

Practical 1

Vasileios Charatsidis

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1 Excercise 1

1.1 Compute (generalized) derivatives of the loss function, with respect to the weights.

$$\mathcal{L} = 0.5 * (y_{out} - y_{gt})^2 \quad (1)$$

$$\text{for } \frac{\partial \mathcal{L}}{\partial W_{out}}$$

$$\frac{\partial \mathcal{L}}{\partial W_{out}} = \frac{\partial \mathcal{L}}{\partial y_{out}} \cdot \frac{\partial y_{out}}{\partial s_{out}} \cdot \frac{\partial s_{out}}{\partial W_{out}} \Rightarrow \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial W_{out}} = (y_{out} - s_{out}) \cdot \frac{\partial f_3(s_{out})}{\partial s_{out}} \cdot \frac{\partial (W_{out} \cdot z_2)}{\partial W_{out}} \Rightarrow$$

$$\frac{\partial \mathcal{L}}{\partial W_{out}} = (y_{out} - s_{out}) \cdot f'_3(s_{out}) \cdot z_2$$

$$\text{for } \frac{\partial \mathcal{L}}{\partial W_2}$$

$$\frac{\partial \mathcal{L}}{\partial W_2} = \frac{\partial \mathcal{L}}{\partial y_{out}} \cdot \frac{\partial y_{out}}{\partial s_{out}} \cdot \frac{\partial s_{out}}{\partial z_2} \cdot \frac{\partial z_2}{\partial s_2} \cdot \frac{\partial s_2}{\partial W_2} \Rightarrow \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial W_2} = (y_{out} - s_{out}) \cdot f'_3(s_{out}) \cdot W_{out} \cdot f'_2(s_2) \cdot z_1$$

$$\text{similarly for } \frac{\partial \mathcal{L}}{\partial W_1}$$

$$\frac{\partial \mathcal{L}}{\partial W_1} = \frac{\partial \mathcal{L}}{\partial y_{out}} \cdot \frac{\partial y_{out}}{\partial s_{out}} \cdot \frac{\partial s_{out}}{\partial z_2} \cdot \frac{\partial z_2}{\partial s_2} \cdot \frac{\partial s_2}{\partial z_1} \cdot \frac{\partial z_1}{\partial s_1} \cdot \frac{\partial s_1}{\partial W_1} \Rightarrow \quad (4)$$

$$\frac{\partial \mathcal{L}}{\partial W_1} = (y_{out} - s_{out}) \cdot f'_3(s_{out}) \cdot W_{out} \cdot f'_2(s_2) \cdot W_2 \cdot f'_1(s_1) \cdot x_{in}$$

1.2 Write down ΔW_k in terms of δ_k

$$\text{We know that: } \delta_k = \frac{\partial \mathcal{L}}{\partial s_k} \text{ and that: } \frac{\partial \mathcal{L}}{\partial s_k} = \frac{\partial \mathcal{L}}{\partial z_k} \cdot \frac{\partial z_k}{\partial s_k}$$

$$\text{So: } \delta_k = \frac{\partial \mathcal{L}}{\partial z_k} \cdot \frac{\partial z_k}{\partial s_k} \quad (5)$$

We can rewrite equations (2), (3) and (4) as:

$$\frac{\partial \mathcal{L}}{\partial W_{out}} = \underbrace{\frac{\partial \mathcal{L}}{\partial z_{out}} \cdot \frac{\partial z_{out}}{\partial s_{out}}}_{\delta_{out}} \cdot \frac{\partial s_{out}}{\partial W_{out}} \Rightarrow$$

$$\Delta W_{out} = \frac{\partial \mathcal{L}}{\partial W_{out}} = \delta_{out} \cdot \frac{\partial s_{out}}{\partial W_{out}} = \delta_{out} \cdot z_2 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial W_2} = \frac{\partial \mathcal{L}}{\partial z_{out}} \cdot \frac{\partial z_{out}}{\partial s_{out}} \cdot \frac{\partial s_{out}}{\partial z_2} \cdot \frac{\partial z_2}{\partial s_2} \cdot \frac{\partial s_2}{\partial W_2} \quad (7)$$

$$\delta_2 = \frac{\partial \mathcal{L}}{\partial s_2} = \frac{\partial \mathcal{L}}{\partial z_{out}} \cdot \frac{\partial z_{out}}{\partial s_{out}} \cdot \frac{\partial s_{out}}{\partial z_2} \cdot \frac{\partial z_2}{\partial s_2} \quad (8)$$

