Mathematical Modeling **Lab**

Consider a partial differential equation (1)

$$u_{xx} + u_{yy} = -1 - u - \lambda u^2 \tag{1}$$

with Dirichlet boundary conditions (2)

$$\begin{aligned} u(0, y) &= 0 \\ u(1, y) &= 0 \\ u(x, 0) &= 0 \\ u(x, 1) &= 0 \end{aligned} \tag{2}$$

Your task is to study the behaviour of the following equation based on different parameter values.

1. (1 point) Derive functional for the equation (1)
2. (5 points) Apply numerical bifurcation analysis with iterative generalised Kantorovich method with only one term to construct bifurcation path. How many qualitatively different solutions are observed in this equation for different parameter values?
3. (2 points) Next apply iterative generalised Kantorovich method with two terms to construct bifurcation path. Does this affect the bifurcation picture of the equation?
4. (2 points) Finally, apply PINN for constructing bifurcation path. Here you probably will need arclength continuation method. About this method you can read in the following article `https://arxiv.org/pdf/2507.09782v1`