LINEAR SYSTEMS AND SIGNAL PROCESSING GATE ASSIGNMENT 2

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https://github.com/VARSHITHAGANJI/EE3900_GATE_ASSIGNMENTS/blob/main/GATE_ASSIGNMENT2/GATE_ASSIGNMENT2.tex

Download all python codes from

https://github.com/VARSHITHAGANJI/EE3900_VECTORS_ASSIGNMENTS/blob/main/GATE_ASSIGNMENT2/code_graphs.py

QUESTION

GATE EC 2008- Q35

Let x(t) be the input and y(t) be the output of a continuous time system. Match the system properties P1, P2, and P3 with the system relations R1, R2, R3, R4.

Properties	Relations
P1: Linear but NOT time-invariant	R1: $y(t) = t^2 x(t)$
P2: Time-invariant but NOT linear	R2: $y(t) = t\|x(t)\|$
P3: Linear and time-invariant	R3: $y(t) = \|x(t)\|$
	R4: $y(t) = x(t-5)$

1) (P1,R1), (P2,R3), (P3,R4)
2) (P1,R2), (P2,R3), (P3,R4)
3) (P1,R3), (P2,R1), (P3,R2)
4) (P1,R1), (P2,R2), (P3,R3)

SOLUTION

Definition 1. *We say that a system is **linear** if and only if it follows the Principle of Superposition, i.e Law of Additivity and Law of Homogeneity.*

Definition 2. *A system is said to be **time invariant** if the output signal does not depend on the absolute time, i.e a time delay on the input signal directly equates to the delay in the output signal.*

Law of Additivity:

Let the three input signals be $x_1(t)$, $x_2(t)$, and $x_1(t)+x_2(t)$ and their corresponding output signals be $y_1(t)$, $y_2(t)$, and $y'(t)$, then: For R1:

$$y_1(t) = t^2 x_1(t) \tag{0.0.1}$$
$$y_2(t) = t^2 x_1(t) \tag{0.0.2}$$
$$y_1(t) + y_2(t) = t^2 (x_1(t) + x_2(t)) \tag{0.0.3}$$
$$y'(t) = t^2 (x_1(t) + x_2(t)) \tag{0.0.4}$$

Clearly, from (0.0.3) and (0.0.4):

$$y'(t) = y_1(t) + y_2(t) \tag{0.0.5}$$

Thus, the Law of Additivity holds.
For R2:

$$y_1(t) = t\,|x_1(t)| \tag{0.0.6}$$
$$y_2(t) = t\,|x_1(t)| \tag{0.0.7}$$
$$y_1(t) + y_2(t) = t\,(|x_1(t)| + |x_2(t)|) \tag{0.0.8}$$
$$y'(t) = t\,(|x_1(t) + x_2(t)|) \tag{0.0.9}$$

Clearly, from (0.0.8) and (0.0.9):

$$y'(t) \leq y_1(t) + y_2(t) \tag{0.0.10}$$

Thus, the Law of Additivity does not hold.
For R3:

$$y_1(t) = |x_1(t)| \tag{0.0.11}$$
$$y_2(t) = |x_1(t)| \tag{0.0.12}$$
$$y_1(t) + y_2(t) = (|x_1(t)| + |x_2(t)|) \tag{0.0.13}$$
$$y'(t) = (|x_1(t) + x_2(t)|) \tag{0.0.14}$$

Clearly, from (0.0.13) and (0.0.14):

$$y'(t) \leq y_1(t) + y_2(t) \tag{0.0.15}$$

Thus, the Law of Additivity does not hold.

For R4:

$$y_1(t) = x_1(t-5) \tag{0.0.16}$$
$$y_2(t) = x_1(t-5) \tag{0.0.17}$$
$$y_1(t) + y_2(t) = (x_1(t-5) + x_2(t-5)) \tag{0.0.18}$$
$$y'(t) = (x_1(t-5) + x_2(t-5)) \tag{0.0.19}$$

Clearly, from (0.0.18) and (0.0.19):

$$y'(t) = y_1(t) + y_2(t) \tag{0.0.20}$$

Thus, the Law of Additivity holds.