$$\text{from (7) and (8) is implied that } \Delta W_2 = \frac{\partial \mathcal{L}}{\partial W_2} = \delta_2 \cdot \frac{\partial s_2}{\partial W_2} = \delta_2 \cdot z_1 \quad (9)$$

similarly for δ_1

$$\frac{\partial \mathcal{L}}{\partial W_1} = \underbrace{\frac{\partial \mathcal{L}}{\partial z_{out}} \cdot \frac{\partial z_{out}}{\partial s_{out}} \cdot \frac{\partial s_{out}}{\partial z_2} \cdot \frac{\partial z_2}{\partial s_2} \cdot \frac{\partial s_2}{\partial z_1} \cdot \frac{\partial z_1}{\partial s_1}}_{\delta_1} \cdot \frac{\partial s_1}{\partial W_1} \Rightarrow$$

$$\text{is implied that } \Delta W_1 = \frac{\partial \mathcal{L}}{\partial W_1} = \delta_1 \cdot \frac{\partial s_1}{\partial W_1} = \delta_1 \cdot x_{in} \quad (10)$$

2 Exercise 2

$$x = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & x_4^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & x_4^{(2)} \end{bmatrix} = \begin{bmatrix} 0.75 & 0.2 & -0.75 & 0.2 \\ 0.8 & 0.05 & 0.8 & -0.05 \end{bmatrix}$$

$$W = \begin{bmatrix} W_{11} & W_{21} & W_{31} \\ W_{12} & W_{22} & W_{32} \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 & 0.0 \\ 0.01 & 0.43 & 0.88 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0.02 \\ 0.03 \\ 0.9 \end{bmatrix}$$

$$y = [y_1 \quad y_2 \quad y_3 \quad y_4] = [1 \quad 1 \quad -1 \quad -1]$$

$$\alpha = 0.05$$

$$s_1 = W^T \cdot x \quad (1)$$

$$z_1 = \text{ReLU}(s_1) \quad (2)$$

$$s_{out} = w^T \cdot z_1 \quad (3)$$

$$y_{out} = z_{out} = \tanh(s_{out}) \quad (4)$$

$$\mathcal{L} = 0.5 \cdot (y_{out} - y)^2 \quad (5)$$

$$\delta_{out} = \frac{\partial \mathcal{L}}{\partial s_{out}} \quad (6)$$

$$\delta_1 = \delta_{out} \cdot w \cdot \frac{\partial f_{(s_1)}}{\partial s_1} \quad (7)$$

$$\Delta w = \delta_{out} \cdot z_1 \quad (8)$$

$$\Delta W = \delta_1 \cdot x_1 \quad (9)$$

$$\Delta W = \delta_1 \cdot x_1 \quad (10)$$

$$W = W - \alpha \cdot \Delta W \quad (11)$$

$$w = w - \alpha \cdot \Delta w \quad (12)$$

If we follow the steps from 1 to 12 with our given input we have done 1 iteration of forward and backward pass.

2.1 iteration 1

$$s_1 = \begin{bmatrix} 0.458 & 0.1205 & -0.442 & 0.1195 \\ 0.869 & 0.1615 & -0.181 & 0.1185 \\ 0.704 & 0.044 & 0.704 & -0.044 \end{bmatrix}$$

$$z_1 = \begin{bmatrix} 0.458 & 0.1205 & 0 & 0.1195 \\ 0.869 & 0.1615 & 0 & 0.1185 \\ 0.704 & 0.044 & 0.704 & 0 \end{bmatrix}$$

$$s_{out} = [0.09859 \quad 0.011215 \quad 0.06336 \quad 0.005945]$$

$$y_{out} = z_{out} = [0.09827181 \quad 0.01121453 \quad 0.06327535 \quad 0.00594493]$$

$$\mathcal{L} = [0.40655687 \quad 0.48884835 \quad 0.56527723 \quad 0.5059626]$$

$$\delta_{out} = [-0.89301989 \quad -0.98866111 \quad 1.05901824 \quad 1.00590938]$$

$$\delta_1 = \begin{bmatrix} -0.0178604 & -0.01977322 & 0 & 0.02011819 \\ -0.0267906 & -0.02965983 & 0 & 0.03017728 \\ -0.08037179 & -0.0889795 & 0.09531164 & 0 \end{bmatrix}$$

$$\Delta w = [-0.4079306 \quad -0.81650279 \quad 0.07336175]$$

$$\Delta W = \begin{bmatrix} -0.01332631 & -0.01998946 & -0.14955847 \\ -0.01628289 & -0.02442433 & 0.00750291 \end{bmatrix}$$

$$w' = [-0.4079306 \quad -0.81650279 \quad 0.07336175]$$

$$W' = \begin{bmatrix} 0.60133263 & 0.70199895 & 0.01495585 \\ 0.01162829 & 0.43244243 & 0.87924971 \end{bmatrix}$$

