Basics of C Programming

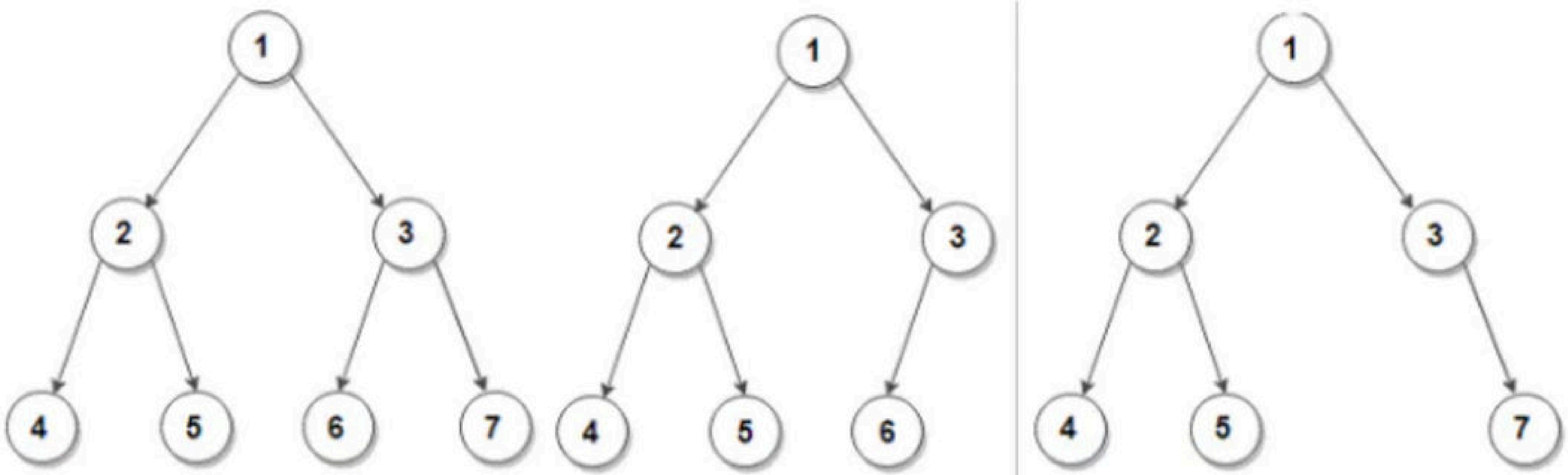
Complete Course on Data Structures for GATE

Sanchit Jain • Lesson 24 • July 15, 2022

Complete Binary Tree

- Consider a binary tree T, the maximum number of nodes at height h is 2^h nodes.

- The binary tree T is said to be complete binary tree, if all its level except possibly the last, have the maximum number of nodes and if all the nodes at the last level appear as far left as possible.

