Introduction To Quantum Hall Effect

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The Classical Hall Effect

The equilibrium of the Hall effect can be described using Ohm's law in convention:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{pmatrix} \begin{pmatrix} J_x \\ 0 \end{pmatrix},$$

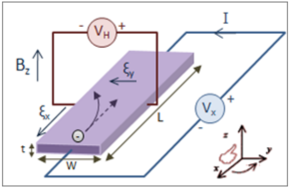


Figure: Classical Hall effect

in which:

$$\rho_{xy} = \frac{E_y}{J_x} = -\frac{B}{ne}, \qquad \rho_{xx} = \frac{E_x}{J_x} = \frac{m}{ne^2\tau}$$

Classically:

$$E_y \propto B \text{ and } E_x \text{ depend on scattering parameter } \tau$$

The Quantum Hall Effect

First introduced in 1980[1] and later on being investigated, the resistance in MOSFET under a strong magnetic field have showing interesting properties:

At certain point:

$$\rho_{xx} = 0.$$

$$\rho_{xy} \sim \frac{1}{\nu}, \quad \nu \in \mathbb{N}$$

Between these points:

$$\rho_{xy} = const.$$

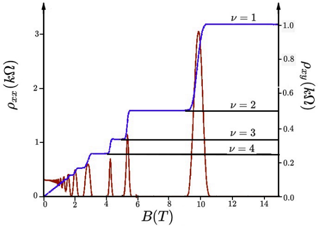


Figure: Quantum Hall Resistance Taken from n.d.

[1] Klitzing, Dorda, and Pepper 1980.

Therefore, we will explain this effect as:

- Why $\rho_{xx} \to 0$ is a peaks at some certain points and 0 otherwise?
- Why these plateux exist?

With a 2D electron gas in an external field field $\mathbf{B} = (0, 0, B)$ & $\mathbf{E} = (E, 0, 0)$, their energy will be collapsed to the Landau levels:

$$E_{\nu,k_y} = \hbar\omega_B\left(\nu + \frac{1}{2}\right) - eE\left(k_y l_B^2 + \frac{eE}{m\omega_B^2}\right) + \frac{m}{2}\frac{E}{B}, \qquad (1)$$

in which

$$\omega_B = \frac{eB}{m}, \quad l_B = \frac{\hbar}{eB}$$

The electron will drift in the O_y direction by:

$$v_y = \frac{1}{\hbar}\frac{\partial E_{\nu,k_y}}{\partial k_y} = -\frac{E}{B} \qquad (2)$$

Classical drift along: $\mathbf{E} \times \mathbf{B}$

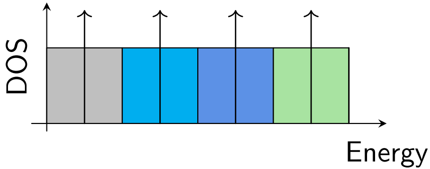


Figure: From constant DOS to Dirac comb

Conductivity

Each filled Landau levels have the degeneracy (in this convention, it's k_y) that when we take over the sum to get:

$$\mathbf{I} = -e\left\langle \dot{\mathbf{x}} \right\rangle = -e\sum_{n=1}^{\nu}\sum_{k_y}\left\langle \psi_{nk_y} \middle| \frac{\hbar}{i}\nabla - \mathbf{A} \middle| \psi_{nk_y} \right\rangle$$

$$\Rightarrow \qquad I_x = 0, \qquad I_y = -\sum_{k_y} e\nu\frac{E}{B} = \frac{e^2\nu E}{2\pi\hbar}$$

Result in:

$$\rho_{xx} = 0, \qquad \rho_{xy} = \frac{2\pi\hbar}{e^2\nu}, \tag{3}$$

in which ν is the total filled number of Landau levels.

But these conduction above not explained everything!

