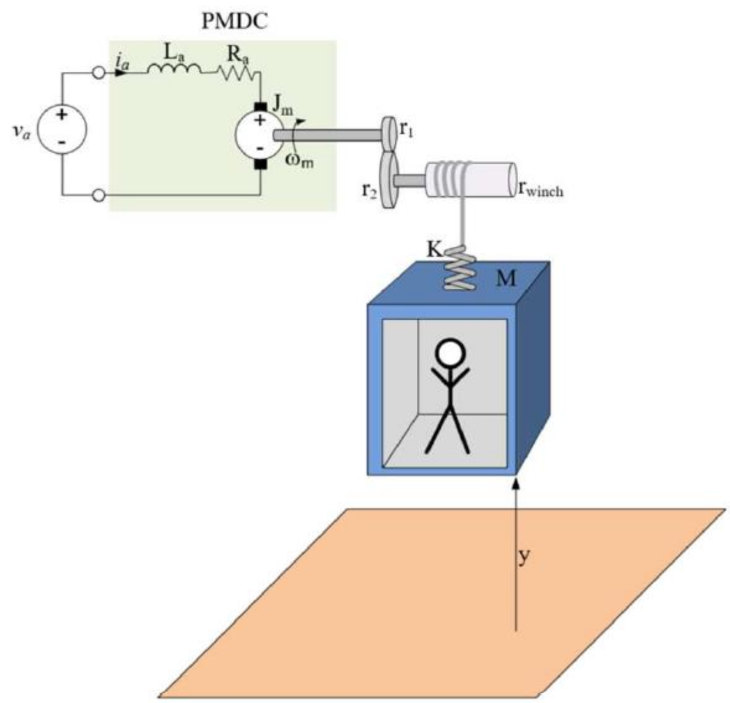
Simulation of an elevator hoist actuated by Permanent Magnet DC Motor



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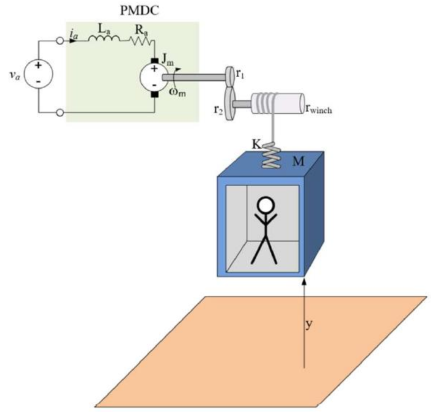
1. Problem Statement

Consider the elevator hoist system shown below. Here $\boldsymbol{V_a}$ is the control input and the vertical position $\boldsymbol{y}$ is the output. Further, $\boldsymbol{M}$ is the total mass of the elevator car and persons, the total rotational inertia is $\boldsymbol{J_m}$, $\boldsymbol{K}$ is the spring constant of the suspension between the car and cable, and $\boldsymbol{K_a}$ the torque constant of the PMDC. Only motion in the vertical direction will be considered. This problem is only considering open-loop dynamics.



Q.1 Derive a system model of first-order differential equations for the elevator hoist system.

- Keep all equations symbolic (do not substitute numeric values yet).
- $\mathbf{V_a}$ is the control input, Vertical position of elevator car - **y** is the system output

Use the following state definitions

- x1 = $\mathbf{i_a}$ = PMDC Armature Current   x2 = $\boldsymbol{\omega_m}$ = PMDC rotational velocity
- x3 = $\boldsymbol{\theta_m}$ = PMDC rotational position   x4 = **v** = Vertical velocity of elevator car
- x5 = **y** = Vertical position of elevator car

Q.2 From the system model of first-order differential equation in a) construct the A, B, C, and D matrices of the form

$$\Delta\dot{\mathbf{X}} = \boldsymbol{A}\,\Delta\boldsymbol{x} + \boldsymbol{B}\,\Delta\boldsymbol{u}$$
$$\Delta\boldsymbol{y} = \boldsymbol{C}\,\Delta\boldsymbol{x} + \boldsymbol{D}\,\Delta\boldsymbol{u}$$

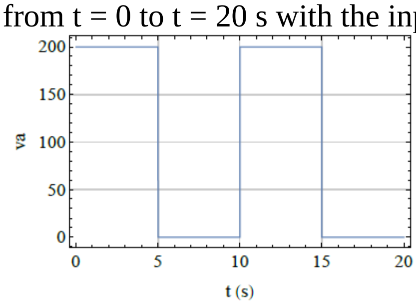
Q.3 From your models in a) and b), use the values below to carry out numeric simulations.

The parameters of the simulations are as follows:

$\mathbf{L_a}$ = 18 mH, $\mathbf{R_a}$ = 0.1 Ω, $\mathbf{K_a}$ = 1 Nm / A, $\mathbf{r_1}$ =5cm, $\mathbf{r_2}$ = 20 cm, $\mathbf{r_w}$ = 15 cm, **M**=1000 kg,

K = 75 000 N / m, $\mathbf{J_m}$ = 5 kg m^2

All states start at zero. Simulate from t = 0 to t = 20 s with the input voltage va shown below.



2. Assumptions & Nomenclatures

While deriving the differential equations of this system and simulating the system, following assumptions and nomenclatures are used.

1. The downward movement of elevator mass is considered as positive.
2. Counter-clockwise rotation of motor shaft is considered as positive.
3. The value of gravity is considered as 9.81.
4. The frictional damping in the motor is neglected.
5. The individual moments of inertia for gear 1, gear 2 and winch are not considered and the motor inertia is considered as the effective moment of inertia.

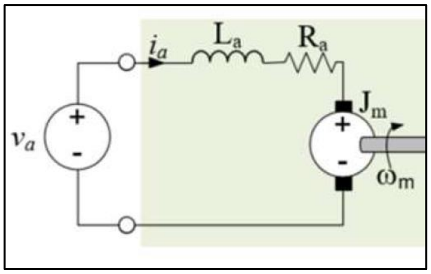
v_a = Voltage input
i_a = Armature Current
R_a = Armature Resistance
L_a = Armature Inductance
K_a = Motor Torque Constant
K_e = Motor Back emf Constant
τ = Torque
J_m = Motor inertia
J_{eff} = Effective moment of inertia
r_1 = radius of gear 1
r_2 = radius of gear 2
N = Gear Ratio
r_w = radius of winch
θ_m = Angular position of driving gear or motor shaft
$\dot{\theta}_m$, ω_m = Angular velocity of motor shaft
$\ddot{\theta}_m$ = Angular acceleration of motor shaft
θ_2 = Angular position of driven gear or winch
$\ddot{\theta}_2$ = Angular acceleration of winch or driven gear shaft
T = Tension in cable
M = Mass of elevator
K = Stiffness of the spring
V = Vertical velocity of elevator car
y = Vertical position of elevator car

3. Derivation of First Order Differential Equations of System

Deriving the first order DEQ of system means representing higher order DEQs in state space form. Thus, we will first derive the original higher order differential equations and then represent them as first order DEQ using state variables as provided in problem.

3.1 Permanent Magnet DC Motor (PMDC) Modeling

Below is the electric schematic of the permanent magnet DC motor which is shown in the problem statement figure.



According to Kirchhoff's voltage law, the summation of voltage around a loop in any circuit is equal to zero.

