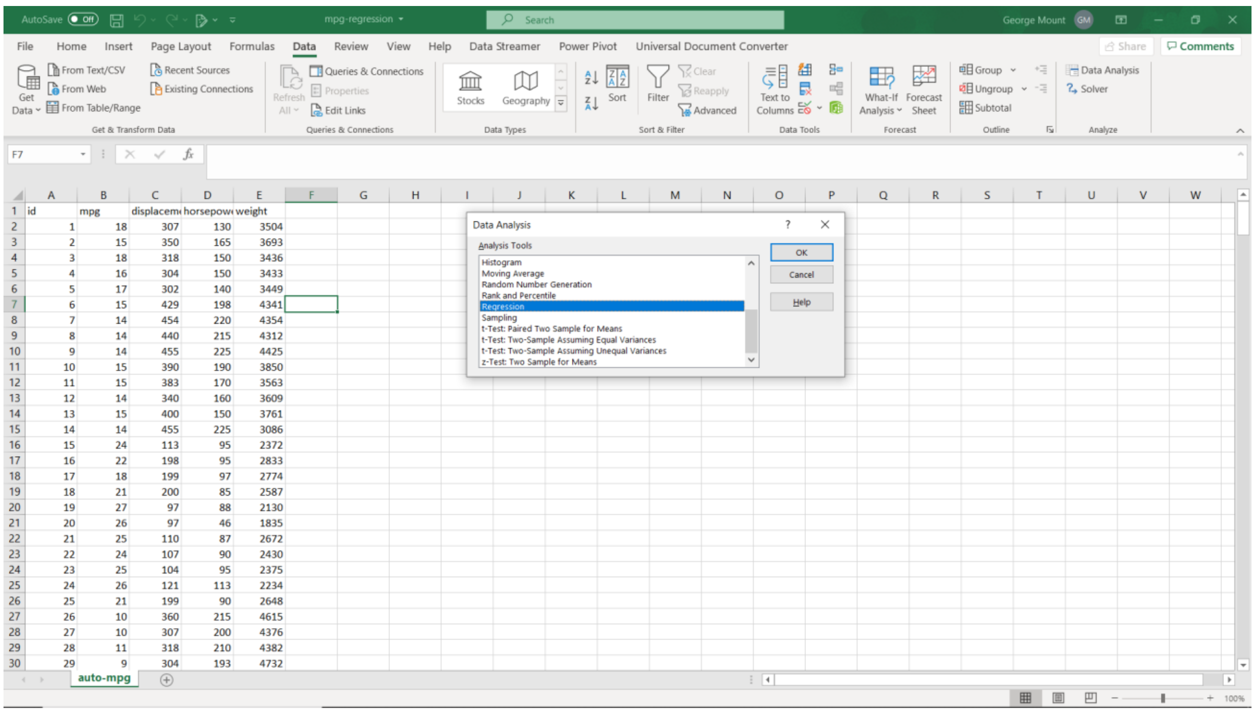
Regression analysis and predictive models: demo notes

