Math computing

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NCERT 9.7.1.5

This question is from class 9 ncert chapter 7.triangles

1. Line l is the bisector of an angle $\angle A$ and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle A$. Show that:

   (a) $\triangle APB \cong \triangle AQB$

   (b) $BP = BQ$ or B is equidistant from the arms of $\angle A$.

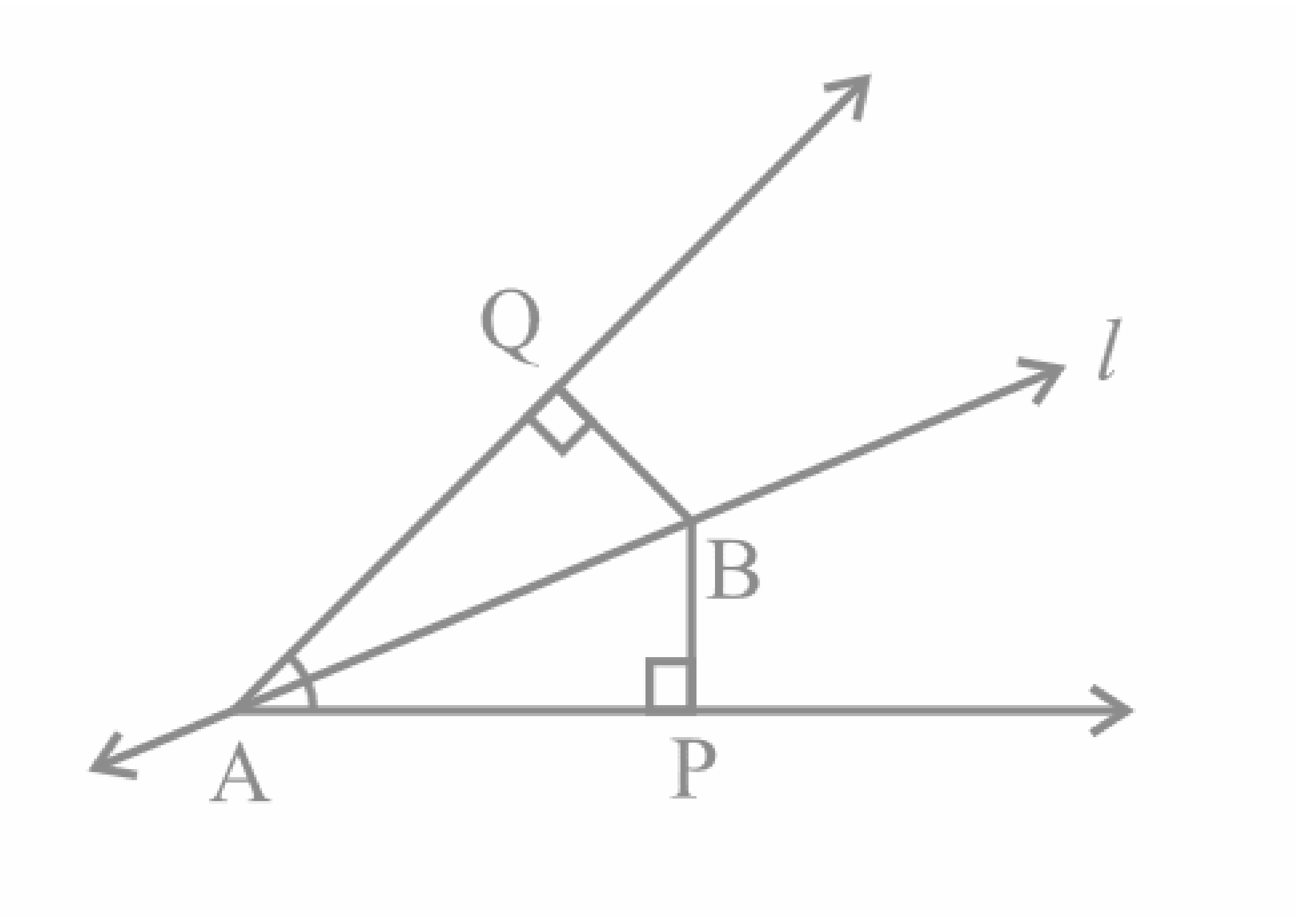

Figure 1: $\triangle AQB \quad and \quad \triangle APB$

1. **Finding foot of perpendiculars P, Q using Eigen Approach:**

   Assuming that the point A be the intersection point of two tangents of a circle with centre B and radius r. Let consider P, Q be the points of contact of two tangents to the circle.

