Discrete Optimization

Integer linear programming formulations of multiple salesman problems and its variations

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Abstract

In this paper, we extend the classical multiple traveling salesman problem (mTSP) by imposing a minimal number of nodes that a traveler must visit as a side condition. We consider single and multidepot cases and propose integer linear programming formulations for both, with new bounding and subtour elimination constraints. We show that several variations of the multiple salesman problem can be modeled in a similar manner. Computational analysis shows that the solution of the multidepot mTSP with the proposed formulation is significantly superior to previous approaches. © 2005 Elsevier B.V. All rights reserved.

Keywords: Integer programming; Multidepot multisalesmen problem; Subtour elimination constraints

1. Introduction

A generalization of the well-known Traveling Salesman Problem is the standard multiple Traveling Salesman Problem (mTSP). The problem can be defined simply as the determination of a set of routes for m salesmen who all start from and return to a single home city (depot). The mTSP finds applications in printing press scheduling [7], crew scheduling [12], mission planning for autonomous mobile robots [1], hot rolling scheduling [16] and interview scheduling [5]. The mTSP is the core of vehicle routing problems (VRPs), which are central to logistics management. Most of the mathematical formulations of VRPs are variations and/or extensions of the mTSP. Thus, a practical mTSP model that is solvable by a commercial code will facilitate the solution of its variations as well those of moderate-sized VRPs.

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There are several integer linear programming formulations (ILPFs) proposed for the mTSP and its variants (see e.g. [13,15,3,9,8,4]). However, to the best of our knowledge, there is no formulation for the mTSP or its variants in which a restriction is imposed on minimal number of nodes visited by any traveler. Such a restriction may frequently arise in real-life applications in the assignment of appropriate workloads to the travelers.

In this paper, integer linear programming formulations of single depot and multidepot mTSPs are given with additional side conditions and new subtour elimination constraints (SECs). The main contribution of this paper is the introduction of new side constraints for the mTSP and its variants, as well as SECs for all the problems considered here. The paper also shows that for various values of the problem's parameters, the proposed formulations reduce to those of some classical routing problems.

The paper is organized as follows. The next section deals with the single depot mTSP and its variants. Section 3 describes the multidepot mTSP and its variants with additional side conditions. Computational analysis is given in Section 4, and the paper ends with concluding remarks in Section 5.

2. Single depot mTSP

2.1. Problem definition and notation

Consider a complete directed graph $G = (V, A)$, where V is the set of n nodes (vertices), A is the set of arcs and $C = (c_{ij})$ is the cost (distance) matrix associated with each arc $(i, j) \in A$. The cost matrix C can be symmetric, asymmetric or Euclidean. Let there be m salesmen located at the depot city 1. Then, the single depot mTSP consists of finding tours for m salesmen such that all start and end at the depot, each other node is located in exactly one tour, the number of nodes visited by a salesman lies within a predetermined interval, and the overall cost of visiting all nodes is minimized.

Let us define x_{ij} as a binary variable equal to 1 if arc (i, j) is in the optimal solution and 0 otherwise. For any traveler, u_i is the number of nodes visited on that traveler's path from the origin up to node i (i.e., the visit number of the ith node). L is the maximum number of nodes a salesman may visit; thus, $1 \leqslant u_i \leqslant L$ for all $i \geqslant 2$. In addition, let K be the minimum number of nodes a salesman must visit, i.e., if $x_{i1} = 1$, then $K \leqslant u_i \leqslant L$ must be satisfied.

2.2. Formulation

We propose the following integer linear programming formulation for the mTSP defined above.

$$\text{minimize} \quad \sum_{(i,j) \in A} c_{ij} x_{ij} \tag{1}$$

$$\text{s.t.} \quad \sum_{j=2}^{n} x_{1j} = m, \tag{2}$$

$$\sum_{j=2}^{n} x_{j1} = m, \tag{3}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 2, \ldots, n, \tag{4}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 2, \ldots, n, \tag{5}$$

$$u_i + (L-2)x_{1i} - x_{i1} \leqslant L-1, \quad i=2,\ldots,n,$$ (6)

$$u_i + x_{1i} + (2-K)x_{i1} \geqslant 2, \quad i=2,\ldots,n,$$ (7)

$$x_{1i} + x_{i1} \leqslant 1, \quad i=2,\ldots,n,$$ (8)

$$u_i - u_j + Lx_{ij} + (L-2)x_{ji} \leqslant L-1, \quad 2 \leqslant i \neq j \leqslant n,$$ (9)

$$x_{ij} \in \{0,1\}, \quad \forall (i,j) \in A.$$ (10)

