Adaptive control of unknown plants using dynamical neural networks

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Adaptive Control of Unknown Plants Using Dynamical Neural Networks

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Abstract— **In this paper, we are dealing with the problem of controlling an unknown nonlinear dynamical system. The algorithm is divided into two phases. First a dynamical neural network identifier is employed to perform "black box" identification and then a dynamic state feedback is developed to appropriately control the unknown system. We apply the algorithm to control the speed of a nonlinearized dc motor, giving in this way an application insight. In the algorithm, not all the plant states are assumed to be available for measurement.**

I. Introduction

ADAPTIVE CONTROL of dynamical systems has been an active area of research since the 1960's. At the beginning, research was focused on the problem of controlling and identifying linear time invariant plants. But it was not until the last decade, that the problem was solved for both continuous and discrete LTI systems [1]–[3].

Recent advances in nonlinear control theory, and in particular feedback linearization techniques [4], [5], has inspired the development of adaptive control schemes for nonlinear plants [6]–[9]. A common assumption made in the above works is that either all or part of the system dynamics are known. Although sometimes it is quite realistic, it constrains considerably the applications field. Furthermore, the most general problem, of controlling a totally unknown plant, cannot even be discussed under these control schemes and assumptions.

An obvious solution to overcome the problem is to introduce identification techniques in the control algorithm. The problem of identification consists of choosing an appropriate identification model and adjusting its parameters such that the response of the model to an input signal approximates the response of the plant under the same input. It has been clear that in control systems theory a mathematical description of a plant is often a prerequisite to analysis and controller design.

Furthermore, it is well known that global stability properties of model reference adaptive systems [10] are guaranteed under the "matching assumption" that the model order is not lower than that of the unknown plant. This restrictive assumption is likely to be violated in applications. Hence, it is important to determine the stability and robustness properties of adaptive schemes with respect to modeling errors.

Ioannou and Kokotovic [11] assumed a separation of time scales between the modeled and unmodeled phenomena and examined the performance of various types of identifiers and adaptive observers, when the order of the model is equal to the slow part of the plant.

Taylor *et al.* [6] and Kanelakopoulos *et al.* [7] examined the stability and robustness properties of nonlinear systems with parametric and dynamic uncertainties. In their work, the true plant is allowed to be of higher order due to unmodeled dynamics. Only the states appearing in the reduced order model are assumed to be available for measurement. Hence, their work appeared as an extension of the robustness analysis of Ioannou and Kokotovic [12], to nonlinear adaptive control.

Recently, neural networks appeared as powerful tools for learning static and dynamic highly nonlinear systems. Due to their massive parallelism, very fast adaptability, and inherent approximation capabilities, neural networks have concentrated a lot of research, especially in the area of identification and control. In the literature one can find interesting works dealing with the subject [13], [14], the feasibility of which has been demonstrated through simulations.

Narendra *et al.* [15] originally formulated the problem. They proposed dynamic backpropagation schemes, which are static backpropagation neural networks, connected either in series or in parallel with linear dynamical systems.

However, in all works mentioned above it was assumed that all states of the unknown plant were available for measurement, and, furthermore, they lack theoretical verification of the results that simulations provide.

In this paper singular perturbation analysis is employed to investigate the stability and robustness properties of dynamical neural network identifier. Various cases that lead to modeling errors are taken into consideration and prove stability and convergence. At the same time internal stability of the identifier is guaranteed. After the identification part has been completed, a dynamic state feedback is developed to appropriately control the unknown system. As will be apparent in the sections that follow, all we need from identification phase is the convergence of the error to zero, since only a rough estimation of the identification parameters is needed to proceed to the control phase. A block diagram of the proposed identification-control architecture is shown in Fig. 1. Finally, we apply our algorithm to control the speed of a nonlinearized dc motor, providing in this way an application insight.

II. Identification

In this section we consider the problem of identifying a continuous time nonlinear dynamic system of the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \tag{1}$$

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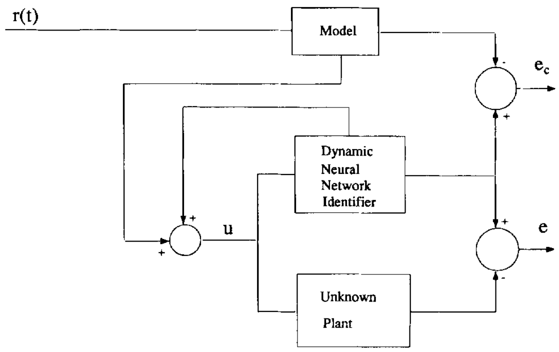

Fig. 1. The identification-control architecture in block diagram form.

where $\mathbf{x} \in \mathcal{M}$, a smooth manifold, the input $\mathbf{u} \in \mathcal{U} \subset \Re^n$ where $\mathcal{U}$ is the class of admissible inputs, $\mathbf{f}$ is a smooth vector called the drift term, $\mathbf{g}$ is a matrix with columns $\mathbf{g_i}$, $i = 1, 2, \cdots, n$ $\mathbf{g} = [\mathbf{g_1 g_2} \cdots \mathbf{g_n}]$, where $\mathbf{g_i}$ are smooth vectorfields and $\mathbf{x}(0) = \mathbf{x_0}$ is the initial condition in $\mathcal{M}$. In examining this problem, we will impose the following assumptions on the system to be approximated.

$A(1)$ Given a class $\mathcal{U}$ of admissible inputs, then for any $u \in \mathcal{U}$ and any finite initial condition, the state trajectories are uniformly bounded for any finite $T > 0$. Hence, $|x(T)| < \infty$.

$(A2)$ $\mathbf{f}, \mathbf{g}$ are continuous with respect to their arguments and satisfy a local Lipschitz condition so that the solution $\mathbf{x}(t)$ to the differential equation (1) is unique for any finite initial condition and $\mathbf{u} \in \mathcal{U}$.

The above assumptions are required to guarantee the existence and uniqueness of solution of (1), for any finite initial condition and $\mathbf{u} \in \mathcal{U}$.

In order to identify the nonlinear dynamical system (1), we employ dynamical neural networks. Dynamical neaural networks are recurrent, fully interconnected nets, containing dynamical elements in their neurons. Therefore, they are described by the following set of differential equations:

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{BWS}(\mathbf{x}) + \mathbf{BW_{n+1}S}'(\mathbf{x})\mathbf{u} \tag{2}$$

where $\hat{\mathbf{x}} \in \mathcal{M}$, the inputs $\mathbf{u} \in \mathcal{U} \subset \Re^n$, $\mathbf{W}$ is a $n \times n$ matrix of synaptic weights $\mathbf{A}$, $\mathbf{B}$ are $n \times n$ diagonal matrices with elements the scalers a_i, b_i, for all $i = 1, 2, 3, \cdots, n$ and $\mathbf{W_{n+1}}$ is a $n \times n$ diagonal matrix of synaptic weights, of the form $\mathbf{W_{n+1}} = \mathrm{diag}[w_{1n+1} w_{2n+1} \cdots w_{nn+1}]$. Finally $\mathbf{S}(\mathbf{x})$ is an n-dimensional vector and $\mathbf{S}'(\mathbf{x})$ is an $n \times n$ diagonal matrix, with elements $s(x_i)$ and $s'(x_i)$ respectively, both smooth, (at least twice differentiable), monotone increasing functions which are usually represented by sigmoids of the form

$$s(x_i) = \frac{k}{1 + e^{-lx_i}}$$
$$s'(x_i) = \frac{k}{1 + e^{-lx_i}} + \lambda$$

for all $i = 1, 2, 3, \cdots, n$ where k, l are parameters representing the bound, (k) and slope, (l), of sigmoid's curvature and $\lambda > 0$ a constant that shifts the sigmoid, such that $s'(x_i) > 0$ for all $i = 1, 2, 3, \cdots, n$. A block diagram of the dynamic neural network used, is shown in Fig. 2.

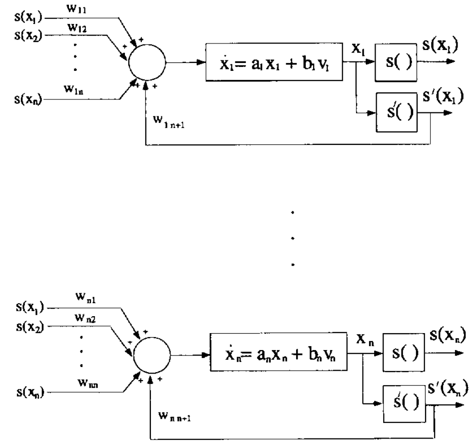

Fig. 2. The dynamic neural network.

A. Parametric Uncertainty

Let us first assume that an exact model of the plant is available, (i.e., we have no modeling error). The purpose of this section is to find a learning law that guarantees stability of the neural network, plus convergence of its output and weights to a desired value. Since we have only parametric uncertainties, we can assume that there exists weight values $\mathbf{W}^\star$, $\mathbf{W}^\star_{\mathbf{n+1}}$ such that the system (1) is completely described by a neural network of the form

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{BW}^\star\mathbf{S}(\mathbf{x}) + \mathbf{BW}^\star_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u} \tag{3}$$

where all matrices are as defined earlier.

Define the error between the identifier states and the real system states as

$$\mathbf{e} = \hat{\mathbf{x}} - \mathbf{x}.$$

Then from (2) and (3) we can obtain the error equation

$$\dot{\mathbf{e}} = \mathbf{Ae} + \mathbf{B}\tilde{\mathbf{W}}\mathbf{S}(\mathbf{x}) + \mathbf{B}\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u} \tag{4}$$

where

$$\tilde{\mathbf{W}} = \mathbf{W} - \mathbf{W}^\star$$
$$\tilde{\mathbf{W}}_{\mathbf{n+1}} = \mathbf{W_{n+1}} - \mathbf{W}^\star_{\mathbf{n+1}}.$$

The Lyapunov synthesis method is used to derive stable adaptive laws. Therefore consider the Lyapunov function candidate

$$\mathcal{V}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}) = \frac{1}{2}\mathbf{e}^T\mathbf{Pe} + \frac{1}{2}tr\{\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}\} + \frac{1}{2}tr\{\tilde{\mathbf{W}}^{\mathbf{T}}_{\mathbf{n+1}}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \tag{5}$$

where $\mathbf{P} > 0$ is chosen to satisfy the Lyapunov equation

$$\mathbf{PA} + \mathbf{A}^T\mathbf{P} = -\mathbf{I}.$$

Observe that since $\mathbf{A}$ is a diagonal matrix, $\mathbf{P}$ can be chosen to be a diagonal matrix too; simplifying in this way, the calculations.