Law of Homogeneity:

Consider an input signal $kx(t)$, where k is any constant. Let the corresponding output be given by $y'(t)$, then:

For R1:

$$y'(t) = t^2 kx(t) \tag{0.0.21}$$
$$= ky(t) \tag{0.0.22}$$

Clearly, from (0.0.22):

$$y'(t) = ky(t) \tag{0.0.23}$$

Thus, the Law of homogeneity holds.

For R2:

$$y'(t) = t|kx(t)| \tag{0.0.24}$$
$$= |k| y(t) \tag{0.0.25}$$

Clearly, from (0.0.25):

$$y'(t) \neq ky(t) \tag{0.0.26}$$

Thus, the Law of homogeneity does not hold.

For R3:

$$y'(t) = |kx(t)| \tag{0.0.27}$$
$$= |k| y(t) \tag{0.0.28}$$

Clearly, from (0.0.28):

$$y'(t) \neq ky(t) \tag{0.0.29}$$

Thus, the Law of homogeneity does not hold.

For R4:

$$y'(t) = kx(t-5) \tag{0.0.30}$$
$$= ky(t) \tag{0.0.31}$$

Clearly, from (0.0.31):

$$y'(t) = ky(t) \tag{0.0.32}$$

Thus, the Law of homogeneity holds.
∴ We get R1 and R4 are linear systems whereas R2 and R3 are non linear systems.

To check time-invariance, we introduce a delay of t_0 in the output and input signal.Let the delayed output signal is $y(t-t_0)$ and the output signal corresponding to the input signal delayed by t_0 is given by $y'(t)$.

For R1:

$$y(t-t_0) = (t-t_0)^2 x(t-t_0) \tag{0.0.33}$$
$$y'(t) = (t)^2 x(t-t_0) \tag{0.0.34}$$

Clearly, from (0.0.33) and (0.0.34):

$$y(t-t_0) \neq y'(t) \tag{0.0.35}$$

Thus, the system is not time invariant.

For R2:

$$y(t-t_0) = (t-t_0)|x(t-t_0)| \tag{0.0.36}$$
$$y'(t) = (t)|x(t-t_0)| \tag{0.0.37}$$

Clearly, from (0.0.37) and (0.0.38):

$$y(t-t_0) \neq y'(t) \tag{0.0.38}$$

Thus, the system is not time invariant.

For R3:

$$y(t-t_0) = |x(t-t_0)| \tag{0.0.39}$$
$$y'(t) = |x(t-t_0)| \tag{0.0.40}$$

Clearly, from (0.0.39) and (0.0.40):

$$y(t-t_0) = y'(t) \tag{0.0.41}$$

Thus, the system is time invariant.

For R4:

$$y(t-t_0) = x(t-t_0-5) \tag{0.0.42}$$
$$y'(t) = x(t-t_0-5) \tag{0.0.43}$$

Clearly, from (0.0.42) and (0.0.43):

$$y(t-t_0) = y'(t) \tag{0.0.44}$$

Thus, the system is time invariant.

$\therefore$ We get R3 and R4 are time invariant systems whereas R1 and R2 are not time invariant.
Tabulating the results,

System	Linear	Time invariant
R1	Yes	No
R2	No	No
R3	No	Yes
R4	Yes	Yes

From the above table , option (1) is correct.
Verification using simple signals.

Fig. 1: Input signals

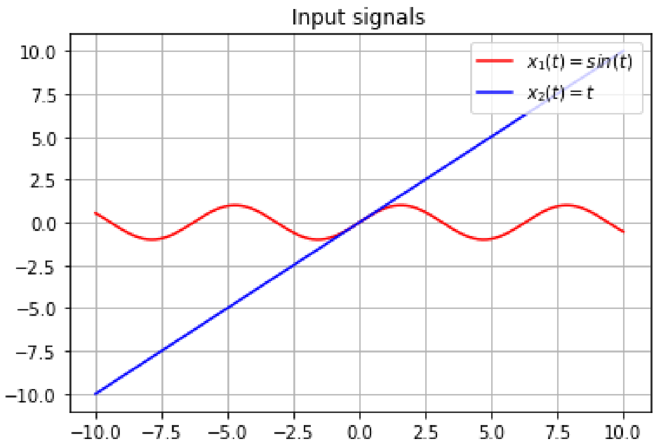

Fig. 2: R1 system: y(t) = t^2x (t)