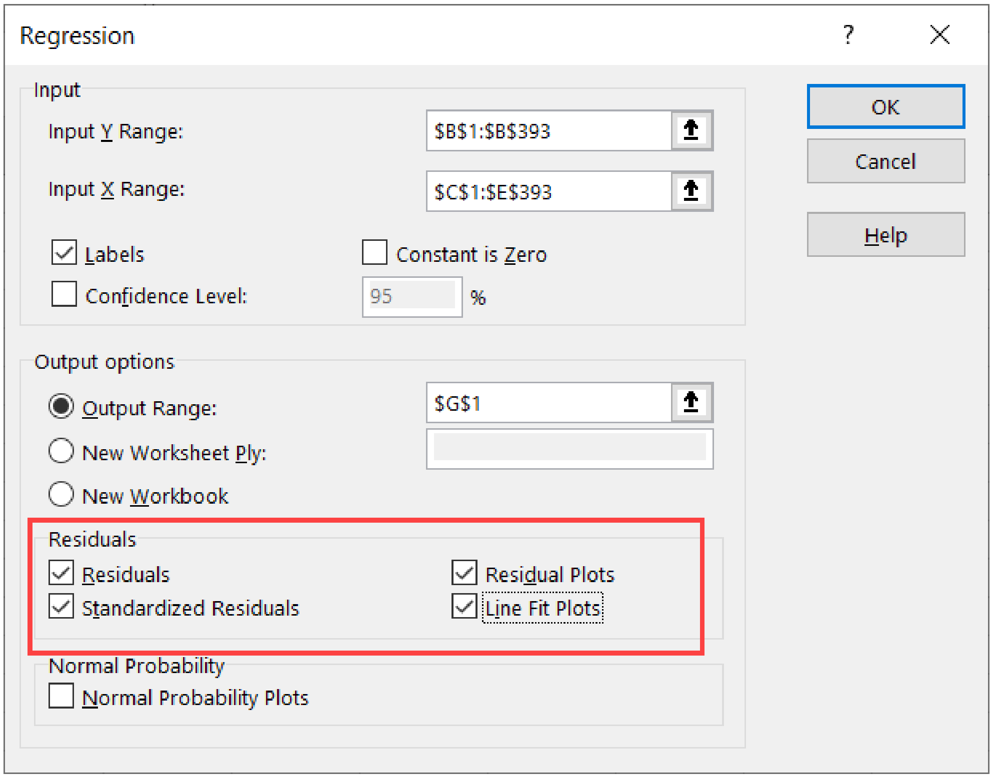
Multiple linear regression

Demo: `mpg-regression.xlsx`

1. Head to Data > Data Analysis > Regression.



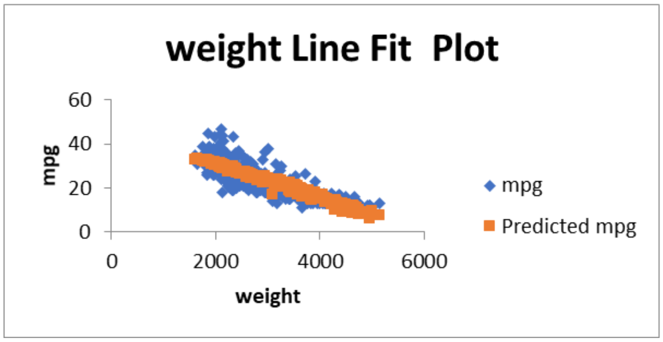
2. The input Y range is our dependent variable, `mpg`. The X range is our three independent variables: `displacement`, `horsepower` and `weight`.
   a. Check all boxes in the Residuals group.

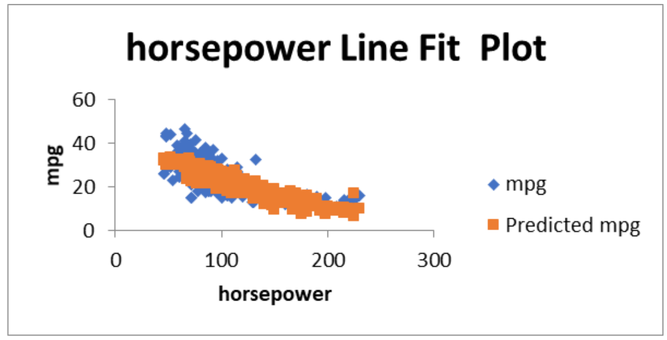


3. We will start our analysis by checking the p-values of our regression, and dropping any non-significant variables.
    a. `displacement` is a non-significant-value, so we will drop it from our next round.

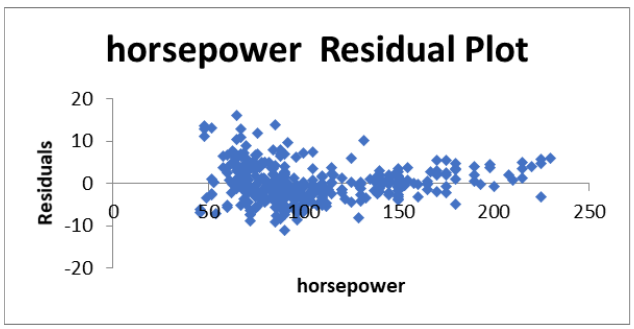
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	44.8559357	1.195920013	37.50747142	4.1075E-131	42.50464109	47.2072303	42.50464109	47.2072303
displacement	-0.005768819	0.00658189	-0.876468422	0.381317748	-0.018709452	0.007171815	-0.018709452	0.007171815
horsepower	-0.041674144	0.012813862	-3.252270314	0.001245063	-0.066867438	-0.016480849	-0.066867438	-0.016480849
weight	-0.005351593	0.000712354	-7.512546915	4.03584E-13	-0.00675215	-0.003951036	-0.00675215	-0.003951036

4. Make a copy of the worksheet and delete over the old output. This time only include horsepower and weight in your X range. Leave the rest of the regression settings as-is.
    a. All variables are now significant 🎉. Let's continue in interpreting the results given by the plots we selected to include.
5. First we get two plots of our independent variables with our actual versus predicted Y variable. We can visually see that there is indeed a line fit into each of the scatterplots. This is an assumption of regression.

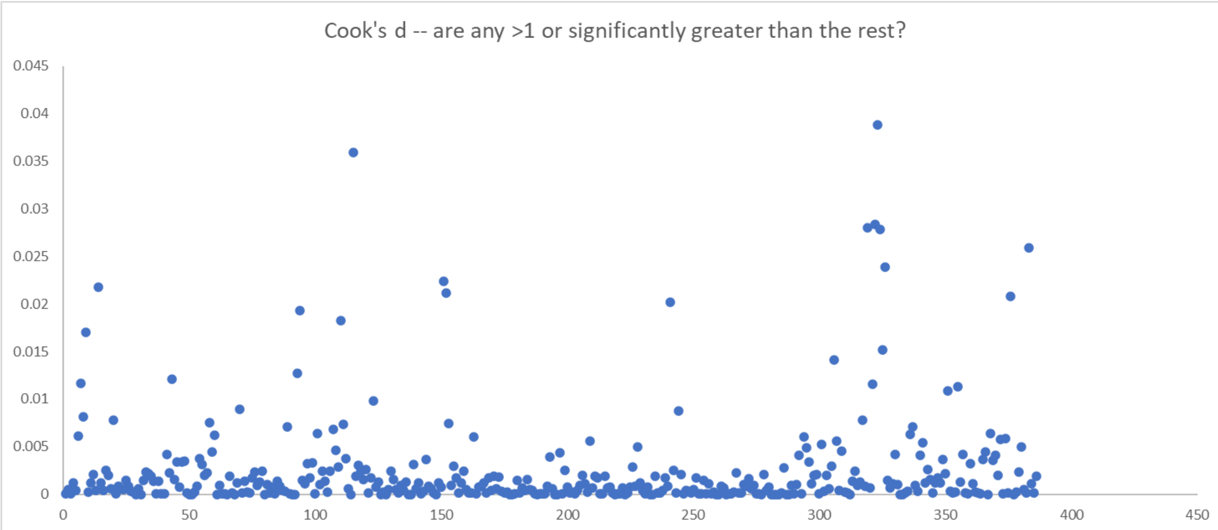




6. We also got two plots of our independent variables against the *residuals*. Remember, these are supposed to look totally *random*, but there is a big clump in the left of each. We may have a problem meeting this assumption.