This formulation is valid when $2 \leqslant K \leqslant \lfloor (n-1)/m \rfloor$ and $L \geqslant K$. When $K \geqslant 4$, constraints (6) and (7) do not allow the situation $x_{1i} = x_{i1} = 1$, i.e., constraint (8) becomes redundant when $K \geqslant 4$. Thus, we need constraint (8) only for the cases $K = 3$ or $K = 2$.

In this formulation, constraints (2) and (3) ensure that exactly m salesmen leave from and return to the depot. Constraints (4) and (5) are the degree constraints. As will be shown in Proposition 1 below, the inequalities given in (6) and (7) serve as upper and lower bound constraints on the number of nodes visited by each salesman, and initialize the value of u_i to 1 if and only if i is the first node on the tour for any salesman. Constraints (6) and (7) we call *bounding constraints*, and these are completely new for the mTSP. Inequality (8) forbids a vehicle from visiting only a single node. The inequalities given in (9) ensure that $u_j = u_i + 1$ if and only if $x_{ij} = 1$. Thus, they prohibit the formation of any subtour between nodes in $V\backslash\{1\}$, so they are the subtour elimination constraints (SECs) of the formulation.

Proposition 1. *Bounding constraints given in* (6) *and* (7) *are valid inequalities for the mTSP for $K \geqslant 2$.*

Proof. Since $x_{1i} = x_{i1} = 1$ is not allowed, then the three remaining cases should be considered:

  (i) If $x_{1i} = x_{i1} = 0$, then (6) and (7) together imply $2 \leqslant u_i \leqslant L-1$.
  (ii) If $x_{1i} = 1$ and $x_{i1} = 0$, then, from (6) and (7) we obtain $u_i \leqslant 1$ and $u_i \geqslant 1$, which implies $u_i = 1$.
  (iii) If $x_{1i} = 0$ and $x_{i1} = 1$, then we obtain $K \leqslant u_i \leqslant L$.

This completes the proof.   □

The following proposition shows the relation between existing and proposed SECs for the mTSP.

Proposition 2. *The constraints*

$$u_i - u_j + Lx_{ij} + (L-2)x_{ji} \leqslant L-1, \quad 2 \leqslant i \neq j \leqslant n,$$ (11)

with $u_i, u_j \in [1, n-1]$ are lifted Kulkarni–Bhave SECs [8] *for the mTSP.*

Proof. Similar to the proof of Proposition 1 in Desrochers and Laporte [2].   □

The bounding constraints (6) and (7) play an important role in the formulation of the mTSP variants described in Section 3.

2.3. Discussion

If there is no restriction on the minimal number of nodes that a salesman may visit, it is sufficient to take the coefficient of x_{i1} in (7) as equal to zero. Also, if the salesmen are allowed to return to the depot after visiting only one node, constraint (8) should be omitted.

For some cases, the lower bound K may be imposed but there might not be any restriction on the upper bound L. In this case, assume that each vehicle except one visits $K(m-1)$ nodes. Then, the remaining

$(n - 1) - K(m - 1)$ nodes have to be visited by the remaining vehicle. Thus, since $K \leqslant \lfloor (n - 1)/m \rfloor$, we may substitute $(n - 1) - (m - 1)K$ in place of L in the formulation.

Observe that if there is no restriction on the lower and upper bounds, it is sufficient to take the coefficient of x_{i1} in (7) as zero, omit inequality (8) and substitute $(n - m)$ in place of L in the formulation. A similar formulation can be found in [14].

To the best of our knowledge, there is no lifted-constraint formulation of the mTSP in which the lifted-constraint formulation of the TSP [2] occurs as a special case. Proposition 3 shows that the proposed formulation has this property.