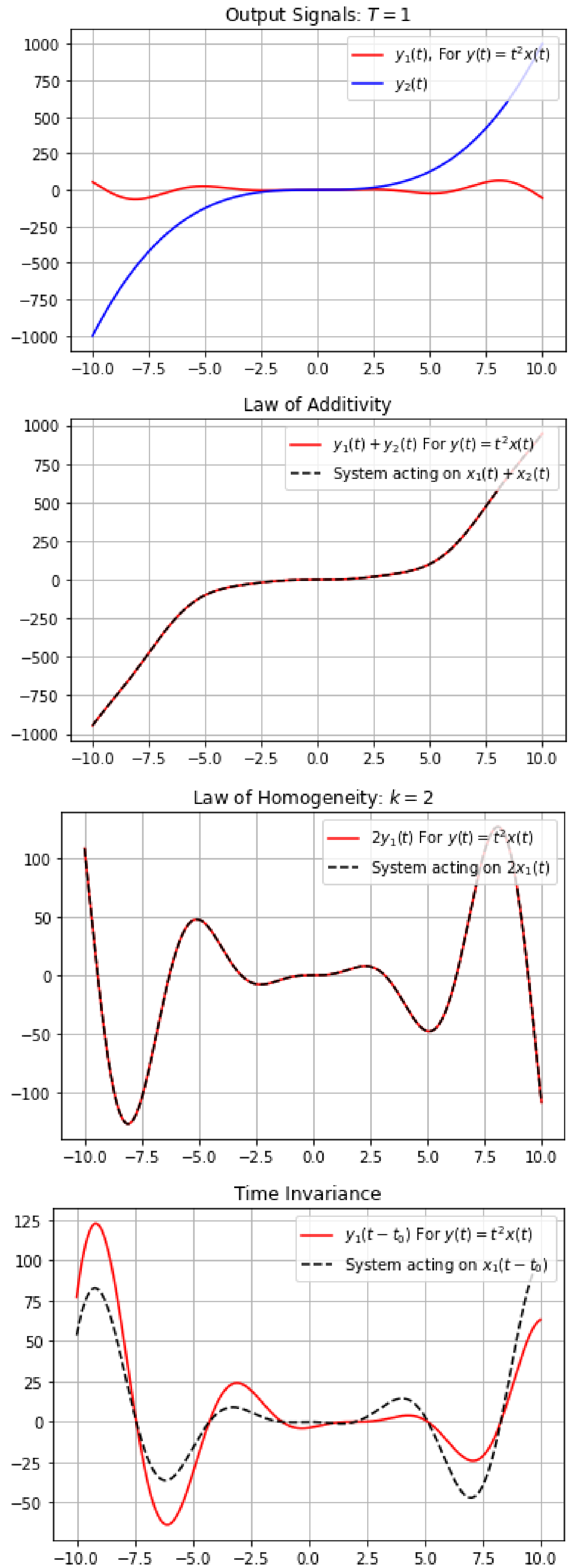

Fig. 3: R2 system: y(t) = t| x (t)|

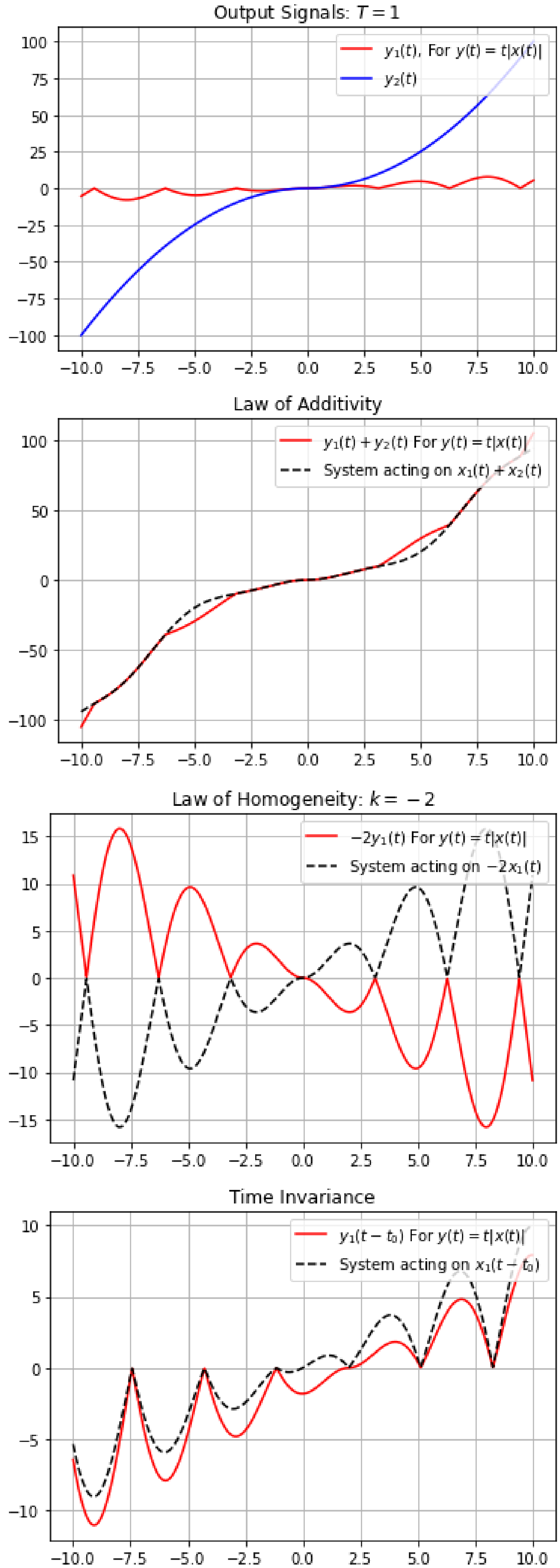