2.2 iteration 2

$$s_1 = \begin{bmatrix} 0.460302 & 0.120848 & -0.441697 & 0.119685 \\ 0.872453 & 0.162022 & -0.180545 & 0.118778 \\ 0.714617 & 0.0469537 & 0.692183 & -0.0409713 \end{bmatrix}$$

$$z_1 = \begin{bmatrix} 0.46030 & 0.120848 & 0 & 0.119685 \\ 0.872453 & 0.162022 & 0 & 0.118778 \\ 0.714617 & 0.0469537 & 0.692183 & 0 \end{bmatrix}$$

$$s_{out} = [0.184466 \quad 0.0293179 \quad 0.0572185 \quad 0.0205376]$$

$$y_{out} = z_{out} = [0.182402 \quad 0.0293095 \quad 0.0571561 \quad 0.0205347]$$

$$\mathcal{L} = [0.334234 \quad 0.47112 \quad 0.55879 \quad 0.520746]$$

$$\delta_{out} = [-0.790397 \quad -0.969857 \quad 1.0537 \quad 1.0201]$$

$$\delta_1 = \begin{bmatrix} -0.0480506 & -0.0589606 & 0 & 0.0620153 \\ -0.088248 & -0.108285 & 0 & 0.113895 \\ -0.0653372 & -0.0801721 & 0.0871031 & 0 \end{bmatrix}$$

$$\Delta w = [-0.358935 \quad -0.725557 \quad 0.118986]$$

$$\Delta W = \begin{bmatrix} -0.035427 & -0.065064 & -0.130365 \\ -0.0444893 & -0.0817074 & 0.0134041 \end{bmatrix}$$

$$w' = [0.0966866 \quad 0.184206 \quad 0.0707652]$$

$$W' = \begin{bmatrix} 0.604875 & 0.708505 & 0.0279923 \\ 0.0160772 & 0.440613 & 0.877909 \end{bmatrix}$$

2.3 iteration 3

$$s_1 = \begin{bmatrix} 0.466518 & 0.121779 & -0.440795 & 0.120171 \\ 0.88387 & 0.163732 & -0.178888 & 0.11967 \\ 0.723322 & 0.0494939 & 0.681333 & -0.038297 \end{bmatrix}$$

$$z_1 = \begin{bmatrix} 0.466518 & 0.121779 & 0 & 0.120171 \\ 0.88387 & 0.163732 & 0 & 0.11967 \\ 0.723322 & 0.0494939 & 0.681333 & 0 \end{bmatrix}$$

$$s_{out} = [0.259106 \quad 0.0454372 \quad 0.0482147 \quad 0.0336629]$$

$$y_{out} = z_{out} = [0.253459 \quad 0.0454059 \quad 0.0481774 \quad 0.0336502]$$

$$\mathcal{L} = [0.278662 \quad 0.455625 \quad 0.549338 \quad 0.534216]$$

$$\delta_{out} = [-0.698582 \quad -0.952626 \quad 1.04574 \quad 1.03248]$$

$$\delta_1 = \begin{bmatrix} -0.0675435 & -0.0921061 & 0 & 0.0998269 \\ -0.128683 & -0.175479 & 0 & 0.190189 \\ -0.0494353 & -0.0674128 & 0.0740024 & 0 \end{bmatrix}$$

$$\Delta w = [-0.317837 \quad -0.649873 \quad 0.160052]$$

$$\Delta W = \begin{bmatrix} -0.0491135 & -0.0935703 & -0.106061 \\ -0.0636314 & -0.12123 & 0.016283 \end{bmatrix}$$

$$w' = [0.12847 \quad 0.249193 \quad 0.05476]$$

$$W' = \begin{bmatrix} 0.609787 & 0.717862 & 0.0385984 \\ 0.0224404 & 0.452736 & 0.876281 \end{bmatrix}$$