

LINEAR SYSTEMS AND SIGNAL PROCESSING

ASSIGNMENT 1

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Download latex codes from

https://github.com/VARSHITHAGANJI/EE3900_VECTORS_ASSIGNMENTS/blob/main/VECTORS_ASSIGNMENT1/VECTORS_ASSIGNMENT1.tex

QUESTION

Vectors 2.10

In each of the following, find the value of k for which the points are collinear

- 1) $\begin{pmatrix} 7 \\ -2 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ k \end{pmatrix}$
- 2) $\begin{pmatrix} 8 \\ 1 \end{pmatrix}, \begin{pmatrix} k \\ -4 \end{pmatrix}, \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

SOLUTION

- 1) Let $\mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ k \end{pmatrix}$

The direction vectors of AB and AC are

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \quad (0.0.1)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ k+2 \end{pmatrix} \quad (0.0.2)$$

If $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a line, AB, AC should have the same direction vector. Hence there exists an α such that

$$\mathbf{B} - \mathbf{A} = \alpha (\mathbf{C} - \mathbf{A}) \quad (0.0.3)$$

Substituting (0.0.1) and (0.0.2) in (0.0.3), we get

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \alpha \begin{pmatrix} -4 \\ k+2 \end{pmatrix} \quad (0.0.4)$$

$$= \begin{pmatrix} -4\alpha \\ \alpha(k+2) \end{pmatrix} \quad (0.0.5)$$

Comparing the vectors

$$-2 = -4\alpha \quad (0.0.6)$$

$$\alpha = \frac{1}{2} \quad (0.0.7)$$

Substituting (0.0.7) in (0.0.4), we get

$$k = 4 \quad (0.0.8)$$

- 2) Let $\mathbf{A} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} k \\ -4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

The direction vectors of AB and AC are

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} k-8 \\ -5 \end{pmatrix} \quad (0.0.9)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -6 \\ -6 \end{pmatrix} \quad (0.0.10)$$

If $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a line, AB, AC should have the same direction vector. Hence there exists an α such that

$$\mathbf{B} - \mathbf{A} = \alpha (\mathbf{C} - \mathbf{A}) \quad (0.0.11)$$

Substituting (0.0.9) and (0.0.10) in (0.0.11), we get

$$\begin{pmatrix} k-8 \\ -5 \end{pmatrix} = \alpha \begin{pmatrix} -6 \\ -6 \end{pmatrix} \quad (0.0.12)$$

$$= \begin{pmatrix} -6\alpha \\ -6\alpha \end{pmatrix} \quad (0.0.13)$$

Comparing the vectors

$$-5 = -6\alpha \quad (0.0.14)$$

$$\alpha = \frac{5}{6} \quad (0.0.15)$$

Substituting (0.0.15) in (0.0.12), we get

$$k = 3 \quad (0.0.16)$$