LINEAR SYSTEMS AND SIGNAL PROCESSING
GATE ASSIGNMENT

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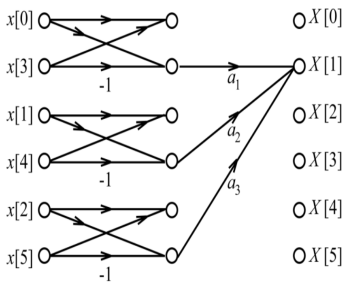
Download latex codes from

https://github.com/VARSHITHAGANJI/EE3900_GATE_ASSIGNMENTS/blob/main/GATE_ASSIGNMENT1/GATE_ASSIGNMENT1.tex

QUESTION

GATE EC-2019 Question 28

Consider a six-point decimation-in-time Fast Fourier Transform (FFT) algorithm, for which the signal-flow graph corresponding to X[1] is shown in the figure. Let $W_6 = \exp\left(-\frac{j2\pi}{6}\right)$. In the figure, what should be the values of the coefficients a_1, a_2, a_3 in terms of W_6 so that X[1] is obtained correctly?



1) $a_1 = -1, a_2 = W_6, a_3 = W_6^2$
2) $a_1 = 1, a_2 = W_6^2, a_3 = W_6$
3) $a_1 = 1, a_2 = W_6, a_3 = W_6^2$
4) $a_1 = -1, a_2 = W_6^2, a_3 = W_6$

SOLUTION

Considering six-point decimation-in-time(DIT) FFT, we have

$$X(k) = \sum_{n=0}^{5} x(n)\, W_6^{kn} \tag{0.0.1}$$

where twiddler factor $W_6 = \exp\left(-\frac{j2\pi}{6}\right)$. For obtaining X[1], put k=1 in (0.0.1)

$$X(1) = x(0)\, W_6^0 + x(1)\, W_6^1 + x(2)\, W_6^2 + x(3)\, W_6^3 + x(4)\, W_6^4 + x(5)\, W_6^5 \tag{0.0.2}$$

By symmetric property of twiddler factor

$$W_N^{k+\frac{N}{2}} = -W_N^k \text{ , where N=6.}$$

Putting k = 0, 1, 2 , we get

$$W_6^3 = -W_6^0 \tag{0.0.3}$$
$$W_6^4 = -W_6^1 \tag{0.0.4}$$
$$W_6^5 = -W_6^2 \tag{0.0.5}$$

respectively.
Substituting (0.0.3), (0.0.4), (0.0.5) in (0.0.1), we get,

$$X(1) = (x(0) - x(3)) + (x(1) - x(4))\, W_6^1 + (x(2) - x(5))\, W_6^2 \tag{0.0.6}$$

From figure, we have

$$X(1) = (x(0) - x(3))\, a_1 + (x(1) - x(4))\, a_2 + (x(2) - x(5))\, a_3$$

Comparing with (0.0.6), we get

$$a_1 = 1, a_2 = W_6, a_3 = W_6^2$$

$\therefore$ Option 3 is correct.