LINEAR SYSTEMS AND SIGNAL PROCESSING QUIZ 2

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Download latex codes from

https://github.com/VARSHITHAGANJI/EE3900_GATE_ASSIGNMENTS/blob/main/QUIZ2/QUIZ2.tex

Download all python codes from

https://github.com/VARSHITHAGANJI/EE3900_GATE_ASSIGNMENTS/blob/main/QUIZ2/quiz2code.py

QUESTION

Problem 3.24(b) Sketch each of the following sequences and determine their $\mathcal{Z}$- transforms, including the region of convergence

(a) $\sum_{k=-\infty}^{\infty} \delta[n-k]$
(b) $\frac{1}{2}\left[e^{j\pi n} + cos\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2} + 2\pi n\right)\right] u[n]$

SOLUTION

(b)
Let

$$a[n] = \frac{1}{2}\left[e^{j\pi n} + \cos\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2} + 2\pi n\right)\right] u[n] \tag{0.0.1}$$

We have

$$e^{j\pi n} = (-1)^n \ \forall n \tag{0.0.2}$$

$$\sin\left(\frac{\pi}{2} + 2\pi n\right) = 1 \ \forall n \tag{0.0.3}$$

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{0.0.4}$$

$$\cos\left(\frac{\pi}{2}n\right) = \begin{cases} 1, & n = 4k,\ k \geq 0 \\ -1, & n = 4k+2,\ k \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{0.0.5}$$

Substituting (0.0.2), (0.0.3),(0.0.4) and (0.0.5) in (0.0.1), we get Let

$$a[n] = \frac{1}{2}\left[(-1)^n + \cos\left(\frac{\pi}{2}n\right) + 1\right] u[n] \tag{0.0.6}$$

$$= \begin{cases} \frac{3}{2}, & n = 4k,\ k \geq 0 \\ \frac{1}{2}, & n = 4k+2,\ k \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{0.0.7}$$

Let the $\mathcal{Z}$-transform of $a[n]$ be $A(z)$.
By definition of $\mathcal{Z}$-transform, we have

$$A(z) = \sum_{n=-\infty}^{\infty} a[n] z^{-n} \tag{0.0.8}$$

As $u[n] = 0 \ \forall n < 0$

$$A(z) = \sum_{n=0}^{\infty} a[n] z^{-n} \tag{0.0.9}$$

Let the sequence be $a_1[n]$ for n = 4k and $a_2[n]$ for n= 4k+2.
Let their corresponding $\mathcal{Z}$-transforms be $A_1(z)$ and $A_2(z)$ respectively.
Take $k = \frac{n}{4}$

$$A_1(z) = \sum_{k=0}^{\infty} a_1[n] z^{-4k} \tag{0.0.10}$$

$$= \sum_{k=0}^{\infty} \frac{3}{2} z^{-4k} \tag{0.0.11}$$

$$= \frac{\frac{3}{2}}{1 - z^{-4}} \tag{0.0.12}$$

ROC is $|z| > 1$.

Take $k = \frac{n-2}{4}$

$$A_2(z) = \sum_{k=0}^{\infty} a_2[n]\, z^{-4k-2} \tag{0.0.13}$$

$$= \sum_{k=0}^{\infty} \frac{1}{2} z^{-2} z^{-4k} \tag{0.0.14}$$

$$= \frac{\frac{1}{2} z^{-2}}{1 - z^{-4}} \tag{0.0.15}$$

ROC is $|z| > 1$.
Since $A(z) = A_1(z) + A_2(z)$,

$$A(z) = \frac{\frac{3}{2} + \frac{1}{2} z^{-2}}{1 - z^{-4}} \tag{0.0.16}$$

ROC is $|z| > 1$.
Solving for poles and zeros of $A(z)$, we get
Zeros $= z = \pm\frac{1}{3} i$
Poles $= z = 1$

Fig. 1: Stem plot of a[n]

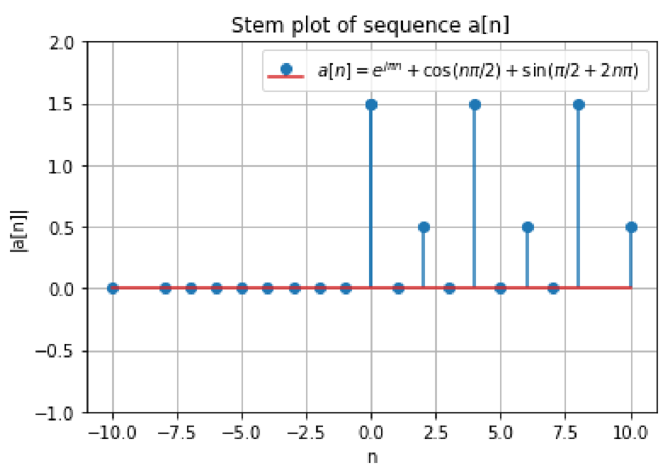

Fig. 2: Poles zero plot of A[z]