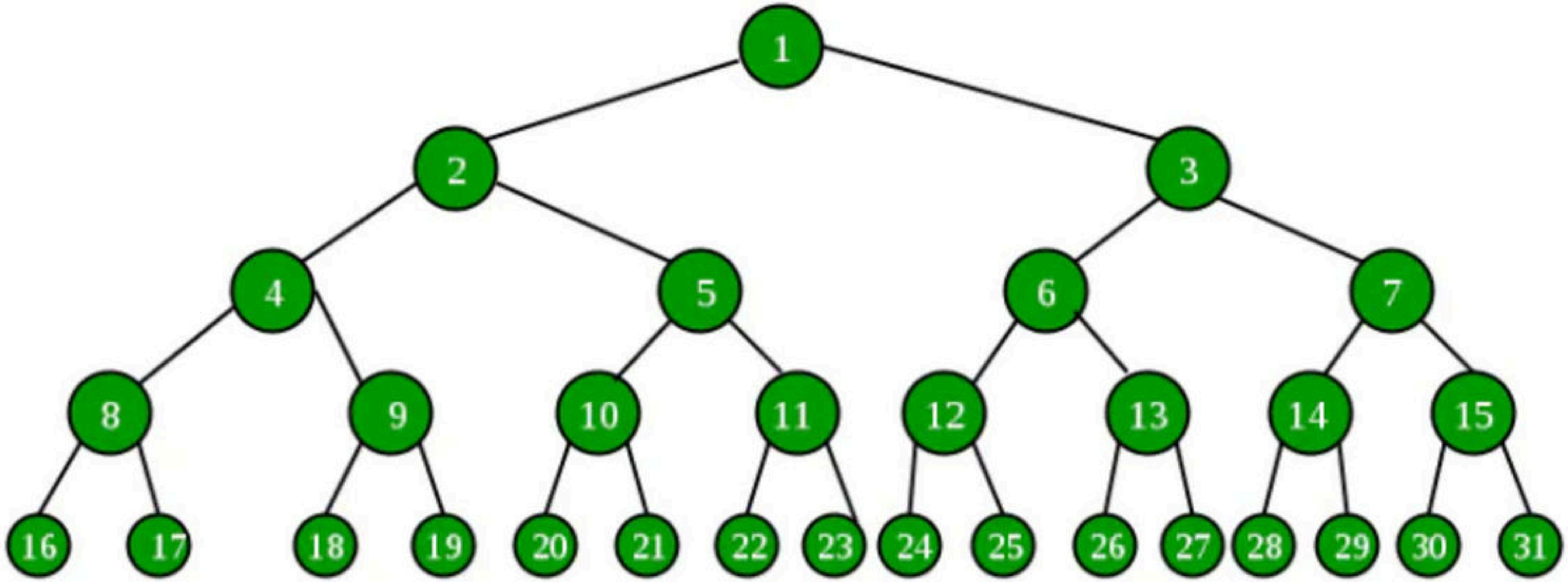


Complete Binary Tree

Not a Complete Binary Tree

- One can easily determine the children and parent of a node k in any complete tree T

- Specially the left and right children of the node K are 2*k, 2*k + 1 and the parent of k is the node lower bound(k/2)



Q A scheme for storing binary trees in an array X is as follows. Indexing of X starts at 1 instead of 0. the root is stored at X[1]. For a node stored at X[i], the left child, if any, is stored in X[2i] and the right child, if any, in X[2i+1]. To be able to store any binary tree on n vertices the minimum size of X should be. **(GATE - 2006) (2 Marks)**

(A) $\log_2 n$ **(B)** n **(C)** $2n + 1$ **(D)** $2^n - 1$

Q Let LASTPOST, LASTIN and LASTPRE denote the last vertex visited in a post order, inorder and preorder traversal, respectively, of a complete binary tree. Which of the following is always true? **(GATE - 2000) (1 Marks)**

(A) LASTIN = LASTPOST

(B) LASTIN = LASTPRE

(C) LASTPRE = LASTPOST

(D) None of the above

Q. What are the children for node 'w' of a complete-binary tree in an array representation? **[Asked in Hexaware 2018]**

a) 2w and 2w+1
b) 2+w and 2-w
c) w+1/2 and w/2
d) w-1/2 and w+1/2

Answer: a

Explanation: The left child is generally taken as 2*w whereas the right child will be taken as 2*w+1 because root node is present at index 0 in the array and to access every index position in the array.

Q If the tree is not a complete binary tree then what changes can be made for easy access of children of a node in the array? **[Asked in Goldman Sachs]**

a) every node stores data saying which of its children exist in the array
b) no need of any changes continue with 2w and 2w+1, if node is at i
c) keep a separate table telling children of a node
d) use another array parallel to the array with tree

