F(s)$ and $G(s)$ respectively.

From Lemma 0.1 we have,

$$G(s) = \frac{1}{s - (a+2)} \qquad (0.0.3)$$

provided Re(s)>a+2.

We have

$$F(s) = e^5 G(s) \qquad (0.0.4)$$
$$= e^5 \left(\frac{1}{s - (a+2)} \right) \qquad (0.0.5)$$

provided Re(s)>a+2.

$\therefore$ Laplace transform of $e^{(a+2)t+5}$ exists for Re(s)>a+2.

Hence option (d) is correct.