According to Ohm's law, the current is proportional to voltage and inversely proportional to resistance giving the relation V = I.R

Voltage drop across resistor = $R_a . i_a$

Voltage drop across inductor = $L_a . \frac{d}{dt}(i_a)$

Voltage loss due to back e.m.f = $K_e . \dot{\theta}_m$ where K_e is back emf constant

Thus, applying KVL and Ohm's law

$$v_a - L_a . \frac{d}{dt}(i_a) - R_a . i_a - K_e . \dot{\theta}_m = 0$$

Rearranging this equation for first derivative of armature current i_a

$$v_a - L_a . \frac{d}{dt}(i_a) - R_a . i_a - K_e . \dot{\theta}_m = 0$$

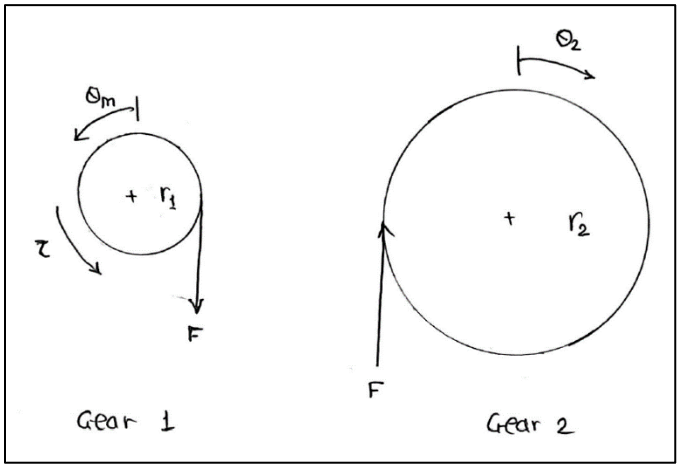
$$\boxed{\frac{d}{dt}(i_a) = \frac{v_a}{L_a} - \frac{R_a}{L_a} . i_a - \frac{K_e}{L_a} . \dot{\theta}_m}$$

Now, we will use this equation later to represent in state variables form.

3.2 Gear Kinematics

If we look from the motor side, small gear which is a driver gear is rotating in anti-clockwise direction and driven gear is rotating in clockwise direction. From this, we can draw free body diagrams as below.

Free Body Diagram of gears:



Here F is the tooth force, τ is the motor torque which is driving small gear having radius r_1 and angular position θ_m. Corresponding values for driven gear are r_2 and θ_2.

Considering summation of moments as zero, we can write

$$F \,. r_1 \; = \; \tau$$

$$F = \frac{\tau}{r_1}$$

<u>Arc length of driving and driven gear must be same that means</u>

$$r_1 \, \theta_1 \; = \; r_2 \, \theta_2$$

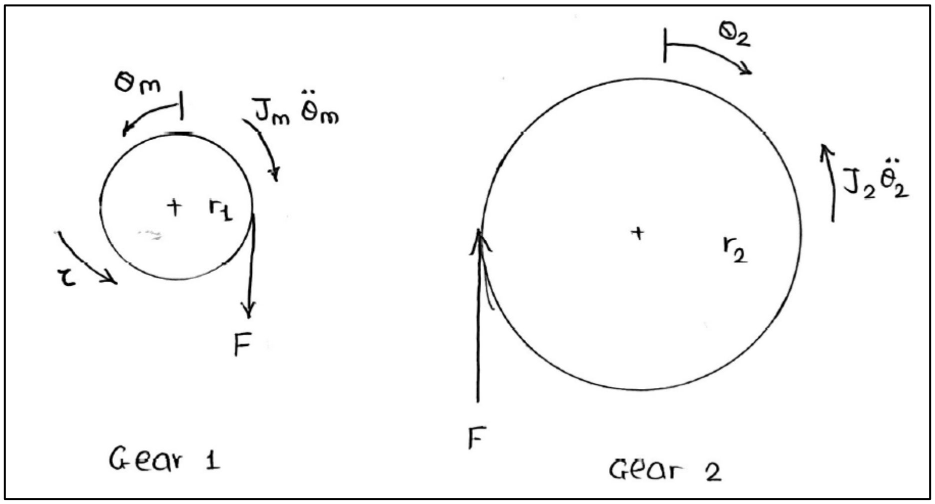
$$\frac{\theta_1}{\theta_2} = \frac{r_2}{r_1} \quad N = Gear\ ratio$$

Thus, <u>the speed is reduced from driving gear to driven gear with a factor of Gear ratio (N)</u>.

We will use this relation while deriving the differential equations further.

3.2 Gear Dynamics

If we draw the free body diagrams again considering the torque and inertia for both of the gears then those would be as follows.



$$J_1\ddot{\theta}_1 + F.r_1 - \tau = 0 \qquad\qquad J_2\ddot{\theta}_2 - F.r_2 = 0$$

$$F.r_1 = \tau - J_1\ddot{\theta}_1 \qquad\qquad J_2\ddot{\theta}_2 = F.r_2$$

$$F = \frac{\tau - J_1\ddot{\theta}_1}{r_1} \qquad\qquad J_2\ddot{\theta}_2 = F.r_2$$

$$J_2\ddot{\theta}_2 = \left(\frac{\tau - J_1\ddot{\theta}_1}{r_1}\right).r_2$$

Replacing $\frac{r_2}{r_1}$ by N and $\ddot{\theta}_1 = N\,\ddot{\theta}_2$ as per derivations in previous step

$$J_2\ddot{\theta}_2 = \tau N - J_1\ddot{\theta}_1 N$$

$$J_2\ddot{\theta}_2 = \tau N - J_1\ddot{\theta}_2 N^2$$

$$(J_1N^2 + J_2)\,\ddot{\theta}_2 = \tau N$$

$$J_{eff}\,\ddot{\theta}_2 = \tau N$$

Where J_{eff} is effective moment of inertia.

3.3 Winch, Elevator Mass & Spring Dynamics

The winch is connected to driven gear and thus rotates in same direction. The rope winded to winch is connected to spring and mass on other side.

Looking from the motor side, the rotation of winch in clockwise direction would cause the mass to go down and thus spring will elongate.

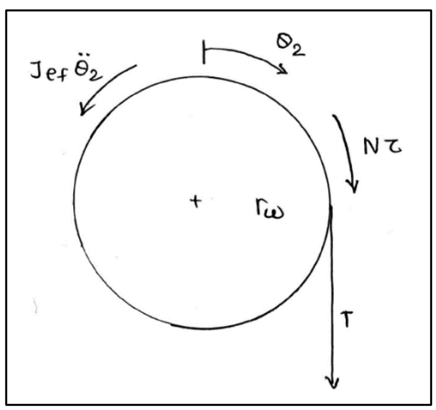
Therefore, in this dynamic, we would consider clockwise rotation and downward movement as positive.

Since there would be the moment of inertia of winch (J_w) as well, it will get added to effective moment of inertia we calculated in previous step as below.

$$J_{eff} = J_m + J_1 N^2 + J_2 + J_w$$

In this problem, since the moment of inertia of gear 1, gear 2 and winch are not provided, we would consider the effective moment of inertia as the moment of inertia of rotor. $J_{eff} = J_m$

Free Body Diagram of winch:



As per Newton's second law

$$-J_m \ddot{\theta}_2 + T \,. r_w + N\,\tau = 0$$

As per Newton's third law, there would be equal but opposite tension acting on the mass and spring system. When mass starts moving downwards then the spring will get elongated for some portion of displacement and then for remaining displacement of mass, spring deflection would remain constant.

