Math computing

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NCERT 9.7.1.5

This question is from class 9 ncert chapter 7.triangles

1. Line l is the bisector of an angle $\angle A$ and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle A$. Show that:
   (a) $\triangle APB \cong \triangle AQB$
   (b) $BP = BQ$ or B is equidistant from the arms of $\angle A$.

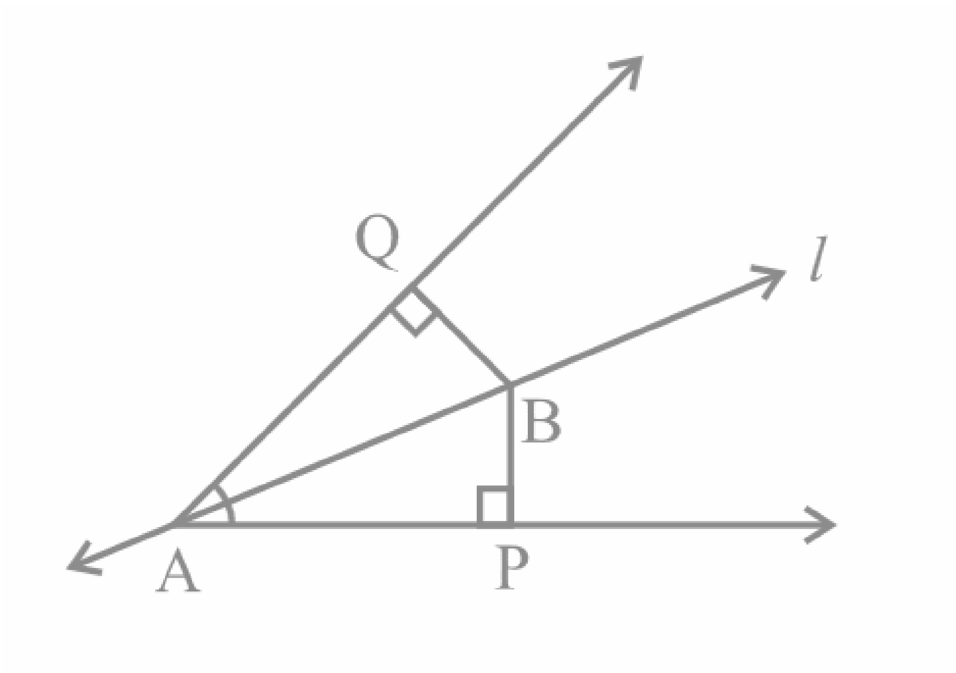

Figure 1: $\triangle AQB \quad and \quad \triangle APB$

Construction steps:

1. (a) Let point A be the reference point whose coordinates are at origin.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

(b) Let the distance between point A and B be x ,and also considering the point B on same axis .

$$\|A - B\| = x \tag{2}$$

(c) So,the coordinates of point B be,

$$\mathbf{B} = \begin{pmatrix} x \\ 0 \end{pmatrix} \tag{3}$$

(d) Let the coordinates of point P be,

$$\mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{4}$$

(e) And,let the coordinates of point Q be,

$$\mathbf{Q} = \begin{pmatrix} c \\ d \end{pmatrix} \tag{5}$$

(f) Let assume the distance between point A and P be r ,and let the line AB makes an angle θ anticlock wise with line AP.

$$\|A - P\| = r \tag{6}$$
$$\angle PAB = \theta \tag{7}$$

(g) Now the coordinates of point P will be,

$$\mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ -r\sin\theta \end{pmatrix} \tag{8}$$

(h) Similarly , let assume the distance between point A and Q also be r , and the line AB makes an angle θ clock wise with line AQ.

$$\|A - Q\| = r \tag{9}$$
$$\angle QAB = \theta \tag{10}$$

(i) Now the coordinates of point Q will be,

$$\mathbf{Q} = \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} \tag{11}$$

(j) Now the coordinates of A,B,P,Q are ,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} \quad = \begin{pmatrix} x \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} r\cos\theta \\ -r\sin\theta \end{pmatrix}, \mathbf{Q} \quad = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} \tag{12}$$

(k) Let assume,

$$x = 5 \tag{13}$$

$$r = 4 \tag{14}$$

$$\theta = 30^\circ \tag{15}$$

(l) on substituting the values ,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} \quad = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 4\cos 30^\circ \\ -4\sin 30^\circ \end{pmatrix}, \mathbf{Q} \quad = \begin{pmatrix} 4\cos 30^\circ \\ 4\sin 30^\circ \end{pmatrix} \tag{16}$$

(m) on calculating we get ,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} \quad = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 3.464101 \\ -2 \end{pmatrix}, \mathbf{Q} \quad = \begin{pmatrix} 3.464101 \\ 2 \end{pmatrix} \tag{17}$$

Joining these points forms the required figure

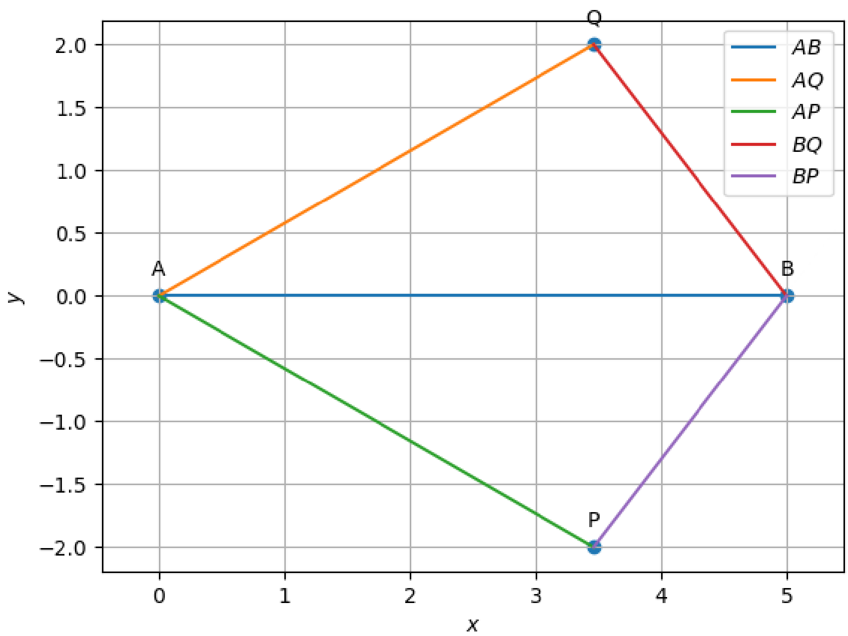

Figure 2: $\triangle APB \quad and \quad \triangle AQB$