Differentiating (5) along the solution of (4) we obtain

$$\dot{\mathcal{V}} = \frac{1}{2}(\dot{\mathbf{e}}^T\mathbf{P}\mathbf{e} + \mathbf{e}^T\mathbf{P}\dot{\mathbf{e}}) + tr\{\dot{\tilde{\mathbf{W}}}^T\tilde{\mathbf{W}}\} + tr\{\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\}$$

or

$$\begin{aligned}\dot{\mathcal{V}} = \frac{1}{2}(&-\mathbf{e}^T\mathbf{e} + \mathbf{S}^T(\mathbf{x})\tilde{\mathbf{W}}^T\mathbf{B}\mathbf{P}\mathbf{e} + \mathbf{u}^T\mathbf{S}'(\mathbf{x})\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{B}\mathbf{P}\mathbf{e}\\ &+ (\mathbf{S}^T(\mathbf{x})\tilde{\mathbf{W}}^T\mathbf{B}\mathbf{P}\mathbf{e})^T + (\mathbf{u}^T\mathbf{S}'(\mathbf{x})(\tilde{\mathbf{W}})_{\mathbf{n+1}}\mathbf{B}\mathbf{P}\mathbf{e})^T)\\ &+ tr\{\dot{\tilde{\mathbf{W}}}^T\tilde{\mathbf{W}}\} + tr\{\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\}.\end{aligned}$$

Now since $\mathbf{S^T}(\mathbf{x})\tilde{\mathbf{W}}^T\mathbf{B}\mathbf{P}\mathbf{e}$, $\mathbf{u}^T\mathbf{S}'(\mathbf{x})\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{B}\mathbf{P}\mathbf{e}$ are scalars

$$\mathbf{S^T}(\mathbf{x})\tilde{\mathbf{W}}^T\mathbf{B}\mathbf{P}\mathbf{e} = (\mathbf{S}^T(\mathbf{x})\tilde{\mathbf{W}}^T\mathbf{B}\mathbf{P}\mathbf{e})^T$$
$$\mathbf{u}^T\mathbf{S}'(\mathbf{x})\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{B}\mathbf{P}\mathbf{e} = (\mathbf{u}^T\mathbf{S}'(\mathbf{x})\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{B}\mathbf{P}\mathbf{e})^T.$$

Therefore $\dot{\mathcal{V}}$ becomes

$$\begin{aligned}\dot{\mathcal{V}} = &-\frac{1}{2}\mathbf{e}^T\mathbf{e} + \mathbf{S}^T(\mathbf{x})\tilde{\mathbf{W}}^T\mathbf{B}\mathbf{P}\mathbf{e} + \mathbf{u}^T\mathbf{S}'(\mathbf{x})\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{B}\mathbf{P}\mathbf{e}\\ &+ tr\{\dot{\tilde{\mathbf{W}}}^T\tilde{\mathbf{W}}\} + tr\{\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\}.\end{aligned} \tag{6}$$

Hence, if we choose

$$tr\{\dot{\tilde{\mathbf{W}}}^T\tilde{\mathbf{W}}\} = -\mathbf{S}^T(\mathbf{x})\tilde{\mathbf{W}}^T\mathbf{B}\mathbf{P}\mathbf{e} \tag{7}$$
$$tr\{\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\} = -\mathbf{u}^T\mathbf{S}'(\mathbf{x})\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{B}\mathbf{P}\mathbf{e} \tag{8}$$

then (6) becomes

$$\dot{\mathcal{V}} = -\frac{1}{2}\mathbf{e}^T\mathbf{e} \tag{9}$$

or

$$\dot{\mathcal{V}} = -\frac{1}{2}\|\mathbf{e}\|^2 \leq 0. \tag{10}$$

From (7) and (8) we obtain learning laws in an element form as

$$\dot{w}_{ij} = -b_i p_i s(x_j) e_i$$
$$\dot{w}_{in+1} = -b_i s'(x_i) p_i u_i e_i$$

for all i, $j = 1, 2, 3, \cdots, n$.

Now we can prove the following theorem.

Theorem 1: Consider the identification scheme (4). The learning law

$$\dot{w}_{ij} = -b_i p_i s(x_j) e_i$$
$$\dot{w}_{in+1} = -b_i s'(x_i) p_i u_i e_i$$

for all $i, j = 1, 2, 3, \cdots, n$ guarantees the following properties

- $\mathbf{e}$, $\hat{\mathbf{x}}$, $\tilde{\mathbf{W}}$, $\tilde{\mathbf{W}}_{n+1} \in L_\infty$, $\mathbf{e} \in L_2$
- $\lim_{t\to\infty} \mathbf{e}(t) = 0$, $\lim_{t\to\infty} \dot{\tilde{\mathbf{W}}}(t) = 0$, $\lim_{t\to\infty} \dot{\tilde{\mathbf{W}}}_{n+1}(t) = 0$

Proof: We have shown that using the learning law

$$\dot{w}_{ij} = -b_i p_i s(x_j) e_i$$
$$\dot{w}_{in+1} = -b_i s(x_i) p_i u_i e_i$$

for all $i, j = 1, 2, 3, \cdots, n \dot{\mathcal{V}}$ becomes

$$\dot{\mathcal{V}} = -\frac{1}{2}\|\mathbf{e}\|^2 \leq 0.$$

Hence, $\mathcal{V} \in L_\infty$, which implies $\mathbf{e}$, $\tilde{\mathbf{W}}$, $\tilde{\mathbf{W}}_{\mathbf{n+1}} \in L_\infty$. Furthermore, $\hat{\mathbf{x}} = \mathbf{e} + \mathbf{x}$ is also bounded. Since $\mathcal{V}$ is a non-increasing function of time and bounded from below, the $\lim_{t\to\infty} \mathcal{V} = \mathcal{V}_\infty$ exists. Therefore by integrating $\dot{\mathcal{V}}$ from 0 to ∞ we have

$$\int_0^\infty \|\mathbf{e}\|^2 dt = 2[\mathcal{V}(0) - \mathcal{V}_\infty] < \infty$$

which implies that $\mathbf{e} \in L_2$. By definition the sigmoid functions $s(x_i)$, $i = 1, 2, \cdots, n$ are bounded for all $\mathbf{x}$ and by assumption all inputs to the neural network are also bounded. Hence from (4) we have that $\dot{\mathbf{e}} \in L_\infty$. Since $\mathbf{e} \in L_2 \cap L_\infty$ and $\dot{\mathbf{e}} \in L_\infty$, using Barbalat's Lemma [19], we conclude that $\lim_{t\to\infty} \mathbf{e}(t) = 0$. Now using the boundedness of $\mathbf{u}$, $\mathbf{S}(\mathbf{x})$, $\mathbf{S}'(\mathbf{x})$ and the convergence of $\mathbf{e}(t)$ to zero, we have that $\dot{\mathbf{W}}$, $\dot{\mathbf{W}}_{\mathbf{n+1}}$ also converges to zero. □

Remark: Under the assumptions of Theorem 1, we cannot conclude anything about the convergence of the weights to their optimal values. In order to guarantee convergence, $\mathbf{S}(\mathbf{x})$, $\mathbf{S}'(\mathbf{x})$, $\mathbf{u}$ need to satisfy a persistancy of excitation condition. A signal $\mathbf{z}(t) \in \Re^n$ is persistently exciting in $\Re^n$ if there exist positive constants β_0, β_1, T such that

$$\beta_0\mathbf{I} \leq \int_t^{t+T} \mathbf{z}(\tau)\mathbf{z}^T(\tau)d\tau \leq \beta_1\mathbf{I} \quad \forall t \geq 0.$$

However, such a condition cannot be verified *a priori* since $\mathbf{S}(\mathbf{x})$ and $\mathbf{S}'(\mathbf{x})$ are nonlinear functions of the state $\mathbf{x}$. □

B. Unmodeled Dynamics Present

In the previous section we assumed that there exist weight values $\mathbf{W}^\star, \mathbf{W}_{\mathbf{n+1}}^\star$ such that a nonlinear dynamical system can be completely described by a neural network of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{W}^\star\mathbf{S}(\mathbf{x}) + \mathbf{B}\mathbf{W}_{\mathbf{n+1}}^\star\mathbf{S}'(\mathbf{x})\mathbf{u}$$

where all matrices are as defined previously. It is well known however, that the model is of lower order than the plant, due to the unmodeled dynamics present in the plant. In the following, we extend our theory within the framework of singular perturbations to include the case where dynamic uncertainties are present. For more details concerning singular perturbation theory, the interested reader is refered to [16]. Now we can assume that the unknown plant can be completely described by

$$\begin{aligned}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} &+ \mathbf{B}\mathbf{W}^\star\mathbf{S}(\mathbf{x}) + \mathbf{B}\mathbf{W}_{\mathbf{n+1}}^\star\mathbf{S}'(\mathbf{x})\mathbf{u}\\ &+ \mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\mathbf{A}_0^{-1}\mathbf{B}_0\mathbf{W}_0\mathbf{u}\\ &+ \mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\mathbf{z}\\ \mu\dot{\mathbf{z}} = \mathbf{A}_0\mathbf{z} &+ \mathbf{B}_0\mathbf{W}_0\mathbf{u}, \quad \mathbf{z} \in \Re^r\end{aligned} \tag{11}$$

where $\mathbf{z}$ is the state of the unmodeled dynamics and $\mu > 0$ a small singular perturbation scalar. If we define the error between the identifier states and the real system states as

$$\mathbf{e} = \hat{\mathbf{x}} - \mathbf{x}$$

then from (2) and (11) we obtain the error equation

$$\begin{aligned} \dot{\mathbf{e}} &= \mathbf{A}\mathbf{e} + \mathbf{B}\tilde{\mathbf{W}}\mathbf{S}(\mathbf{x}) + \mathbf{B}\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u} - \mathbf{F} \\ &\quad \times (\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\mathbf{A}_0^{-1}\mathbf{B}_0\mathbf{W}_0\mathbf{u} - \mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\mathbf{z} \\ \mu\dot{\mathbf{z}} &= \mathbf{A}_0\mathbf{z} + \mathbf{B}_0\mathbf{W}_0\mathbf{u}, \quad \mathbf{z} \in \Re^r \end{aligned} \tag{12}$$

where $\mathbf{F}(\mathbf{x},\mathbf{W},\mathbf{W}_{n+1})$, $\mathbf{B}_0\mathbf{W}_0\mathbf{u}$, $\mathbf{B}\tilde{\mathbf{W}}\mathbf{S}(\mathbf{x})$, $\mathbf{B}\tilde{\mathbf{W}}_{n+1}\,\mathbf{S}'(\mathbf{x})\mathbf{u}$ are bounded and differentiable with respect to their arguments for every $\tilde{w} \in B_{\tilde{w}}$ a ball in $\Re^{n\times n}$, $\tilde{w}_{n+1} \in B_{\tilde{w}_{n+1}}$ a ball in $\Re^n$ and all $x \in B_x$ a ball in $\Re^n$. We further assume that the unmodeled dynamics are asymptotically stable for all $x \in B_x$. In other words we assume that there exist a constant $\nu > 0$ such that

$$Re\lambda\{\mathbf{A}_0\} \le -\nu < 0.$$

Note that $\dot{\mathbf{z}}$ is large since μ is small, hence the unmodeled dynamics are fast. For a singular perturbation from $\mu > 0$ to $\mu = 0$ we obtain

$$\mathbf{z} = -\mathbf{A}_0^{-1}\mathbf{B}_0\mathbf{W}_0\mathbf{u}.$$

Since the unmodeled dynamics are asymptotically stable, the existence of $\mathbf{A}_0^{-1}$ is assured. As it is well known from singular perturbation theory, we express the state $\mathbf{z}$ as

$$\mathbf{z} = \mathbf{h}(\mathbf{x}, \eta) + \eta \tag{13}$$

where $\mathbf{h}(\mathbf{x}, \eta)$ is defined as the quasi-steady-state of $\mathbf{z}$ and η as its fast transient. In our case

$$\mathbf{h}(\mathbf{x}, \eta) = -\mathbf{A}_0^{-1}\mathbf{B}_0\mathbf{W}_0\mathbf{u}.$$