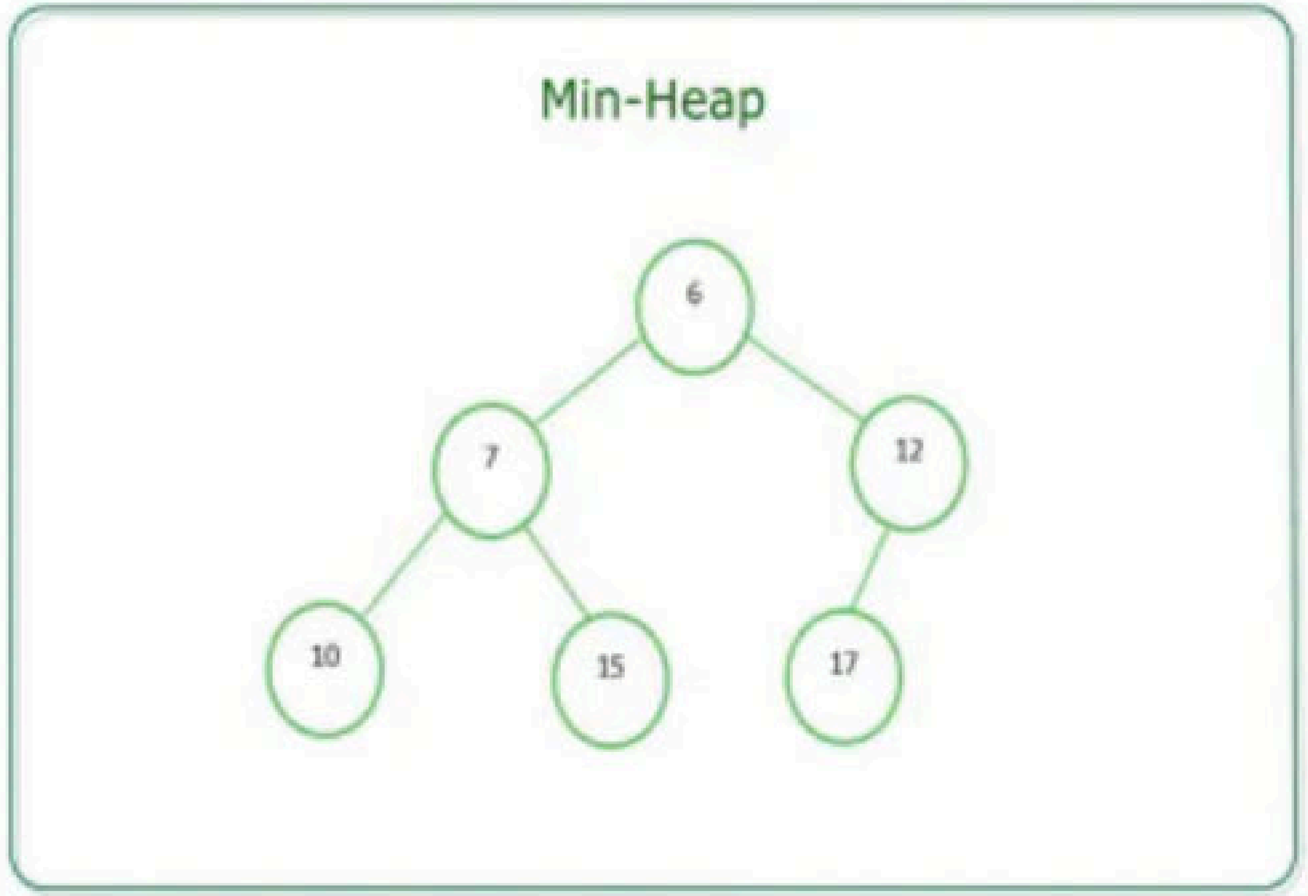
Answer: a

Explanation: Array cannot represent arbitrary shaped trees. It can only be used in case of complete trees. If every node stores data saying that which of its children exists in the array then elements can be accessed easily.