Proposition 3. *The above formulation reduces to the Desrochers–Laporte formulation* [2] *for the TSP when* $m = 1$.

Proof. When $m = 1$, this means that we are dealing with the classical TSP. In this case, the lower bound K and the upper bound L must be equal to $n - 1$. If we substitute these values in (6), (7) and (9), we obtain the Desrochers–Laporte SECs [2] given for the TSP (reduced forms of (6) and (7) render (8) redundant). □

The above discussion shows that the proposed formulation represents all the cases for the single depot mTSP. We extend this formulation to include the multidepot cases in the following section.

3. Multidepot mTSP

3.1. Problem definition and notation

The multidepot mTSP (MmTSP) is a generalization of the single depot mTSP, such that there is more than one depot and a number of salesmen at each depot. Let the node set be partitioned such that $V = D \cup V'$, where the first d nodes of V are depot set D, there are m_i salesmen located at depot i initially and the total number of salesmen is m. Also, let $V' = \{d + 1, d + 2, \ldots, n\}$ be the set of customer nodes. Let the x_{ij}'s, L, K and the u_i's be defined as before.

The MmTSP consists of finding tours for all the salesmen such that all customers are visited exactly once, the number of customers visited by a salesman lies between a predetermined interval and the total cost of all the tours is minimized. If the problem is to determine a total of m tours such that the salesmen must return to their original depots, this is referred to as the *fixed destination MmTSP*. On the other hand, if the salesmen do not have to return to their original depots but the number of salesmen at each depot should remain the same at the end as it was in the beginning, we have the *nonfixed destination MmTSP*. We will formulate both problems in the following subsections.

3.2. Nonfixed destination MmTSP

Although the nonfixed destination characteristic is not explicitly stated in their paper, the first integer linear programming formulation for the nonfixed destination MmTSP with $O(n^2)$ binary variables and $O(n^2)$ constraints seems to be due to Kulkarni and Bhave [8]. There also exists a correction to this formulation by Laporte [10], who shows that this formulation is not adequate to represent a fixed destination situation. Another work regarding the nonfixed destination MmTSP is due to GuoXing [17], who presents a complete transformation of the problem to the standard TSP. Thus, the resulting problem can be solved by any algorithm devised for the TSP. This transformation makes copies of depots and results in an expanded graph.

In the above mentioned formulations, no lower bounds are imposed for the number of nodes a salesman visits. We now propose the following formulation for the nonfixed MmTSP with new side conditions:

$$\text{minimize} \quad \sum_{(i,j)\in A} c_{ij} x_{ij} \tag{12}$$

$$\text{s.t.} \quad \sum_{j\in V'} x_{ij} = m_i, \quad i \in D, \tag{13}$$

$$\sum_{i\in V'} x_{ij} = m_j, \quad j \in D, \tag{14}$$

$$\sum_{i\in V} x_{ij} = 1, \quad j \in V', \tag{15}$$

$$\sum_{j\in V} x_{ij} = 1, \quad i \in V', \tag{16}$$

$$u_i + (L-2)\sum_{k\in D} x_{ki} - \sum_{k\in D} x_{ik} \leqslant L-1, \quad i \in V', \tag{17}$$

$$u_i + \sum_{k\in D} x_{ki} + (2-K)\sum_{k\in D} x_{ik} \geqslant 2, \quad i \in V', \tag{18}$$

$$x_{ki} + x_{ik} \leqslant 1, \quad k \in D, \ i \in V', \tag{19}$$

$$u_i - u_j + Lx_{ij} + (L-2)x_{ji} \leqslant L-1, \quad i \neq j; \ i,j \in V', \tag{20}$$

$$x_{ij} \in \{0,1\} \quad \forall i,j \in V. \tag{21}$$

In this formulation, for each $i \in D$, m_i outward and m_i inward arcs are guaranteed by (13) and (14). Eqs. (15) and (16) are the degree constraints for the customer nodes. Constraints (17) and (18) impose bounds on the number of nodes a salesman visits together with initializing the value of the u_i's as 1 if i is the first node visited on the tour. Constraint (19) prohibit a salesman from serving only a single customer. Finally, constraint (20) are SECs in that they break all subtours between customer nodes.

