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# Chapter 1

## Triangle

Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}, \quad (1.1)$$

### 1.1. Vectors

### 1.2. Median

### 1.3. Altitude

1.3.1.  $\mathbf{D}_1$  is a point on  $BC$  such that

$$AD_1 \perp BC \quad (1.3.1.1)$$

and  $AD_1$  is defined to be the altitude. Find the normal vector of  $AD_1$ .

**Solution:** Given

$$\mathbf{A} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (1.3.1.2)$$

$$\mathbf{B} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \quad (1.3.1.3)$$

$$\mathbf{C} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \quad (1.3.1.4)$$

The normal vector of  $AD_1$  is orthogonal to  $AD_1$  and hence parallel to  $BC$

Direction vector  $\mathbf{m}_{BC}$

$$= \mathbf{C} - \mathbf{B} \quad (1.3.1.5)$$

$$= \begin{pmatrix} -4 \\ -3 \end{pmatrix} - \begin{pmatrix} 3 \\ -5 \end{pmatrix} \quad (1.3.1.6)$$

$$m_{BC} = \begin{pmatrix} -7 \\ 2 \end{pmatrix} \quad (1.3.1.7)$$

Normal vector of  $AD_1$  is

$$\mathbf{n} = \begin{pmatrix} -7 \\ 2 \end{pmatrix} \quad (1.3.1.8)$$

1.3.2. Find the equation of  $AD_1$ .

**Solution:** from (1.3.1.8)

$$\mathbf{n} = \begin{pmatrix} -7 \\ 2 \end{pmatrix} \quad (1.3.2.1)$$

The equation of  $AD_1$  is

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \quad (1.3.2.2)$$

$$\Rightarrow \begin{pmatrix} -7 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -7 & 2 \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (1.3.2.3)$$

$$\begin{pmatrix} -1 & 2 \end{pmatrix} \mathbf{x} = 11 \quad (1.3.2.4)$$

see Fig. 1.1

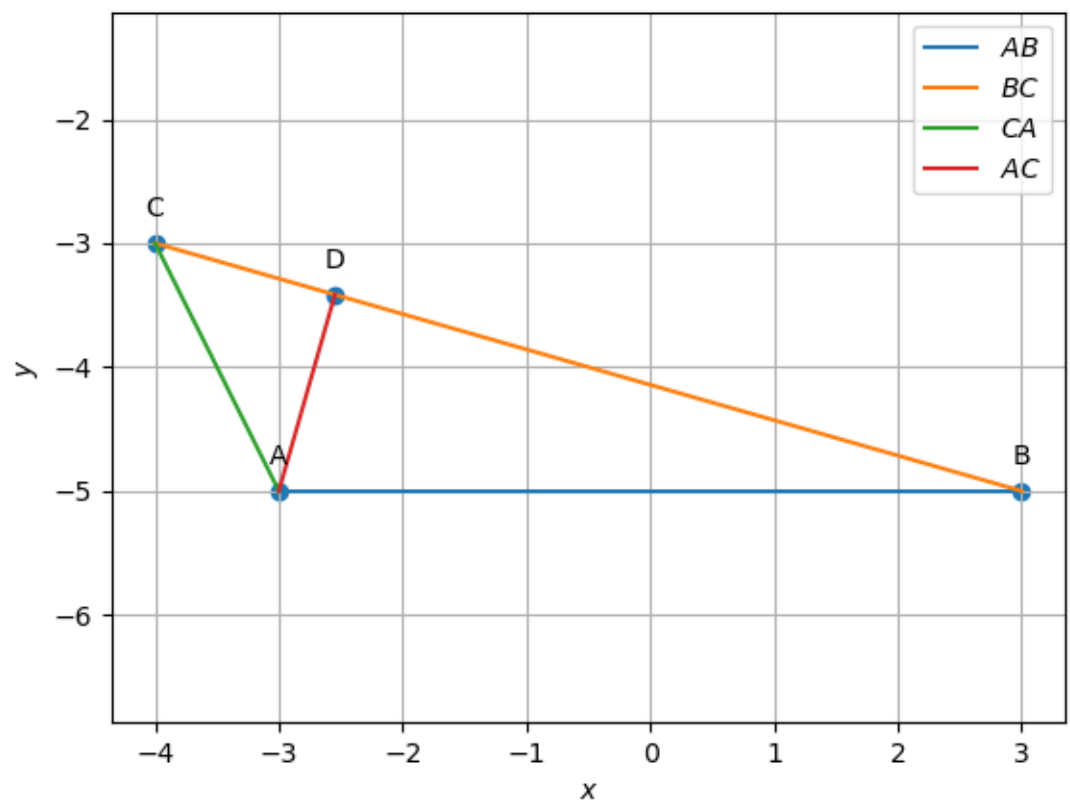


Figure 1.1: Line AD

1.3.3. Find the equations of the altitudes  $BE_1$  and  $CF_1$  to the sides  $AC$  and  $AB$  respectively.

**Solution:** Given

$$\mathbf{A} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (1.3.3.1)$$

$$\mathbf{B} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \quad (1.3.3.2)$$

$$\mathbf{C} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \quad (1.3.3.3)$$

