Calculation of the Linear-Absorption Spectrum of an Ideal Two-Dimensional System of $\mathrm{MoS_2}$

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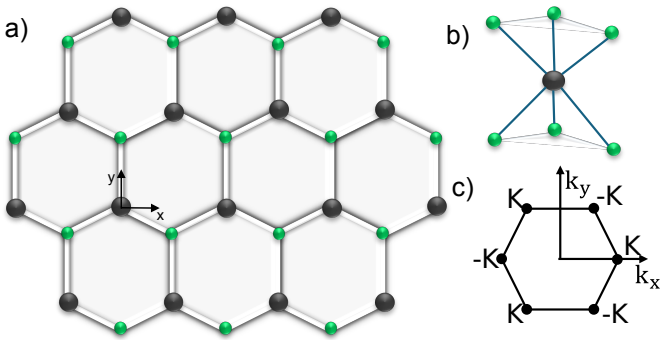
Outline

Transition Metal Dichalcogenides Monolayer

Group VI-B Transition Metal Dichalcogenides (TMD) are compound semiconductors of the type MX_2. M is Transition Metal atom (black dots), X are Dichalcogenides atom (green dots):



- They stable in both mono- and few-layer in the air at room temperature.
- They are semiconductor with a direct band gap.
- Strong spin-orbit Coupling (SOC) in TMD monolayers leads to spin splitting of hundreds meV.

Exciton Binding Energy In TMD

Why calculate the linear absorption spectrum?
$\rightarrow$ Exciton Binding Energy

- TMD is a low-dimension material, create huge binding energy in compare with bulk semiconductor $\rightarrow$ huge effect on phenomenon $\Rightarrow$ need to be calculated and taken into account in any further many-body calculation.
- Early theories predict binding energy too large $(0.5 - 1$ eV) in compare with experiment $(0.2 - 0.5$ eV) $\Rightarrow$ need precise calculation in agree with experiment.
- Other theory only fitting bandstructure around high symmetry point such as K/K', not on entire BZ $\Rightarrow$ Tight-Binding method for fitting on entire BZ.

Tight-Binding (TB) wave function have the form of Linear combination of Atomic Orbital (LCAO):

$$|\psi_{\lambda\mathbf{k}}(\mathbf{r})\rangle = \sum_{\alpha} c_{\lambda\alpha}(\mathbf{k}) \sum_{\mathbf{R}} e^{i\mathbf{k}\mathbf{R}} |\phi_{\alpha}(\mathbf{r}-\mathbf{R})\rangle \tag{1}$$

Time-independence independence-electron approximation Schrödinger equation with LCAO included:

$$H_{1e} \sum_{\alpha} c_{\lambda\alpha}(\mathbf{k}) \sum_{\mathbf{R}} e^{i\mathbf{k}\mathbf{R}} |\phi_{\alpha}(\mathbf{r}-\mathbf{R})\rangle = \varepsilon_{\lambda}(\mathbf{k}) \sum_{\alpha} c_{\lambda\alpha}(\mathbf{k}) \sum_{\mathbf{R}} e^{i\mathbf{k}\mathbf{R}} |\phi_{\alpha}(\mathbf{r}-\mathbf{R})\rangle$$

Multiply with $\langle\phi_{\beta}|$ on the left and take integral over $\mathbf{r}$

$$\sum_{\alpha} [H^{TB}_{\beta\alpha}(\mathbf{k}) - \varepsilon_{\lambda}(\mathbf{k})\delta_{\beta\alpha}] c_{\lambda\alpha}(\mathbf{k}) = 0. \tag{2}$$

In which the Tight-binding Hamiltonian matrix elements:

$$H^{TB}_{\beta\alpha}(\mathbf{k}) = \sum_{\mathbf{R}} \langle\phi_{\beta}(\mathbf{r})| H_{1e} |\phi_{\alpha}(\mathbf{r}-\mathbf{R})\rangle \tag{3}$$

Use basic functions of d-type Orbitals:

$$|\phi_1\rangle = d_{z^2}, |\phi_2\rangle = d_{xy}, |\phi_3\rangle = d_{x^2-y^2}.$$

Three-band TB Hamiltonian with SOC has the form:

$$H^{TB}_{6\times 6}(\mathbf{k}) = \begin{bmatrix} H^{TB}_{3\times 3}(\mathbf{k}) + \gamma L_z & 0 \\ 0 & H^{TB}_{3\times 3}(\mathbf{k}) - \gamma L_z \end{bmatrix}, \quad L_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{bmatrix} \tag{4}$$