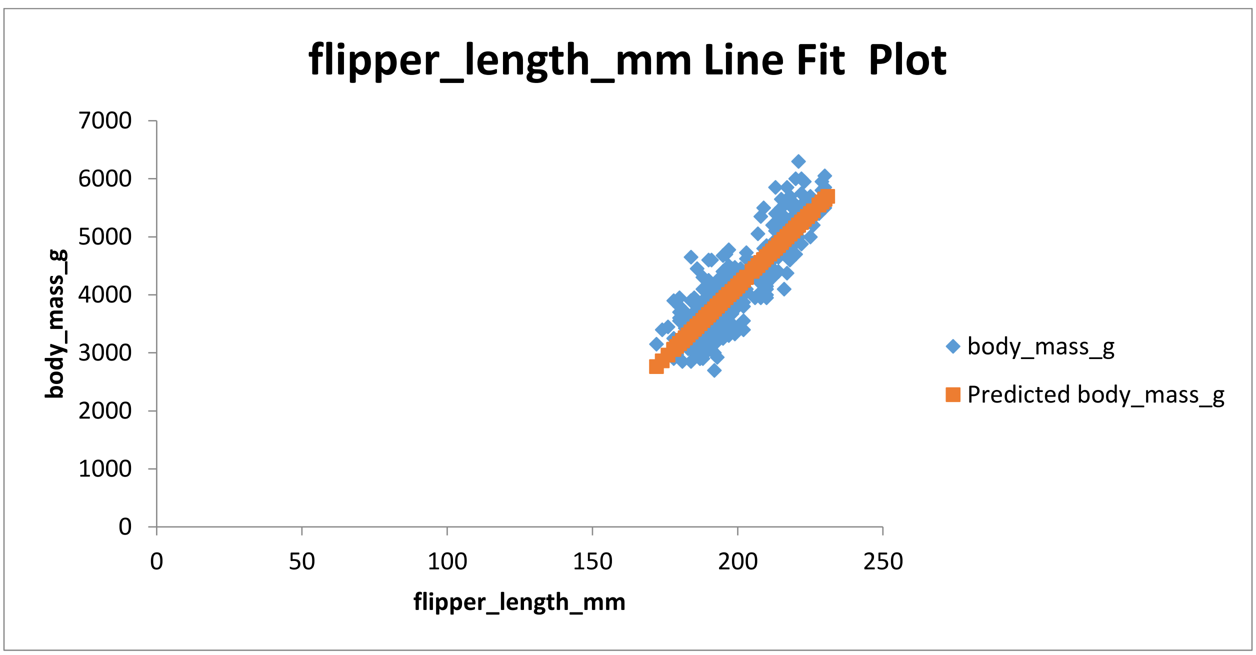


7. Finally, we want to check for influential cases. This is possible to do in Excel but takes some heavy set-up. Un-hide the `influential-cases` worksheet in your workbook.
    a. I have calculated a measure called Cook's D to check for influential cases. Generally a Cook's D greater than 1 signals an influential case, however if there are any much different than the others, those could be considered influential.
        i. In this case, it may be worth identifying two cases as possible influential cases.