Symbol	Value	Description
$\mathbf{A}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	coordinates of vertex A
$\mathbf{B}$	$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$	centre of circle B
r	2	radius of circle

Table 1: parameters for eigen approach

   (a) let ,consider $h = A$ and $u = -B$

$$\mathbf{h} = \mathbf{A} \tag{1}$$
$$\mathbf{u} = -\mathbf{B} \tag{2}$$
$$\mathbf{h} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{3}$$
$$\mathbf{u} = \begin{pmatrix} c \\ d \end{pmatrix} \tag{4}$$
$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{5}$$
$$\mathbf{f} = \|u\|^2 - r^2 \tag{6}$$

   (b) finding the circle equation:

$$\mathbf{gh} = \mathbf{h^T V h} + 2\mathbf{u^T h} + f \tag{7}$$
$$\mathbf{gh} = \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} + 2 \begin{pmatrix} c & d \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} + f \tag{8}$$
$$\tag{9}$$

   (c) from Table 1 and (1),

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{10}$$
$$\mathbf{u} = \begin{pmatrix} -3 \\ -3 \end{pmatrix} \tag{11}$$
$$r = 2 \tag{12}$$

   (d) on substituting these in (6) and (8) we get,

$$\mathbf{f} = 14.000000 \tag{13}$$
$$\mathbf{gh} = \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} -3 & -3 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 14.0 \tag{14}$$
$$\mathbf{gh} = 14.000000 \tag{15}$$

   (e) Finding the sigma matrix :

$$\mathbf{\Sigma} = (\mathbf{Vh} + \mathbf{u})(\mathbf{Vh} + \mathbf{u})^T - \mathbf{ghV} \tag{16}$$
$$\mathbf{\Sigma} = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} \right) \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} \right)^T - \mathbf{gh} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{17}$$

on substituting we get

$$\boldsymbol{\Sigma} = \left(\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}-3\\-3\end{pmatrix}\right)\left(\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}-3\\-3\end{pmatrix}\right)^T - 14.0\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{18}$$

$$= \begin{pmatrix}-5.000000 & 9.000000\\9.000000 & -5.000000\end{pmatrix} \tag{19}$$

(f) Finding eigen values and eigen vectors for (19) ,we get

$$\lambda = \begin{pmatrix}4\\-14\end{pmatrix} \tag{20}$$

$$\lambda_{\mathbf{1}} = 4 \tag{21}$$

$$\lambda_{\mathbf{2}} = -14 \tag{22}$$

$$\mathbf{P} = \begin{pmatrix}-0.707107 & -0.707107\\-0.707107 & 0.707107\end{pmatrix} \tag{23}$$

(g) Finding The direction vectors of the tangents from a point h to the circle

$$\mathbf{m} = \mathbf{P}\begin{pmatrix}\sqrt{|\lambda_2|}\\\pm\sqrt{|\lambda_1|}\end{pmatrix} \tag{24}$$

$$= \begin{pmatrix}-0.707107 & -0.707107\\-0.707107 & 0.707107\end{pmatrix}\begin{pmatrix}\sqrt{|-14|}\\\pm\sqrt{|4|}\end{pmatrix} \tag{25}$$

on solving we get

$$\mathbf{m_1} = \begin{pmatrix}-4.059965\\-1.231538\end{pmatrix} \tag{26}$$

$$\mathbf{m_2} = \begin{pmatrix}-1.231538\\-4.059965\end{pmatrix} \tag{27}$$

(h) Finding the contact points

$$\mathbf{x} = \mathbf{h} + \mu\mathbf{m} \tag{28}$$

Here

$$\mu = -\frac{\mathbf{m^T}\left(\mathbf{Vh}+u\right)}{\mathbf{m^TVm}} \tag{29}$$

$$\mu_{\mathbf{1}} = -\frac{\mathbf{m_1^T}\left(\mathbf{Vh}+u\right)}{\mathbf{m_1^TVm_1}} \tag{30}$$

$$\mu_{\mathbf{2}} = -\frac{\mathbf{m_2^T}\left(\mathbf{Vh}+u\right)}{\mathbf{m_2^TVm_2}} \tag{31}$$

$$\mu_{\mathbf{1}} = -\frac{\begin{pmatrix}-4.059965 & -1.231538\end{pmatrix}\left(\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}-3\\-3\end{pmatrix}\right)}{\begin{pmatrix}-4.059965 & -1.231538\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}-4.059965\\-1.231538\end{pmatrix}} \tag{32}$$