Substituting (13) into (12) we obtain the singularly perturbed model as

$$\begin{aligned} \dot{\mathbf{e}} &= \mathbf{A}\mathbf{e} + \mathbf{B}\tilde{\mathbf{W}}\mathbf{S}(\mathbf{x}) + \mathbf{B}\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u} \\ &\quad - \mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\eta \\ \mu\dot{\eta} &= \mathbf{A}_0\eta - \mu\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u}) \end{aligned} \tag{14}$$

where we define

$$\begin{aligned} \dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u}) &= \frac{\partial \mathbf{h}}{\partial \mathbf{e}}\dot{\mathbf{e}} + \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{W}}}\dot{\tilde{\mathbf{W}}} \\ &\quad + \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{W}}_{\mathbf{n+1}}}\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}} + \frac{\partial \mathbf{h}}{\partial \mathbf{u}}\dot{\mathbf{u}}. \end{aligned}$$

Notice, however, that in the control case, $\mathbf{u}$ is a function of $\mathbf{e}$, $\tilde{\mathbf{W}}$, $\tilde{\mathbf{W}}_{\mathbf{n+1}}$ therefore making $\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u})$ to be equal to

$$\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u}) = \frac{\partial \mathbf{h}}{\partial \mathbf{e}}\dot{\mathbf{e}} + \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{W}}}\dot{\tilde{\mathbf{W}}} + \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{W}}_{\mathbf{n+1}}}\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}.$$

Remark: $\mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})$ $\mathbf{A}_0^{-1}$ $\mathbf{B}_0\mathbf{W}_0\mathbf{u}$, $\mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{n+1})z$ in (11) can be viewed as correction terms in the input vectorfields and in the drift term of

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{W}^{*}\mathbf{S}(\mathbf{x}) + \mathbf{B}\mathbf{W}^{*}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u}$$

in the sense that the unknown system can now be described by a neural network plus the correction terms. □

C. Robustness of the Neural Network Identifier

In this subsection we proceed to the stability analysis of the neural network identification scheme when the unmodeled dynamics considered in the previous subsection are present. Before proceeding any further we need to prove the following lemma.

Lemma 1: It is true that $\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u})$ is bounded by

$$\|\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u})\| \le \rho_1\|\mathbf{e}\| + \rho_2\|\eta\|$$

provided that the following inequalities hold:

$$\begin{aligned} \|\mathbf{h}_{\mathbf{w}}\dot{\tilde{\mathbf{W}}}\| &\le k_0\|\mathbf{e}\| \\ \|\mathbf{h}_{\mathbf{w}_{n+1}}\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}\| &\le k_1\|\mathbf{e}\| \\ \|\mathbf{h}_{\mathbf{e}}\mathbf{B}\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u}\| &\le k_2\|\mathbf{e}\| \\ \|\mathbf{h}_{\mathbf{e}}\mathbf{B}\tilde{\mathbf{W}}\mathbf{S}(\mathbf{x})\| &\le k_3\|\mathbf{e}\| \\ \|\mathbf{h}_{\mathbf{e}}\mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\| &\le \rho_2 \\ \|\mathbf{h}_{\mathbf{e}}\mathbf{A}\mathbf{e}\| &\le k_4\|\mathbf{e}\| \\ \|\mathbf{h}_{\mathbf{u}}\dot{\mathbf{u}}\| &\le k_5\|\mathbf{e}\| \end{aligned}$$

and

$$\rho_1 = k_0 + k_1 + k_2 + k_3 + k_4 + k_5$$

Proof: Differentiating $\mathbf{h}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u})$ we obtain

$$\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u}) = \mathbf{h}_{\mathbf{e}}\dot{\mathbf{e}} + \mathbf{h}_{\tilde{\mathbf{W}}}\dot{\tilde{\mathbf{W}}} + \mathbf{h}_{\tilde{\mathbf{W}}_{n+1}}\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}} + \mathbf{h}_{\mathbf{u}}\dot{\mathbf{u}}$$

or

$$\begin{aligned} \dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u}) &= \mathbf{h}_{\mathbf{e}}(\mathbf{A}\mathbf{e} + \mathbf{B}\tilde{\mathbf{W}}\mathbf{S}(\mathbf{x}) \\ &\quad + \mathbf{B}\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u} \\ &\quad - \mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}}), \eta) + \mathbf{h}_{\tilde{\mathbf{W}}}\dot{\tilde{\mathbf{W}}} \\ &\quad + \mathbf{h}_{\tilde{\mathbf{W}}_{n+1}}\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}} + \mathbf{h}_{\mathbf{u}}\dot{\mathbf{u}}. \end{aligned}$$

Therefore

$$\begin{aligned} \|\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u})\| &\le \|\mathbf{h}_{\mathbf{e}}\mathbf{A}\mathbf{e}\| + \|\mathbf{h}_{\mathbf{e}}\mathbf{B}\tilde{\mathbf{W}}\mathbf{S}(\mathbf{x})\| \\ &\quad + \|\mathbf{h}_{\mathbf{e}}\mathbf{B}\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u}\| \\ &\quad + \|\mathbf{h}_{\mathbf{e}}\mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}}), \eta\| \\ &\quad + \|\mathbf{h}_{\tilde{\mathbf{W}}}\dot{\tilde{\mathbf{W}}}\| + \|\mathbf{h}_{\tilde{\mathbf{W}}_{n+1}}\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}\| \\ &\quad + \|\mathbf{h}_{\mathbf{u}}\dot{\mathbf{u}}\| \\ &\le k_4\|\mathbf{e}\| + k_3\|\mathbf{e}\| + k_2\|\mathbf{e}\| \\ &\quad + \|\mathbf{h}_{\mathbf{e}}\mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\|\|\eta\| \\ &\quad + k_0\|\mathbf{e}\| + k_1\|\mathbf{e}\| + k_5\|\mathbf{e}\| \\ &\le k_4\|\mathbf{e}\| + k_3\|\mathbf{e}\| + k_2\|\mathbf{e}\| \\ &\quad + \rho_2\|\eta\| + k_0\|\mathbf{e}\| \\ &\quad + k_1\|\mathbf{e}\| + k_5\|\mathbf{e}\|. \end{aligned}$$

Hence,

$$\dot{\mathbf{h}}(\mathbf{e},\tilde{\mathbf{W}},\tilde{\mathbf{W}}_{\mathbf{n+1}},\eta,\mathbf{u}) \leq \rho_1\|\mathbf{e}\| + \rho_2\|\eta\|$$

which concludes the proof. □

We are now able to prove the following theorem

Theorem 2: The equilibrium of the singularly perturbed model is asymptotically stable for all

$$\mu \in \left(0, \frac{1}{c_1c_2 + 2c_3}\right)$$

and an estimate of its region of attraction is

$$S = \{\mathbf{e},\tilde{\mathbf{W}},\tilde{\mathbf{W}}_{\mathbf{n+1}},\eta : \mathcal{V}(\mathbf{e},\tilde{\mathbf{W}},\tilde{\mathbf{W}}_{\mathbf{n+1}},\eta) \leq c\}$$

where c is the largest constant such that the set$\{\mathbf{e},\tilde{\mathbf{W}},\tilde{\mathbf{W}}_{\mathbf{n+1}}$: $\mathcal{V}(\mathbf{e},\tilde{\mathbf{W}},\tilde{\mathbf{W}}_{\mathbf{n+1}},\mathbf{0}) \leq c\}$ is contained to $B_e \times B_w \times B_{w_{n+1}}$. Furthermore, the following properties are guaranteed:

- $\mathbf{e},\hat{\mathbf{x}},\eta,\mathbf{W},\mathbf{W}_{\mathbf{n+1}} \in L_\infty$, $\mathbf{e},\eta \in L_2$
- $\lim_{t\to\infty}\mathbf{e}(t) = 0$, $\lim_{t\to\infty}\eta(t) = 0$
- $\lim_{t\to\infty}\dot{\tilde{\mathbf{W}}}(t) = 0$, $\lim_{t\to\infty}\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}(t) = 0$

Proof: Let's take the Lyapunov function candidate

$$\begin{aligned}\mathcal{V}(\mathbf{e},\tilde{\mathbf{W}},\tilde{\mathbf{W}}_{\mathbf{n+1}},\eta) &= \frac{1}{2}c_1\mathbf{e}^T\mathbf{P}\mathbf{e} \\ &\quad + \frac{1}{2}c_2\eta^T\mathbf{P_0}\eta + \frac{1}{2}c_1 tr\{\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}\} \\ &\quad + \frac{1}{2}c_1 tr\{\tilde{\mathbf{W}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \end{aligned} \tag{15}$$

where $\mathbf{P},\mathbf{P_0} > 0$ are chosen to satisfy the Lyapunov equation

$$\mathbf{PA} + \mathbf{A}^T\mathbf{P} = -\mathbf{I}$$
$$\mathbf{P_0A_0} + \mathbf{A_0}^T\mathbf{P_0} = -\mathbf{I}.$$

Observe that (15) is a weighted sum of a slow and a fast part. Taking the time derivative of (15) and using the learning law

$$\dot{w}_{ij} = -b_ip_is(x_j)e_i$$
$$\dot{w}_{in+1} = -b_is'(x_i)p_iu_ie_i$$

for all $i = 1, 2, \cdots, n$, we obtain, as in a previous subsection, that

$$\begin{aligned}\dot{\mathcal{V}} &= -\frac{c_1}{2}\|\mathbf{e}\|^2 - \frac{c_2}{2\mu}\|\eta\|^2 - c_1\mathbf{e}^T\mathbf{PF}(\mathbf{x},\mathbf{W},\mathbf{W}_{\mathbf{n+1}})\eta \\ &\quad - c_2\eta^T\mathbf{P_0}\dot{\mathbf{h}}(\mathbf{e},\tilde{\mathbf{W}},\mathbf{W}_{\mathbf{n+1}},\eta,\mathbf{u}) \\ &\leq -\frac{c_1}{2}\|\mathbf{e}\|^2 - \frac{c_2}{2\mu}\|\eta\|^2 + \|c_1\mathbf{e}^T\mathbf{PF}(\mathbf{x},\mathbf{W},\mathbf{W}_{\mathbf{n+1}})\eta \\ &\quad + c_2\eta^T\mathbf{P_0}\dot{\mathbf{h}}(\mathbf{e},\tilde{\mathbf{W}},\tilde{\mathbf{W}}_{\mathbf{n+1}},\eta,\mathbf{u})\|\end{aligned}$$

and by employing Lemma 1 we obtain

$$\begin{aligned}\dot{\mathcal{V}} &\leq -\frac{c_1}{2}\|\mathbf{e}\|^2 - \frac{c_2}{2\mu}\|\eta\|^2 + c_1\|\mathbf{e}^T\mathbf{PF}(\mathbf{x},\mathbf{W},\mathbf{W}_{\mathbf{n+1}})\|\|\eta\| \\ &\quad + c_2\|\eta\mathbf{P_0}\|(\rho_1\|\mathbf{e}\| + \rho_2\|\eta\|)\end{aligned}$$

which finally takes the form

$$\dot{\mathcal{V}} \leq -\frac{c_1}{2}\|\mathbf{e}\|^2 - c_2\left(\frac{1}{2\mu} - c_3\right)\|\eta\|^2 + c_1c_2\|\mathbf{e}\|\|\eta\| \tag{16}$$

provided that the following inequalities hold:

$$\|\mathbf{PF}(\mathbf{x},\mathbf{W},\mathbf{W}_{\mathbf{n+1}})\| \leq c_2$$
$$\|\mathbf{P_0}\|\rho_1 \leq c_1$$
$$\|\mathbf{P_0}\|\rho_2 \leq c_3.$$