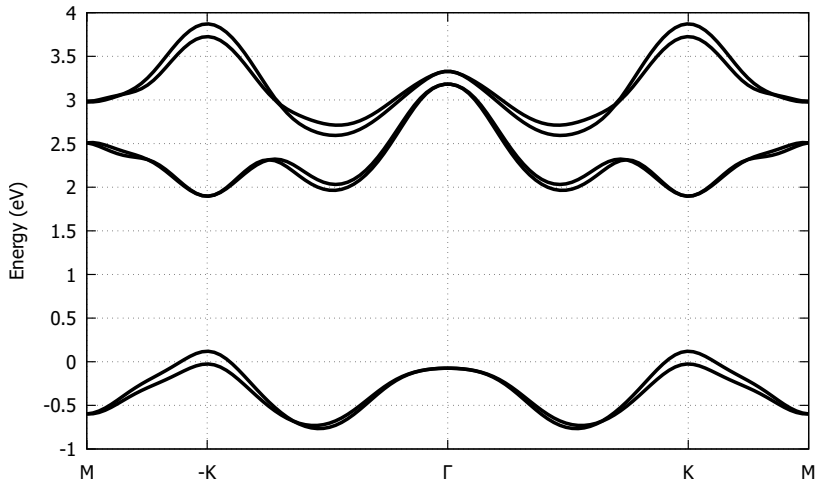


Figure: MoS_2 Bandstructure calculated from 3-band TB models[1]

Multiband velocity gauge semiconductor Bloch equations (SBE) with Coulomb interaction in Hatree Fock approximation:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\lambda\lambda'}(\mathbf{k},t) = &-\frac{i}{\hbar}(\varepsilon_\lambda(\mathbf{k})-\varepsilon_{\lambda'}(\mathbf{k}))\rho_{\lambda\lambda'}(\mathbf{k}) \\ &- i\sum_\mu(\Omega_{\lambda\mu}(\mathbf{k})\rho_{\mu\lambda'}(\mathbf{k},t)-\rho_{\lambda\mu}(\mathbf{k},t)\Omega_{\mu\lambda'}(\mathbf{k})) \\ &+\frac{\rho_{\lambda\lambda'}(\mathbf{k},t)}{T_2}(1-\delta_{\lambda\lambda'}),\end{aligned} \tag{5}$$

where

$$\Omega_{\mu\nu}(\mathbf{k}) = \frac{1}{\hbar}\left(\frac{e}{m}\mathbf{A}(t)\mathbf{p}_{\mu\nu}(\mathbf{k}) - \sum_{\alpha\beta\mathbf{q}} W^{\alpha\mu\beta\nu}_{\mathbf{k},\mathbf{k}+\mathbf{q},\mathbf{q}}\rho_{\beta\alpha}(\mathbf{k}+\mathbf{q})\right), \tag{6}$$

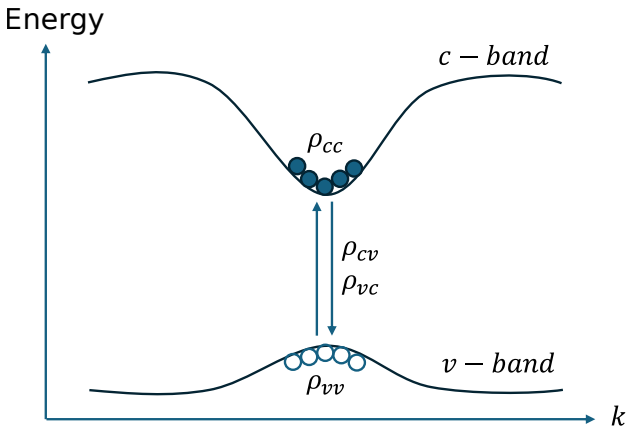
$$\mathbf{p}_{\mu\nu}(\mathbf{k}) = \frac{m}{\hbar}\sum_{\alpha,\beta} c^*_{\mu\alpha}\nabla_{\mathbf{k}}H^{TB}_{\alpha\beta}(\mathbf{k})c_{\nu\beta}(\mathbf{k}) \tag{7}$$

For $\mu \neq \nu$:

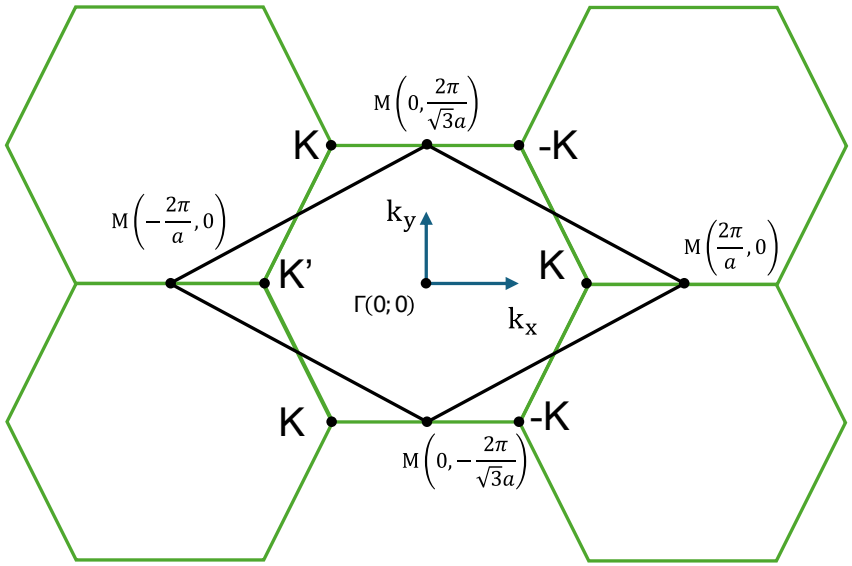
$$\vec{\xi}_{\mu\nu}(\mathbf{k}) = \frac{-i\hbar}{m} \frac{\mathbf{p}_{\mu\nu}(\mathbf{k})}{\varepsilon_{\mu}(\mathbf{k}) - \varepsilon_{\nu}(\mathbf{k})} \quad (8)$$

Time-dependence interband polarization density:

$$\begin{aligned} \mathbf{P}(t) =& \frac{e}{L^2} \sum_{\mathbf{k}} \mathrm{Tr} \left[\vec{\xi}(\mathbf{k}) \rho(\mathbf{k}, t) \right] \\ =& \frac{e}{L^2} \sum_{\mathbf{k}\lambda\lambda'} \vec{\xi}_{\lambda\lambda'}(\mathbf{k}) \rho_{\lambda'\lambda}(\mathbf{k}, t) \end{aligned} \quad (9)$$

