LINEAR SYSTEMS AND SIGNAL PROCESSING GATE ASSIGNMENT 1

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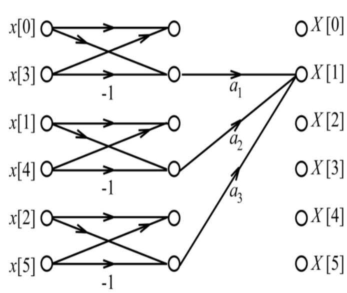
Download latex codes from

https://github.com/VARSHITHAGANJI/EE3900_GATE_ASSIGNMENTS/blob/main/GATE_ASSIGNMENT1/GATE_ASSIGNMENT1.tex

QUESTION

GATE EC-2019 Question 28

Consider a six-point decimation-in-time Fast Fourier Transform (FFT) algorithm, for which the signal-flow graph corresponding to X[1] is shown in the figure. Let $W_6 = \exp\left(-\frac{j2\pi}{6}\right)$. In the figure, what should be the values of the coefficients a_1, a_2, a_3 in terms of W_6 so that X[1] is obtained correctly?



1) $a_1 = -1, a_2 = W_6, a_3 = W_6^2$
2) $a_1 = 1, a_2 = W_6^2, a_3 = W_6$
3) $a_1 = 1, a_2 = W_6, a_3 = W_6^2$
4) $a_1 = -1, a_2 = W_6^2, a_3 = W_6$

SOLUTION

Considering six-point DFT, we have

$$\begin{pmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \end{pmatrix} = \begin{pmatrix} W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5 \\ W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5 \\ W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5 \\ W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5 \\ W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5 \\ W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5 \end{pmatrix} \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \end{pmatrix} \tag{0.0.1}$$

where twiddler factor $W_6 = \exp\left(-\frac{j2\pi}{6}\right)$.
By symmetric property of twiddler factor

$$W_N^{k+\frac{N}{2}} = -W_N^k \text{ , where N=6.}$$

Putting k = 0, 1, 2 , we get

$$W_6^3 = -W_6^0 \tag{0.0.2}$$
$$W_6^4 = -W_6^1 \tag{0.0.3}$$
$$W_6^5 = -W_6^2 \tag{0.0.4}$$

respectively.
Obtaining X[1] by substituting (0.0.2), (0.0.3), (0.0.4) in (0.0.1), we get,

$$X[1] = \begin{pmatrix} W_6^0 & W_6^1 & W_6^2 & -W_6^0 & -W_6^1 & -W_6^2 \end{pmatrix} \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \end{pmatrix} \tag{0.0.5}$$

$$= \begin{pmatrix} W_6^0 & W_6^1 & W_6^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \end{pmatrix} \tag{0.0.6}$$

$$= \begin{pmatrix} W_6^0 & W_6^1 & W_6^2 \end{pmatrix} \begin{pmatrix} x[0] - x[3] \\ x[1] - x[4] \\ x[2] - x[5] \end{pmatrix} \tag{0.0.7}$$

$$= \begin{pmatrix} W_6^0 \\ W_6^1 \\ W_6^2 \end{pmatrix}^\top \begin{pmatrix} x[0] - x[3] \\ x[1] - x[4] \\ x[2] - x[5] \end{pmatrix} \tag{0.0.8}$$

From the signal flow graph, we have

$$X[1] = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}^\top \begin{pmatrix} x[0] - x[3] \\ x[1] - x[4] \\ x[2] - x[5] \end{pmatrix} \tag{0.0.9}$$

Comparing (0.0.9) with (0.0.8), we get

$$a_1 = 1, a_2 = W_6, a_3 = W_6^2 \tag{0.0.10}$$

$\therefore$ Option 3 is correct.