Homework №3

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1 Let $x_1, \ldots, x_n$ be a sample from a uniform distribution Unif $(0, \theta)$. Denote by Z the following:

$$Z = x_{max}/\theta, x_{\max} = \max(x_1, \ldots, x_n)$$

- Prove that random variable Z has the following distribution function

$$F_Z(z) = \begin{cases} 0, & \textbf{for } z < 0 \\ z^n, & \textbf{for } z \in [0, 1] \\ 1, & \textbf{for } z > 1 \end{cases}$$

- For a fixed θ find the lower boundary $L_{\textbf{low}}$ of a confidence interval for a parameter θ, such as for given $1 - \alpha$

$$P(\theta \geq L_{\textbf{low}}) = 1 - \alpha.$$

1.1 Distribution

First of all let's denote that $\xi = Uniform(0, \theta) => F_\xi(x) = \frac{x}{\theta}$
$F_Z(z) = P(Z < z) = P(max(x_1, ..., x_n)/\theta < z) = P(max(x_1, ..., x_n) < \theta z) = P(x_1 < z\theta) \cdot ... \cdot P(x_n < z\theta) = P^n(\xi < z\theta) = F_\xi^n(z\theta) = z^n$
Taking in account that objects in a sample are independent we can rewrite the following as: $P(max(x_1, ..., x_n) < z) = P(x_1 < z) \cdot ... \cdot P(x_n < z)$

1.2 Interval

Using the found distribution function let's find the left-sided confidence interval for our variable:
$P(Z < \phi) = \gamma = 1 - \alpha = F_Z(\phi) = \phi^n => 1- = \phi^n => \phi = \sqrt[n]{(1 - \alpha)}$
Let's rewrite the confidence interval:
$Z < \phi$
$Z < \sqrt[n]{(1 - \alpha)}$
$\frac{x_{max}}{\theta} < \sqrt[n]{(1 - \alpha)}$
$x_{max} < \theta \sqrt[n]{(1 - \alpha)}$
$\theta > \frac{x_{max}}{\sqrt[n]{(1-\alpha)}}$
So, we get the that $L_{low} = \frac{x_{max}}{\sqrt[n]{(1-\alpha)}}$