From this, we can write the equation for arc length travelled by winch would be

$$r_w \theta_2 = y - x$$

The tension in the rope would be equal to the spring force.

$$T = k.x$$

Putting the equation of spring displacement,

$$T = k.(y - \ r_w\theta_2)$$

Putting equation for T in original equation of torque summation

$$-J_m\ddot{\theta_2} \ + \ k.(y - \ r_w\theta_2).r_w + \ N\,\tau = 0$$

$$-J_m\ddot{\theta_2} \ + k\ y\,r_w - k\ \ r_w{}^2\ \ \theta_2 + \ N\,\tau = 0$$

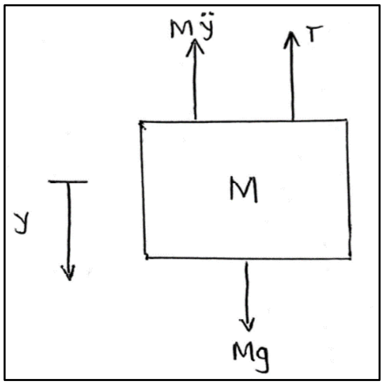
$$J_m\ddot{\theta_2} \ = \ k\ y\,r_w - k\ \ r_w{}^2\ \ \theta_2 + \ N\,\tau$$

$$\ddot{\theta_2} \ = \ \frac{k\ y\,r_w - k\ \ r_w{}^2\ \ \theta_2 + \ N\,\tau}{J_m}$$

$$\ddot{\theta_m} \ = \ \frac{k\ y\,r_w\,N - k\ \ r_w{}^2\ \ \theta_m + \ N^2\,\tau}{J_m}$$

$$\boxed{\ddot{\theta_m} \ = \ \frac{N^2\ \ K_a}{J_m}\ (\,i_a) - \frac{k\ \ r_w{}^2}{J_m}\ (\theta_m)\ + \ \frac{k\,r_w\,N}{J_m}\ (y)}$$

Free Body Diagram of mass:



As per Newton’s second law, looking at the free body diagram, summation of all forces must be zero.

$$M\ddot{y} = T - Mg$$

$$\ddot{y} = \ g - \frac{T}{M}$$

Putting equation for T which is derived in previous step

$$\ddot{y} = g - \frac{k\ (y - r_w \theta_2)}{M}$$

$$\ddot{y} = g - \frac{k\ (y)}{M} + \frac{k\ r_w\ (\theta_2)}{M}$$

$$\boxed{\ddot{y} = g - \frac{k\ (y)}{M} + \frac{k\ r_w\ (\theta_1)}{M\ N}}$$

This is the equation of motion for the elevator car.

We have definitions of state variables, input and output as below from the problem statement

States:	Output:	input:
$x_1 = i_a$ $x_2 = \dot{\theta}_m$ $x_3 = \theta_m$ $x_4 = \dot{y}$ $x_5 = y$	$y = x_5$	$u = v_a$

Thus, we can write the state derivatives as below

$$\dot{x_1} = \dot{i_a}$$
$$\dot{x_2} = \ddot{\theta_m}$$
$$\dot{x_3} = \dot{\theta_m}$$
$$\dot{x_4} = \ddot{y}$$
$$\dot{x_5} = \dot{y}$$

Using the definitions of state-space variables mentioned above we can re-write our original differential equations as below.

$$\dot{i_a} = \frac{v_a}{L_a} - \frac{R_a}{L_a}.i_a - \frac{K_e}{L_a}.\dot{\theta}_m$$

$$\dot{x_1} = \frac{u}{L_a} - \frac{R_a}{L_a}.x_1 - \frac{K_e}{L_a}.x_2$$

$$\ddot{\theta_m} = \frac{N^2\ K_a}{J_m}\ (i_a) - \frac{k\ r_w{}^2}{J_m}\ (\theta_m) + \frac{k\, r_w\, N}{J_m}\ (y)$$

$$\dot{x_2} = \frac{N^2\ K_a}{J_m}(x_1) - \frac{k\ {r_w}^2}{J_m}(x_3) + \frac{k\, r_w\, N}{J_m}(x_5)$$

$$\ddot{y} = g - \frac{k\,(y)}{M} + \frac{k\ r_w\,(\theta_1)}{M\ N}$$

$$\dot{x_4} = g + \frac{k\ r_w\,(x_3)}{M\ N} - \frac{k\,(x_5)}{M}$$

3.4 First order differential equations of system

So, from this, our first order differential equations of the system can be written as.

$$\dot{x_1} = \frac{1}{L_a}(u) - \frac{R_a}{L_a}.(x_1) - \frac{K_e}{L_a}.(x_2)$$

$$\dot{x_2} = \frac{N^2\ K_a}{J_m}(x_1) - \frac{k\ {r_w}^2}{J_m}(x_3) + \frac{k\, r_w\, N}{J_m}(x_5)$$

$$\dot{x_3} = x_2$$

$$\dot{x_4} = g + \frac{k\ r_w\,(x_3)}{M\ N} - \frac{k\,(x_5)}{M}$$

$$\boldsymbol{\dot{x_5}} = \boldsymbol{x_4}$$

4. 1 Linearization of differential equations

The differential equations which we have derived in previous step are non-linear. As asked in the problem statement, we are going to linearize them and then represent in state-space form as below.

$$\Delta\dot{\mathbf{X}} = \boldsymbol{A}\,\Delta\boldsymbol{x} + \boldsymbol{B}\,\Delta\boldsymbol{u}$$
$$\Delta\boldsymbol{y} = \boldsymbol{C}\,\Delta\boldsymbol{x} + \boldsymbol{D}\,\Delta\boldsymbol{u}$$

Let's follow the steps involved in the linearization.

Step 1 – Set all time derivatives ($\dot{x}$) equal to zero and then replace state and input variables with nominal variables such as $\boldsymbol{x}$ by $\boldsymbol{x_0}$ & $\boldsymbol{u}$ by $\boldsymbol{u_0}$.

Thus, above equations can be written then as

$$\dot{x_1} = 0 = \frac{1}{L_a}(u_0) - \frac{R_a}{L_a}.(x_{1,0}) - \frac{K_e}{L_a}.(x_{2,0})$$

$$\dot{x_2} = 0 = \frac{N^2\ K_a}{J_m}(x_{1,0}) - \frac{k\ r_w{}^2}{J_m}(x_{3,0}) + \frac{k\,r_w\,N}{J_m}(x_{5,0})$$

$$\dot{x_3} = 0 = x_{2,0}$$

$$\dot{x_4} = 0 = g + \frac{k\ r_w}{M\ N}(x_{3,0}) - \frac{k}{M}(x_{5,0})$$

$$\boldsymbol{\dot{x_5} = 0 = x_{4,0}}$$

From this, we can now establish certain relationships and values of equilibrium points which will be used in later step. These are

$$\boxed{\boldsymbol{x_{2,0} = 0 \qquad x_{4,0} = 0, \qquad x_{1,0} = \frac{u_0}{R_a}}}$$

$$\boldsymbol{\frac{N^2\ K_a}{J_m}\left(\frac{u_0}{R_a}\right) - \frac{k\ r_w{}^2}{J_m}(x_{3,0}) + \frac{k\,r_w\,N}{J_m}(x_{5,0}) = 0}$$

And we also have,

$$\boldsymbol{g + \frac{k\ r_w}{M\ N}(x_{3,0}) - \frac{k}{M}(x_{5,0}) = 0}$$

If we multiply this equation on both sides with $\boldsymbol{\frac{M\,r_w\,N}{J_m}}$ then we get first equation. That means, first equation is multiple of second by this factor.

Then solving these two equations we get,

$$\boxed{\boldsymbol{x_{3,0} = 0 \qquad x_{5,0} = 0}}$$

Step 2 – Rewrite original dynamic equations by replacing all the variables with the sum of equilibrium points and the perturbed quantity and use the relationship created in first step to remove all $\boldsymbol{x_0}$ $\boldsymbol{u_0}$ variables.

