

Problem 1 Mathematical Formation of the Question:

Maximum Weight Matching

Objective :

$$\text{maximize } \sum_{v \in E} w_v x_v \quad (\text{maximize weight} \times \text{edge selection})$$

Constraints : $x_v \in \{0, 1\}$ (every edge is considered or not)

1) $\sum_{\substack{j \in 2^{\text{nd}} \text{ set} \\ i \in 1^{\text{st}} \text{ set}}} x_{ij} = 1$ (Sum of all edges on a vertex in first set should be 1)

2) For perfect matching (nodes = even) (optional if perfect matching is needed)

$$\sum_{\substack{j \in 2^{\text{nd}} \text{ set} \\ i \in 1^{\text{st}} \text{ set}}} x_{ji} = 1$$

Decision Variable :

x_v : decision variable for every edge $v \in E$

The above formulas shows the formation of the maximum weight matching problem.

The objective is to select the edges such that the sum of the weights of those edges is maximized.

The first constraint states that the sum of all edges passing through a node in set 1 (smaller set of the bipartite graph if uneven or the first set if equal) should be 1.

The second constraint is optional. It is only for perfect matching (if we want perfect matching in maximum weighted bipartite matching problem as it is not mentioned in the assignment question) and it is only possible when the user enters even number of nodes (So both sets will have equal nodes). If we don't want perfect matching, we can avoid this constraint and continue with simple matching (Commenting the cell where this part is implemented in the code). The second constraint states that the sum of all edges passing through a node in set 2 should be 1.

The decision variable states whether we have selected the edge to get maximum weighted bipartite matching or not. It is binary, either 0 or 1.