Homework №4

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1 Assume there is a disease X, and it is known that 70% of patients recover within one week. Your lab developed a treatment, and during the studies 80 out of 100 patients in the test group recovered after one week. Is it appropriate to assume that the treatment actually works, and the higher rate is not just a result of randomness? Can you reject the null hypothesis? What if 740 out of 1000 patients in the test group recovered after one week?

Let's write down the null and alternative hypothesis: $H_0 : p_{treatment} = 0.7$
$H_1 : p_{treatment} > 0.7$
Let's use the asymptotic criteria which we can get using the central limit theorem.
$t(x_1, ..., x_n) = \sqrt{n} \cdot \frac{\overline{X} - p}{\sqrt{p(1-p)}} \rightarrow^D_{n\to\infty} N(0, 1)$
Let's use the confidence value of $\alpha = 0.01$. Taking in account that we use one-sided criteria we actually need the quantile of a standard normal distribution of level $1-\alpha = 0.99$ wihch is equal to 2.326.

1.1 80 out of 100

Now let's evaluate whether we can reject the zero hypothesis in case of 80 successes out of 100:
$r = \sqrt{100} \cdot \frac{0.8-0.7}{\sqrt{0.7 \cdot 0.3}} = 2.1822$
$S_{0.01} = (2.326, \infty)$ - critical area
$r \notin S_{0.01} =>$ we cannot reject the null hypothesis.

1.2 740 out of 1000

Now let's evaluate whether we can reject the zero hypothesis in case of 740 successes out of 1000:
$r = \sqrt{1000} \cdot \frac{0.74-0.7}{\sqrt{0.7 \cdot 0.3}} = 2.7603$
$S_{0.01} = (2.326, \infty)$ - critical area
$r \in S_{0.01} =>$ we can reject the null hypothesis.