Fig. 4: R3 system: y(t) = | x (t)|

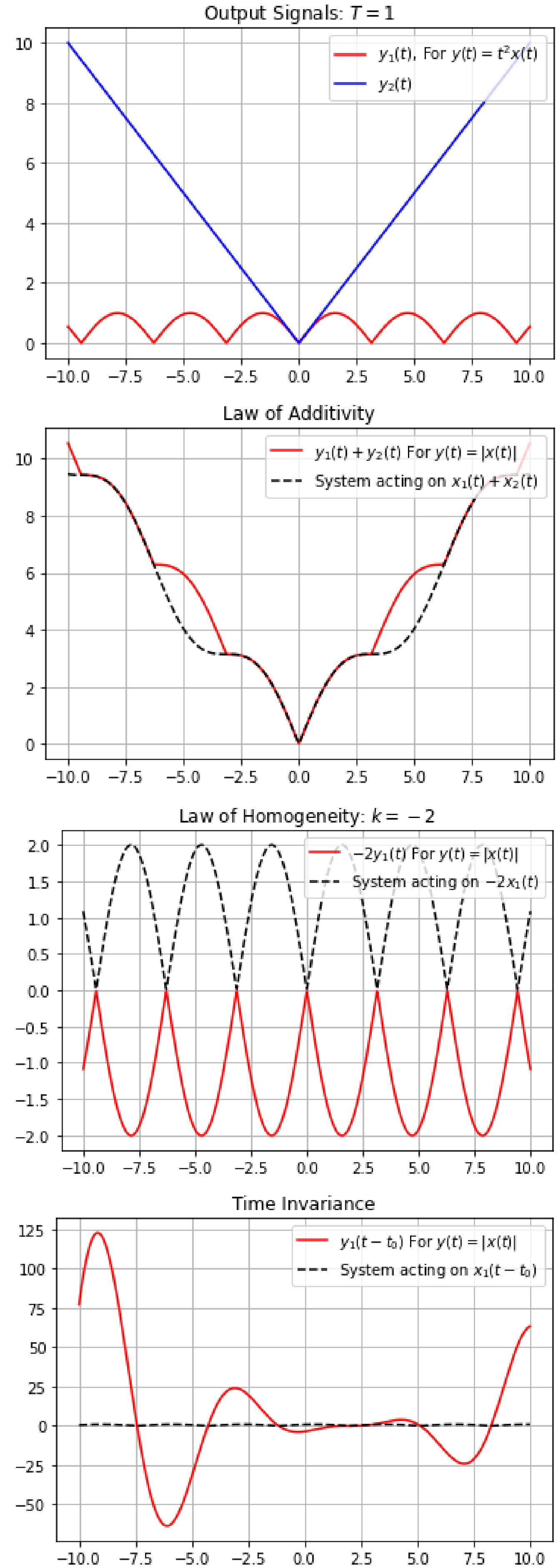

Fig. 5: R4 system: y(t) = x (t-5)