Therefore

$$\dot{\mathcal{V}} \leq -\begin{bmatrix}\|\mathbf{e}\| & \|\eta\|\end{bmatrix}\begin{bmatrix}\frac{c_1}{2} & -\frac{c_1c_2}{2} \\ -\frac{c_1c_2}{2} & c_2(\frac{1}{2\mu} - c_3)\end{bmatrix}\begin{bmatrix}\|\mathbf{e}\| \\ \|\eta\|\end{bmatrix}. \tag{17}$$

The 2×2 matrix in (17) is positive definite, when

$$\mu < \frac{1}{c_1c_2 + 2c_3}$$

Then $\dot{\mathcal{V}}$ is negative semidefinite. Since $\dot{\mathcal{V}} \leq 0$ we conclude that $\mathcal{V} \in L_\infty$, which implies $\mathbf{e},\eta,\tilde{\mathbf{W}},\tilde{\mathbf{W}}_{\mathbf{n+1}} \in L_\infty$. Furthermore, $\hat{\mathbf{x}} = \mathbf{e} + \mathbf{x}$, $\mathbf{W} = \tilde{\mathbf{W}} + \mathbf{W}^*$, $\mathbf{W}_{\mathbf{n+1}} = \tilde{\mathbf{W}}_{\mathbf{n+1}} + \mathbf{W}^*_{\mathbf{n+1}}$ are also bounded. Since $\mathcal{V}$ is a non-increasing function of time and bounded from below, the $\lim_{t\to\infty}\mathcal{V} = \mathcal{V}_\infty$ exists. Therefore by integreting $\dot{\mathcal{V}}$ from 0 to ∞ we have

$$\begin{aligned}&\frac{c_1}{2}\int_0^\infty \|\mathbf{e}\|^2 dt + c_2\left(\frac{1}{2\mu} - c_3\right)\int_0^\infty \|\eta\|^2 dt - c_1c_2 \\ &\quad \int_0^\infty \|\mathbf{e}\|\|\eta\| dt \leq [V(0) - V_\infty] < \infty\end{aligned}$$

which implies that $\mathbf{e}$, $\eta \in L_2$. Furthermore

$$\begin{aligned}\dot{\mathbf{e}} &= \mathbf{Ae} + \mathbf{B}\tilde{\mathbf{W}}\mathbf{S}(\mathbf{x}) + \mathbf{B}\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u} \\ &\quad - \mathbf{F}(\mathbf{x},\mathbf{W},\mathbf{W}_{\mathbf{n+1}})\eta \\ \mu\dot{\eta} &= \mathbf{A_0}\eta - \mu\dot{\mathbf{h}}(\mathbf{e},\tilde{\mathbf{W}},\tilde{\mathbf{W}}_{\mathbf{n+1}},\eta,\mathbf{u}).\end{aligned}$$

Since $\mathbf{u}$, $\mathbf{A_0}$, $\dot{\mathbf{h}}(\mathbf{e},\tilde{\mathbf{W}},\tilde{\mathbf{W}}_{\mathbf{n+1}},\eta,\mathbf{u})$ are bounded, $\dot{\mathbf{e}} \in L_\infty$ and $\dot{\eta} \in L_\infty$. Since $\mathbf{e} \in L_2 \cap L_\infty, \eta \in L_2 \cap L_\infty$, using Barbalat's Lemma we conclude that $\lim_{t\to\infty}\mathbf{e}(t) = 0$, $\lim_{t\to\infty}\eta(t) = 0$. Now using the boundedness of $\mathbf{u},\mathbf{S}(\mathbf{x}),\mathbf{S}'(\mathbf{x})$ and the convergence of $\mathbf{e}(t)$ to zero, we have that $\dot{\tilde{\mathbf{W}}}$, $\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}$ also converges to zero. □

Remark: Again from the above analysis we cannot conclude anything about the convergence of the weights to their optimal values. The Remark for Theorem 1 provides some details concerning the problem.

III. CONTROL

In this section we investigate the tracking problem. The unknown nonlinear dynamical system is identified by a dynamic neural network, and then it is driven to follow the response of a known system. The purpose of the identification stage is to provide adequate initial values for the control stage, therefore leading to better transient response of the error.

A. Parametric Uncertainty

In this section we assume that the unknown system can be modeled by a dynamical neural network of the form

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{BW}^*\mathbf{S}(\mathbf{x}) + \mathbf{BW}^*_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u} \tag{18}$$

where all matrices are as defined previously. Define the error between the identifier states and the real system states as

$$\mathbf{e} = \hat{\mathbf{x}} - \mathbf{x}$$

then from (2) and (18) we obtain the error equation

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{B}\tilde{\mathbf{W}}\mathbf{S}(\mathbf{x}) + \mathbf{B}\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u} \tag{19}$$

where

$$\tilde{\mathbf{W}} = \mathbf{W} - \mathbf{W}^{\star}$$
$$\tilde{\mathbf{W}}_{\mathbf{n+1}} = \mathbf{W}_{\mathbf{n+1}} - \mathbf{W}^{\star}_{\mathbf{n+1}}.$$

In the tracking problem we want the real system states to follow the states of a reference model. Let the reference model be described by

$$\dot{\mathbf{x}}_{\mathbf{m}} = \mathbf{A}_{\mathbf{m}}\mathbf{x}_{\mathbf{m}} + \mathbf{B}_{\mathbf{m}}\mathbf{r}$$

where $\mathbf{x}_{\mathbf{m}} \in \Re^n$ are the model states, $\mathbf{r} \in \Re^n$ inputs and $\mathbf{A}_{\mathbf{m}}$, $\mathbf{B}_{\mathbf{m}}$ are constant matrices of appropriate dimensions. Define the error between the identifier states and the model states as

$$\mathbf{e}_{\mathbf{c}} = \hat{\mathbf{x}} - \mathbf{x}_{\mathbf{m}}. \tag{20}$$

Differentiating (20) we obtain

$$\dot{\mathbf{e}}_{\mathbf{c}} = \dot{\hat{\mathbf{x}}} - \dot{\mathbf{x}}_{\mathbf{m}}$$

or

$$\begin{aligned}\dot{\mathbf{e}}_{\mathbf{c}} = {} & \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{W}\mathbf{S}(\mathbf{x}) + \mathbf{B}\mathbf{W}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})\mathbf{u} \\ & -\mathbf{A}_{\mathbf{m}}\mathbf{x}_{\mathbf{m}} - \mathbf{B}_{\mathbf{m}}\mathbf{r}.\end{aligned} \tag{21}$$

Taking $\mathbf{u}$ to be equal to

$$\begin{aligned}\mathbf{u} = {} & -[\mathbf{B}\mathbf{W}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x})]^{-1}[\mathbf{A}\mathbf{x}_{\mathbf{m}} + \mathbf{B}\mathbf{W}\mathbf{S}(\mathbf{x}) \\ & -\mathbf{A}_{\mathbf{m}}\mathbf{x}_{\mathbf{m}} - \mathbf{B}_{\mathbf{m}}\mathbf{r}]\end{aligned} \tag{22}$$

and substituting it to (21) we finally obtain

$$\dot{\mathbf{e}}_{\mathbf{c}} = \mathbf{A}\mathbf{e}_{\mathbf{c}}. \tag{23}$$

The Lyapunov synthesis method is again used to derive stable adaptive laws. Therefore, if we take the Lyapunov function candidate

$$\begin{aligned}\mathcal{V}(\mathbf{e}, \mathbf{e}_{\mathbf{c}}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}) = {} & \frac{1}{2}\mathbf{e}^T\mathbf{P}\mathbf{e} + \frac{1}{2}\mathbf{e}_{\mathbf{c}}^T\mathbf{P}\mathbf{e}_{\mathbf{c}} + \frac{1}{2}tr\{\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}\} \\ & + \frac{1}{2}tr\{\tilde{\mathbf{W}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\}\end{aligned}$$

where $\mathbf{P} > 0$ is chosen to satisfy the Lyapunov equation

$$\mathbf{P}\mathbf{A} + \mathbf{A}^T\mathbf{P} = -\mathbf{I}$$

we obtain, (following the same procedure as in Section II-A), that the learning laws

$$\dot{w}_{ij} = -b_i p_i s(x_j) e_i$$
$$\dot{w}_{in+1} = -b_i s'(x_i) p_i u_i e_i$$

for all $i, j = 1, 2, 3, \cdots, n$.

Make

$$\dot{\mathcal{V}} = -\frac{1}{2}\|\mathbf{e}\|^2 - \frac{1}{2}\|\mathbf{e}_{\mathbf{c}}\|^2 \le 0.$$

Furthermore, it is trivial to verify that the learning laws above can be written in matrix form as

$$\dot{\mathbf{W}} = -\mathbf{E}\mathbf{B}\mathbf{P}\mathbf{S}_0$$
$$\dot{\mathbf{W}}_{\mathbf{n+1}} = -\mathbf{B}\mathbf{P}\mathbf{S}'\mathbf{U}\mathbf{E}$$

where all matrices are defined as follows:

$$\mathbf{P} = \mathrm{diag}[p_1, p_2, \cdots, p_n]$$
$$\mathbf{B} = \mathrm{diag}[b_1, b_2, \cdots, b_n]$$
$$\mathbf{E} = \mathrm{diag}[e_1, e_2, \cdots, e_n]$$
$$\mathbf{U} = \mathrm{diag}[u_1, u_2, \cdots, u_n]$$
$$\mathbf{S}_0 = \begin{bmatrix} s(x_1) & \cdots & s(x_n) \\ \vdots & & \vdots \\ s(x_1) & \cdots & s(x_n) \end{bmatrix}.$$

