

Western Michigan University  
Electrical and Computer Engineering Department  
ECE4550/ECE5550-Digital Signal Processing

**Final Exam**

**SPRING  
2024**

*Show all your work!  
Failure to show your  
work steps will result  
in a major loss of your*

*Due 11 pm on  
Monday 4/22/2024*

**Answer all problems as best as you can. Write clearly!**

*Submit one pdf file for all your work. Number your pages and  
upload to eLearning dropbox. Image files must be converted to  
pdf before uploading*

Name (last name 1<sup>st</sup>): .....

*This cover sheet must be  
scanned and used as a  
cover sheet with your  
submission*

<b>PROBLEM 1 (25)</b>	
<b>PROBLEM 2 (25)</b>	
<b>PROBLEM 3 (30)</b>	
<b>TOTAL (100)</b>	

*Make sure to clearly label all  
axes of all your graph*

### PROBLEM #1 (25 Points) Filter Design/Analysis (Chapter 5 content)

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Design a filter that completely blocks the frequency of  $\omega_o = \frac{\pi}{3}$ , by placing its poles and zeros in the z-plane. The designed filter should yield real output, given the input is real. Clearly and show all your work, answer the following:

- 1) Draw poles and zeros of the filter in the Z-plane, clearly showing magnitude and phase.
- 2) Write the expression for the transfer function,  $H(z)$  associated with the filter you designed per your pole-zero plot. You can use  $K=1$  for a scaling constant.
- 3) Write an expression for  $H(\omega)$ , and give its magnitude and phase.
- 4) Plot the magnitude of  $H(\omega)$  (**not the Bode plot**), that is  $|H(\omega)|$  versus frequency  $\omega$ . **You can use any software.**
- 5) Discuss this filter stability in sight of your choice of its poles and zeros.
- 6) Determine the output  $y(n)$ , if the input signal to the filter is given by

$$x(n) = 6 + 3 \cos\left(\frac{\pi}{3}n\right) + \sin\left(\frac{\pi}{6}n + \frac{\pi}{2}\right) + 2 \cos\left(\frac{\pi}{2}n\right) \quad -\infty < n < \infty$$

- 7) What type of a filter can this be used for, and why?
- 8) Based on the filter's type, explain the relation between  $y(n)$  and  $x(n)$  in part 6.

## PROBLEM #2 (25 points) FIR DESIGN USING WINDOWING METHOD

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Use the specifications of the following ideal filter to design a BP FIR Filter, answer the following questions:

$$H_{IDEAL}(F) = \begin{cases} 1 & 10 \leq |F| \leq 15 \\ 0 & \text{otherwise} \end{cases}$$

With sample frequency  $F_s = 50\text{kHz}$  and all given  $F$  values above are in kHz.

And using,

$$W_{hamming}(n) = \begin{cases} 0.54 - 0.46 \cos\left(\frac{2\pi n}{M}\right) & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

- 1) Sketch the frequency response for the desired filter using your hand drawing.
- 2) Using the above window, determine the impulse response of FIR filter which approximates this frequency response.
- 3) Compute the first 4 coefficients of the impulse response  $h(n)$  coefficients.
- 4) Use MATLAB to plot both the desired  $h(n)$ , and the frequency response magnitude and phase of the filter you designed.
- 5) Comment on the expected performance of the filter you designed, and verify that it holds.
- 6) Comment on the nature of the filters phase.
- 7) Discuss how this window would be a better choice over the rectangular window.

**PROBLEM #3 (30 Points) IIR Design Methods Using Analog Filters.**

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Consider the following filter  $H(s)$

$$H(s) = \frac{1}{s}$$

Using  $F_s = 10\text{Hz}$ , design an IIR filter using these methods.

- *Method 1: Impulse Invariant*
- *Method 2: Bilinear Transformation*

*In your design, address these questions*

- 1) Give the model for  $H(z)$  and  $H(\omega)$  for each method.
- 2) Sketch the magnitude frequency response of  $H(\Omega)$ ,  $H_i(\omega)$  *of each of the methods listed above*.  
You can use any tools.
- 3) Discuss the performance of both designs in how they approximate the analog filter for certain frequencies, that is discuss filter performance for low range frequencies, and high frequencies in comparison to the analog filter.
- 4) What type of a filter is this? Explain your answer.
- 5) Does your answer in part 4 align with the advantages and disadvantages of these two design methods? Keep your answer short and precise statements.
- 6) Discuss warping effect on  $H_2(\omega)$  if any.