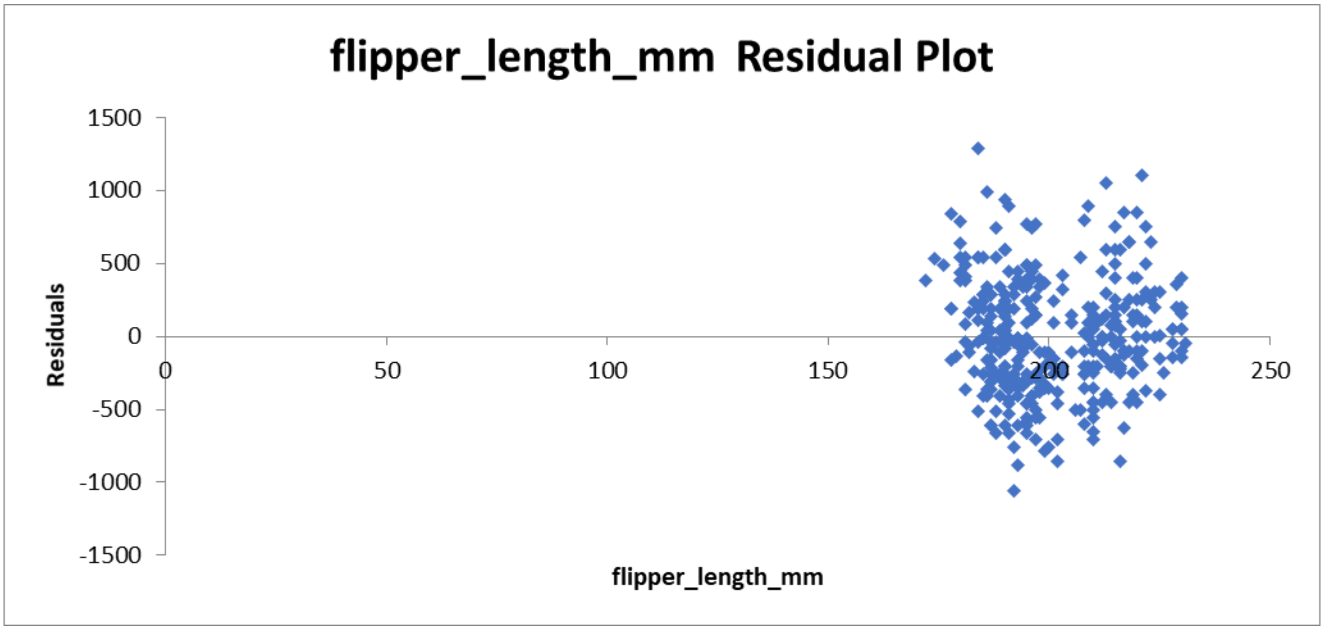


Drill: `penguins-linear.xlsx`

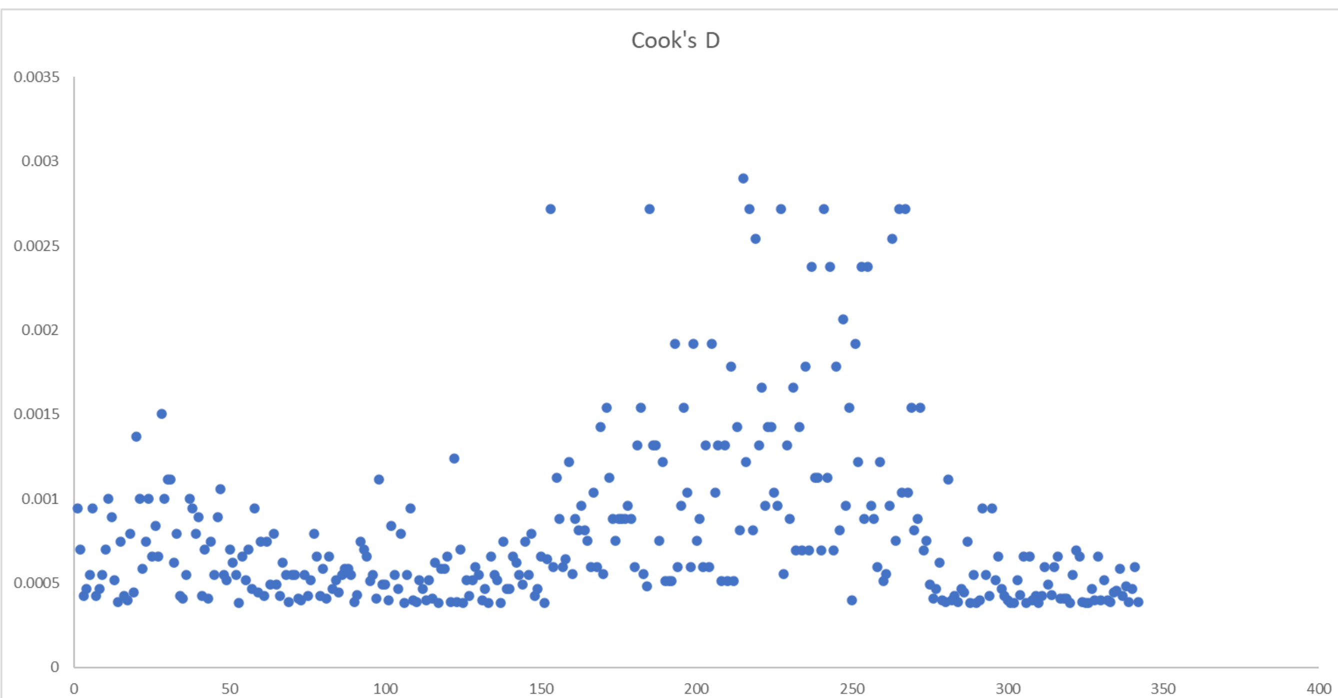
1. In this case, culmen length and culmen depth are not significant, so our only independent variable is flipper length. It's no longer multiple linear regression, but we can still check the same types of diagnostics.
   a. Linearity checks out from the scatter plot.



   b. There is no pattern in the residuals.



c. While none of the Cook's d values are over 1, it appears that some group of observations do have more influence over the curve than others. There are some ways to investigate that are outside the scope of this course.



Demo: `mpg-regression-diagnostics.xlsx`

1. Because this dataset uses multiple independent variables, we should use the *adjusted* r-square. That is available in the ToolPak results.
    a. An adjusted r-square means that 70% of the variance in Y is explained by our X's.

SUMMARY OUTPUT		
Regression Statistics		
Multiple R	0.840461346	
R Square	0.706375274	
Adjusted R Square	0.704865635	<- 70% of the variance in Y is explained by X's
Standard Error	4.240169468	
Observations	392	

2. We can also use the estimated coefficients from our output to make point predictions. Use the intercept and slopes to predict a value of Y given some X's:

	F	G	H	I	J	K	L	M
1		SUMMARY OUTPUT						
2								
3		*Regression Statistics*						
4		Multiple R	0.840461346					
5		R Square	0.706375274			200	3000	
6		Adjusted R Square	0.704865635	<- 70% of the variance in Y is explained by X's		18.79716613	<- What is the expected MPG of a car weighing 3,000 pounds that has 200 horsepower?	
7		Standard Error	4.240169468			=H17+(K5*H18)+(L5*H19)		
8		Observations	392					
9								
10		ANOVA						
11			*df*	*SS*	*MS*	*F*	*Significance F*	
12		Regression	2	16825.14803	8412.574016	467.9101535	3.0596E-104	
13		Residual	389	6993.845437	17.97903711			
14		Total	391	23818.99347				
15								
16			*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
17		Intercept	45.64021084	0.793195833	57.5396503	2.3171E-192	44.08072353	47.19969815
18		horsepower	-0.047302863	0.011085086	-4.267252507	2.48848E-05	-0.069097042	-0.025508684
19		weight	-0.005794157	0.000502327	-11.53463263	1.12436E-26	-0.006781773	-0.004806542
20								
21								

Drill: `penguin-linear-diagnostics.xlsx`

1. Follow the same steps as above. Notice that since this time we only have one independent variable, the r-square and adjusted r-square are the same.

