LINEAR SYSTEMS AND SIGNAL PROCESSING ASSIGNMENT 5

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Download latex codes from

https://github.com/VARSHITHAGANJI/EE3900_VECTORS_ASSIGNMENTS/blob/main/QUADRATIC_FORMS_ASSIGNMENT5/QUADRATIC_FORMS_ASSIGNMENT5.tex

Download all python codes from

https://github.com/VARSHITHAGANJI/EE3900_VECTORS_ASSIGNMENTS/blob/main/QUADRATIC_FORMS_ASSIGNMENT5/plot_code.py

QUESTION

Quadratic Forms 2.6

Find the area lying in the first quadrant and bounded by the circle $\mathbf{x}\mathbf{x}^\top = 4$ and the lines x = 0 and x = 2.

SOLUTION

The general equation of a circle is

$$\mathbf{x}\mathbf{x}^\top - 2\mathbf{O}^\top\mathbf{x} + \left\|O\right\|^2 - r^2 = 0 \tag{0.0.1}$$

Given equation of the circle is

$$\mathbf{x}\mathbf{x}^\top = 4 \tag{0.0.2}$$

Comparing (0.0.2) with (0.0.1), we get

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.3}$$

$$r = 2 \tag{0.0.4}$$

Given lines are

$$L_1 : \mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.0.5}$$

$$L_2 : \mathbf{x} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.0.6}$$

where α and β are real numbers. We know the points of intersection of the line

$$L : \mathbf{x} = \mathbf{q} + \mu\mathbf{m} \tag{0.0.7}$$

with the circle in (0.0.2) is given by

$$\mathbf{x_i} = \mathbf{q} + \mu_i\mathbf{m} \tag{0.0.8}$$

where

$$\mu_i = \frac{1}{\mathbf{m}^\top\mathbf{I}\mathbf{m}} \left(-\mathbf{m}^\top\left(\mathbf{I}\mathbf{q} + \mathbf{O}\right) \pm \sqrt{\left[-\mathbf{m}^\top\left(\mathbf{I}\mathbf{q} + \mathbf{O}\right)\right]^2 - \left(\mathbf{q}^\top\mathbf{I}\mathbf{q} + \mathbf{O}^\top\mathbf{q} - r^2\right)\left(\mathbf{m}^\top\mathbf{I}\mathbf{m}\right)}\right)$$

Solving for α and β, we get

$$\alpha = \pm 2 \qquad \beta = 0 \tag{0.0.9}$$

Points of intersection of line L_1 are $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ and line L_2 is $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

The angle made by lines L_1 and L_2 with the x axis i.e $\begin{pmatrix} 0 & 1 \end{pmatrix}\mathbf{x} = 0$ is

$$\cos\theta = \frac{\begin{pmatrix} 1 \\ 0 \end{pmatrix}^\top \begin{pmatrix} 0 & 1 \end{pmatrix}}{\left\|\begin{pmatrix} 1 & 0 \end{pmatrix}\right\|\left\|\begin{pmatrix} 0 & 1 \end{pmatrix}\right\|} \tag{0.0.10}$$

$$= 0 \tag{0.0.11}$$

$$\implies \theta = 90^\circ \tag{0.0.12}$$

The area of sector thus obtained is

$$\frac{\theta^\circ}{360^\circ}\pi r^2 = \frac{90^\circ}{360^\circ}\pi r^2 \tag{0.0.13}$$

$$= \frac{\pi}{4}2^2 \tag{0.0.14}$$

$$= \pi \tag{0.0.15}$$

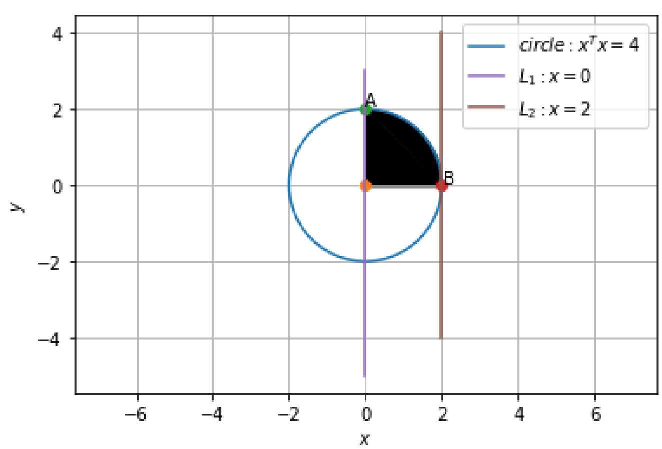

Fig. 1: Plotting the region bounded by circle and lines in first quadrant