This means that replacing

$x = x_0 + \Delta x, \quad u = u_0 + \Delta u$

<u>The time derivative of the equilibrium points would be zero as they are constant terms</u>

So, differentiating above equations would give us

$\dot{x} = \dot{\Delta x}, \quad \dot{u} = \dot{\Delta u}$

Let's rewrite original dynamic equations then

The equation

$\dot{x_1} = \frac{1}{L_a}(u_0) - \frac{R_a}{L_a}.(x_{1,0}) - \frac{K_e}{L_a}.(x_{2,0})$ would become

$$\Delta\dot{x}_1 = \frac{1}{L_a}(u_0) - \frac{R_a}{L_a}.(x_{1,0} + \Delta x_1) - \frac{K_e}{L_a}.(x_{2,0} + \Delta x_2)$$

Putting value of $x_{1,0}$ obtained in last step

$$\Delta\dot{x}_1 = \frac{1}{L_a}(u_0) - \frac{R_a}{L_a}.(\frac{u_0}{R_a} + \Delta x_1) - \frac{K_e}{L_a}.(x_{2,0} + \Delta x_2)$$

which would then resolve after cancelling of equal terms as below

$$\boxed{\Delta\dot{x}_1 = \frac{1}{L_a}(\Delta u) - \frac{R_a}{L_a}(\Delta x_1) - \frac{K_e}{L_a}(\Delta x_2)}$$

And we would follow similar procedure for remaining equations which would then become

$$\Delta\dot{x}_2 = \frac{N^2\ K_a}{J_m}(x_{1,0} + \Delta x_1) - \frac{k\ r_w{}^2}{J_m}\left(x_{3,0} + \Delta x_3\right) + \frac{k\, r_w\, N}{J_m}\left(x_{5,0}\right)$$

Putting values of $\boldsymbol{x_{1,0}}$, $\boldsymbol{x_{3,0}}$, $\boldsymbol{x_{5,0}}$ obtained in last step

$$\Delta\dot{x}_2 = \frac{N^2\ K_a}{J_m}\left(\frac{u_0}{R_a} + \Delta x_1\right) - \frac{k\ r_w{}^2}{J_m}(0 + \Delta x_3) + \frac{k\, r_w\, N}{J_m}(0 + \Delta x_5)$$

which would then resolve as

$$\boxed{\boldsymbol{\Delta\dot{x}_2 = \frac{N^2\ K_a}{J_m}(\Delta x_1) - \frac{k\ r_w{}^2}{J_m}(\Delta x_3) + \frac{k\, r_w\, N}{J_m}(\Delta x_5)}}$$

$\Delta\dot{x}_3 = x_{2,0} + \Delta x_2$ but $x_{2,0} = 0$ as calculated in first step. So substituting that

$$\boxed{\Delta\dot{x}_3 = \Delta x_2}$$

$$\Delta\dot{\boldsymbol{x}}_4 \;=\; \boldsymbol{g} + \frac{\boldsymbol{k}\ \boldsymbol{r}_w}{\boldsymbol{M}\ \boldsymbol{N}}\left(\boldsymbol{x}_{3,0} + \Delta\boldsymbol{x}_3\right) - \frac{\boldsymbol{k}}{\boldsymbol{M}}\left(\boldsymbol{x}_{5,0} + \Delta\boldsymbol{x}_5\right)$$

Putting values of $\boldsymbol{x}_{3,0}$ & $\boldsymbol{x}_{5,0}$ obtained in last step, we can resolve as

$$\boxed{\Delta\dot{\boldsymbol{x}}_4 \;=\; = \;\frac{\boldsymbol{k}\ \boldsymbol{r}_w}{\boldsymbol{M}\ \boldsymbol{N}}(\Delta\boldsymbol{x}_3) - \frac{\boldsymbol{k}}{\boldsymbol{M}}(\Delta\boldsymbol{x}_5)}$$

$\Delta\dot{x}_5 \;=\; x_{4,0} + \Delta x_{42}$ but $x_{4,0} = 0$ as calculated in first step. So, substituting that

$$\boxed{\Delta\dot{x}_5 \;=\; \Delta x_4}$$

We can write from these equations 5x1 matrices of $\Delta\boldsymbol{x}$ and $\Delta\dot{\boldsymbol{x}}$ as

$$[\Delta\dot{x}] = \begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \\ \Delta\dot{x}_3 \\ \Delta\dot{x}_4 \\ \Delta\dot{x}_5 \end{bmatrix} \qquad [\Delta x] = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \\ \Delta x_5 \end{bmatrix}$$

4.2 Construction of State-Space matrices (A, B, C & D)

We do have original first order non-linear differential equations from which we can write function f(x,u) for state derivatives and function g(x) for output as below.

$$\dot{x} = f(x,u) = \begin{bmatrix} \frac{1}{L_a}(u) - \frac{R_a}{L_a}(x_1) - \frac{K_e}{L_a}(x_2) \\ \frac{N^2\ K_a}{J_m}(x_1) - \frac{k\ r_w{}^2}{J_m}(x_3) + \frac{k\, r_w\, N}{J_m}(x_5) \\ \Delta x_2 \\ g + \frac{k\ r_w}{M\, N}(x_3) - \frac{k}{M}(x_5) \\ x_4 \end{bmatrix}$$

$$y = g(x) = x_5$$

Matrices A, B, C & D can be computed from these equations by taking partial derivative of this matrix with respect to every state variable which is done below.

$\mathbf{A} = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{R_a}{L_a} & -\frac{K_e}{L_a} & 0 & 0 & 0 \\ \frac{N^2\ K_a}{J_m} & 0 & -\frac{k\ r_w{}^2}{J_m} & 0 & \frac{k\, r_w\, N}{J_m} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{k\ r_w}{M\, N} & 0 & -\frac{k}{M} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$
$\mathbf{B} = \frac{\partial f}{\partial u} = \begin{bmatrix} -\frac{1}{L_a} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
$\mathbf{C} = \frac{\partial g}{\partial x} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
$\mathbf{D} = \frac{\partial g}{\partial u} = \begin{bmatrix} 0 \end{bmatrix}$

And thus, the state space representation can be given as below

$$[\dot{\Delta x}] = \begin{bmatrix} \frac{R_a}{L_a} & -\frac{K_e}{L_a} & 0 & 0 & 0 \\ \frac{N^2 \ K_a}{J_m} & 0 & -\frac{k \ r_w{}^2}{J_m} & 0 & \frac{k \, r_w \, N}{J_m} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{k \ r_w}{M \, N} & 0 & -\frac{k}{M} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} [\Delta x] + \begin{bmatrix} -\frac{1}{L_a} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta u$$

$$\Delta y = [0 \ \ 0 \ \ 0 \ \ 0 \ \ 1] \ [\Delta x] + [0] \ \Delta u$$

5. System Simulation in MATLAB-SIMULINK

Now, we will simulate this dynamic system in MATLAB-SIMULINK as per the parameters given in the problem statement.

For this, we need to enter all the parameters in MATLAB script, create, A,B,C & D matrices in the workspace and utilize those in linearized state space block of Simulink. We would also need to create the pulse input in the Simulink.