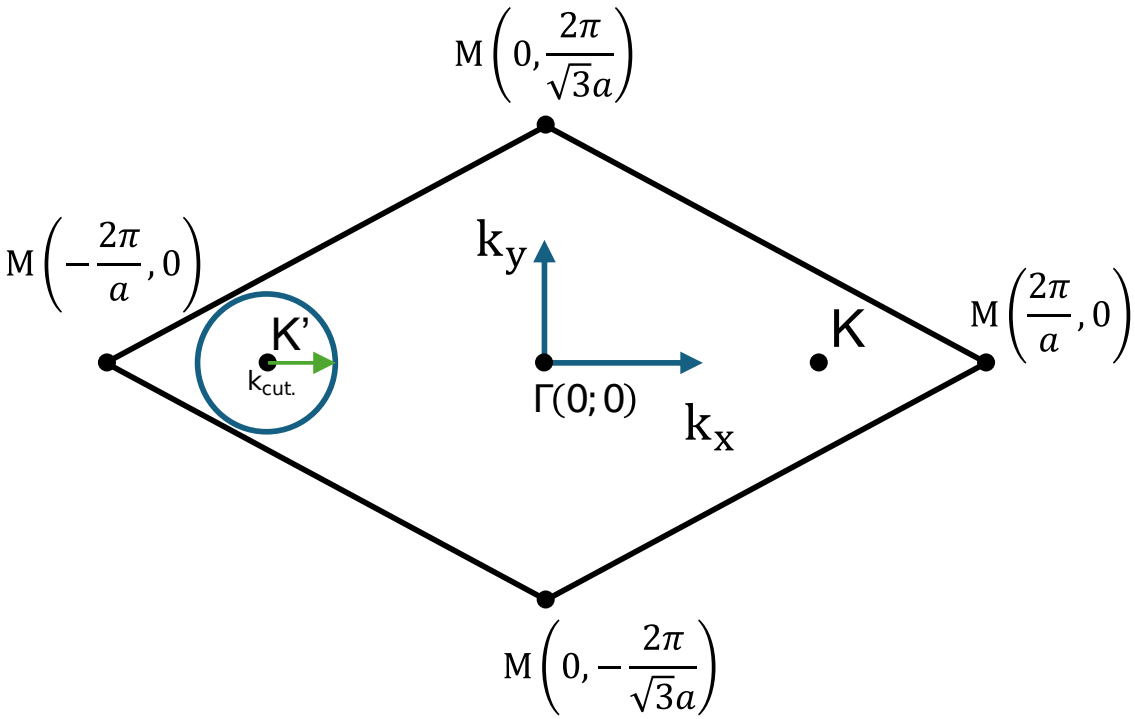


Numerical Sum Over k-Space



$$\sum_{\mathbf{k}} \ldots \rightarrow \frac{L^2}{4\pi^2} \int \int_{BZ} dk_x dk_y \ldots \tag{10}$$

k-Cutoff



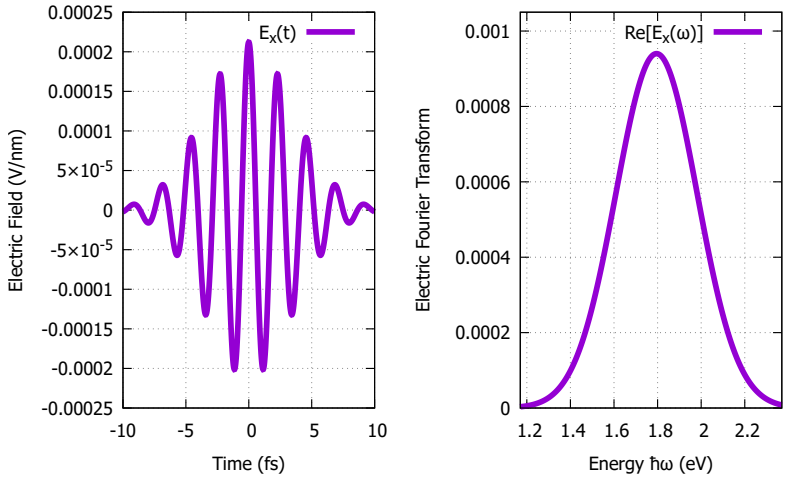
$$W^{\alpha\mu\beta\nu}_{\mathbf{k},\mathbf{k}',\mathbf{q}} \approx W^{\alpha\mu\beta\nu}_{\mathbf{k},\mathbf{k}',\mathbf{q}} \theta(k_{cut.} - |\mathbf{k} - \mathbf{k}_{K'}|)\theta(k_{cut.} - |\mathbf{k}' - \mathbf{k}_{K'}|). \qquad (11)$$

Electromagnetic Field

Electromagnetic has Gaussian form:

$$\mathbf{E}(t) = \mathbf{E}_0 \cos(\omega t) e^{-\frac{t^2}{\tau_L^2}} \qquad (12)$$

- small E_0 : $\rho_{cc}(\mathbf{k}) \to 0$
- $\hbar\omega = E_{gapp.} \to$ linear absorption
- small τ_L for better range around ω



Absorption obtain by[2]:

$$\alpha(\omega) \propto \frac{P(\omega)}{E(\omega)} \qquad (13)$$

Experiment measure:

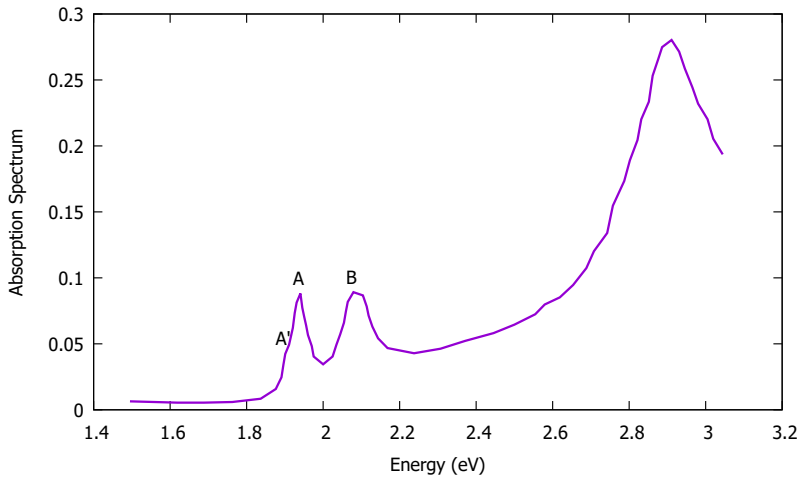
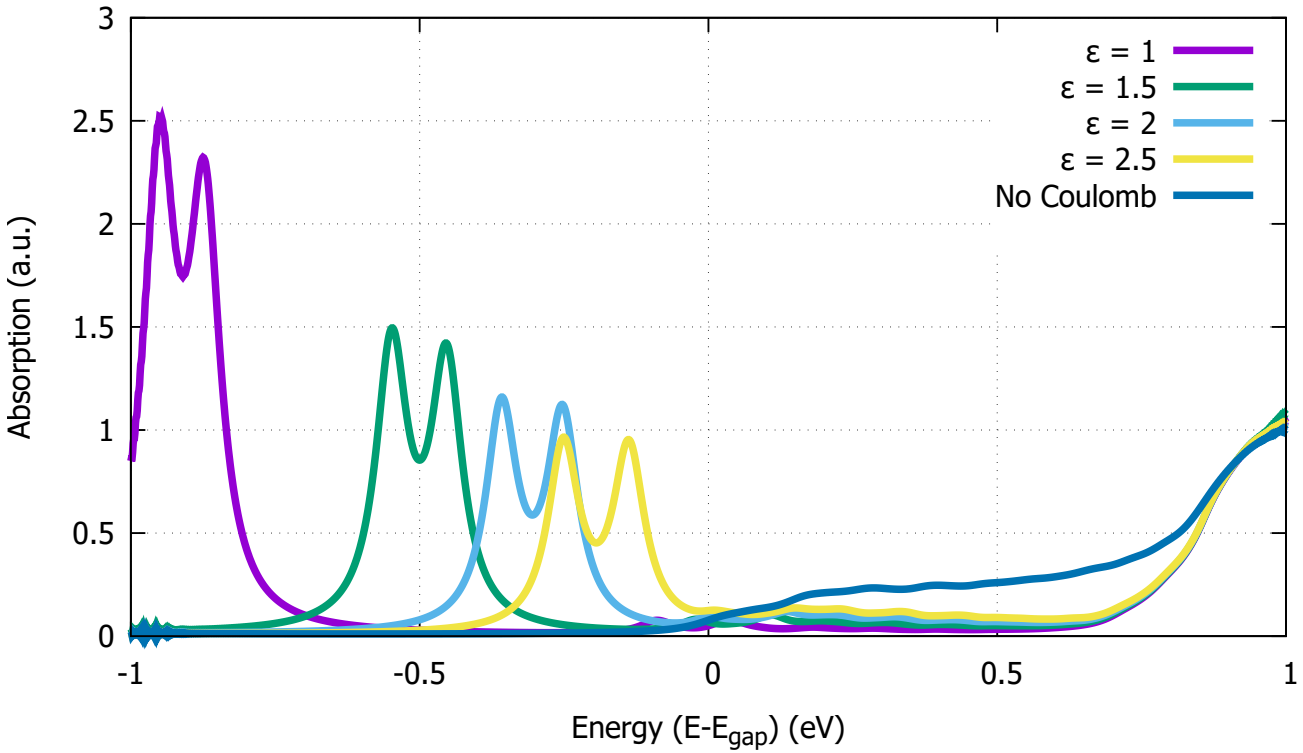
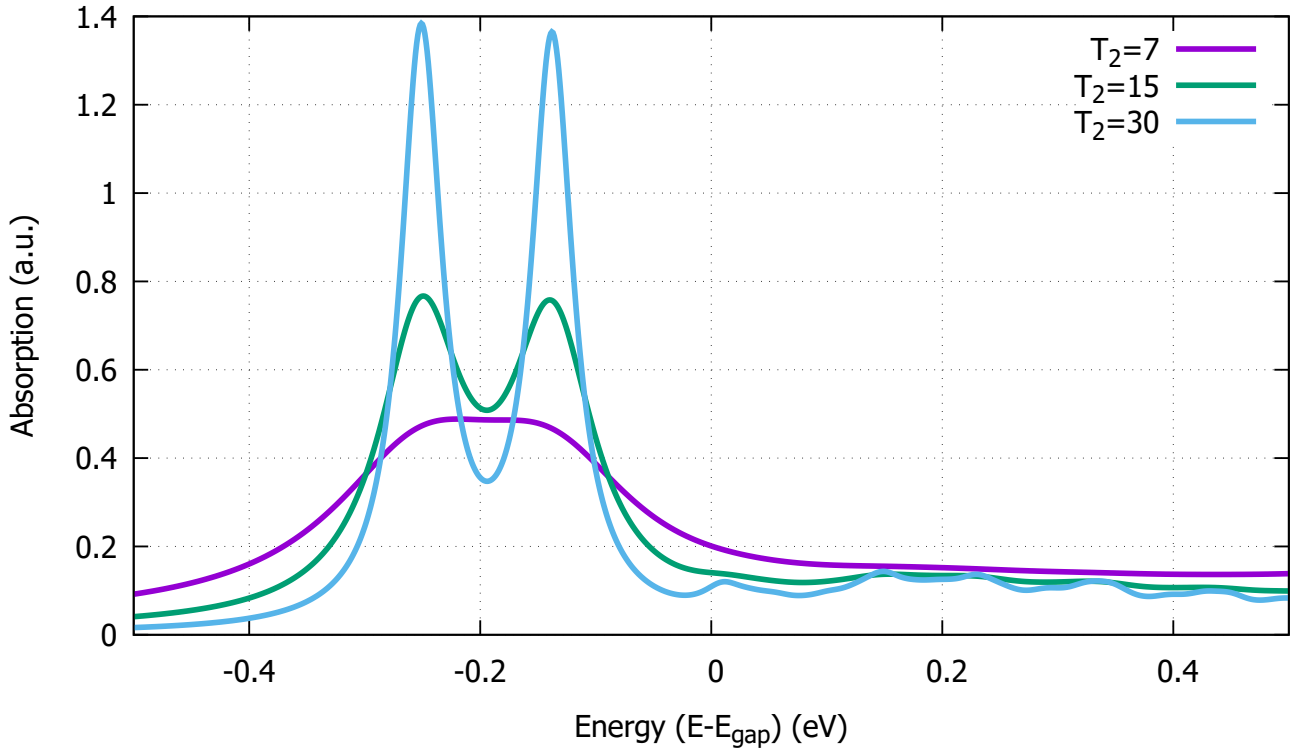


