NCERT 9.7.1.5

This Question is from ncert class 9 chapter 7.Triangles

1. Line l is the bisector of an angle $\angle A$ and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle A$. Show that:

   (a) $\triangle APB \cong \triangle AQB$

   (b) $BP = BQ$ or B is equidistant from the arms of $\angle A$.

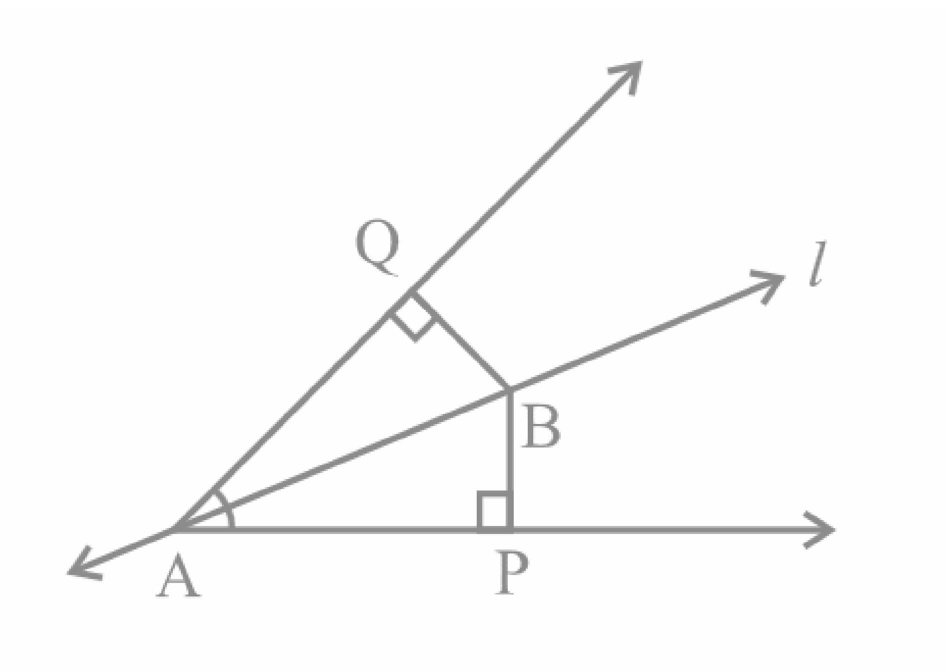

Figure 1: $\triangle AQB \quad and \quad \triangle APB$

solution:

Given l is the angular bisector of $\angle QAP$,so $\angle QAB = \angle PAB$ and

BP,BQ are perpendicular bisectors so angle =90° . let

$$\angle QAB = \angle PAB = \theta = 30^\circ \tag{1}$$

$$AP = r = 4 \tag{2}$$

$$AB = 5 \tag{3}$$

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} r\cos(\theta) \\ -r\sin(\theta) \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} r\cos(\theta) \\ r\sin(\theta) \end{pmatrix} \tag{4}$$

$$\tag{5}$$

1. (a) By the definition of AAS (Angle Angle Side) congruency rule ,If two triangles have two equal angles and a pair of corresponding sides are equal then the two triangles said to be congruent. Here

$$\angle QAB = \angle PAB = 30^\circ \quad (l \text{ is angular bisector of } \angle QAP) \tag{6}$$

$$\angle AQB = \angle APB = 90^\circ \quad (BP \text{ and } BQ \text{ are perpendicular bisectors}) \tag{7}$$

$$AB = 5 \quad \text{(common side for two triangles)} \tag{8}$$

$\therefore$ By AAS congruency rule $\triangle APB \cong \triangle AQB$

(b)