5.1 MATLAB Code:

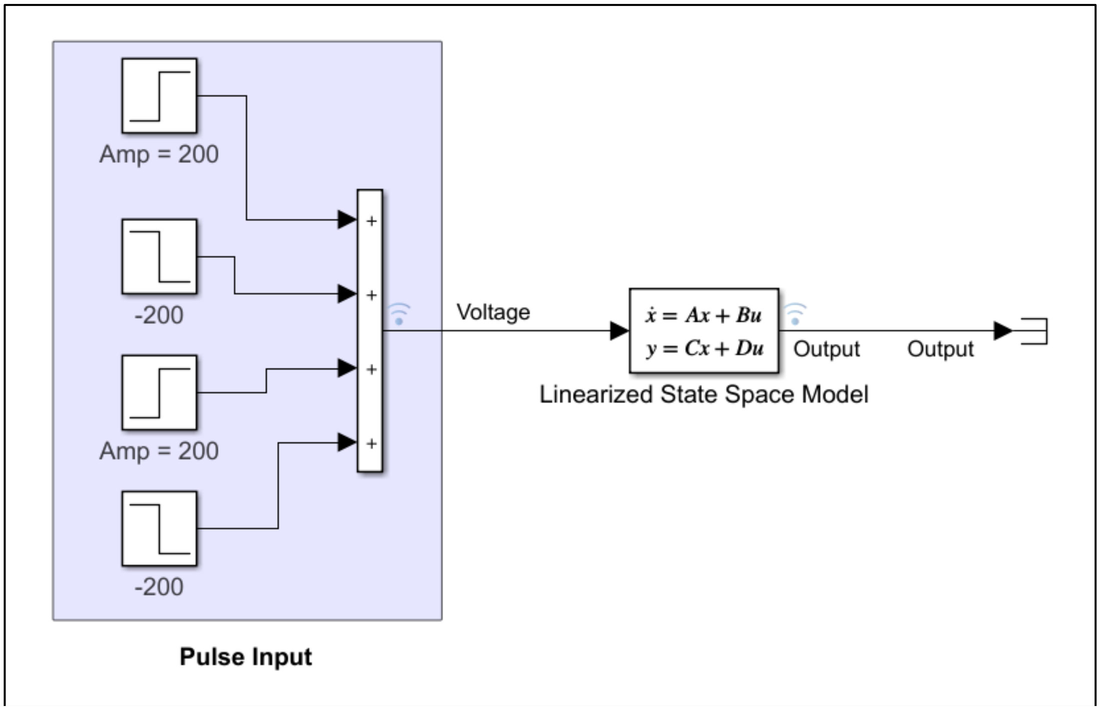
The MATLAB script is written for setting up the required parameters and state-space matrices which is shown below.

```
%% Clean Up
clearvars;  % Clear the workspace
clc;  % Clear the command window
close('all');  % Close the open figures
Simulink.sdi.clear;  % Clear the simulink data inspector


%% Simulation parameters
simPrm.solTyp = 'Fixed-step';  % Solver type
simPrm.sol    = 'ode3';  % Solver type
simPrm.dt  = 0.001; % Integration step size
simPrm.tEnd = 20; % Simulation end time
simPrm.t = 0:simPrm.dt:simPrm.tEnd;  %simulation time goes from 0 to 20


%% Physical Constancts
g  = 10;   % grav acceleration in m/s^2


%% Problem Statement Parameters
% Motor
Motor.Ka =  1; % Torque Constant(Nm/A)
Motor.Ra =  0.1;   % Armature resistance(ohm)
Motor.La =  0.018; % Inductance (H)
Motor.Jm = 5;  % rotational moment of Inertia(Kg-m2)
Motor.Ke = Motor.Ka;  %  Back emf constant= Torque constant


% Gear & Winch
Gear.R1 =  0.05; % Driving gear radius (m)
Gear.R2 =  0.2;   % Driven gear radius (m)
Gear.Rw =  0.15; % Winch radius (m)
```

```
% Spring and Mass
Load.M = 1000; % Mass of elevator car (Kg)
Load.K = 75000; % Stiffness of suspension spring (N/m)


%% Calculated Parameters
calc.N = Gear.R2/Gear.R1; % Gear Ratio

% Below parameters are the coefficient going in A matrix.
% These are calculated prior to make A matrix easy.

calc.Z1 = -Motor.Ra/Motor.La;
calc.Z2 = -Motor.Ke/Motor.La;
calc.Z3 = (Motor.Ka*calc.N*calc.N)/Motor.Jm;
calc.Z4 = -(Load.K*Gear.Rw*Gear.Rw)/Motor.Jm;
calc.Z5 = Load.K*calc.N*Gear.Rw/Motor.Jm;
calc.Z6 = (Load.K*Gear.Rw)/(Load.M*calc.N);
calc.Z7 = -Load.K/Load.M;


%% Matrices

A=[calc.Z1 calc.Z2     0     0   0
   calc.Z3    0     calc.Z4  0 calc.Z5
      0       1        0     0   0
      0       0     calc.Z6  0 calc.Z7
      0       0        0     1   0     ]; % A Matrix

B=[1/Motor.La; 0; 0; 0; 0]; % B Matrix


% I have deliberately increased the size of original C matrix by adding Identity
matrix and D matrix in order to capture the states as well from the output of
linearized state space block of Simulink. This does not affect the original
results.

C=[0 0 0 0 1
   1 0 0 0 0
   0 1 0 0 0
   0 0 1 0 0
   0 0 0 1 0
   0 0 0 0 1]; % C Matrix


D=[0;0;0;0;0;0]; % D Matrix

%% Simulate the math model
set_param('HW_1_VM','SolverType',simPrm.solTyp); % set this solver type in simulink

set_param('HW_1_VM','Solver',simPrm.sol); % set this integration method in simulink

SimOut.Ode3 = sim('HW_1_VM','SignalLoggingName','sdata'); % Simulate the math model
and save the data
```

5.2 Simulink Model:

With the parameters set up in workspace using setup script, those are then used in Simulink linearized state space model which would give output as all states and the output of system. That is position of the elevator. Below is the snapshot of the Simulink model. The output from this block would contain six elements – 5 states and 1 output of the system.



5.2 Plots of the results:

In order to create the plots of all state variables and the output, I have extended the original MATLAB script the description of which is given below.

```
%% Extract Data for plotting
Voltage = 1;  % Variable for extracting the data of voltage signal
Pos_NL = 2;
Output = 3;  % Variable for extracting the output data


% Results
Results.Pos = SimOut.sdata{3}.Values.Data(:,1); % Linear model position
Results.Current = SimOut.sdata{3}.Values.Data(:,2); % Armature current   values
Results.AngVel = SimOut.sdata{3}.Values.Data(:,3);  % Angular Velocity values
Results.AngPos = SimOut.sdata{3}.Values.Data(:,4);  % Angualr Position values
Results.Vel = SimOut.sdata{3}.Values.Data(:,5);   % Velocity of elevator values
Results.PosNL = SimOut.sdata{2}.Values.Data(:,1);  % Non Linear model position
%% Plot the results
figure(1);
plot(simPrm.t,Results.Pos,'r','LineWidth',0.8);
hold on
xticks(0:1:20);  % Give the limits of both axis
yticks(0:5:80);
title("Displacement of Elevator car actuated by PMDC"); % Give the title
xlabel('Time (s)');  % Give X label as Time in seconds
ylabel('Displacement (m)');  % Give Y label as displacement in meters


figure(2);
plot(simPrm.t,Results.Current,'b','LineWidth',0.8);
hold on
xticks(0:1:20);  % Give the limits of both axis
yticks(-1200:200:1200);
title("Armature current of PMDC vs Time"); % Give the title
xlabel('Time (s)');  % Give X label as Time in seconds
ylabel('Current (A)');  % Give Y label as displacement in meters


figure(3);
plot(simPrm.t,Results.AngVel,'b','LineWidth',0.8);
hold on
xticks(0:1:20);  % Give the limits of both axis
yticks(-2:0.5:5);
title("Angular Velocity of DC Motor shaft vs Time"); % Give the title
xlabel('Time (s)');  % Give X label as Time in seconds
ylabel('Angular Velocity (rad/s)');  % Give Y label as displacement in meters


figure(4);
plot(simPrm.t,Results.AngPos,'b','LineWidth',0.8);
```

```
hold on
xticks(0:1:20);  % Give the limits of both axis
yticks(0:5:40);
title("Angular Velocity of DC Motor shaft vs Time"); % Give the title
xlabel('Time (s)');  % Give X label as Time in seconds
ylabel('Angular Position (rad)');  % Give Y label as displacement in meters


figure(5);
plot(simPrm.t,Results.Vel,'b','LineWidth',0.8);
hold on
xticks(0:1:20);  % Give the limits of both axis
yticks(-3:0.5:10);
title("Translational Velocity of Elevator hoist vs Time"); % Give the title
xlabel('Time (s)');  % Give X label as Time in seconds
ylabel('Velocity (m/s)');  % Give Y label as displacement in meters


figure(6);
plot(simPrm.t,Results.Pos,'r','LineWidth',0.8);
hold on
plot(simPrm.t,Results.PosNL,'k','LineWidth',0.01);
xticks(0:1:20);  % Give the limits of both axis
yticks(0:10:120);
title("Displacement of Elevator car actuated by PMDC"); % Give the title
xlabel('Time (s)');  % Give X label as Time in seconds
ylabel('Displacement (m)');  % Give Y label as displacement in meters
legend('Linear Model','Non Linear Model'); % Legends
```

