

LINEAR SYSTEMS AND SIGNAL PROCESSING

QUIZ 2

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Download latex codes from

https://github.com/VARSHITHAGANJI/EE3900_GATE_ASSIGNMENTS/blob/main/QUIZ2/QUIZ2.tex

Download all python codes from

https://github.com/VARSHITHAGANJI/EE3900_GATE_ASSIGNMENTS/blob/main/QUIZ2/quiz2code.py

Substituting (0.0.2), (0.0.3), (0.0.4) and (0.0.5) in (0.0.1), we get Let

$$a[n] = \frac{1}{2} \left[(-1)^n + \cos\left(\frac{\pi}{2}n\right) + 1 \right] u[n] \quad (0.0.6)$$

$$= \begin{cases} \frac{3}{2}, & n = 4k, k \geq 0 \\ \frac{1}{2}, & n = 4k + 2, k \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (0.0.7)$$

Let the \mathcal{Z} -transform of $a[n]$ be $A(z)$.
By definition of \mathcal{Z} -transform, we have

$$A(z) = \sum_{n=-\infty}^{\infty} a[n] z^{-n} \quad (0.0.8)$$

As $u[n] = 0 \forall n < 0$

$$A(z) = \sum_{n=0}^{\infty} a[n] z^{-n} \quad (0.0.9)$$

Let the sequence be $a_1[n]$ for $n = 4k$ and $a_2[n]$ for $n = 4k+2$.

Let their corresponding \mathcal{Z} -transforms be $A_1(z)$ and $A_2(z)$ respectively.

Take $k = \frac{n}{4}$

$$A_1(z) = \sum_{k=0}^{\infty} a_1[n] z^{-4k} \quad (0.0.10)$$

$$= \sum_{k=0}^{\infty} \frac{3}{2} z^{-4k} \quad (0.0.11)$$

$$= \frac{\frac{3}{2}}{1 - z^{-4}} \quad (0.0.12)$$

ROC is $|z| > 1$.

QUESTION

Problem 3.24(b) Sketch each of the following sequences and determine their \mathcal{Z} -transforms, including the region of convergence

- (a) $\sum_{k=-\infty}^{\infty} \delta[n-k]$
 (b) $\frac{1}{2} \left[e^{j\pi n} + \cos\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2} + 2\pi n\right) \right] u[n]$

SOLUTION

(b)

Let

$$a[n] = \frac{1}{2} \left[e^{j\pi n} + \cos\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2} + 2\pi n\right) \right] u[n] \quad (0.0.1)$$

We have

$$e^{j\pi n} = (-1)^n \quad \forall n \quad (0.0.2)$$

$$\sin\left(\frac{\pi}{2} + 2\pi n\right) = 1 \quad \forall n \quad (0.0.3)$$

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (0.0.4)$$

$$\cos\left(\frac{\pi}{2}n\right) = \begin{cases} 1, & n = 4k, k \geq 0 \\ -1, & n = 4k + 2, k \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (0.0.5)$$

Take $k = \frac{n-2}{4}$

$$A_2(z) = \sum_{k=0}^{\infty} a_2[n] z^{-4k-2} \quad (0.0.13)$$

$$= \sum_{k=0}^{\infty} \frac{1}{2} z^{-2} z^{-4k} \quad (0.0.14)$$

$$= \frac{\frac{1}{2} z^{-2}}{1 - z^{-4}} \quad (0.0.15)$$

ROC is $|z| > 1$.

Since $A(z) = A_1(z) + A_2(z)$,

$$A(z) = \frac{\frac{3}{2} + \frac{1}{2} z^{-2}}{1 - z^{-4}} \quad (0.0.16)$$

ROC is $|z| > 1$.