To apply the control law (22), we have to assure the existence of $(\mathbf{B}\mathbf{W}_{\mathbf{n+1}}\mathbf{S}'(\mathbf{x}))^{-1}$. Since $\mathbf{B}$, $\mathbf{W}_{\mathbf{n+1}}$ and $\mathbf{S}'(\mathbf{x})$ are diagonal matrices and b_i, $s'(x_i) \neq 0$, $\forall i = 1, 2, \cdots, n$ all we need to establish is $w_{in+1}(t) \neq 0$, $\forall t \ge 0$, $\forall i = 1, 2, \cdots, n$. Hence $\mathbf{W}_{\mathbf{n+1}}(t)$ is confined through the use of a projection algorithm [1], [17], [18] to the set $\mathcal{W} = \{\mathbf{W}_{\mathbf{n+1}}: \|\tilde{\mathbf{W}}_{\mathbf{n+1}}\| \le w_m\}$ where w_m is a positive constant. Furthermore, $\tilde{\mathbf{W}}_{\mathbf{n+1}} = \mathbf{W}_{\mathbf{n+1}} - \mathbf{W}^{\star}_{\mathbf{n+1}}$ and $\mathbf{W}^{\star}_{\mathbf{n+1}}$ contains the initial values of $\mathbf{W}_{\mathbf{n+1}}$ that identification provides. In particular, the standard adaptive laws are modified to

$$\dot{\mathbf{W}} = -\mathbf{E}\mathbf{B}\mathbf{P}\mathbf{S}_0$$
$$\dot{\mathbf{W}}_{\mathbf{n+1}} = \begin{cases} -\mathbf{B}\mathbf{P}\mathbf{S}'\mathbf{U}\mathbf{E} \\ \qquad \text{if } \mathbf{W}_{\mathbf{n+1}} \in \mathcal{W} \text{ or } \{\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\| = w_m \\ \qquad \text{and } tr\{-\mathbf{B}\mathbf{P}\mathbf{S}'\mathbf{U}\mathbf{E}\tilde{\mathbf{W}}_{n+1}\} \le 0\} \\ -\mathbf{B}\mathbf{P}\mathbf{S}'\mathbf{U}\mathbf{E} + tr\{B\mathbf{P}\mathbf{S}'\mathbf{S}\mathbf{U}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ \quad \left(\frac{1+\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|}{w_m}\right)^2 \tilde{\mathbf{W}}_{\mathbf{n+1}} \\ \qquad \{\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\| = w_m \text{ and} \\ \qquad tr\{-\mathbf{B}\mathbf{P}\mathbf{S}'\mathbf{U}\mathbf{E}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} > 0\}. \end{cases}$$

Therefore, if the initial weights are chosen such that $\|\tilde{\mathbf{W}}(\mathbf{0})_{\mathbf{n+1}}\| \le w_m$, then we have that $\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\| \le w_m$ for all $t \ge 0$. This can be readily established by noting that whenever $\|\tilde{\mathbf{W}}(\mathbf{t})_{\mathbf{n+1}}\| = w_m$ then

$$\frac{d\|\tilde{\mathbf{W}}(t)_{n+1}\|^2}{dt} \le 0 \tag{24}$$

which implies that the weights $\mathbf{W}_{\mathbf{n+1}}$ are directed toward the inside or the ball $\{\mathbf{W}_{\mathbf{n+1}}: \|\tilde{\mathbf{W}}_{\mathbf{n+1}}\| \le w_m\}$. A proof of the inequality (24), can be found in the Appendix. Now we can prove the following theorem.

Theorem 3: Consider the control scheme (19), (22), (23). The learning law

$$\dot{\mathbf{W}} = -\mathbf{E}\mathbf{B}\mathbf{P}\mathbf{S}_0$$
$$\dot{\mathbf{W}}_{\mathbf{n+1}} = \begin{cases} -\mathbf{B}\mathbf{P}\mathbf{S}'\mathbf{U}\mathbf{E} \\ \qquad \text{if } \mathbf{W}_{\mathbf{n+1}} \in \mathcal{W} \text{ or } \{\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\| = w_m \\ \qquad \text{and } tr\{-\mathbf{B}\mathbf{P}\mathbf{S}'\mathbf{U}\mathbf{E}\tilde{\mathbf{W}}_{n+1}\} \le 0\} \\ -\mathbf{B}\mathbf{P}\mathbf{S}'\mathbf{U}E + tr\{B\mathbf{P}\mathbf{S}'\mathbf{S}\mathbf{U}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ \quad \left(\frac{1+\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|}{w_m}\right)^2 \tilde{\mathbf{W}}_{\mathbf{n+1}} \\ \qquad \{\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\| = w_m \text{ and} \\ \qquad tr\{-\mathbf{B}\mathbf{P}\mathbf{S}'\mathbf{U}\mathbf{E}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} > 0\} \end{cases}$$

guarantees the following properties:

- $\mathbf{e}$, $\mathbf{e_c}$, $\hat{\mathbf{x}}$, $\tilde{\mathbf{W}}$, $\tilde{\mathbf{W}}_{\mathbf{n+1}} \in L_\infty$, $\quad \mathbf{e}$, $\mathbf{e_c} \in L_2$
- $\lim_{t\to\infty} \mathbf{e}(t) = 0, \quad \lim_{t\to\infty} \mathbf{e_c}(t) = 0$
- $\lim_{t\to\infty} \dot{\tilde{\mathbf{W}}}(t) = 0, \quad \lim_{t\to\infty} \dot{\tilde{\mathbf{W}}}_{n+1}(t) = 0$

Proof: With the adaptive laws mentioned above, $\dot{\mathcal{V}}$ becomes

$$\begin{aligned}\dot{\mathcal{V}} &= -\frac{1}{2}\|\mathbf{e}\|^2 - \frac{1}{2}\|\mathbf{e_c}\|^2 + I_n tr\{tr\{\mathbf{BPS'UE\tilde{W}_{n+1}}\} \\ &\quad \left(\frac{1+\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|}{w_m}\right)^2 \tilde{\mathbf{W}}^T_{\mathbf{n+1}}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ &\le -\frac{1}{2}\|\mathbf{e}\|^2 - \frac{1}{2}\|\mathbf{e_c}\|^2 + I_n tr\{\mathbf{BPS'UE\tilde{W}_{n+1}}\} \\ &\quad \left(\frac{1+\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|}{w_m}\right)^2 tr\{\tilde{\mathbf{W}}^T_{\mathbf{n+1}}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ &\le -\frac{1}{2}\|\mathbf{e}\|^2 - \frac{1}{2}\|\mathbf{e_c}\|^2 + I_n tr\{\mathbf{BPS'UE\tilde{W}_{n+1}}\} \\ &\quad \left(\frac{1+w_m}{w_m}\right)^2 \|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|^2 \\ &\le -\frac{1}{2}\|\mathbf{e}\|^2 - \frac{1}{2}\|\mathbf{e_c}\|^2 + I_n tr\{\mathbf{BPS'UE\tilde{W}_{n+1}}\} \\ &\quad (1+w_m)^2\end{aligned}$$

where I_n is an indicator function defined as $I_n = 1$ if the conditions $\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\| = w_m$ and $tr\{-\mathbf{BPS'UE\tilde{W}_{n+1}}\} > 0$ are satisfied. Now since $tr\{\mathbf{BPS'UE\tilde{W}_{n+1}}\} \le 0$ then $I_n tr\{\mathbf{BPS'UE\tilde{W}_{n+1}}\} \times (1+w_m)^2 < 0$. Hence, $\dot{\mathcal{V}} \le 0$. Therefore, the additional terms introduced by the projection can only make $\dot{\mathcal{V}}$ more negative. Since $\dot{\mathcal{V}}$ is negative semidefinite we have that $\mathcal{V} \in L_\infty$, which implies $\mathbf{e}, \mathbf{e_c}$, $\tilde{\mathbf{W}}$, $\tilde{\mathbf{W}}_{\mathbf{n+1}} \in L_\infty$. Furthermore, $\hat{\mathbf{x}} = \mathbf{e} + \mathbf{x}$ is also bounded. Since $\mathcal{V}$ is a non-increasing function of time and bounded from below, the $\lim_{t\leftarrow\infty} \mathcal{V} = \mathcal{V}_\infty$ exist. Therefore by integrating $\dot{\mathcal{V}}$ from 0 to ∞ we have

$$\begin{aligned}&\int_0^\infty \frac{1}{2}(\|\mathbf{e}\|^2 + \|\mathbf{e_c}\|^2)dt \\ &\quad - I_n(1+w_m)^2 \int_0^\infty tr\{\mathbf{BPS'UE\tilde{W}_{n+1}}\}dt \\ &\quad \le [\mathcal{V}(O) - \mathcal{V}_\infty] < \infty\end{aligned}$$

which implies that $\mathbf{e}, \mathbf{e_c} \in L_2$. By definition the sigmoid functions $\mathbf{S(x)}$, $\mathbf{S'(x)}$ are bounded for all $\mathbf{x}$ and by assumption all inputs to the reference model are also bounded. Hence from (22) we have that $\mathbf{u}$ is bounded and from (19), (23) $\dot{\mathbf{e}}, \dot{\mathbf{e}}_\mathbf{c} \in L_\infty$. Since $\mathbf{e}$, $\mathbf{e_c} \in L_2 \cap L_\infty$ and $\dot{\mathbf{e}}, \dot{\mathbf{e}}_\mathbf{c} \in L_\infty$, using Barbalat's Lemma [19], we conclude that $\lim_{t\to\infty} \mathbf{e}(t) = \lim_{t\to\infty} \mathbf{e_c}(t) = 0$. Now using the boundedness of $\mathbf{u}$, $\mathbf{S(x)}$, $\mathbf{S'(x)}$ and the convergence of $\mathbf{e}(t)$ to zero, we have that $\dot{\tilde{\mathbf{W}}}, \dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}$ also converges to zero.

□

Remark: The analysis above implies that the projection modification guarantees boundedness of the weights, without affecting the rest of the stability properties established in the absence of projection.

B. Parametric Plus Dynamic Uncertainties

In this subsection we examine a more general case where parametric and dynamic uncertainties are present. To analyze the problem, the complete singular perturbation model (14) is used. Therefore, the control scheme is now described by the following set of nonlinear differential equations:

$$\begin{aligned}\dot{\mathbf{e}} &= \mathbf{Ae} + \mathbf{B\tilde{W}S(x)} + \mathbf{B\tilde{W}_{n+1}S'(x)u} \\ &\quad - \mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{n+1})\eta \\ \dot{\mathbf{e}}_\mathbf{c} &= A\mathbf{e_c} \\ \mu\dot{\eta} &= \mathbf{A}_0\eta - \mu\dot{\mathbf{h}}(e, \tilde{W}, \tilde{W}_{\mathbf{n+1}}, \eta) \\ \mathbf{u} &= -[\mathbf{BW_{n+1}S'(x)}]^{-1}[\mathbf{Ax_m} + \mathbf{BWS(x)} \\ &\quad - \mathbf{A_m x_m} - \mathbf{B_m r}]\end{aligned} \tag{25}$$

Before proceeding any further, we need to prove the following lemma.