Figure: Measure Absorption Spectrum of MoS_2 at $T = 5K$ extracted from [3]

- Two resonance labeled by A and B is Exciton peak. Show band split by SOC
- Small trion peak near A label by A'



- Choosing the ε for fitting with experiment.
- For 3-band TB model: $\varepsilon \in (1.5, 2.5)$ is good with Exciton binding energy $E_{bind.} = 0.2 - 0.5eV$



- Choosing the T_2 for clearer Exciton peak.
- The bigger T_2, the clearer main Exciton peaks $\rightarrow$ confirm.
- At $T_2 = 30fs$ show other smaller exciton peaks $\rightarrow$ predict.

Summary:

- From three-band TB + SBE $\rightarrow$ Linear Absorption Spectrum
- Confirm on Exciton binding energy in monolayer $\mathrm{MoS_2}$ in contract with experiment, predict smaller exciton peaks.

Further research:

- High Harmonic Generation
- High-order Side-band Generation
- Photovoltaic effect

[1] G.-B. Liu, W.-Y. Shan, Y. Yao, W. Yao, and D. Xiao, "Three-band tight-binding model for monolayers of group-VIB transition metal dichalcogenides," *Physical Review B*, vol. 88, p. 085433, Aug. 2013. Publisher: American Physical Society.

[2] H. Haug and S. W. Koch, *Quantum Theory Of The Optical And Electronic Properties Of Semiconductors (5th Edition)*. World Scientific Publishing Company, Jan. 2009. Google-Books-ID: 1J1IDQAAQBAJ.

[3] C. Zhang, H. Wang, W. Chan, C. Manolatou, and F. Rana, "Absorption of light by excitons and trions in monolayers of metal dichalcogenide $\mathrm{Mo}{\mathrm{S}}_{2}$: Experiments and theory," *Physical Review B*, vol. 89, p. 205436, May 2014. Publisher: American Physical Society.