Observe that there are $O(n^2)$ binary variables and $O(n^2)$ constraints in this formulation. It is easily seen that the MmTSP model reduces to that of the mTSP when D contains only one node, i.e., when D is a singleton.

3.3. Fixed destination MmTSP

To the best of the authors' knowledge, there exist no formulations for the fixed destination MmTSP in the literature. However, there exist some studies for the fixed destination multidepot VRP which include integer linear programming formulations. Some of these formulations are polynomial in size (see [6,8]), while others include an exponential number of constraints (see [11]). These formulations, as a special case, reduce to the fixed destination MmTSP.

The model presented above for the nonfixed destination MmTSP can be converted to the fixed destination MmTSP with $O(dn^2)$ binary variables and $O(n^2)$ constraints as follows. We first define the following binary variable.

$$x_{ijk} = \begin{cases} 1, & \text{if a traveler belonging to the } k\text{th depot goes from } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}$$

u_i's, c_{ij}'s, D, V and V' are defined as previously. We can now state the related model for the fixed destination MmTSP as follows:

$$\text{minimize} \quad \sum_{k \in D} \sum_{j \in V'} (c_{kj} x_{kjk} + c_{jk} x_{jkk}) + \sum_{k \in D} \sum_{i \in V'} \sum_{j \in V'} c_{ij} x_{ijk} \tag{22}$$

$$\text{s.t.} \quad \sum_{j \in V'} x_{kjk} = m_k, \quad k \in D, \tag{23}$$

$$\sum_{k \in D} x_{kjk} + \sum_{k \in D} \sum_{i \in V'} x_{ijk} = 1, \quad j \in V', \tag{24}$$

$$x_{kjk} + \sum_{i \in V'} x_{ijk} - x_{jkk} - \sum_{i \in V'} x_{jik} = 0, \quad k \in D, \ j \in V', \tag{25}$$

$$\sum_{j \in V'} x_{kjk} - \sum_{j \in V'} x_{jkk} = 0, \quad k \in D, \tag{26}$$

$$u_i + (L-2) \sum_{k \in D} x_{kik} - \sum_{k \in D} x_{ikk} \leqslant L-1, \quad i \in V', \tag{27}$$

$$u_i + \sum_{k \in D} x_{kik} + (2-K) \sum_{k \in D} x_{ikk} \geqslant 2, \quad i \in V', \tag{28}$$

$$\sum_{k \in D} x_{kik} + \sum_{k \in D} x_{ikk} \leqslant 1, \quad i \in V', \tag{29}$$

$$u_i - u_j + L \sum_{k \in D} x_{ijk} + (L-2) \sum_{k \in D} x_{jik} \leqslant L-1, \quad i \neq j, \ i,j \in V', \tag{30}$$

$$x_{ijk} \in \{0,1\}, \quad i,j \in V, \ k \in D. \tag{31}$$

In this formulation, constraints (23) ensure that exactly m_k salesmen depart from each depot $k \in D$. Constraints (24) ensure that each customer is visited exactly once. Route continuity for customer nodes and depot nodes is represented respectively by constraints (25) and (26). Constraints (27)–(30) play the same role as in the nonfixed destination case, i.e., they are the bounding constraints and the SECs of the formulation.

3.4. Variants

The MmTSP model given above for the nonfixed destination MmTSP can be easily adapted to other special types of problems that arise in vehicle routing. We will describe two of these problems in the following subsections.

3.4.1. Multiple departures single destination mTSP

In this section we consider a simpler variant of the MmTSP which can be a useful modeling approach for school bus routing type problems. In such problems, there are buses that depart from multiple depots and arrive at a single destination (the school), while each remaining node is visited exactly once by any bus to pick up the students located at these nodes. This variant can be formally stated as follows: Let D, K, L, m and the m_i's be defined as above. Suppose that each salesman starts his journey from any node of D, and that all finish their journey at some destination node denoted by 0 not in V. This problem consists of finding tours (paths in a sense) for all the salesmen such that all customers are visited exactly once and the total cost of all the tours is minimized. To the best of our knowledge, there is no such generalization in the related literature. We refer to this problem as the *multiple departures single destination* mTSP (MDmTSP).

