LINEAR SYSTEMS AND SIGNAL PROCESSING ASSIGNMENT 1

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Download latex codes from

https://github.com/VARSHITHAGANJI/EE3900_VECTORS_ASSIGNMENTS/blob/main/VECTORS_ASSIGNMENT1/VECTORS_ASSIGNMENT1.tex

Download all python codes from

https://github.com/VARSHITHAGANJI/EE3900_VECTORS_ASSIGNMENTS/blob/main/VECTORS_ASSIGNMENT1/VEC1_CODE.py

QUESTION

Vectors 2.10

In each of the following, find the value of k for which the points are collinear

1) $\begin{pmatrix}7\\-2\end{pmatrix}, \begin{pmatrix}5\\1\end{pmatrix}, \begin{pmatrix}3\\k\end{pmatrix}$
2) $\begin{pmatrix}8\\1\end{pmatrix}, \begin{pmatrix}k\\-4\end{pmatrix}, \begin{pmatrix}2\\-5\end{pmatrix}$

SOLUTION

1) Let $\mathbf{A} = \begin{pmatrix}7\\-2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}5\\1\end{pmatrix}, \mathbf{C} = \begin{pmatrix}3\\k\end{pmatrix}$

The direction vectors of AB and AC are

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix}-2\\3\end{pmatrix} \tag{0.0.1}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix}-4\\k+2\end{pmatrix} \tag{0.0.2}$$

$$\mathbf{M} = \begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix}^{\top} \tag{0.0.3}$$

Substituting (0.0.1) and (0.0.2) in (0.0.3), we get

$$\mathbf{M} = \begin{pmatrix}-2 & 3\\-4 & k+2\end{pmatrix} \tag{0.0.4}$$

We know that if $rank(\mathbf{M}) = 1$, the points are collinear.

Finding the rank of the matrix in the problem,

$$\mathbf{M} = \begin{pmatrix}-2 & 3\\-4 & k+2\end{pmatrix} \overset{R_2 \to R_2 - 2R_1}{\longleftrightarrow} \begin{pmatrix}-2 & 3\\0 & k-4\end{pmatrix} \tag{0.0.5}$$

Since $rank(\mathbf{M}) = 1$, the number of non zero rows left after doing row operations should be equal to 1.

Since row 1 in (0.0.5) is non zero, elements row 2 should be equal to 0.

$$\therefore k = 4 \tag{0.0.6}$$

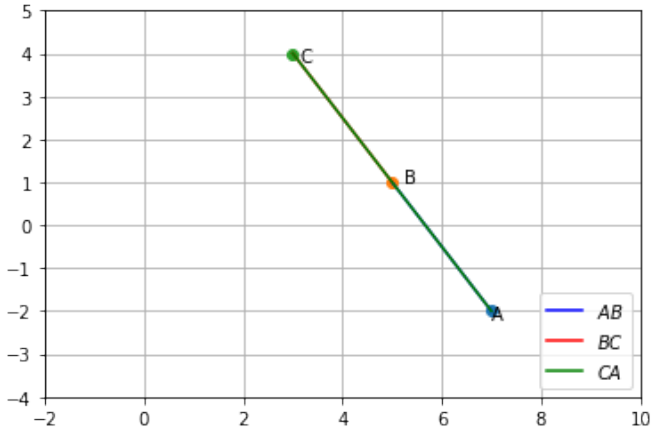

Fig. 1: Plot of the line

2) Let $\mathbf{A} = \begin{pmatrix}8\\1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}k\\-4\end{pmatrix}, \mathbf{C} = \begin{pmatrix}2\\-5\end{pmatrix}$

The direction vectors of AB and AC are

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix}k-8\\-5\end{pmatrix} \tag{0.0.7}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix}-6\\-6\end{pmatrix} \tag{0.0.8}$$

$$\mathbf{M} = \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix}^{\top} \tag{0.0.9}$$

Substituting (0.0.7) and (0.0.8) in (0.0.9), we get

$$\mathbf{M} = \begin{pmatrix} k-8 & -5 \\ -6 & -6 \end{pmatrix} \tag{0.0.10}$$

We know that if $rank\,(\mathbf{M}) = 1$, the points are collinear.
Finding the rank of the matrix in the problem,

$$\mathbf{M} = \begin{pmatrix} k-8 & -6 \\ -5 & -6 \end{pmatrix} \overset{R_2 \to 5R_2 - 6R_1}{\longleftrightarrow} \begin{pmatrix} k-8 & -5 \\ 18-6k & 0 \end{pmatrix} \tag{0.0.11}$$

Since $rank\,(\mathbf{M}) = 1$, the number of non zero rows left after doing row operations should be equal to 1.
Since row 1 in (0.0.11) is non zero for any value of k , elements row 2 should be equal to 0.

$$\therefore k = 3 \tag{0.0.12}$$

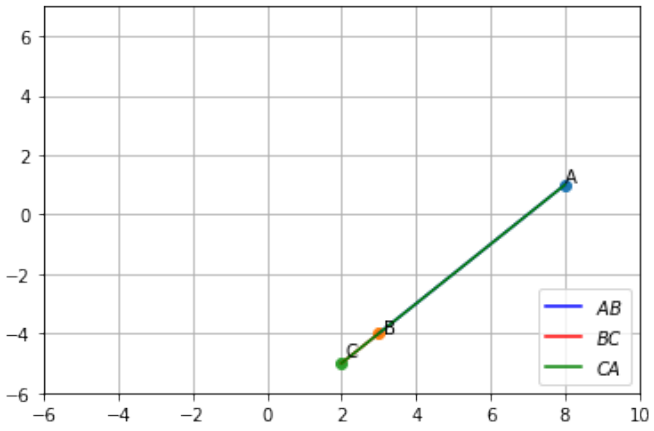

Fig. 2: Plot of the line