Lemma 2: It is true that $\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u})$ is bounded by

$$\|\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u})\| \le \rho_1\|\mathbf{e}\| + \rho_2\|\eta\|$$

provided that the following inequalities hold:

$$\begin{aligned}\|\mathbf{h_W}\dot{\tilde{\mathbf{W}}}\| &\le k_0\|\mathbf{e}\| \\ \|\mathbf{h_{W_{n+1}}}\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}\| &\le k_1\|\mathbf{e}\| \\ \|\mathbf{h_e B\tilde{W}_{n+1}S'(x)u}\| &\le k_2\|\mathbf{e}\| \\ \|\mathbf{h_e B\tilde{W}S(x)}\| &\le k_3\|\mathbf{e}\| \\ \|\mathbf{h_e F}(\mathbf{x}, \mathbf{W}, \mathbf{W_{n+1}})\| &\le \rho_2 \\ \|\mathbf{h_e Ae}\| &\le k_4\|\mathbf{e}\|\end{aligned}$$

and

$$\rho_1 = k_0 + k_1 + k_2 + k_3 + k_4.$$

Proof: Differentiating $\mathbf{h}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u})$ we obtain

$$\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta) = \mathbf{h_e}\dot{\mathbf{e}} + \mathbf{h_{\tilde{W}}}\dot{\tilde{\mathbf{W}}} + \mathbf{h_{\tilde{W}_{n+1}}}\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}$$

or

$$\begin{aligned}\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta) &= \mathbf{h_e}(\mathbf{Ae} + \mathbf{B\tilde{W}S(x)} \\ &\quad + \mathbf{BW_{n+1}\tilde{S}'(x)u} \\ &\quad - \mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W_{n+1}})\eta) + \mathbf{h_{\tilde{W}}}\dot{\tilde{\mathbf{W}}} \\ &\quad + \mathbf{h_{\tilde{W}_{n+1}}}\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}.\end{aligned}$$

Therefore

$$\begin{aligned}\|\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u})\| &\le \|\mathbf{h_e Ae}\| + \|\mathbf{h_e B\tilde{W}S(x)}\| \\ &\quad + \|\mathbf{h_e B\tilde{W}_{n+1}S'(x)u}\| \\ &\quad + \|\mathbf{h_e F}(\mathbf{x}, \mathbf{W}, \mathbf{W_{n+1}})\eta\| \\ &\quad + \|\mathbf{h_{\tilde{W}}}\dot{\tilde{\mathbf{W}}}\| + \|\mathbf{h_{\tilde{W}_{n+1}}}\dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}\| \\ &\le k_4\|\mathbf{e}\| + k_3\|\mathbf{e}\| + k_2\|\mathbf{e}\| \\ &\quad + \|\mathbf{h_e F}(\mathbf{x}, \mathbf{W}, \mathbf{W_{n+1}})\|\|\eta\| \\ &\quad + k_0\|\mathbf{e}\| + k_1\|\mathbf{e}\| \\ &\le k_4\|\mathbf{e}\| + k_3\|\mathbf{e}\| + k_2\|\mathbf{e}\| \\ &\quad + \rho_2\|\eta\| + k_0\|\mathbf{e}\| + k_1\|\mathbf{e}\|.\end{aligned}$$

Hence,

$$\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta, \mathbf{u}) \leq \rho_1\|\mathbf{e}\| + \rho_2\|\eta\|$$

which concludes the proof. □

We are now able to prove the following theorem

Theorem 4: The control scheme (25), is asymptotically stable for all

$$\mu \in (0, \mu_0)$$

where $\mu_0 = \frac{1}{2}(\frac{1}{2\gamma_1\gamma_2+\gamma_3})$.

Furthermore, the learning law

$$\dot{\mathbf{W}} = -\mathbf{EBPS}_0$$

$$\dot{\mathbf{W}}_{\mathbf{n+1}} = \begin{cases} -\mathbf{BPS'UE} \\ \qquad \text{if } \mathbf{W}_{\mathbf{n+1}} \in \mathcal{W} \text{ or } \{\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\| = w_m \\ \qquad \text{and } tr\{-\mathbf{BPS'UE}\tilde{W}_{n+1}\} \leq 0\} \\ -\mathbf{BPS'UE} + tr\{B\mathbf{PS'SU}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ \quad \left(\frac{1+\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|}{w_m}\right)^2 \tilde{\mathbf{W}}_{\mathbf{n+1}} \\ \qquad \{\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\| = w_m \text{ and} \\ \qquad tr\{-\mathbf{BPS'UE}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} > 0\} \end{cases}$$

guarantees the following properties:

- $\mathbf{e}, \mathbf{e_c}, \eta, \hat{\mathbf{x}}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}} \in L_\infty$, $\mathbf{e}, \mathbf{e_c}, \eta \in L_2$
- $\lim_{t\to\infty} \mathbf{e}(t) = 0$, $\lim_{t\to\infty} \mathbf{e_c}(t) = 0$, $\lim_{t\to\infty} \eta(t) = 0$
- $\lim_{t\to\infty} \dot{\tilde{\mathbf{W}}}(t) = 0$, $\lim_{t\to\infty} \dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}(t) = 0$

Proof: Let's take the Lyapunov function candidate

$$\begin{aligned} \mathcal{V}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta) = \frac{1}{2}\gamma_1\mathbf{e}^T\mathbf{Pe} + \frac{1}{2}\gamma_1\mathbf{e_c}^T\mathbf{Pe_c} + \frac{1}{2}\gamma_2\eta^T\mathbf{P_0}\eta \\ + \frac{1}{2}\gamma_1 tr\{\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}\} \\ + \frac{1}{2}\gamma_1 tr\{\tilde{\mathbf{W}}^T_{\mathbf{n+1}}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \end{aligned} \tag{26}$$

where $\mathbf{P}$, $\mathbf{P_0} > 0$ are chosen to satisfy the Lyapunov equation

$$\mathbf{PA} + \mathbf{A}^T\mathbf{P} = -\mathbf{I}$$
$$\mathbf{P_0A_0} + \mathbf{A_0^T P_0} = -\mathbf{I}.$$

Observe that (26) is a weighted sum of a slow and fast part. Taking the time derivative of (26) we obtain

$$\begin{aligned} \dot{\mathcal{V}} =& \frac{\gamma_1}{2}(\dot{\mathbf{e}}^T\mathbf{Pe} + \mathbf{e}^T\mathbf{P}\dot{\mathbf{e}}) + \frac{\gamma_1}{2}(\dot{\mathbf{e}}_\mathbf{c}^T\mathbf{Pe_c} + \mathbf{e_c^T}\mathbf{P}\dot{\mathbf{e}}_\mathbf{c}) \\ &+ \frac{\gamma_2}{2}(\dot{\eta}^T\mathbf{P_0}\eta + \eta^T\mathbf{P_0}\dot{\eta}) \\ &+ \gamma_1 tr\{\dot{\tilde{\mathbf{W}}}^T\tilde{\mathbf{W}}\} + \gamma_2 tr\{\dot{\tilde{\mathbf{W}}}^T_{\mathbf{n+1}}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \end{aligned}$$

or

$$\begin{aligned} \dot{\mathcal{V}} = & \frac{\gamma_1}{2}(-\mathbf{e}^T\mathbf{e} + \mathbf{S}^T(\mathbf{x})\tilde{\mathbf{W}}^T\mathbf{BPe} + \mathbf{u}^T\mathbf{S}'(\mathbf{x})\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{BPe} \\ & - \eta^T F^T(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\mathbf{Pe}) \\ & + \frac{\gamma_1}{2}((\mathbf{S}^T(\mathbf{x})\tilde{\mathbf{W}}^T\mathbf{BPe})^T + (\mathbf{u}^T\mathbf{S}'(\mathbf{x})\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{BPe})^T \\ & - (\eta^T\mathbf{F}^T(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\mathbf{Pe})^T) \\ & - \frac{\gamma_1}{2}\mathbf{e_c^T}\mathbf{e_c} + \gamma_1 tr\{\dot{\tilde{\mathbf{W}}}^T\tilde{\mathbf{W}}\} + \gamma_2 tr\{\dot{\tilde{\mathbf{W}}}^T_{\mathbf{n+1}}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ & + \frac{\gamma_2}{2}\left(-\frac{1}{\mu}\eta^T\eta - (\dot{\mathbf{h}}^T(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta\right) \\ & \mathbf{P_0}\eta - (\dot{\mathbf{h}}^T(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta)\mathbf{P_0}\eta)^T). \end{aligned}$$

Now, since $\mathbf{S^T}(\mathbf{x})\tilde{\mathbf{W}}^T$ $\mathbf{BPe}$, $\mathbf{u}^T\mathbf{S}'$ $(\mathbf{x})\tilde{\mathbf{W}}_{\mathbf{n+1}}$ $\mathbf{BPe}$, η^T $\mathbf{F}^T(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})$ $\mathbf{Pe}$, $\dot{\mathbf{h}}^T$ $(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta)$ $\mathbf{P_0}$ η, are scalars, we have

$$\mathbf{S}^T(\mathbf{x})\tilde{\mathbf{W}}^T\mathbf{BPe} = (\mathbf{S}^T(\mathbf{x})\tilde{\mathbf{W}}^T\mathbf{BPe})^T$$
$$\mathbf{u}^T\mathbf{S}'(\mathbf{x})\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{BPe} = (\mathbf{u}^T\mathbf{S}'(\mathbf{x})\tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{BPe})^T$$
$$\eta^T\mathbf{F}^T(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\mathbf{Pe} = (\eta^T\mathbf{F}^T(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\mathbf{Pe})^T$$
$$\dot{\mathbf{h}}^T(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta)\mathbf{P_0}\eta = (\dot{\mathbf{h}}^T(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta)\mathbf{P_0}\eta)^T.$$

Therefore $\dot{\mathcal{V}}$ becomes

$$\begin{aligned} \dot{\mathcal{V}} = & \frac{\gamma_1}{2}(-\|\mathbf{e}\|^2 + 2\mathbf{S}^T(\mathbf{x})\tilde{\mathbf{W}}\mathbf{BPe} + 2\mathbf{u}^T\mathbf{S}'(\mathbf{x}), \tilde{\mathbf{W}}_{\mathbf{n+1}}\mathbf{BPe} \\ & - 2\eta^T\mathbf{F}^T(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\mathbf{Pe}) \\ & + \frac{\gamma_2}{2}\left(-\frac{1}{\mu}\|\eta\|^2 - 2\eta^T\mathbf{P_0}\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta)\right) \\ & - \frac{\gamma_1}{2}\|\mathbf{e_c}\|^2 + \gamma_1 tr\{\dot{\tilde{\mathbf{W}}}^T\tilde{\mathbf{W}}\} \\ & + \gamma_2 tr\{\dot{\tilde{\mathbf{W}}}^T_{\mathbf{n+1}}\tilde{\mathbf{W}}_{\mathbf{n+1}}\}. \end{aligned} \tag{27}$$

If we apply the learning laws we have obtained in the previous subsection, (27) becomes

$$\begin{aligned} \dot{\mathcal{V}} = & -\frac{\gamma_1}{2}\|\mathbf{e}\|^2 - \frac{\gamma_1}{2}\|\mathbf{e_c}\|^2 - \frac{\gamma_2}{2\mu}\|\eta\|^2 \\ & - \gamma_1\eta^T\mathbf{F}^T(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\mathbf{Pe} \\ & - \gamma_2\dot{\mathbf{h}}^T(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta)\mathbf{P_0}\eta \\ & + I_n tr\{tr\{\mathbf{BPS'UE}\tilde{\mathbf{W}}_{\mathbf{n+1}}\}\left(\frac{1+\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|}{w_m}\right)^2 \\ & \tilde{\mathbf{W}}^T_{\mathbf{n+1}}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ \leq & -\frac{\gamma_1}{2}\|\mathbf{e}\|^2 - \frac{\gamma_1}{2}\|\mathbf{e_c}\|^2 - \frac{\gamma_2}{2\mu}\|\eta\|^2 + \gamma_1\|\eta\| \\ & \|\mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\|\|\mathbf{P}\|\|\mathbf{e}\| \\ & + \gamma_2\|\dot{\mathbf{h}}(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}}, \eta)\|\|\mathbf{P_0}\|\|\eta\| \\ & + I_n tr\{\mathbf{BPS'UE}\tilde{\mathbf{W}}_{\mathbf{n+1}}\}\left(\frac{1+w_m}{w_m}\right)^2\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|^2. \end{aligned}$$