For the MDmTSP, the degree constraints of node 0 will be

$$\sum_{i \in V'} x_{i0} = m \tag{32}$$

and the degree constraints for the customer nodes are written as follows:

$$\sum_{j \in V' \cup \{0\}} x_{ij} = 1, \quad i \in V'. \tag{33}$$

The bounding constraints are written as follows:

$$u_i + (L-2) \sum_{k \in D} x_{ki} - x_{i0} \leqslant L - 1, \quad i \in V', \tag{34}$$

$$u_i + \sum_{k \in D} x_{ki} + (2-K)x_{i0} \geqslant 2, \quad i \in V' \tag{35}$$

and the restriction for single customer visits becomes

$$\sum_{k \in D} x_{ki} + x_{i0} \leqslant 1, \quad \forall i \in V'. \tag{36}$$

So, the integer programming formulation of the MDmTSP is given as follows: $\text{Min} \sum_{i=1}^{n} \sum_{j=0}^{n} c_{ij} x_{ij}$: s.t. (13), (15), (20), (21), (32), (33)–(36).

The formulation presented for the MDmTSP may be easily transformed to the case when m salesmen start their trip at a node 0 and m_i of them finish at the ith node of D. This can be referred to as the *single departure multiple destination* mTSP.

3.4.2. The multiple Hamiltonian path problem

We now consider another case. Let D contain only one depot (node 1) and m salesmen start their trip from node 1 and finish at node 0. We will have, in a sense, the *multiple Hamiltonian path problem* (mHPP). When such a case arises, V' reduces to $\{2,3,\ldots,n\}$ and the degree constraints of nodes 0 and 1 will be as follows:

$$\sum_{i \in V'} x_{1i} = m, \tag{37}$$

$$\sum_{i \in V'} x_{i0} = m \tag{38}$$

and bounding constraints reduce to

$$u_i + (L-2)x_{1i} - x_{i0} \leqslant L - 1, \quad i \in V', \tag{39}$$

$$u_i + x_{1i} + (2-K)x_{i0} \geqslant 2, \quad i \in V'. \tag{40}$$

Thus, the integer programming formulation of the mHPP will be as follows: $\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$: s.t. (15), (20), (21), (33), (36)–(40).

Observe that when the starting node is 1 and the ending node is 0 and there are $n-1$ customers, the mHPP model reduces to the integer linear programming formulation of the Hamiltonian path problem if $m = 1$ (obviously, L and K would be the same and equal to $n-1$ in this case).

4. Computational experiments

In this section, we describe our experience with some computational experiments done using the formulations proposed in this paper. In the two following subsections, computational results on the standard mTSP and the MmTSP are presented, respectively.

4.1. Experiments with the mTSP formulation

In this section, we present our results for the mTSP formulation defined by (1)–(10), since all the other formulations presented in this paper are derived from this formulation. For this purpose, we have randomly generated (asymmetric) instances with $n = 30, 50, 70$, $m = 3, 4, 5$ using various values for the parameters K and L. The elements of the distance matrix have been randomly chosen from the interval $[1, 100]$. The optimization package CPLEX 6.0 has been used to solve each formulation on a Pentium III 1400 MHz PC running Linux. We summarize the results below.

- For problems with $n = 30$, the parameters were chosen from the following intervals: $3 \leqslant K \leqslant 5$, $6 \leqslant L \leqslant 15$. Over a total of 18 instances, the average solution time to solve these instances to optimality was 13.06 CPU seconds.
- For problems with $n = 50$, the parameters were chosen from the following intervals: $3 \leqslant K \leqslant 5$, $10 \leqslant L \leqslant 25$. Over a total of 18 instances, the average solution time to solve these instances to optimality was 158.33 CPU seconds.
- For problems with $n = 70$, the parameters were chosen from the following intervals: $3 \leqslant K \leqslant 5$, $15 \leqslant L \leqslant 30$. Over a total of 18 instances, the average solution time to solve these instances to optimality was 747.17 CPU seconds.

The above results demonstrate that it is possible to solve moderate-sized instances to optimality using the formulation presented in this paper. In addition, the formulation easily permits one to change the parameters K and L of an instance without a significant increase in computation time.