$$\begin{aligned}
\|\mathbf{B}-\mathbf{Q}\| &= \sqrt{(\mathbf{B}-\mathbf{Q})^{\top}(\mathbf{B}-\mathbf{Q})} && (9)\\
\mathbf{B}-\mathbf{Q} &= \begin{pmatrix} 5 \\ 0 \end{pmatrix} - \begin{pmatrix} r\cos(\theta) \\ r\sin(\theta) \end{pmatrix} && (10)\\
\mathbf{B}-\mathbf{Q} &= \begin{pmatrix} 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 3.464101 \\ 2 \end{pmatrix} && (11)\\
\mathbf{B}-\mathbf{Q} &= \begin{pmatrix} 1.535899 \\ -2 \end{pmatrix} && (12)\\
(\mathbf{B}-\mathbf{Q})^{\top} &= \begin{pmatrix} 1.535899 \\ -2 \end{pmatrix}^{\top} = \begin{pmatrix} 1.535899 & -2 \end{pmatrix} && (13)\\
(\mathbf{B}-\mathbf{Q})^{\top}(\mathbf{B}-\mathbf{Q}) &= \begin{pmatrix} 1.535899 & -2 \end{pmatrix} \begin{pmatrix} 1.535899 \\ -2 \end{pmatrix} && (14)\\
&= 2.3589857 + 4 && (15)\\
&= 6.3589857 && (16)\\
\sqrt{(\mathbf{B}-\mathbf{Q})^{\top}(\mathbf{B}-\mathbf{Q})} &= \sqrt{6.3589857} && (17)\\
\implies \|\mathbf{B}-\mathbf{Q}\| &= \mathbf{BQ} = 2.5217029 && (18)
\end{aligned}$$

$$\|\mathbf{B}-\mathbf{P}\| = \sqrt{(\mathbf{B}-\mathbf{P})^{\top}(\mathbf{B}-\mathbf{P})} \tag{19}$$

$$\mathbf{B}-\mathbf{P} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} - \begin{pmatrix} r\cos(\theta) \\ -r\sin(\theta) \end{pmatrix} \tag{20}$$

$$\mathbf{B}-\mathbf{P} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 3.464101 \\ -2 \end{pmatrix} \tag{21}$$

$$\mathbf{B}-\mathbf{P} = \begin{pmatrix} 1.535899 \\ 2 \end{pmatrix} \tag{22}$$

$$(\mathbf{B}-\mathbf{P})^{\top} = \begin{pmatrix} 1.535899 \\ 2 \end{pmatrix}^{\top} = \begin{pmatrix} 1.535899 & 2 \end{pmatrix} \tag{23}$$

$$(\mathbf{B}-\mathbf{P})^{\top}(\mathbf{B}-\mathbf{P}) = \begin{pmatrix} 1.535899 & 2 \end{pmatrix} \begin{pmatrix} 1.535899 \\ 2 \end{pmatrix} \tag{24}$$

$$= 2.3589857 + 4 \tag{25}$$

$$= 6.3589857 \tag{26}$$

$$\sqrt{(\mathbf{B}-\mathbf{P})^{\top}(\mathbf{B}-\mathbf{P})} = \sqrt{6.3589857} \tag{27}$$

$$\implies \|\mathbf{B}-\mathbf{P}\| = \mathbf{BP} = 2.5217029 \tag{28}$$

$\therefore BP = BQ$ Hence proved

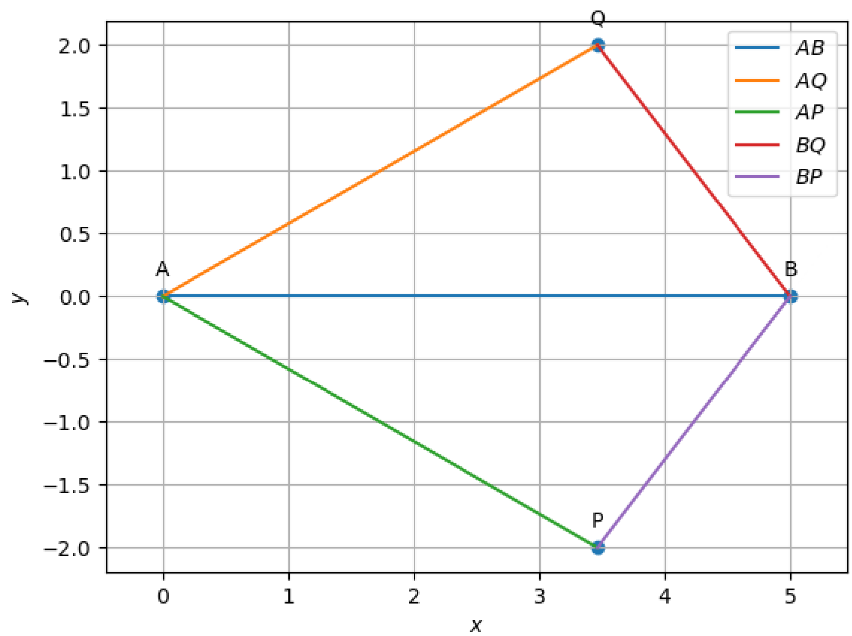

Figure 2: $\triangle APB \quad and \quad \triangle AQB$