Employing Lemma 2 we obtain

$$\begin{aligned} \dot{\mathcal{V}} \leq & -\frac{\gamma_1}{2}\|\mathbf{e}\|^2 - \frac{\gamma_1}{2}\|\mathbf{e_c}\|^2 - \frac{\gamma_2}{2\mu}\|\eta\|^2 \\ & + \gamma_1\|\eta\|\|\mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\|\|\mathbf{P}\|\|\mathbf{e}\| \\ & + \gamma_2(\rho_1\|\mathbf{e}\| + \rho_2\|\eta\|)\|\mathbf{P_0}\|\|\eta\| \\ & + I_n tr\{tr\{\mathbf{BPS'UE}\tilde{\mathbf{W}}_{\mathbf{n+1}}\}(1+w_m)^2 \\ \leq & -\frac{\gamma_1}{2}\|\mathbf{e}\|^2 - \frac{\gamma_1}{2}\|\mathbf{e_c}\|^2 - \frac{\gamma_2}{2\mu}\|\eta\|^2 \\ & + 2\gamma_1\gamma_2\|\mathbf{e}\|\|\eta\| + \gamma_2\gamma_3\|\eta\|^2 \\ & + I_n tr\{\mathbf{BPS'UE}\tilde{\mathbf{W}}_{\mathbf{n+1}}\}(1+w_m)^2 \end{aligned}$$

provided that

$$\|\mathbf{F}(\mathbf{x}, \mathbf{W}, \mathbf{W}_{\mathbf{n+1}})\|\|\mathbf{P}\| \leq \gamma_2$$
$$\rho_1\|\mathbf{P_0}\| \leq \gamma_1$$
$$\rho_2\|\mathbf{P_0}\| \leq \gamma_3$$

which finally takes the form

$$\begin{aligned}\dot{\mathcal{V}} \leq &-\frac{\gamma_1}{2}\|\mathbf{e}\|^2-\frac{\gamma_1}{2}\|\mathbf{e_c}\|^2\\&-\gamma_2\left(\frac{1}{2\mu}-\gamma_3\right)\|\eta\|^2+2\gamma_1\gamma_2\|\mathbf{e}\|\|\eta\|\\&+I_n tr\{\mathbf{BPS'UE\tilde{W}_{n+1}}\}(1+w_m)^2\end{aligned}$$

or

$$\begin{aligned}\dot{\mathcal{V}} \leq &-[\|\mathbf{e}\| \quad \|\eta\| \quad \|\mathbf{e}_c\|]\\&\begin{bmatrix}\frac{\gamma_1}{2} & -\gamma_1\gamma_2 & 0\\ -\gamma_1\gamma_2 & \gamma_2\left(\frac{1}{2\mu}-\gamma_3\right) & 0\\ 0 & 0 & \frac{\gamma_1}{2}\end{bmatrix}\begin{bmatrix}\|\mathbf{e}\|\\ \|\eta\|\\ \|\mathbf{e_c}\|\end{bmatrix}\\&+I_n tr\{\mathbf{BPS'UE\tilde{W}_{n+1}}\}(1+w_m)^2.\end{aligned} \tag{28}$$

The 3×3 matrix in (28) is positive definite when

$$\mu \leq \frac{1}{2}\left(\frac{1}{2\gamma_1\gamma_2+\gamma_3}\right)$$

Then $\dot{\mathcal{V}}$ is negative semidefinite. Since $\dot{\mathcal{V}} \leq 0$ we conclude that $\mathcal{V} \in L_\infty$, which implies $\mathbf{e}, \mathbf{e_c}, \eta, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}} \in L_\infty$. Furthermore, $\hat{\mathbf{x}} = \mathbf{e}+\mathbf{x}, \mathbf{W} = \tilde{\mathbf{W}}+\mathbf{W}^\star$, $\mathbf{W_{n+1}} = \tilde{\mathbf{W}}_{\mathbf{n+1}} + \mathbf{W}^\star_{\mathbf{n+1}}$ are also bounded. Since $\mathcal{V}$ is a non-increasing function of time and bounded from below, the $\lim_{t\to\infty}\mathcal{V} = \mathcal{V}_\infty$ exists. Therefore by integrating $\dot{\mathcal{V}}$ from 0 to ∞ we have

$$\begin{aligned}&\frac{\gamma_1}{2}\int_0^\infty \|\mathbf{e}\|^2 dt+\gamma_2\left(\frac{1}{2\mu}-\gamma_3\right)\int_0^\infty \|\eta\|^2 dt\\&+\frac{\gamma_1}{2}\int_0^\infty \|\mathbf{e_c}\|^2 dt - 2\gamma_1\gamma_2\int_0^\infty \|\mathbf{e}\|\|\eta\| dt\\&-I_n(1+w_m)^2\int_0^\infty tr\{\mathbf{BPS'UE\tilde{W}_{n+1}}\}dt\\&\leq [V(0)-V_\infty] < \infty\end{aligned}$$

which implies that $\mathbf{e}, \mathbf{e_c}, \eta \in L_2$. Furthermore

$$\begin{aligned}\dot{\mathbf{e}} &= \mathbf{Ae}+\mathbf{B\tilde{W}S'(x)}+\mathbf{B\tilde{W}_{n+1}S'(x)u}\\&\quad -\mathbf{F(x,W,W_{n+1})}\eta\\ \dot{\mathbf{e}}_\mathbf{c} &= \mathbf{Ae_c}\\ \mu\dot{\eta} &= \mathbf{A_0}\eta-\mu\dot{\mathbf{h}}(\mathbf{e},\tilde{\mathbf{W}},\tilde{\mathbf{W}}_\mathbf{n+1},\eta)\\ \mathbf{u} &= -[\mathbf{BW_{n+1}S'(x)}]^{-1}[\mathbf{Ax_m}+\mathbf{BWS(x)}\\&\quad -\mathbf{A_m x_m}-\mathbf{B_m r}].\end{aligned}$$

Since $\mathbf{u}, \mathbf{A_0}, \dot{\mathbf{h}}$ $(\mathbf{e}, \tilde{\mathbf{W}}, \tilde{\mathbf{W}}_{\mathbf{n+1}})$, $\mathbf{S(x)}, \mathbf{S'(x)}$ are bounded, $\dot{\mathbf{e}}, \dot{\mathbf{e}}_c, \dot{\eta} \in L_\infty$. Since $\mathbf{e}, \mathbf{e_c}, \eta \in L_2 \cap L_\infty$, using Barbalat's Lemma we conclude that $\lim_{t\to\infty}\mathbf{e}(t) = 0$, $\lim_{t\to\infty}\mathbf{e_c}(t) = 0$, $\lim_{t\to\infty}\eta(t) = 0$. Now using the boundedness of $\mathbf{u}, \mathbf{S(x)}, \mathbf{S'(x)}$ and the convergence of $\mathbf{e}(t)$ to zero, we have that $\dot{\tilde{\mathbf{W}}}, \dot{\tilde{\mathbf{W}}}_{\mathbf{n+1}}$ also converges to zero. □

IV. Application to Speed Control of DC Motors

In this section we apply our theory to solve the problem of controlling the speed of dc motor. A separately excited dc motor is described by a set of three nonlinear differential equations; but traditionally, the nonlinear model is linearized and reduced by considering the field excitation constant. However, this method of linearization and reduction can not be used when one has to alter the field excitation to fulfill additional requirements imposed to the control system, as for example in the loss minimization problem, [20].

A. DC Motor

A separately excited dc motor can be described by the following set of normalized nonlinear differential equations [21]–[24]:

$$\begin{aligned}T_a\frac{dI_a}{dt} &= -I_a-\Phi\Omega+V_a\\ T_m\frac{d\Omega}{dt} &= \Phi I_a-K_0\Omega-m_L\\ T_f\frac{d\Phi}{dt} &= -I_f+V_f\\ \Phi &= \frac{\alpha I_f}{1+\beta I_f}\end{aligned} \tag{29}$$

where

$T_a = \dfrac{L_a}{R_a}$: electric time constant

$T_m = \dfrac{J\Omega_n}{T_{Ln}}$: mechanic time constant

$T_f = \dfrac{N_f\Phi_n}{V_{fn}}$: field time constant

$K_0 = \dfrac{B_f\Omega_n}{T_{Ln}}$: normalized viscous friction coefficient

α, β: parameters

m_L: a speed independent component of the applied load torque

with

Ω_n: nominaal on load speed

Φ_n: nominal flux

$V_n = c\Phi_n\Omega_n$: nominal armature voltage

$I_{an} = \dfrac{V_{an}}{R_a}$: extrapolated stalled rotor current at nominal voltage

$T_{Ln} = c\Phi_n I_{an}$: extrapolated stalled rotor torque

$I_{fn} = \dfrac{V_{fn}}{R_f}$: nominal field current at nominal field voltage and

c: a constant
R_a: rotor resistance
R_f: field resistance
L_a: rotor self-inductance
L_f: stator self-inductance
N_f: number of turns, stator winding
J: moment of inertia of the motor load system, referred to the motor shaft
V_a: armature voltage
V_f: field voltage
I_a: armature current
I_f: field current
B: viscous friction coefficient
Ω: angular speed
Φ: magnetic flux.

The state equations for this system may be written in the familiar form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

where the states are chosen to be the stator flux, the armature current, and the angular speed, $\mathbf{x} = [I_a \Omega \Phi]^T$. As control inputs we have the armature and the field voltage, $\mathbf{u} = [V_a V_f]^T$. With this choice, we have

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} -\frac{1}{T_a}x_1 - \frac{1}{T_a}x_2x_3 \\ \frac{1}{T_m}x_1x_3 - \frac{K_0}{T_m}x_2 - \frac{m_L}{T_m} \\ -\frac{1}{T_f}\frac{x_3}{\alpha - \beta x_3} \end{bmatrix}$$

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} \frac{1}{T_a} & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_m} \end{bmatrix}.$$

B. The Algorithm

Traditionally, the angular velocity of a dc motor is controlled with changes in its armature voltage, while keeping constant the field excitation. In our method, the magnetic flux is not necessarily constant, leading in this way to a nonlinear model for our motor. Our algorithm can be separated into two phases. Since we begin with the hypothesis that we know nothing about the dc motor, it is logical to perform identification first. The role of identification is twofold. First, it gives us an insight about the system and the green light to proceed to the control phase. If identification fails, the system under consideration can not be controlled by our algorithm. Second, it provides the control phase with appropriate initial conditions, thus improving the transient behavior of our system.

