

# VCU Discrete Mathematics Seminar

## *An Introduction to the Critical Group of a Graph*

**Jonathan Gerhard**  
**(VCU!)**

Wednesday, Sept. 24  
1:00-1:50 EDT

**In person** in 4145 Harris Hall. And a Zoom option:

<https://vcu.zoom.us/j/81475528886>  
password=graphs2357



This is a talk based on a research project done as a Sophomore at JMU, so it will assume no prior knowledge, with the hope of being accessible to anyone interested.

The critical group appears in many contexts, but the one I'll focus on for this talk will be from the viewpoint of a fun game on a graph called "chip-firing". A configuration of a graph is an assignment of some number of chips to each vertex, and we can fire a vertex by sending 1 chip along each edge connected to it. We consider two configurations equivalent if we can get from one to the other through a series of chip-firing. The basic question is: Up to this equivalence, how many configurations can a graph  $G$  have?

In this talk, we will explore various examples of critical groups and briefly discuss their many connections to other fields of mathematics, including an appearance of an interesting poset, a beautiful analogue of the Riemann-Roch theorem, and a clever connection to rational points on an elliptic curve.

For the DM seminar schedule, see:

<https://go.vcu.edu/discrete>