$$\mu_{\mathbf{2}} = -\frac{\begin{pmatrix}-1.231538 & -4.059965\end{pmatrix}\left(\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}-3\\-3\end{pmatrix}\right)}{\begin{pmatrix}-1.231538 & -4.059965\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}-1.231538\\-4.059965\end{pmatrix}} \tag{33}$$

on solving we get,

$$\mu_{\mathbf{1}} = -0.881917 \tag{34}$$

$$\mu_{\mathbf{2}} = -0.881917 \tag{35}$$

(i) now for contact points P and Q

$$\mathbf{P} = \mathbf{h} + \mu_{\mathbf{1}}\mathbf{m_1} \tag{36}$$

$$\mathbf{Q} = \mathbf{h} + \mu_{\mathbf{2}}\mathbf{m_2} \tag{37}$$

$$\mathbf{P} = \begin{pmatrix}0\\0\end{pmatrix} + \left(-0.881917\right)\begin{pmatrix}-4.059965\\-1.231538\end{pmatrix} \tag{38}$$

$$\mathbf{Q} = \begin{pmatrix}0\\0\end{pmatrix} + \left(-0.881917\right)\begin{pmatrix}-1.231538\\-4.059965\end{pmatrix} \tag{39}$$

$$\mathbf{P} = \begin{pmatrix}3.580552\\1.086114\end{pmatrix} \tag{40}$$

$$\mathbf{Q} = \begin{pmatrix}1.086114\\3.580552\end{pmatrix} \tag{41}$$

(j) As the lines BP and BQ are radius of circle ,So the lengths of BP and BQ are equal

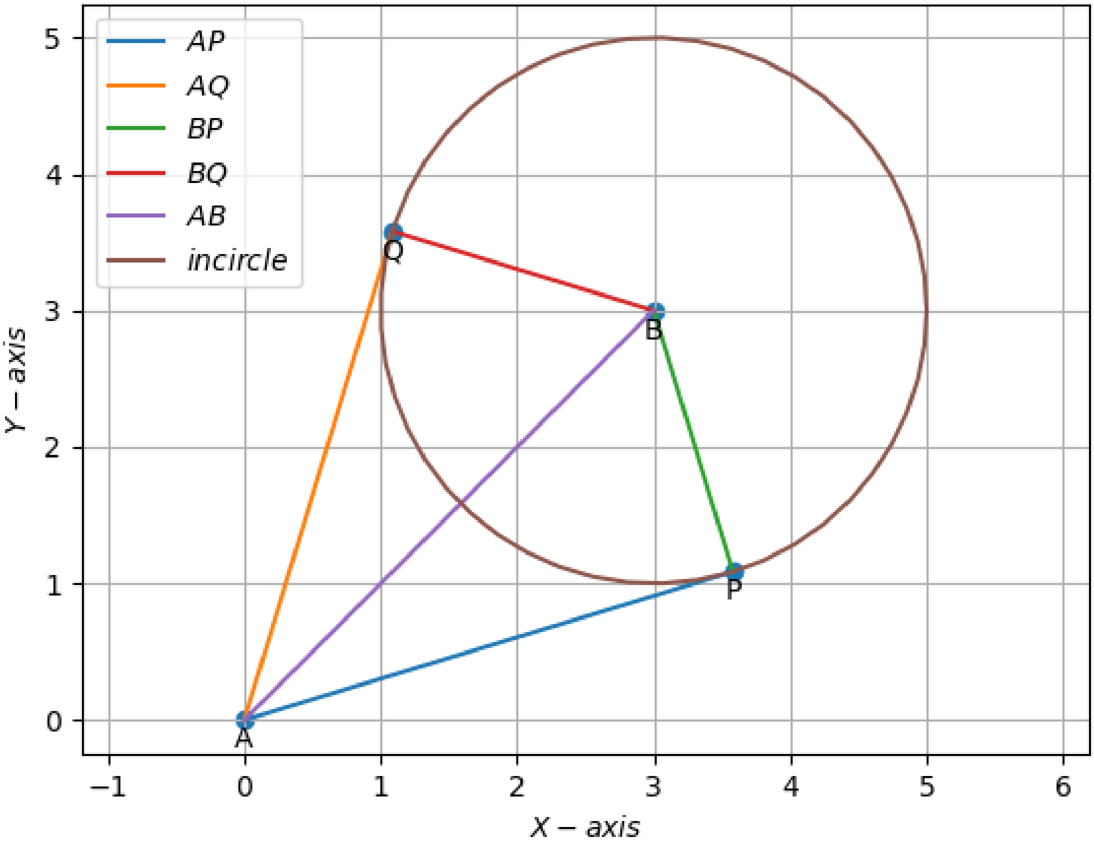

Figure 2: Contact points P and Q