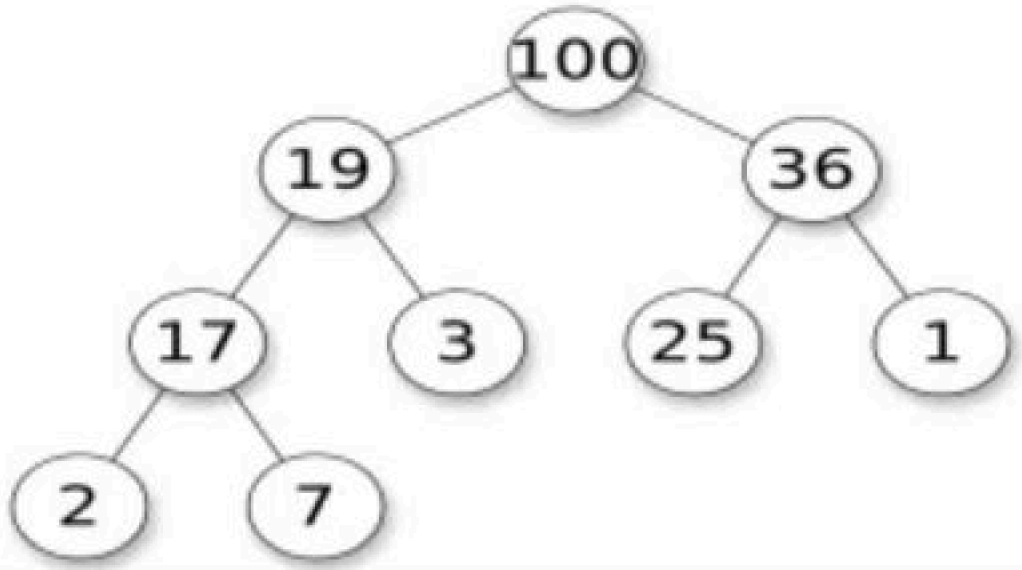
Break

Heap

- Suppose H is a complete binary tree with n elements, H is called a Heap, if each node N of H has following properties:
  - The value of N is greater than to the value at each of the children of N then it is called Max heap.
  - A min heap is defined as the value at N is less than the value at any of the children of N.