	A	B	C	D	E
1	SUMMARY OUTPUT				
2					
3	*Regression Statistics*				
4	Multiple R		0.871201767		
5	R Square		0.758992519	<= 76% of variability in Y is caused by X	
6	Adjusted R Square		0.758283674		
7	Standard Error		394.2781775		
8	Observations		342		
9					

Interaction terms

Demo: `airquality-interaction.xlsx`

It's not a bad idea to run the regression *without* the interaction terms at first to establish a baseline.

1. We will set up our interaction term in column E by multiplying columns C and D.
2. For our baseline (the DV on the two IV's), about 50% of the variability in Y is explained.

	A	B	C	D	E
1	id	Ozone	Temp	Solar.R	temp * solar
2	1	41	67	190	12730
3	2	36	72	118	8496
4	3	12	74	149	11026
5	4	18	62	313	19406
6	7	23	65	299	19435
7	8	19	59	99	5841
8	9	8	61	19	1159
9	12	16	69	256	17664
10	13	11	66	290	19140
11	14	14	68	274	18632
12	15	18	58	65	3770
13	16	14	64	334	21376
14	17	34	66	307	20262
15	18	6	57	78	4446
16	19	30	68	322	21896
17	20	11	62	44	2728
18	21	1	59	8	472
19	22	11	73	320	23360
20	23	4	61	25	1525
21	24	32	61	92	5612

	G	H	I	J	K	L	M	N	O
1	SUMMARY OUTPUT								
2									
3	*Regression Statistics*								
4	Multiple R	0.71436457							
5	R Square	0.510316738							
6	Adjusted R Square	0.50124853							
7	Standard Error	23.50026724							
8	Observations	111							
9									
10	ANOVA								
11		*df*	*SS*	*MS*	*F*	*Significance F*			
12	Regression	2	62157.5534	31078.7767	56.27536418	1.80063E-17			
13	Residual	108	59644.35651	552.2625603					
14	Total	110	121801.9099						
15									
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
17	Intercept	-145.7031551	18.44671758	-7.898595208	2.52933E-12	-182.2677495	-109.1385607	-182.2677495	-109.1385607
18	Temp	2.278466835	0.24599582	9.262217671	2.21556E-15	1.790860443	2.766073227	1.790860443	2.766073227
19	Solar.R	0.057109594	0.025718852	2.220534321	0.028470633	0.006130367	0.10808882	0.006130367	0.10808882

3. If we add the interaction term, our adjusted R-square increases to .54. Is it worth adding an extra variable?

	A	B	C	D	E	F
16	19	30	68	322	21896	
17	20	11	62	44	2728	
18	21	1	59	8	472	
19	22	11	73	320	23360	
20	23	4	61	25	1525	
21	24	32	61	92	5612	
22	28	23	67	13	871	
23	29	45	81	252	20412	
24	30	115	79	223	17617	
25	31	37	76	279	21204	
26	38	29	82	127	10414	
27	40	71	90	291	26190	
28	41	39	87	323	28101	
29	44	23	82	148	12136	
30	47	21	77	191	14707	
31	48	37	72	284	20448	
32	49	20	65	37	2405	
33	50	12	73	120	8760	
34	51	13	76	137	10412	
35	62	135	84	269	22596	
36	63	49	85	248	21080	
37	64	32	81	236	19116	
38	66	64	83	175	14525	
39	67	40	83	314	26062	
40	68	77	88	276	24288	
41	69	97	92	267	24564	
42	70	97	92	272	25024	
43	71	85	89	175	15575	
44	73	10	73	264	19272	
45	74	27	81	175	14175	
46	76	7	80	48	3840	
47	77	48	81	260	21060	

	G	H	I	J	K	L	M	N	O	P	Q	R	S
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*				
17	Intercept	-145.7031551	18.44671758	-7.898595208	2.52933E-12	-182.2677495	-109.1385607	-182.2677495	-109.1385607				
18	Temp	2.278466835	0.24599582	9.262217671	2.21556E-15	1.790860443	2.766073227	1.790860443	2.766073227				
19	Solar.R	0.057109594	0.025718852	2.220534321	0.028470633	0.006130367	0.10808882	0.006130367	0.10808882				
20													
21	SUMMARY OUTPUT												
22													
23	*Regression Statistics*												
24	Multiple R	0.742486766											
25	R Square	0.551286598											
26	Adjusted R Square	0.538705848											
27	Standard Error	22.60058501											
28	Observations	111											
29													
30	ANOVA												
31		*df*	*SS*	*MS*	*F*	*Significance F*							
32	Regression	3	67147.76055	22382.58685	43.81985304	1.49244E-18							
33	Residual	107	54654.14936	510.7864426									
34	Total	110	121801.9099										
35													
36		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*				
37	Intercept	-36.89906236	39.07007244	-0.944432914	0.347076192	-114.3509242	40.55279952	-114.3509242	40.55279952				
38	Temp	0.760655388	0.540163008	1.408196002	0.161971967	-0.310154776	1.831465552	-0.310154776	1.831465552				
39	Solar.R	-0.547794547	0.195103591	-2.807711251	0.005931117	-0.934564661	-0.161024433	-0.934564661	-0.161024433				
40	temp * solar	0.008274977	0.002647447	3.125644678	0.002284303	0.003026723	0.013523232	0.003026723	0.013523232				

airquality

Linear regression with categorical IV's

Demo: `mpg-dummy.xlsx`

1. We want to model the influence of weight and origin on mpg. Because origin is categorical, we will create dummy variables to include as IV's.
    a. Origin takes three values: USA, Europe and Asia. We will encode Europe as one dummy variable, and Asia another. USA is implied when both dummies are set to zero. We will not include this column in our regression since it's all zeros.