5.2.1 Elevator position (Output and fifth state variable) vs Time

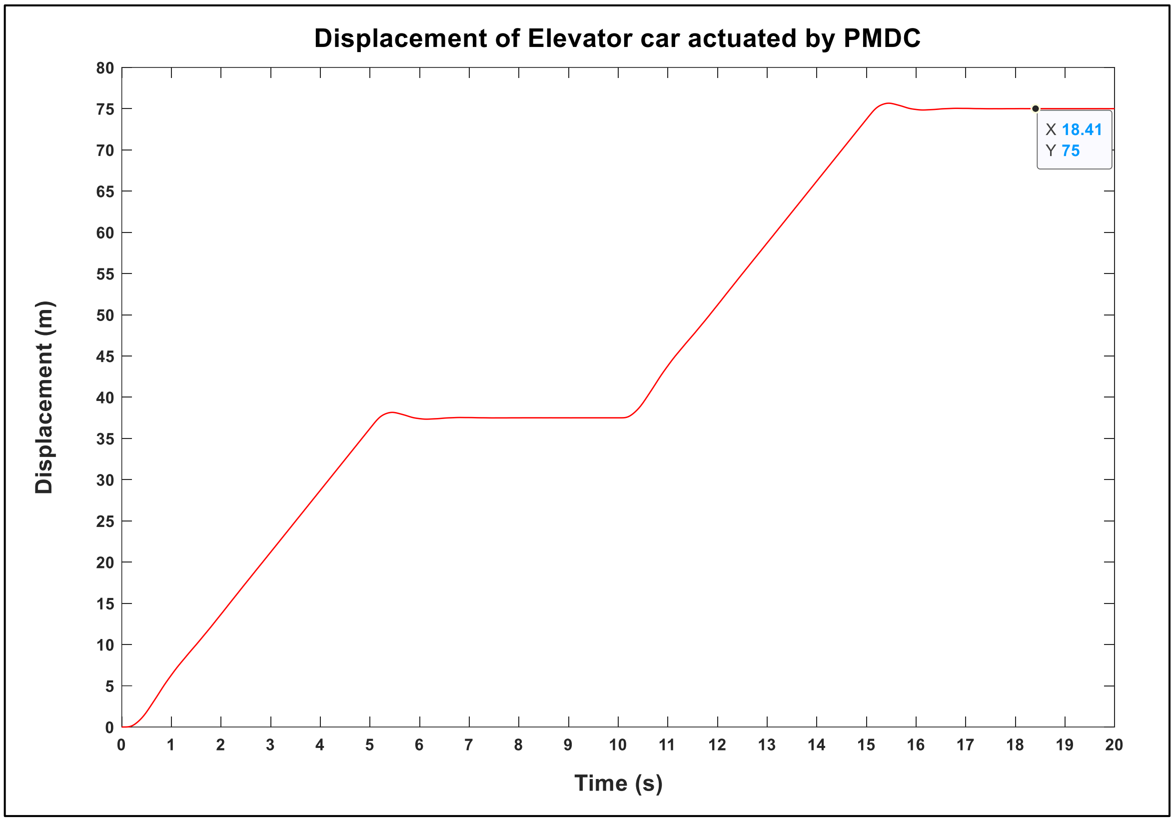


Figure 1

The above figure shows the displacement of elevator with respect to the time. When the voltage is made zero in the input pulse, the elevator stops at that position and again starts lowering when the voltage is switched to 200 volts. The total displacement of elevator is 75 m.

5.2.2 Armature current (first state variable) vs Time

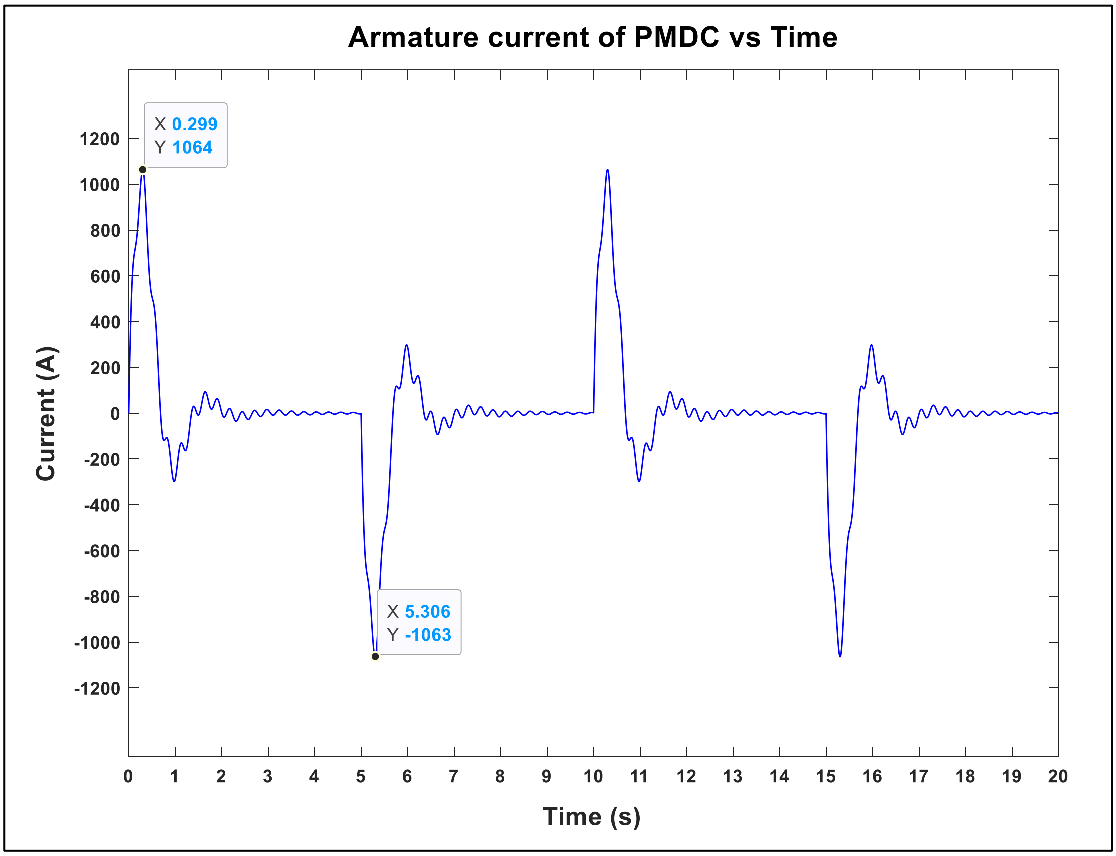


Figure 2

The above figure shows the armature current of motor with respect to the time. The maximum current drawn from the motor is 1064 Amperes.