Revisit the calculation of (7) from (2) with a more generalize approach give:

$$v_y = -\frac{1}{eB}\frac{\partial V(x)}{\partial x} \qquad (4)$$

$$\sigma_{xy} = \frac{e}{EL_x}\int \frac{dk}{2\pi} v_y(x)$$

$$= \frac{e}{\cancel{EL_x}} \frac{\cancel{V\Big|_{x=x_{min}}^{x=x_{max}}}}{2\pi\hbar} = \frac{e^2}{2\pi\hbar} \qquad (5)$$

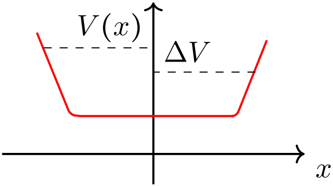


Figure: Difference in Fermi energy at both sides Edge's potential $V(x)$

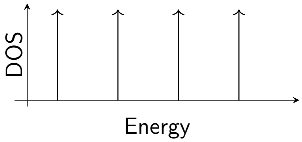
$\Rightarrow$ As long as the $\partial_x V(x)$ smooth enough, only the difference of the edges create the quantize value!

But, how we can got the round (not sharps) as in fig. 2?

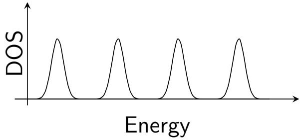
$\Rightarrow$ Turn out the disorder (impurity) play as another key role!

Disorder causes:

- Perturbation $\rightarrow$ Broader side (peaks!).
- Catch the localized state $\rightarrow$ Plateux!



(a) Without disorder



(b) with disorder

The Impurity act as a perturbation, causing the broad edge of the energy.

Sample too perfect? $\rightarrow$ flat spectrum

But:

Too much disorder $\rightarrow$ not recognize the peaks!

From the calculation of the Landau levels (1), there's the degeneracy k_y in each Landau levels ν:

- Decrease B → more bands filled
- Total number of electrons: constant

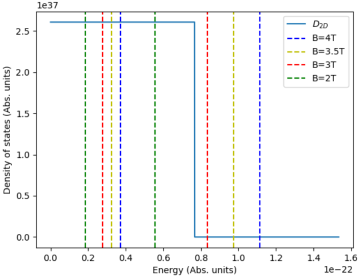


Figure: Illustrated number of filled levels when decrease B

From the calculation of the Landau levels (1), there's the degeneracy k_y in each Landau levels ν:

- Decrease B → more bands filled
- Total number of electrons: constant

But:

- Same filled levels ν: $N_e \propto B$.

Where do the electrons go?
They're still there!
(just not in the bands!)

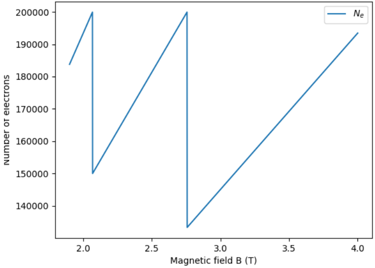


Figure: Illustrated Number of electrons when decrease B

Disorder causes:

- Perturbation → Broader side (peaks!).
- Catch the localized state → Plateux!

The impurity → broad peaks → higher or lower energy than the center→ localized by the impurity

Localized state don't contribute in conduction → plateux

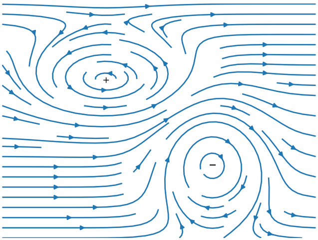


Figure: Movement of center of mass localized under impurity's maximum + or minimum −.

When decrease B but not filled the next level yet:

→ The electrons will populate the localized states!

The Quantum Hall Effect

So, let summary two main aspects:

- Edge states make sure quantized values.
- Impurity create the peaks and the plateux.

Partly filled levels?
→ Impurity create scattering inside level → longitude peaks.

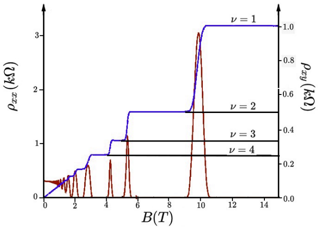


Figure: Quantum Hall Resistance Taken from n.d.

Reference:

Advanced Quantum Mechanics II PHYS 40202 — oer.physics.manchester.ac.uk (n.d.). https://oer.physics.manchester.ac.uk/AQM2/Notes/Notes-4.4.html. [Accessed 15-02-2025].

Klitzing, K. v., G. Dorda, and M. Pepper (1980). "New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance". In: *Phys. Rev. Lett.* 45 (6), pp. 494–497.