	A	B	C	D	E	F	G
1			=IF(F3="Europe",1,0)	=IF(F3="Asia",1,0)	=IF(F3="USA",0,0)		
2	mpg	weight	origin_europe	origin_asia	origin_usa	origin	car name
3	18	3504	0	0	0	USA	chevrolet chevelle malibu
4	15	3693	0	0	0	USA	buick skylark 320
5	18	3436	0	0	0	USA	plymouth satellite
6	16	3433	0	0	0	USA	amc rebel sst
7	17	3449	0	0	0	USA	ford torino
8	15	4341	0	0	0	USA	ford galaxie 500
9	14	4354	0	0	0	USA	chevrolet impala
10	14	4312	0	0	0	USA	plymouth fury iii
11	14	4425	0	0	0	USA	pontiac catalina
12	15	3850	0	0	0	USA	amc ambassador dpl
13	15	3563	0	0	0	USA	dodge challenger se
14	14	3609	0	0	0	USA	plymouth 'cuda 340

2. Run the regression from the ToolPak, exporting the results to cell K1 of the same worksheet. You will get the following results.
    a. These value give the coefficients to include with our dummy-coded variables.
        i. Because the dummy-code for USA was left to zero, this becomes our reference category. We see here from the p-values that European cars do not have a significantly higher mileage from American cars, but Japanese cars do.
            1. However, we cannot keep some dummy variables and drop others. That would lead to inaccurate comparisons across groups. Since this p-value is so close to .05, I will decide to keep it in the model.

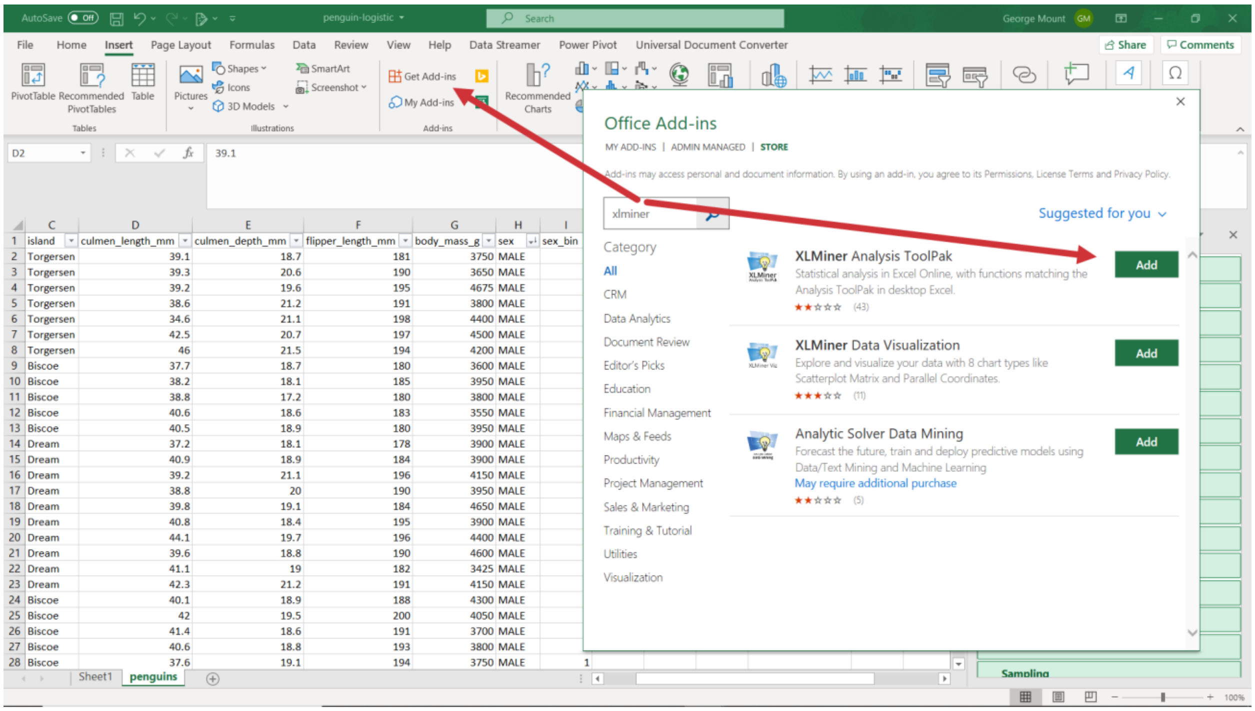
	K	L	M	N	O	P	Q	R	S
1	SUMMARY OUTPUT								
2						Weight	origin_europe	origin_asia	mpg_pred
3	*Regression Statistics*				American	3000	0	0	
4	Multiple R	0.837558007			European	3000	1	0	
5	R Square	0.701503415			Asian	3000	0	1	
6	Adjusted R Square	0.699230599							
7	Standard Error	4.286477069							
8	Observations	398							
9									
10	ANOVA								
11		*df*	*SS*	*MS*	*F*	*Significance F*			
12	Regression	3	17013.26452	5671.088175	308.6493667	4.8646E-103			
13	Residual	394	7239.310953	18.37388567					
14	Total	397	24252.57548						
15									
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
17	Intercept	43.69585641	1.104363368	39.56655724	2.5821E-139	41.5246745	45.86703833	41.5246745	45.86703833
18	weight	-0.007023439	0.000318398	-22.05865536	8.40858E-71	-0.007649411	-0.006397467	-0.007649411	-0.006397467
19	origin_europe	1.215471794	0.652373629	1.863152862	0.063184619	-0.067096849	2.498040436	-0.067096849	2.498040436
20	origin_asia	2.355434709	0.662030569	3.557894182	0.000419392	1.053880491	3.656988926	1.053880491	3.656988926
21									

3. We can now use these coefficients to make point predictions in the range O2:S5. Notice that we could technically include each of the dummy-coded variables in our equations; but the non-necessary coefficients are multiplied by zero.