5.2.3 Angular Velocity of motor shaft (second state variable) vs Time

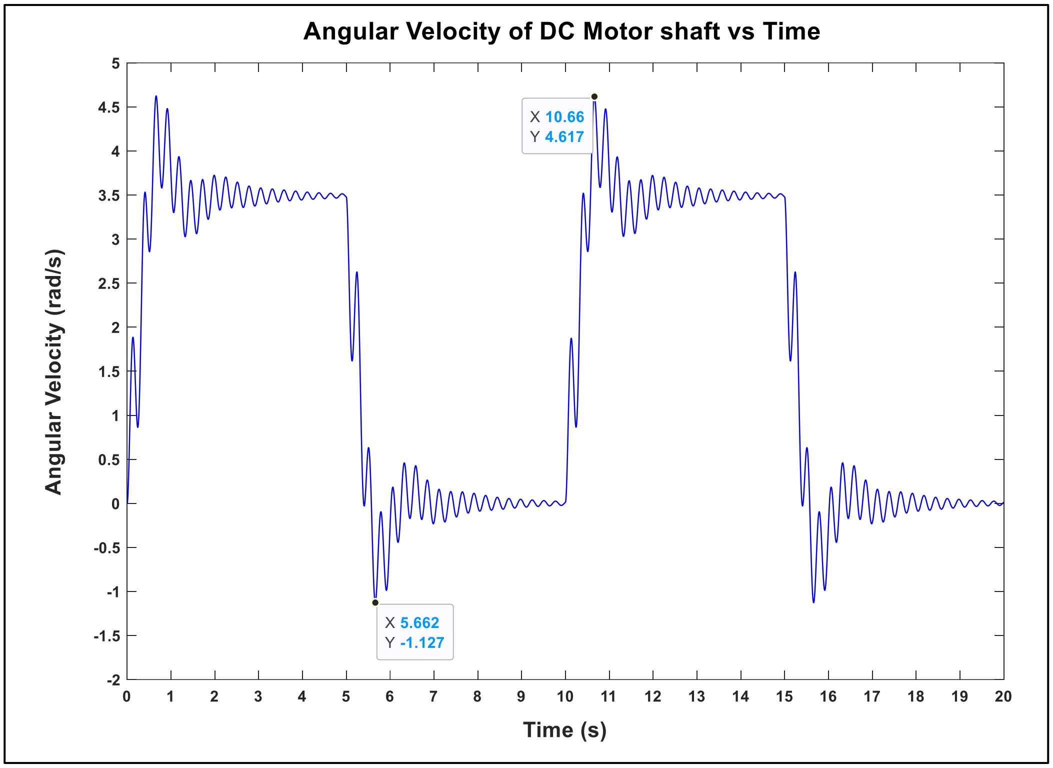


Figure 3

The above figure shows the Angular Velocity of DC motor shaft with respect to the time. The angular velocity follows the pulse durations of voltage input. Maximum angular velocity is 4.617 rad/s.

5.2.4 Angular position of motor shaft (third state variable) vs Time

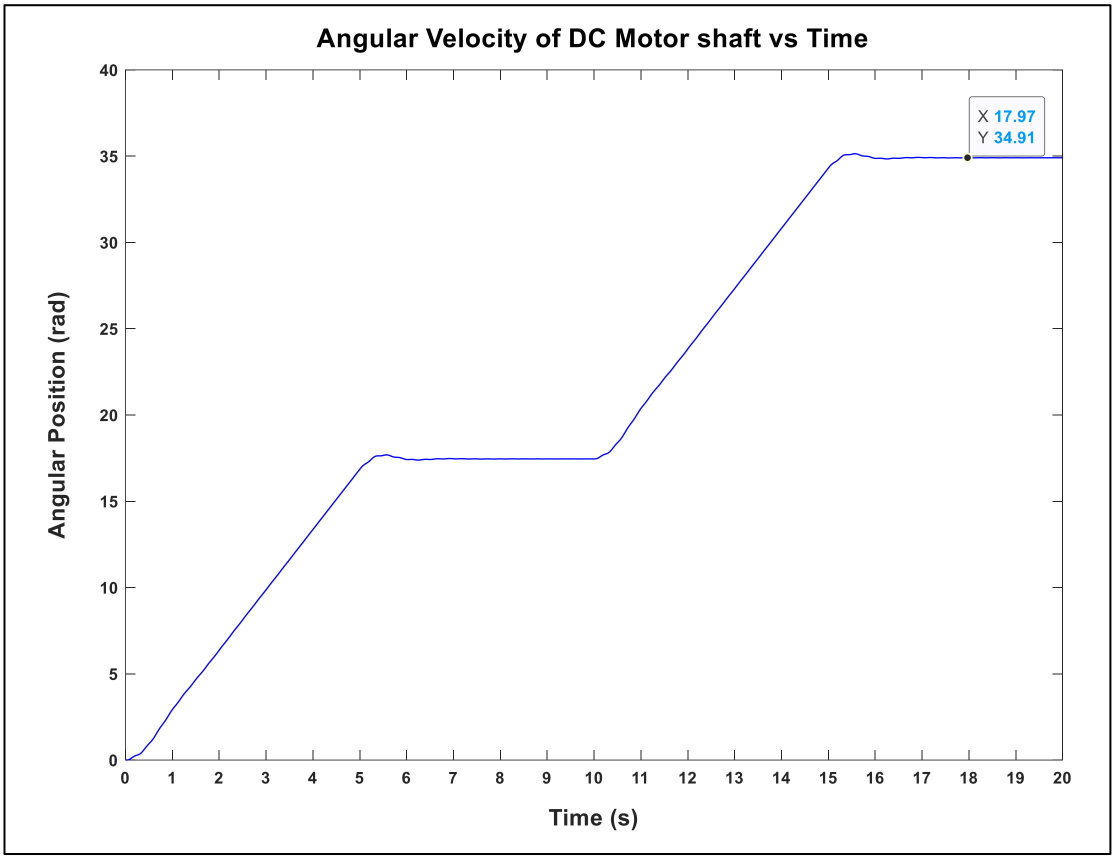


Figure 4

The above figure shows the Angular position of DC motor shaft with respect to the time. The angular position follows the position of elevator mass. Maximum angular rotation is 35 rad.