Direction vector

$$\mathbf{m}_{AB} = \mathbf{B} - \mathbf{A} \quad (1.3.3.4)$$

$$= \begin{pmatrix} 3 \\ -5 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (1.3.3.5)$$

$$= \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (1.3.3.6)$$

$$\mathbf{m}_{AC} = \mathbf{C} - \mathbf{A} \quad (1.3.3.7)$$

$$= \begin{pmatrix} -4 \\ -3 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (1.3.3.8)$$

$$= \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (1.3.3.9)$$

$$(1.3.3.10)$$

Normal vector of  $BE_1$  is orthogonal to  $BE_1$  and hence parallel to  $AC$

and normal vector of  $CF_1$  is orthogonal to  $CF_1$  and hence parallel to  $AB$

$$\mathbf{n}_{BE_1} = \mathbf{m}_{AC} \quad (1.3.3.11)$$

$$= \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (1.3.3.12)$$

$$\mathbf{n}_{CF_1} = \mathbf{m}_{AB} \quad (1.3.3.13)$$

$$= \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (1.3.3.14)$$

$$(1.3.3.15)$$

Equation of line is represented by :

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{p}) = 0 \quad (1.3.3.16)$$

(a) The equation of line  $CF_1$

$$\mathbf{n}_{CF_1}^\top (\mathbf{x} - \mathbf{C}) = 0 \quad (1.3.3.17)$$

$$\mathbf{n}_{CF_1}^\top \mathbf{x} = \mathbf{n}_{CF_1}^\top \mathbf{C} \quad (1.3.3.18)$$

$$\begin{pmatrix} 6 \\ 0 \end{pmatrix}^\top \mathbf{x} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}^\top \begin{pmatrix} -4 \\ -3 \end{pmatrix} \quad (1.3.3.19)$$

$$\begin{pmatrix} 6 & 0 \end{pmatrix} \mathbf{x} = -24 \quad (1.3.3.20)$$



(b) The equation of line  $BE_1$

$$\mathbf{n}_{BE_1}^\top (\mathbf{x} - \mathbf{B}) = 0 \quad (1.3.3.21)$$

$$\mathbf{n}_{CF_1}^\top \mathbf{x} = \mathbf{n}_{BE_1}^\top \mathbf{B} \quad (1.3.3.22)$$

$$\begin{pmatrix} -1 \\ 2 \end{pmatrix}^\top \mathbf{x} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}^\top \begin{pmatrix} 3 \\ -5 \end{pmatrix} \quad (1.3.3.23)$$