	K	L	M	N	O	P	Q	R	S	T
1	SUMMARY OUTPUT									
2						Weight	origin_europe	origin_asia	mpg_pred	
3	*Regression Statistics*				American	3000	0	0	22.62553953	=L17+((P3*L18)+(Q3*L19)+(R3*L20))
4	Multiple R	0.837558007			European	3000	1	0	23.84101132	=L17+(P4*L18)+(Q4*L19)+(R4*L20)
5	R Square	0.701503415			Asian	3000	0	1	24.98097424	=L17+(P5*L18)+(L19*Q5)+(R5*L20)
6	Adjusted R Square	0.699230599								
7	Standard Error	4.286477069								
8	Observations	398								
9										
10	ANOVA									
11		*df*	*SS*	*MS*	*F*	*Significance F*				
12	Regression	3	17013.26452	5671.088175	308.6493667	4.8646E-103				
13	Residual	394	7239.310953	18.37388567						
14	Total	397	24252.57548							
15										
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*	
17	Intercept	43.69585641	1.104363368	39.56655724	2.5821E-139	41.5246745	45.86703833	41.5246745	45.86703833	
18	weight	-0.007023439	0.000318398	-22.05865536	8.40858E-71	-0.007649411	-0.006397467	-0.007649411	-0.006397467	
19	origin_europe	1.215471794	0.652373629	1.863152862	0.063184619	-0.067096849	2.498040436	-0.067096849	2.498040436	
20	origin_asia	2.355434709	0.662030569	3.557894182	0.000419392	1.053880491	3.656988926	1.053880491	3.656988926	
21										
22										

Logistic regression

0. The Analysis ToolPak does not include logistic regression, so you need to install XLMiner. Fortunately this is free and requires no external downloads: go to **Insert > Get Add-Ins > XLMiner Analysis ToolPak.**



Demo: `occupancy.xlsx`

1. Select your Y range (`Occupancy`) and X range (`Temperature`, `Humidity`, and `Light`).
   a. The range selector tool in XLMiner is terribly user-unfriendly. It may be easier to grab a smaller range and fill it out by typing.
2. We will get some familiar output expressed in intercepts and p-values. All of our X's are significant.

	H	I	J	K	L	M	N	O	P	Q
1	SUMMARY OUTPUT									
2										
3	Regression Statistics									
4	Chi Square	7305.64272								
5	Residual Dev.	1114.657625								
6	# of iterations	10								
7	Observations	8143								
8										
9		Coefficients	Standard Error	P-value	Odd Ratio	Lower 95%	Upper 95%	Lower 95%	Upper 95%	
10	Intercept	0.47203998	2.337356777	0.839952208	1.603261	0.016423	156.5186	0.016423	156.5186	
11	Temperature	-0.586523047	0.110473391	1.1012E-07	0.556258	0.447962	0.690735	0.447962	0.690735	
12	Humidity	0.145701588	0.017123159	1.75391E-17	1.156851	1.11867	1.196335	1.11867	1.196335	
13	Light	0.024478643	0.0009229	5.179E-155	1.024781	1.022929	1.026636	1.022929	1.026636	
14										

Drill: `penguin-logistic.xlsx`

1. XLMiner requires that all variables be *numeric.* This means that the `MALE`/`FEMALE` labels for `sex` should be converted into 0's and 1's.

I2 =IF(H2="MALE",0,1)

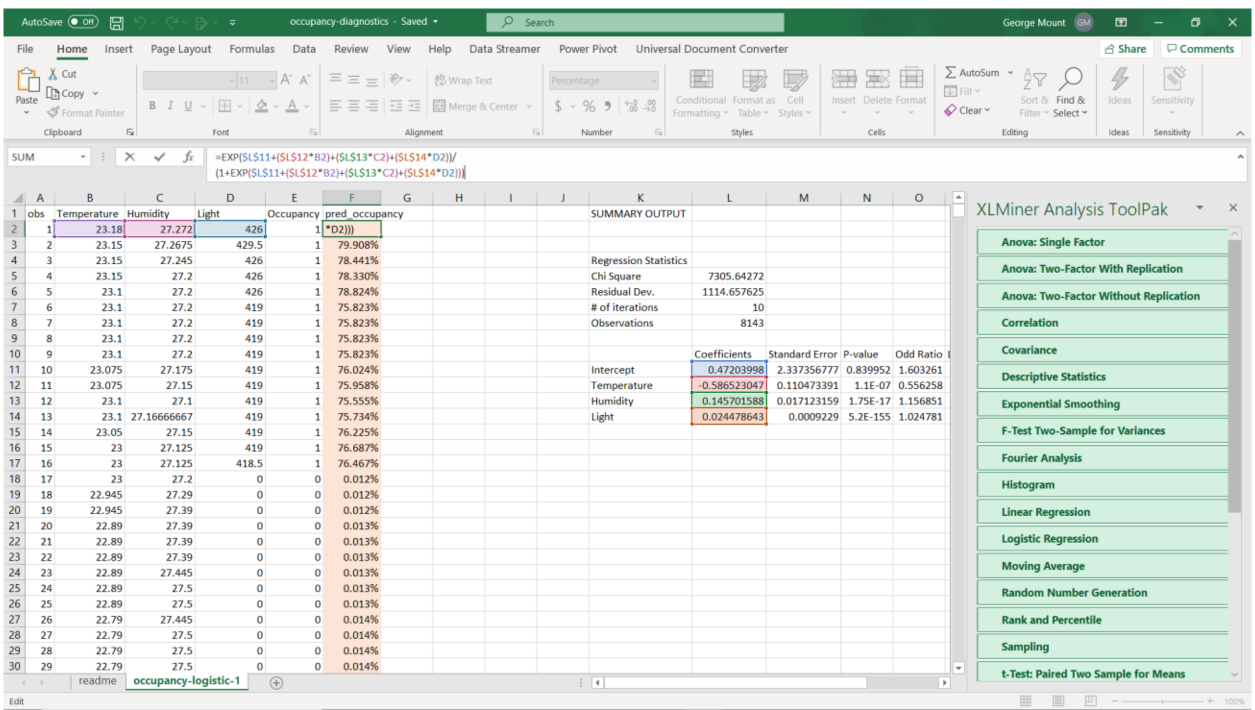
	D	E	F	G	H	I	J
1	culmen_length_mm	culmen_depth_mm	flipper_length_mm	body_mass_g	sex	sex_bin	
2	39.1	18.7	181	3750	MALE	0	
3	39.3	20.6	190	3650	MALE	0	
4	39.2	19.6	195	4675	MALE	0	
5	38.6	21.2	191	3800	MALE	0	
6	34.6	21.1	198	4400	MALE	0	
7	42.5	20.7	197	4500	MALE	0	
8	46	21.5	194	4200	MALE	0	

