Pauli Hamiltonian

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Since we are working with the charge in the interaction between the 1/2 charge and the external field, we need to use the Pauli Equation instead of the original Schrödinger equation. In general, it reads:

$$\left[\frac{1}{2m}(\vec{\sigma}(\vec{p}-q\vec{A}))^2+q\phi\right]|\psi\rangle = i\hbar\frac{\partial|\psi\rangle}{\partial t} \tag{1}$$

To separate the spinor part and the radius part, we use the Pauli vector identity:

$$(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b}+i\vec{\sigma}\cdot(\vec{a}\times\vec{b}), \tag{2}$$

which in turn, implies: $\sigma_j\sigma_k = \delta_{jk}I + i\varepsilon_{jkl}\sigma_l$. Also, since $\vec{p}\propto\nabla$ and $\nabla\times\vec{A}=\vec{B}$, then the standard Pauli equation will be:

$$\left[\frac{1}{2m}\left[(\vec{p}-q\vec{A})^2-q\hbar\sigma\cdot\vec{B}\right]+q\phi\right]|\psi\rangle = i\hbar\frac{\partial|\psi\rangle}{\partial t} \tag{3}$$