LINEAR SYSTEMS AND SIGNAL PROCESSING ASSIGNMENT 5

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Download latex codes from

https://github.com/VARSHITHAGANJI/EE3900_VECTORS_ASSIGNMENTS/blob/main/QUADRATIC_FORMS_ASSIGNMENT5/QUADRATIC_FORMS_ASSIGNMENT5.tex

Download all python codes from

https://github.com/VARSHITHAGANJI/EE3900_VECTORS_ASSIGNMENTS/blob/main/QUADRATIC_FORMS_ASSIGNMENT5/plot_code.py

QUESTION

Quadratic Forms 2.6

Find the area lying in the first quadrant and bounded by the circle $\mathbf{x}\mathbf{x}^\top = 4$ and the lines x = 0 and x = 2.

SOLUTION

The general equation of a circle is

$$\mathbf{x}\mathbf{x}^\top - 2\mathbf{O}^\top\mathbf{x} + \left\|O\right\|^2 - r^2 = 0 \tag{0.0.1}$$

Given equation of the circle is

$$\mathbf{x}\mathbf{x}^\top = 4 \tag{0.0.2}$$

Comparing (0.0.2) with (0.0.1), we get

$$\mathbf{O} = \begin{pmatrix}0\\0\end{pmatrix} \tag{0.0.3}$$

$$r = 2 \tag{0.0.4}$$

Given lines are

$$L_1 : \begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = 0 \tag{0.0.5}$$

$$L_2 : \begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = 2 \tag{0.0.6}$$

The angle made by lines L_1 and L_2 with the x axis i.e $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = 0$ is

$$\cos\theta = \frac{\begin{pmatrix}1\\0\end{pmatrix}^\top\begin{pmatrix}0 & 1\end{pmatrix}}{\left\|\begin{pmatrix}1 & 0\end{pmatrix}\right\|\left\|\begin{pmatrix}0 & 1\end{pmatrix}\right\|} \tag{0.0.7}$$

$$= 0 \tag{0.0.8}$$

$$\implies \theta = 90^\circ \tag{0.0.9}$$

The area of sector thus obtained is

$$\frac{\theta^\circ}{360^\circ}\pi r^2 = \frac{90^\circ}{360^\circ}\pi r^2 \tag{0.0.10}$$

$$= \frac{\pi}{4}2^2 \tag{0.0.11}$$

$$= \pi \tag{0.0.12}$$

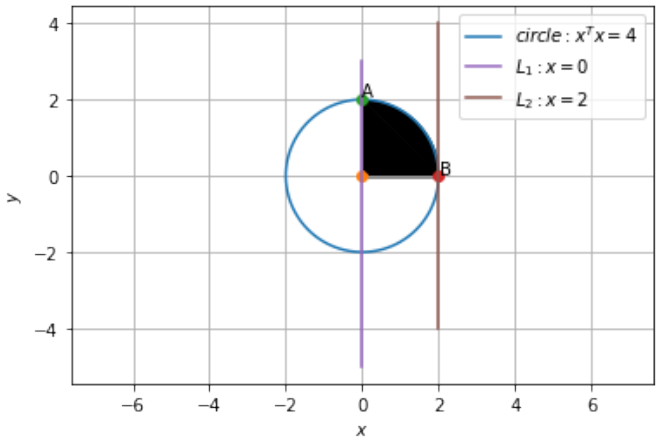

Fig. 1: Plotting the region bounded by circle and lines in first quadrant