Phase 1 (Identification): In order to identify a dc motor, we consider a neural network identifier

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{BWS}(\mathbf{x}) + \mathbf{BW_{n+1}u}$$

with $n = 2$, (since we assume that we can not measure the magnetic flux), and two inputs which represent the armature

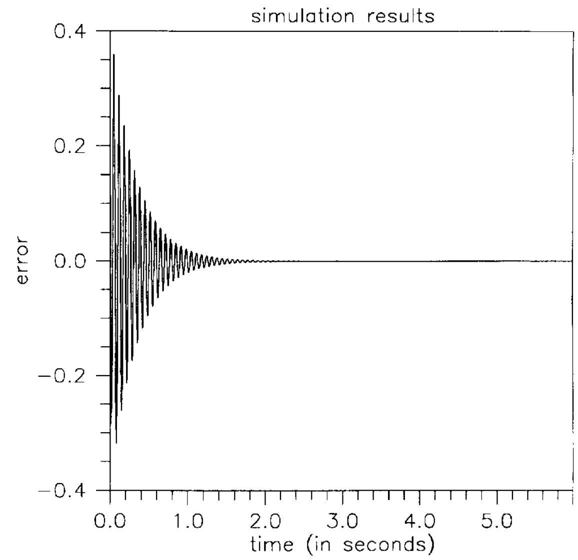

Fig. 3. The evolution of the error $e_1(t)$ with time $u_1 = u_2 = 1 + 0.8\sin(0.001t)$.

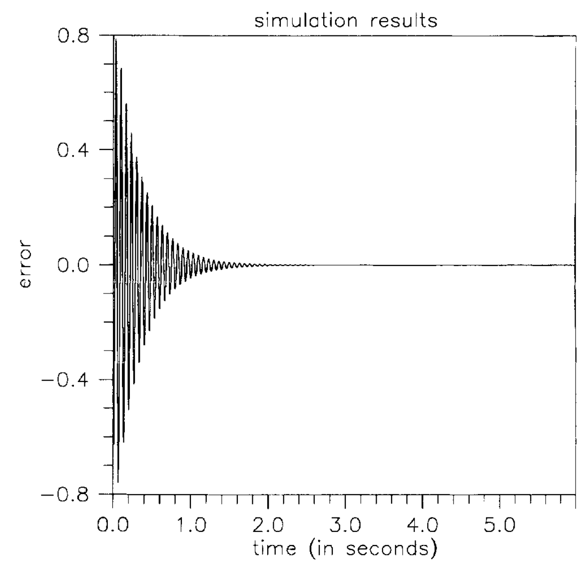

Fig. 4. The evolution of the error $e_2(t)$ with time $u_1 = u_2 = 1 + 0.8\sin(0.001t)$.

and field voltages respectively. The weights W, W_{n+1} are adjusted according to a linear law of the form

$$\dot{w}_{ij} = -b_i p_i s(x_j) e_i$$
$$\dot{w}_{in+1} = -b_i p_i u_i e_i$$

for all $i = 1, 2$.

Observe that we have simplified the identifier structure by omitting $\mathbf{S}'(\mathbf{x})$. However, (as can be seen from Figs. 3 and 4), such a simplification does not destroy the identification capabilities of our neural network and furthermore, leads to a simpler control law.

TABLE I

variable	init. value
$w_{11}(t)$	0.037802
$w_{12}(t)$	0.236919
$w_{13}(t)$	−0.135972
$w_{21}(t)$	0.067767
$w_{22}(t)$	0.281422
$w_{23}(t)$	−0.116260
$\Phi(t)$	0.98

Phase 2 (Control): To keep the speed of a dc motor constant we apply the control law

$$u_1 = -\frac{\Sigma_{i=1}^{2} b_1 w_{1i} s(x_i) - 10_v}{b_1 w_{1n+1}}$$
$$u_2 = 1.$$

Note that by $u_2 = 1$ we mean that we keep the field voltage constant while v is a step input. Concurrently, the weights are adjusted as in the identification phase for $i = 1$. Furthermore, the error is now defined as $e_1 = \hat{x}_1 - x_2$.

C. Simulation Results

We simulate a 1 KW dc motor with a normalized model as in (29) with

$$\frac{1}{T_a} = 148.88$$
$$\frac{1}{T_m} = 42.91$$
$$\frac{K_0}{T_m} = .0129$$
$$T_f = 31.88$$
$$m_L = 0$$
$$\alpha = 2.6$$
$$\beta = 1.6.$$

Our two-stage algorithm, as derived in Subsection IV-B, was applied. In the identification phase, we used the dynamic neural network shown previously with parameters

$$a_i = 15$$
$$b_i = 65$$

for all $i = 1, 2$, while the parameters for the sigmoid functions were

$$k = 1.2$$
$$l = 1.0.$$

The inputs were chosen to be

$$u_1 = u_2 = 1 + 0.8\sin(0.001t).$$

All initial values were set to zero, except that of the magnetic flux which was taken to be equal to 0.98. Figures 3 and 4 give the evolution of the errors e_1 and e_2 respectively.

In the control phase, we applied the dynamic state feedback obtained in the previous subsection, with the initial values that can be seen in Table I and a step input $v = 0.6$. Figure 5 shows how speed is varied with time. It is well known that in order for a dc motor to be functional, field excitation has to maintain its nominal value during operation. Therefore, the large transients obtained in Fig. 5 are expected, since in our case, magnetic flux varies below its nominal value. However, the transient behavior of our system can be sufficiently improved, if we lower the value of the parameter k of the sigmoid function. Same results can be obtained for the armature curent, the evolution of which is presented in Fig. 6.

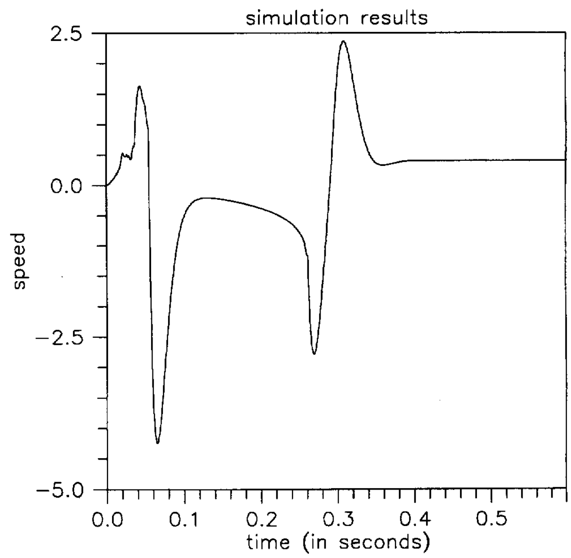

Fig. 5. Motor speed with time ($v = 0.6$).

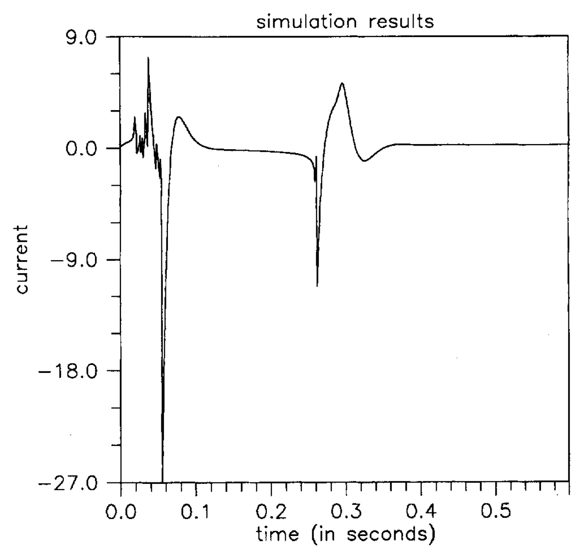

Fig. 6. Armature current with time ($v = 0.6$).

V. Conclusion

The purpose of this paper is to adaptively control unknown nonlinear dynamical systems, using dynamical neural networks. Since the plant is unknown, a two-step algorithm is considered. In step one, a dynamical neural network identifier is employed to perform "black box" identification. Many cases that lead to modeling errors, i.e., parameteric and dynamic uncertainties, are taken into consideration. Convergence of the identification error and weights (under a sufficiency of excitation condition), to zero and to a constant value respectively, is proved. Since our main concern lies on the control problem, only a rough estimation of the region where the weights should belong is needed to proceed to the control phase of our algorithm, in which a dynamic state feedback is developed such that the outputs of the unknown plant follow the outputs of a reference model. Convergence of the error to zero and boundedness of all signals in the closed loop is again proved. Finally, only the states of the unknown plant which are related to the reduced order model, are assumed to be available for measurement.

Appendix: Proof of Inequality (23)

It is true that

$$\frac{d(\|\tilde{\mathbf{W}}_{\mathbf{n+1}}(t)\|^2)}{dt} = \frac{dtr\{\tilde{\mathbf{W}}_{\mathbf{n+1}}^T(t)\tilde{\mathbf{W}}_{\mathbf{n+1}}(t)\}}{dt} = tr\{\dot{\mathbf{W}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}(t)\}.$$

Employing the modified adaptive law we obtain

$$tr\{\dot{\mathbf{W}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}(t)\} = tr\{(-\mathbf{BPSUE})^T\tilde{\mathbf{W}}_{\mathbf{n+1}} + tr\{\mathbf{BPSUE}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \left(\frac{1+\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|}{w_m}\right)^2 \tilde{\mathbf{W}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\}.$$

Since **BPSUE** is a diagonal matrix $(\mathbf{BPSUE})^T = \mathbf{BPSUE}$, therefore

$$\begin{aligned} tr\{\dot{\mathbf{W}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}(t)\} &= tr\{-\mathbf{BPSUE}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ &\quad + tr\{tr\{\mathbf{BPSUE}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \left(\frac{1+\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|}{w_m}\right)^2 \tilde{\mathbf{W}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ &\le tr\{(-\mathbf{BPSUE})^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ &\quad + tr\{\mathbf{BPSUE}\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \left(\frac{1+\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|}{w_m}\right)^2 tr\{\tilde{\mathbf{W}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \end{aligned}$$

which finally takes the form

$$\begin{aligned} tr\{\dot{\mathbf{W}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}(t)\} \le & tr\{(-\mathbf{BPSUE})^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ & + tr\{(\mathbf{BPSUE})\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ & \left(\frac{1+\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|}{w_m}\right)^2 \\ & \|\tilde{\mathbf{W}}_{\mathbf{n+1}}\|^2. \end{aligned}$$

Now since $tr\{(-\mathbf{BPSUE})^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\} > 0$ and $\|\tilde{\mathbf{W}}_{\mathbf{n+1}}\| = w_m$ we obtain

$$\begin{aligned} tr\{\dot{\mathbf{W}}_{\mathbf{n+1}}^T\tilde{\mathbf{W}}_{\mathbf{n+1}}(t)\} \le & tr\{(-\mathbf{BPSUE})^T\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ & + tr\{(\mathbf{BPSUE})\tilde{\mathbf{W}}_{\mathbf{n+1}}\} \\ & (1+w_m)^2 < 0. \end{aligned}$$

Hence, since $(1+w_m)^2 > 1$ we finally obtain $\frac{d\|\tilde{\mathbf{W}}_{\mathbf{n+1}}(t)\|^2}{dt} \le 0$. □

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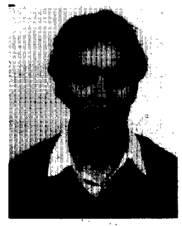

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