5.2.5 Velocity of elevator (fourth state variable) vs Time

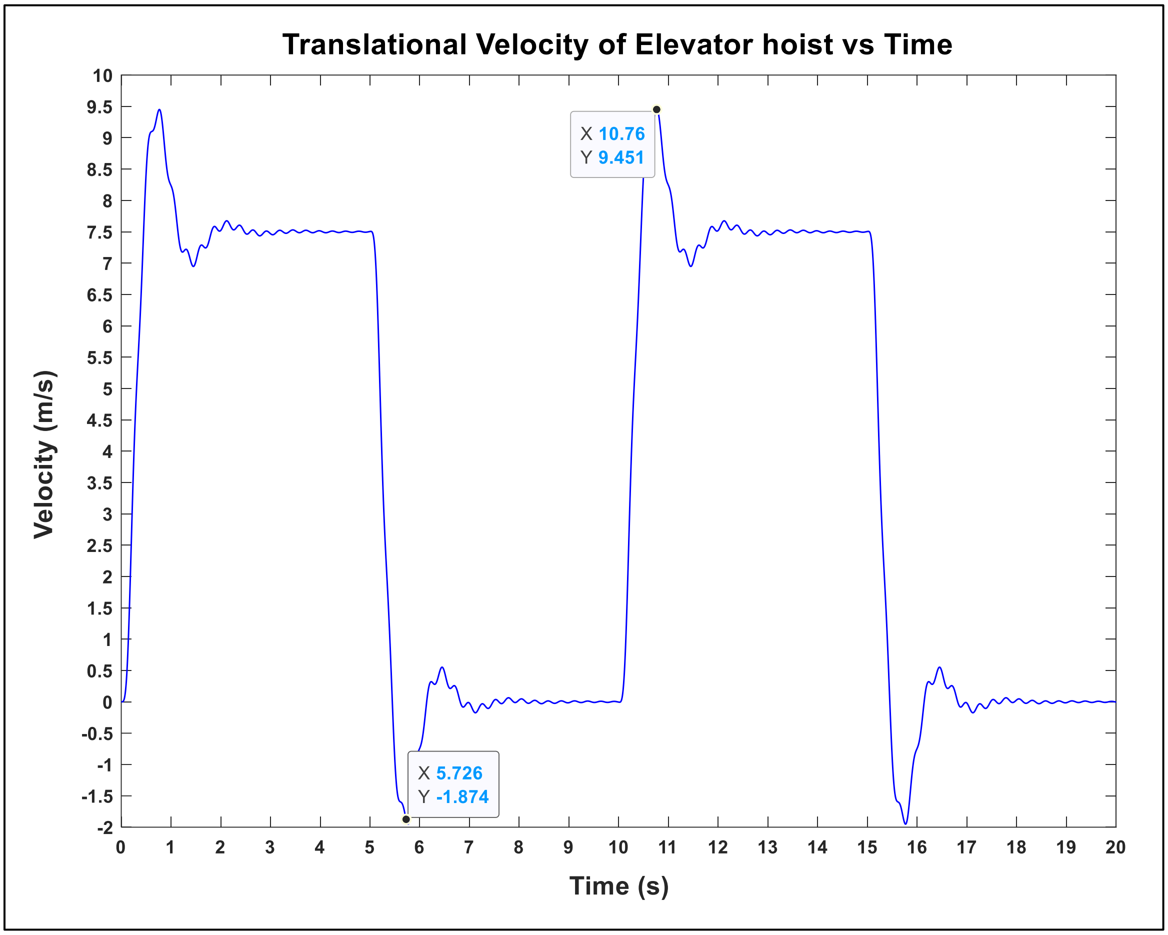


Figure 5

The above figure shows the variation of linear velocity of mass with respect to the time. The angular velocity follows the pulse profile of voltage input. Maximum velocity is 9.435 m/s.

5.3 Numerical Integration method and step size

One of the more critical challenges in solving the system of ordinary differential equations lies in determining the optimum integration step size and integration method. As we go on reducing the step size and high order integration method, with no doubt improve the accuracy of the solution, however that is achieved by compromising the computational time of the solution. Thus, there has to be trade-off between accuracy of the solution and the computational time. Since we are simulating the linear system, we can take smaller step size as compared to what we take usually for non-linear system. Also, the integration method of higher order can be used. In order to trade off between the computational time and accuracy, we have selected following parameters.

Numerical Integration Method	**Ode3**
Integration Step Size	**0.001**

5.4 Comparison with Non-linear response

The system was first linearized and then simulated. Thus, while linearizing the constant parameters in the state equations are eliminated. Thus, they are not playing the role in output of system. However, it would be interesting to examine how those parameters change the output and let's see that in the form of the plot.

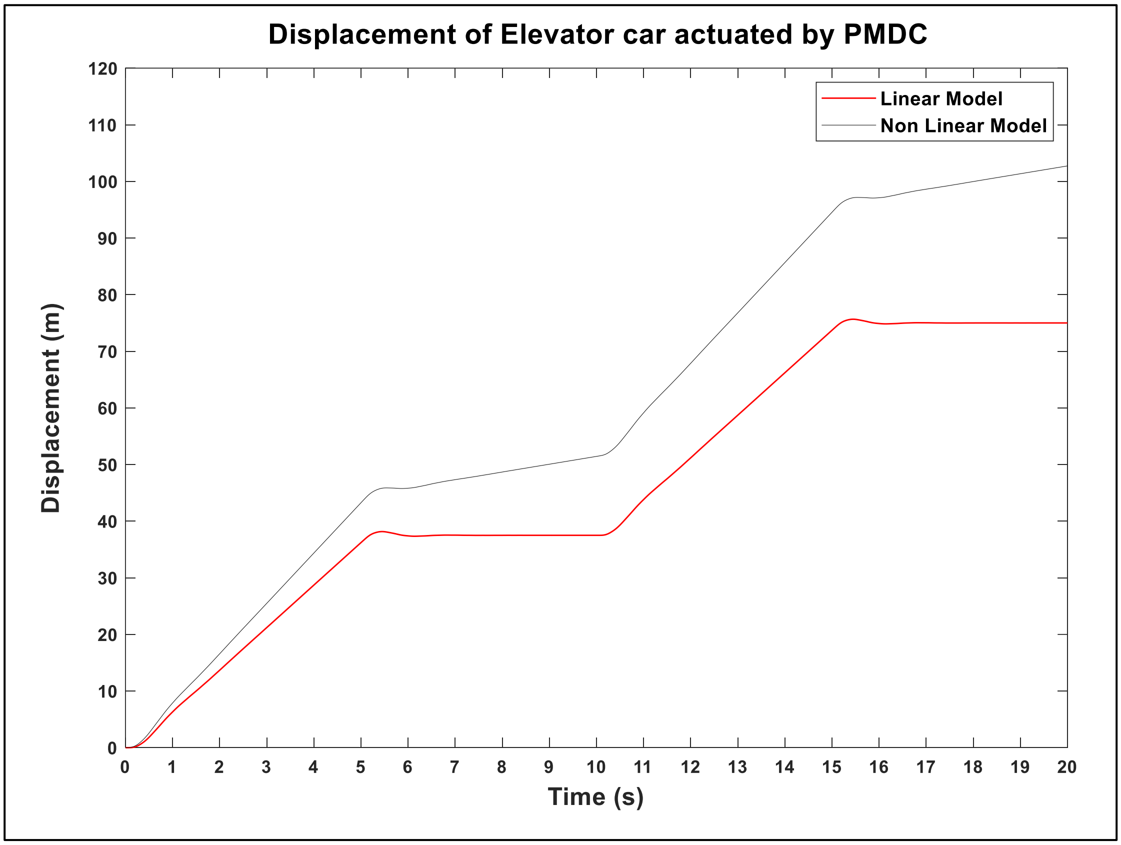


Figure 6

The above graph shows the comparison of output from linear model and non-linear model. The output plotted here is the displacement of elevator car with respect to time. It can be seen that non-linear model has output result values greater than linear model values. This is due the fact that non-linear model is considering the constant term 'g – gravitational acceleration' in the output response of the system.