2. **Focus in Early Grades: Materials do not assess any of the following topics before the grade level indicated.**

Table 2

Topic	**Grade Introduced in the Standards**
Probability, including chance, likely outcomes, probability models.	**7**
Statistical distributions, including center, variation, clumping, outliers, mean, median, mode, range, quartiles, and **statistical association or trends**, including two-way tables, bivariate measurement data, scatter plots, trend line, line of best fit, correlation.	**6**
Similarity, congruence, or geometric transformations.	**8**
Symmetry of shapes, including line/reflection symmetry, rotational symmetry.	**4**

As the second column indicates, the Standards as a whole do include the topics in Table 2—they are not being left out. However, in the coherent progression of the Standards, these topics first appear at later grades in order to establish focus. Thus, in aligned materials there are no chapter tests, unit tests, or other such assessment components that make students or teachers responsible for any of the above topics before the grade in which they are introduced in the Standards. (One way to meet this criterion is for materials to omit these topics entirely prior to the indicated grades.)

3. **Focus and Coherence through Supporting Work: Supporting content enhances focus and coherence simultaneously by engaging students in the major work of the grade.** For example, materials for K–5 generally treat data displays as an occasion for solving grade-level word problems using the four operations (see 3.MD.3);[12] materials for grade 7 take advantage of opportunities to use probability to support ratios, proportions, and percents. (This criterion does not apply in the case of targeted supplemental materials or other tools that do not include supporting content.)

4. **Rigor and Balance: Materials and tools reflect the balances in the Standards and help students meet the Standards' rigorous expectations, by (all of the following, in the case of comprehensive materials; at least one of the following for supplemental or targeted resources):**

    a. **Developing students' conceptual understanding of key mathematical concepts, especially where called for in specific content standards or cluster headings.** Materials amply feature high-quality conceptual problems and questions. This includes brief conceptual problems with low computational difficulty (e.g., 'Find a number greater than 1/5 and less than 1/4'); brief

[12] For more information about this example, see Table 1 in the *Progression* for K-3 Categorical Data and 2-5 Measurement Data, http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf. More generally, the *PARCC Model Content Frameworks* give examples in each grade of how to improve focus and coherence by linking supporting topics to the major work.

conceptual questions (e.g., 'If the divisor does not change and the dividend increases, what happens to the quotient?'); and problems that involve identifying correspondences across different mathematical representations of quantitative relationships.[13] Classroom discussion about such problems can offer opportunities to engage in mathematical practices such as constructing and critiquing arguments (MP.3). In the materials, conceptual understanding is attended to most thoroughly in those places in the content standards where explicit expectations are set for understanding or interpreting. Such problems and activities center on fine-grained mathematical concepts–place value, the whole-number product $a \times b$, the fraction a/b, the fraction product $(a/b) \times q$, expressions as records of calculations, solving equations as a process of answering a question, etc. Conceptual understanding of key mathematical concepts is thus distinct from applications or fluency work, and these three aspects of rigor must be balanced as indicated in the Standards.

b. **Giving attention throughout the year to individual standards that set an expectation of procedural skill and fluency**. The Standards are explicit where fluency is expected. Materials in grades K–6 help students make steady progress throughout the year toward fluent (accurate and reasonably fast) computation, including knowing single-digit products and sums from memory (see, e.g., 2.OA.2 and 3.OA.7). Progress toward these goals is interwoven with students' developing conceptual understanding of the operations in question.[14] Manipulatives and concrete representations such as diagrams that enhance conceptual understanding are connected to the written and symbolic methods to which they refer (see, e.g., 1.NBT). As well, purely procedural problems and exercises are present. These include cases in which opportunistic strategies are valuable—e.g., the sum 698 + 240 or the system $x + y = 1$, $2x + 2y = 3$—as well as an ample number of generic cases so that students can learn and practice efficient algorithms (e.g., the sum 8767 + 2286). Methods and algorithms are general and based on principles of mathematics, not mnemonics or tricks.[15] Materials attend most thoroughly to those places in the content standards where explicit expectations are set for fluency. In higher grades, algebra is the language of much of mathematics. Like learning any language, we learn by using it. Sufficient practice with algebraic operations is provided so as to make realistic the attainment of the Standards as a whole; for example, fluency in algebra can help students get past the need to manage computational details so that they can observe structure (MP.7) and express regularity in repeated reasoning (MP.8).

c. **Allowing teachers and students using the materials as designed to spend sufficient time working with engaging applications, without losing focus on the major work of each grade.** Materials in grades K–8 include an ample number of single-step and multi-step contextual problems that develop the mathematics of the grade, afford opportunities for practice, and

[13] Note that for ELL students, multiple representations also serve as multiple access paths.

[14] For more about how students develop fluency in tandem with understanding, see the *Progressions* for Operations and Algebraic Thinking, http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf and for Number and Operations in Base Ten, http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf.

[15] Non-mathematical approaches (such as the "butterfly method" of adding fractions) compromise focus and coherence and displace mathematics in the curriculum (cf. 5.NF.1). For additional background on this point, see the remarks by Phil Daro excerpted at http://vimeo.com/achievethecore/darofocus and/or the full video, available at http://commoncoretools.me/2012/05/21/phil-daro-on-learning-mathematics-through-problem-solving/.

engage students in problem solving. Materials for grades 6–8 also include problems in which students must make their own assumptions or simplifications in order to model a situation mathematically. Applications take the form of problems to be worked on individually as well as classroom activities centered on application scenarios. Materials attend thoroughly to those places in the content standards where expectations for multi-step and real-world problems are explicit. Students learn to use the content knowledge and skills specified in the content standards in applications, with particular stress on applying major work, and a preference for the more fundamental techniques from additional and supporting work. Modeling builds slowly across K–8, and applications are relatively simple in earlier grades. Problems and activities are grade-level appropriate, with a sensible tradeoff between the sophistication of the problem and the difficulty or newness of the content knowledge the student is expected to bring to bear.

Additional aspects of the Rigor and Balance Criterion:
(1) *The three aspects of rigor are not always separate in materials.* (Conceptual understanding and fluency go hand in hand; fluency can be practiced in the context of applications; and brief applications can build conceptual understanding.)

(2) *Nor are the three aspects of rigor always together in materials.* (Fluency requires dedicated practice to that end. Rich applications cannot always be shoehorned into the mathematical topic of the day. And conceptual understanding will not always come along for free unless explicitly taught.)

(3) Digital and online materials with no fixed lesson flow or pacing plan are not designed for superficial browsing but rather should be designed to instantiate the Rigor and Balance criterion.

5. **Consistent Progressions: Materials are consistent with the progressions in the Standards, by (all of the following):**

   a. **Basing content progressions on the grade-by-grade progressions in the Standards.** Progressions in materials match well with those in the Standards. Any discrepancies in content progressions enhance the required learning in each grade and are clearly aimed at helping students meet the Standards as written, rather than setting up competing requirements or effectively rewriting the standards. Comprehensive materials do not introduce gaps in learning by omitting any content that is specified in the Standards.

   The basic model for grade-to-grade progression involves students making tangible progress during each given grade, as opposed to substantially reviewing then marginally extending from previous grades. Remediation may be necessary, particularly during transition years, and resources for remediation may be provided, but previous-grades review is clearly identified as such to the teacher, and teachers and students can see what their specific responsibility is for the current year.

   Digital and online materials that allow students and/or teachers to navigate content across grade levels promote the Standards' coherence by tracking the structure and progressions in the Standards. For example, such materials might link problems and concepts so that teachers and students can browse a progression.