Cap 13

Variational Approximation Methods

Questão 10

Construct a three-term Rayleigh-Ritz approximate solution to

$$\begin{aligned} u_{xx} + u_{yy} &= x; x, y \in \Omega \\ u &= 0; x, y \in \partial\Omega \\ u_x(0, y) &= 0; 0 < y < 100 \\ u_y(x, 0) &= 0; 0 < x < 100 \end{aligned}$$

Base functions:

$$\begin{aligned} \phi_1 &= 10 - (x^2 + y^2)^{1/2} \\ \phi_2 &= 10 * x - x * (x^2 + y^2)^{1/2} \\ \phi_3 &= 10 * y - y * (x^2 + y^2)^{1/2} \end{aligned}$$

Solution

Numerical Integration Method Used

The numerical integration use to solve this problem is very naive. Basicly, a mesh is computed for each function to be integrated over the domain and then all the elements of this mesh are summed. The resolution is then used to correct the scaling of the integral. The function used to do this is **sum**.

There are other approches that can be used to solve this problem. One is to use the **quad** functions that use Gauss quadrature to compute the integral. The problem with this is that it needs a function handler as an input. As the Rayleigh-Ritz method needs the gradient of the base functions, one needs to compute manually all the gradient of these base functions. If one does not bother with this, as these functions are know **a priori**, this is a good approch.

Boundaries

Other problem with the current solution is that the boundary is very different from the specified in the original problem:

$$x^2 + x^2 < 100, x, y > 0$$

As oppose to the one use in the solution:

$$x < 10, y < 10$$