2. `flipper_length_mm` is not significant, so drop it and re-run.

	K	L	M	N	O	P	Q	R	S
1	SUMMARY OUTPUT								
2									
3	Regression Statistics								
4	Chi Square	302.6055688							
5	Residual Dev.	159.0034261							
6	# of iterations	8							
7	Observations	333							
8									
9		Coefficients	Standard Error	P-value	Odd Ratio	Lower 95%	Upper 95%	Lower 95%	Upper 95%
10	Intercept	56.11740399	8.369320872	2.01227E-11	2.35223E+24	1.77E+17	3.13E+31	1.77E+17	3.13E+31
11	culmen_length_mm	-0.107629548	0.047967294	0.024844556	0.897960186	0.817386	0.986477	0.817386	0.986477
12	culmen_depth_mm	-2.031515596	0.249465002	3.84064E-16	0.13113662	0.080423	0.21383	0.080423	0.21383
13	flipper_length_mm	0.032474395	0.034755396	0.350113129	1.033007443	0.964983	1.105827	0.964983	1.105827
14	body_mass_g	-0.005512026	0.000823488	2.17886E-11	0.994503137	0.992899	0.99611	0.992899	0.99611
15									
16									
17									

Demo: `occupancy-diagnostics.xlsx`

1. The equation to find the probability is a doozy, but this comes from the logit equation. Read through it to interpret!



2. We can assume that any probability greater than 50% is a “yes,” otherwise “no.” Find this predicted outcome in column G.

G2 =IF(F2>0.5,1,0)

	A	B	C	D	E	F	G	H	I
1	obs	Temperature	Humidity	Light	Occupancy	pred_occupancy	pred_occupied?	pred_correct?	
2	1	23.18	27.272	426	1	78.209%	1	TRUE	
3	2	23.15	27.2675	429.5	1	79.908%	1	TRUE	
4	3	23.15	27.245	426	1	78.441%	1	TRUE	
5	4	23.15	27.2	426	1	78.330%	1	TRUE	
6	5	23.1	27.2	426	1	78.824%	1	TRUE	
7	6	23.1	27.2	419	1	75.823%	1	TRUE	

3. Now we can simply find a TRUE/FALSE result as to whether the predicted outcome is the same as the actual one.

H2 =E2=G2

	A	B	C	D	E	F	G	H	I
1	obs	Temperature	Humidity	Light	Occupancy	pred_occupancy	pred_occupied?	pred_correct?	
2	1	23.18	27.272	426	1	78.209%	1	TRUE	
3	2	23.15	27.2675	429.5	1	79.908%	1	TRUE	
4	3	23.15	27.245	426	1	78.441%	1	TRUE	
5	4	23.15	27.2	426	1	78.330%	1	TRUE	
6	5	23.1	27.2	426	1	78.824%	1	TRUE	
7	6	23.1	27.2	419	1	75.823%	1	TRUE	
8	7	23.1	27.2	419	1	75.823%	1	TRUE	

4. We can now calculate a basic predictive accuracy measure for this model. 98.7% isn't too bad for a first pass!

	K	L	M	N	O	P
14						
15						
16	Number observations	8143	=COUNT(A2:A8144)			
17	Number observations predicted correctly?	8039	=COUNTIF(H2:H8144,"TRUE")			
18	Pred. accuracy	98.7%	=L17/L16			
19						

5. Now that we have set up our equations in the table, we can easily plug in any point-estimates to predict a "yes" or "no" outcomes

a. For temperature 25, humidity 30 and light 400, it's a close call! Maybe making binary predictions isn't so straightforward after all.

	A	B	C	D	E	F	G	H	I
1	obs	Temperature	Humidity	Light	Occupancy	pred_occupancy	pred_occupied?	pred_correct?	
2		25	30	400		49.287%	0		
3	1	23.18	27.272	426	1	78.209%	1	TRUE	
4	2	23.15	27.2675	429.5	1	79.908%	1	TRUE	
5	3	23.15	27.245	426	1	78.441%	1	TRUE	

Drill: `penguin-logistic-diagnostics.xlsx`

1. Follow the same steps as above. See if you can write the logit curve equation on your own!