$$\begin{pmatrix} -1 & 2 \end{pmatrix} \mathbf{x} = -13 \quad (1.3.3.24)$$

see Fig. 1.2

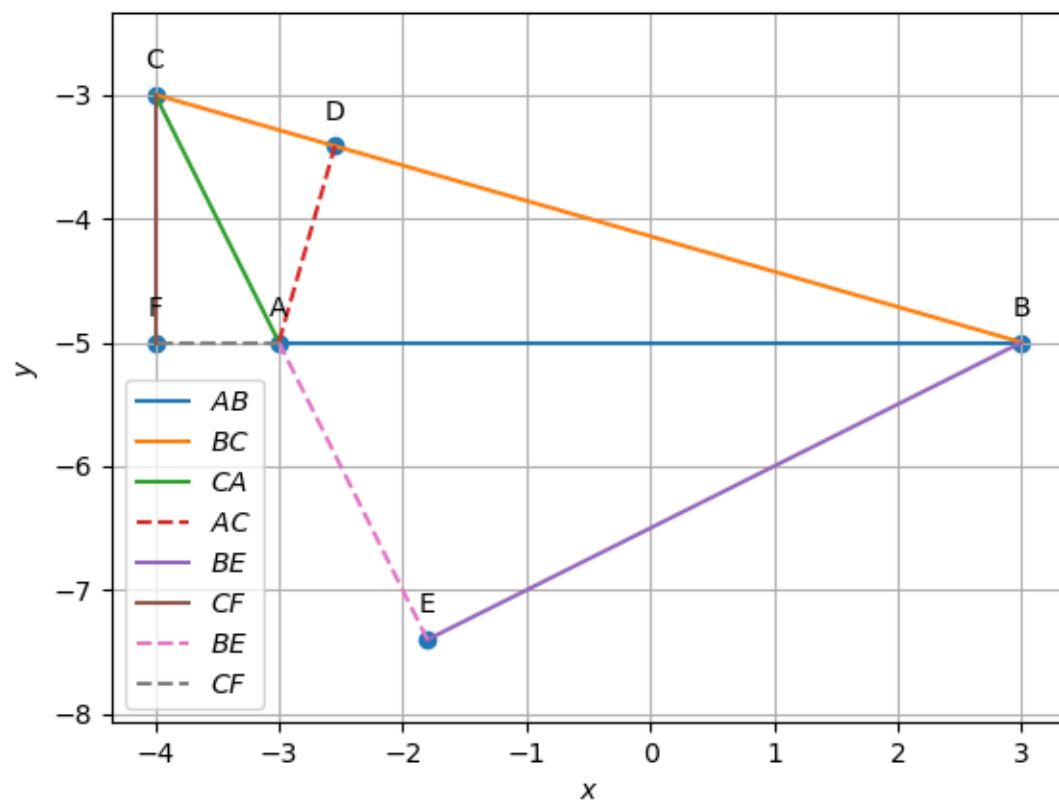


Figure 1.2: Lines  $\mathbf{BE_1}$  and  $\mathbf{CF_1}$

1.3.4. Find the intersection  $\mathbf{H}$  of  $BE_1$  and  $CF_1$ .

**Solution:** Equation of  $\mathbf{BE_1}$

$$\begin{pmatrix} -1 & 2 \end{pmatrix} \mathbf{x} = -13 \quad (1.3.4.1)$$

Equation of  $\mathbf{CF}_1 //$

$$\begin{pmatrix} 6 & 0 \end{pmatrix} \mathbf{x} = -24 \quad (1.3.4.2)$$

Therefore ,we need to solve the following equation to get  $\mathbf{H}$  :

$$\begin{pmatrix} -1 & 2 \\ 6 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -13 \\ -24 \end{pmatrix} \quad (1.3.4.3)$$

which can be solved as

$$\begin{pmatrix} -1 & 2 & -13 \\ 6 & 0 & -24 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow -R_1} \begin{pmatrix} 1 & -2 & 13 \\ 6 & 0 & -24 \end{pmatrix} \quad (1.3.4.4)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 6R_1} \begin{pmatrix} 1 & -2 & 13 \\ 0 & 12 & -102 \end{pmatrix} \quad (1.3.4.5)$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{12}} \begin{pmatrix} 1 & -2 & 13 \\ 0 & 1 & \frac{-17}{2} \end{pmatrix} \quad (1.3.4.6)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + 2R_2} \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & \frac{-17}{2} \end{pmatrix} \quad (1.3.4.7)$$

yielding

$$\mathbf{H} = \frac{-1}{2} \begin{pmatrix} 8 \\ 17 \end{pmatrix}, \quad (1.3.4.8)$$

See Fig. 1.3

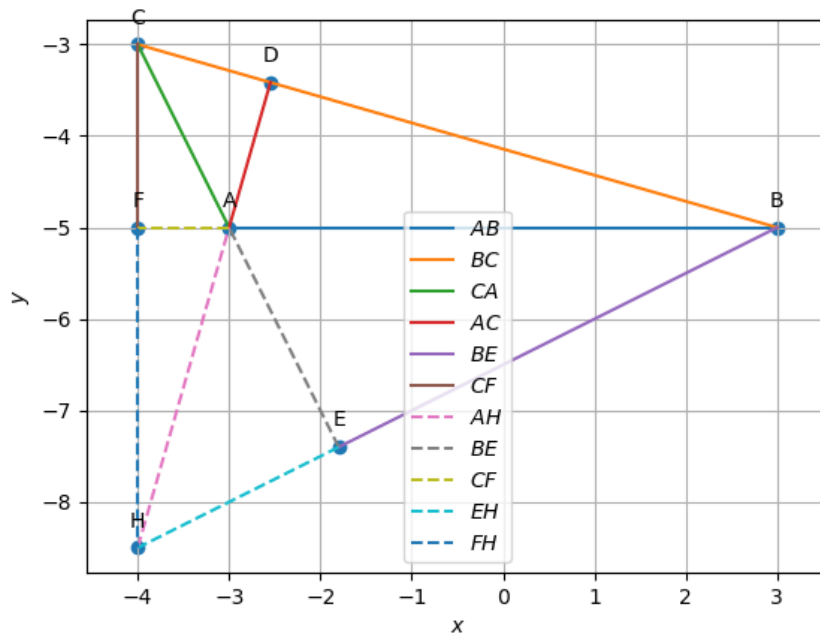


Figure 1.3: Intersection point  $\mathbf{H}$  of altitudes  $BE_1$  and  $CF_1$  plotted using python