4.2. Experiments with the nonfixed destination MmTSP formulation

In this section, we compare the results of our formulation proposed for the nonfixed destination MmTSP (defined by (12)–(21)) to those obtained via Kulkarni and Bhave's formulation [8] (denoted by *KB*) and those obtained via GuoXing's transformation procedure [17]. The transformation procedure (denoted by *TR*) consists of transforming the nonfixed destination MmTSP to the standard TSP and solving the latter problem. The resulting TSPs have been solved using the relevant formulation suggested by Desrochers and Laporte [2].

Testing was performed on randomly generated instances with the number of nodes chosen as 100 and 120, the number of depots chosen as 3, 4 and 5, and up to 2 salesmen at each depot. The first three columns of Table 1 show 12 configurations of these parameters. For each configuration, 30 instances have been generated randomly, using the same settings as described in Section 4.1. In order to be able to compare our formulation with the existing formulations, we set $L = (n - d) - (m - 1)$ and imposed no lower bounds.

We present the results of the experiments in Table 1, where n refers to the size of the problem, d refers to the number of depots, and s refers to the number of salesmen at each depot. The columns labeled *TR*, *KB*, and *M* show the average CPU time (in seconds) required by each formulation, calculated across 30 instances for each configuration. A 3-hours limit was imposed on the solution time for each formulation. We denote by "*" the sets which include instances that could not be solved to optimality within the given time limit. The last two columns present the average reduction in solution times obtained by using the proposed formulation *M* as opposed to using the transformation procedure *TR* or the formulation *KB*, respectively. These entries are calculated as

$$M/TR = \frac{t(TR) - t(M)}{t(TR)} * 100 \tag{41}$$

Table 1
Results of comparisons for the nonfixed destination MmTSP

n	d	s	TR	KB	M	M/TR (%)	M/KB (%)
100	3	1	274.63	334.37	48.23	82.44	85.58
100	4	1	330.23	533.47	40.20	87.83	92.46
100	5	1	294.17	293.67	22.30	92.42	92.41
120	3	1	816.87	1176.20	97.07	88.12	91.75
120	4	1	706.03	1131.47	57.40	91.87	94.93
120	5	1	530.77	358.60	50.00	90.58	86.06
100	3	2	496.50	493.07	17.33	96.51	96.49
100	4	2	770.93	82.30	18.80	97.56	77.16
100	5	2	431.28	23.83	16.56	96.16	30.51
120	3	2	1512.10[*]	603.77	97.53	93.55	83.85
120	4	2	1427.10[*]	647.23	55.70	96.10	91.39
120	5	2	846.27[*]	265.03	45.30	94.65	82.91

[*] Indicates that the set contains some instances that could not be solved within the 3 hours limit.

and

$$M/KB = \frac{t(KB) - t(M)}{t(KB)} * 100, \tag{42}$$

respectively, where $t(\cdot)$ denotes the time required by the formulation $(\cdot)$.

The results in Table 1 show that the proposed formulation can solve all the instances to optimality, requiring significantly less solution times than those of *TR* and *KB*. In fact, as the last two columns indicate, the savings in solution times can be quite high (up to 97%). An important conclusion which may be drawn from these results is that it is preferable to solve the original MmTSP rather than to transform it into a TSP and solve the resulting problem. This result parallels those of Laporte and Nobert [9] and Gavish and Srikanth [4], who concluded that the transformed problems are much harder to solve than the original ones for the standard mTSP.

5. Conclusion

In this paper, we present an integer linear programming formulation for a more general case of the mTSP. We show that the formulation with additional restrictions may easily be adapted to special cases and multidepot situations.

Imposing a minimal number of nodes that a traveler should visit allows the decision maker to assign appropriate work loads to each traveler. Postoptimality analysis can be conducted simply by varying the parameters of the problem and observing the changes in the solution values. Computational analysis shows that moderate-sized instances can easily be solved to optimality via the proposed formulations.

Imposing lower bounds is also very important in real-life problems of vehicle routing. The modeling approach presented in this paper may be applied to certain types of VRPs, and this is currently being studied.

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