1.3.5. Verify that

$$(\mathbf{A} - \mathbf{H})^\top (\mathbf{B} - \mathbf{C}) = 0 \quad (1.3.5.1)$$

**Solution:**

$$\mathbf{A} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (1.3.5.2)$$

$$\mathbf{B} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \quad (1.3.5.3)$$

$$\mathbf{C} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \quad (1.3.5.4)$$

$$\mathbf{H} = \frac{-1}{2} \begin{pmatrix} 8 \\ 17 \end{pmatrix} \quad (1.3.5.5)$$

$$\mathbf{A} - \mathbf{H} = \frac{1}{2} \begin{pmatrix} 2 \\ 7 \end{pmatrix}, \mathbf{B} - \mathbf{C} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \quad (1.3.5.6)$$

$$\Rightarrow (\mathbf{A} - \mathbf{H})^\top (\mathbf{B} - \mathbf{C}) = \frac{1}{2} \begin{pmatrix} 2 & 7 \end{pmatrix} \begin{pmatrix} 7 \\ -2 \end{pmatrix} = 0 \quad (1.3.5.7)$$

see Fig. 1.4

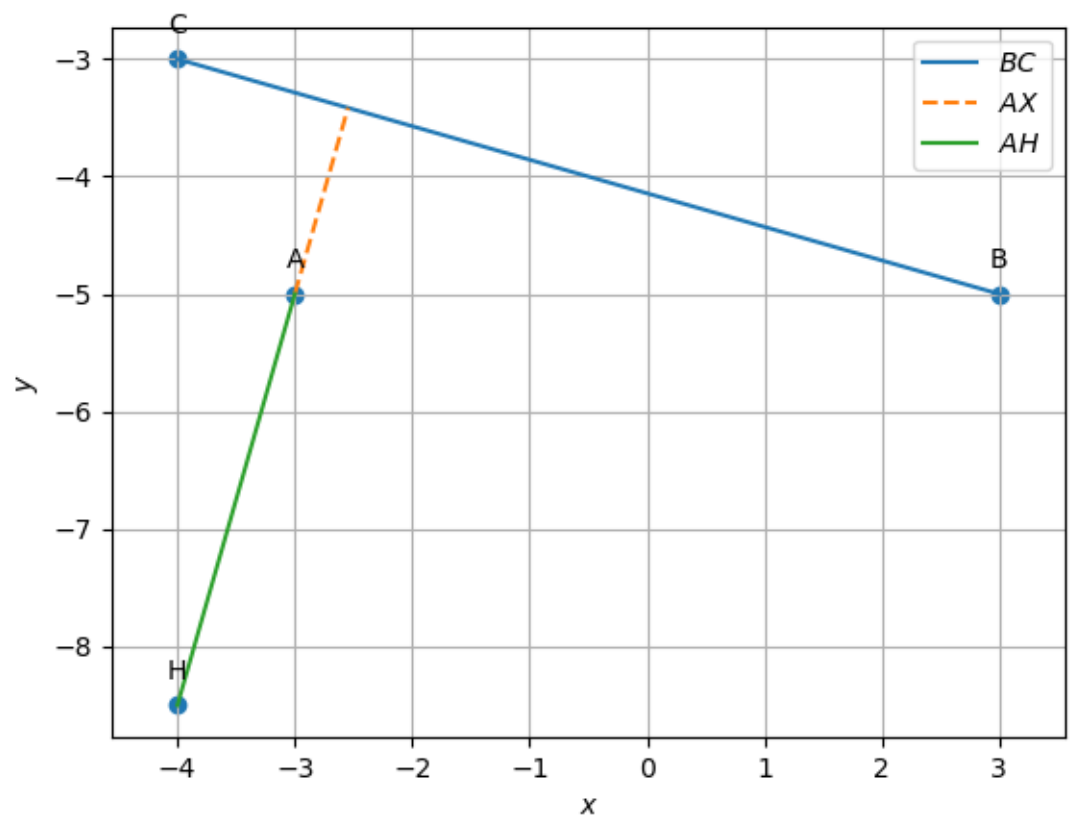


Figure 1.4: Plot of points A,B,C and H