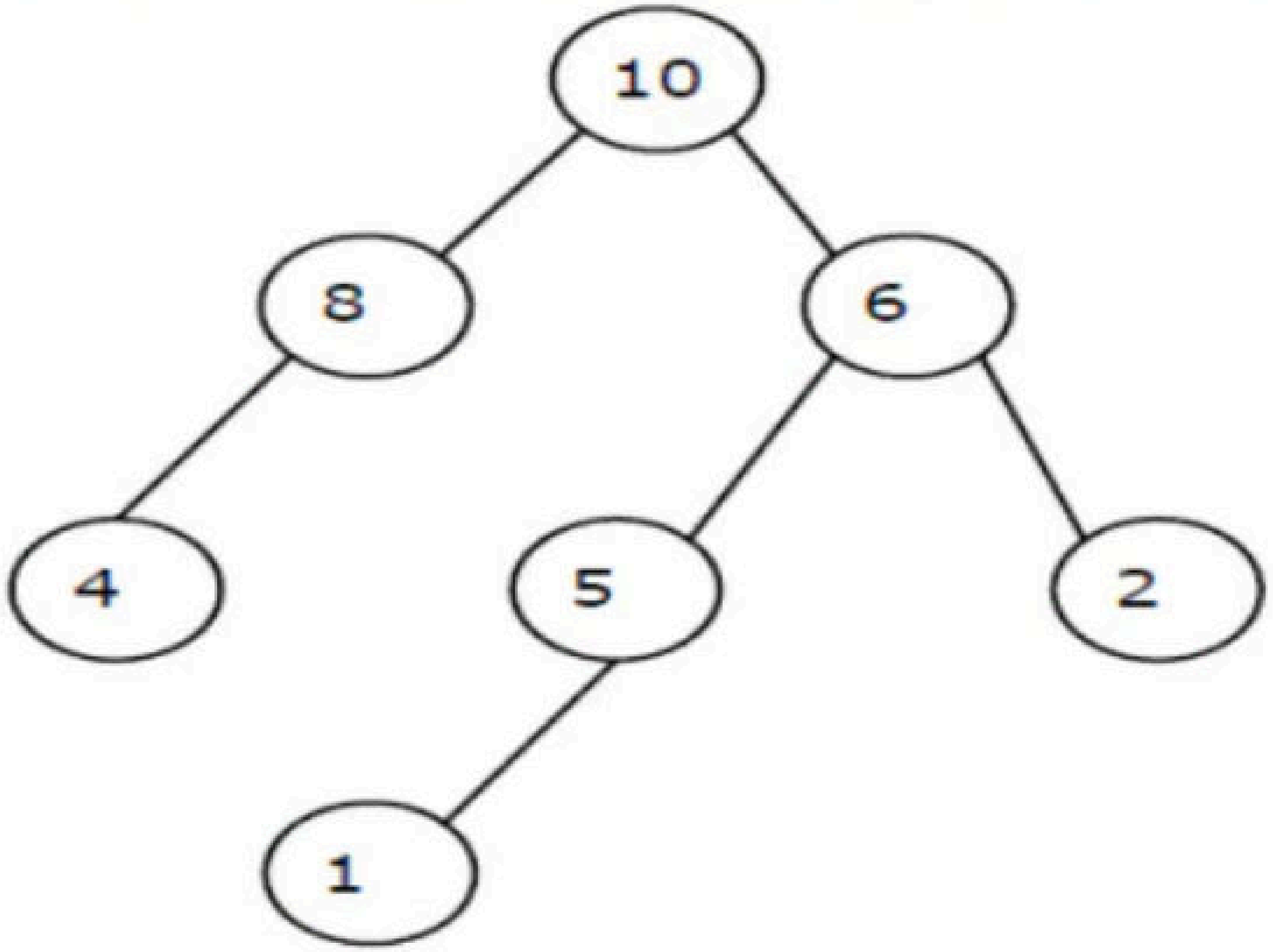


Tree representation

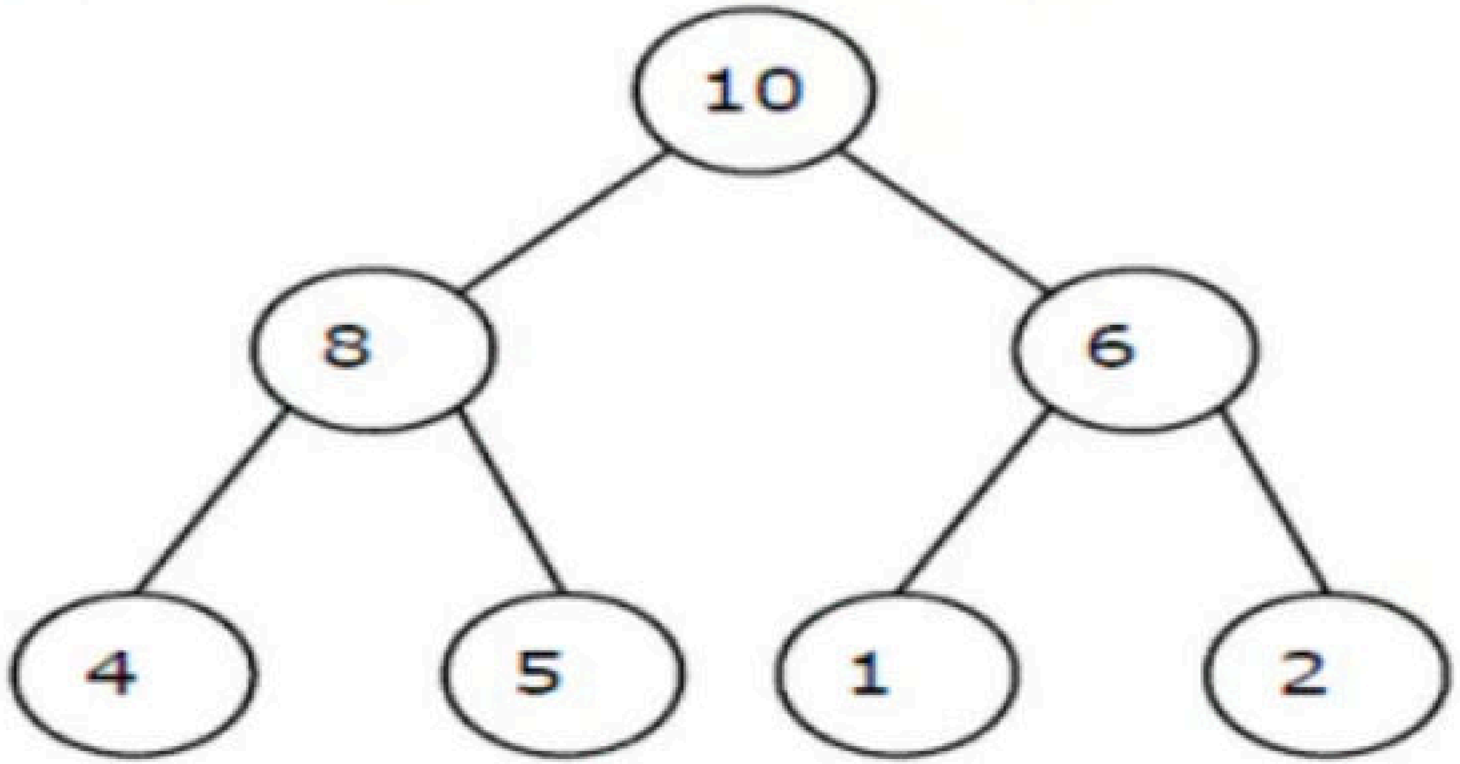


Q A max-heap is a heap where the value of each parent is greater than or equal to the values of its children. Which of the following is a max-heap? **(GATE - 2011) (2 Marks)**

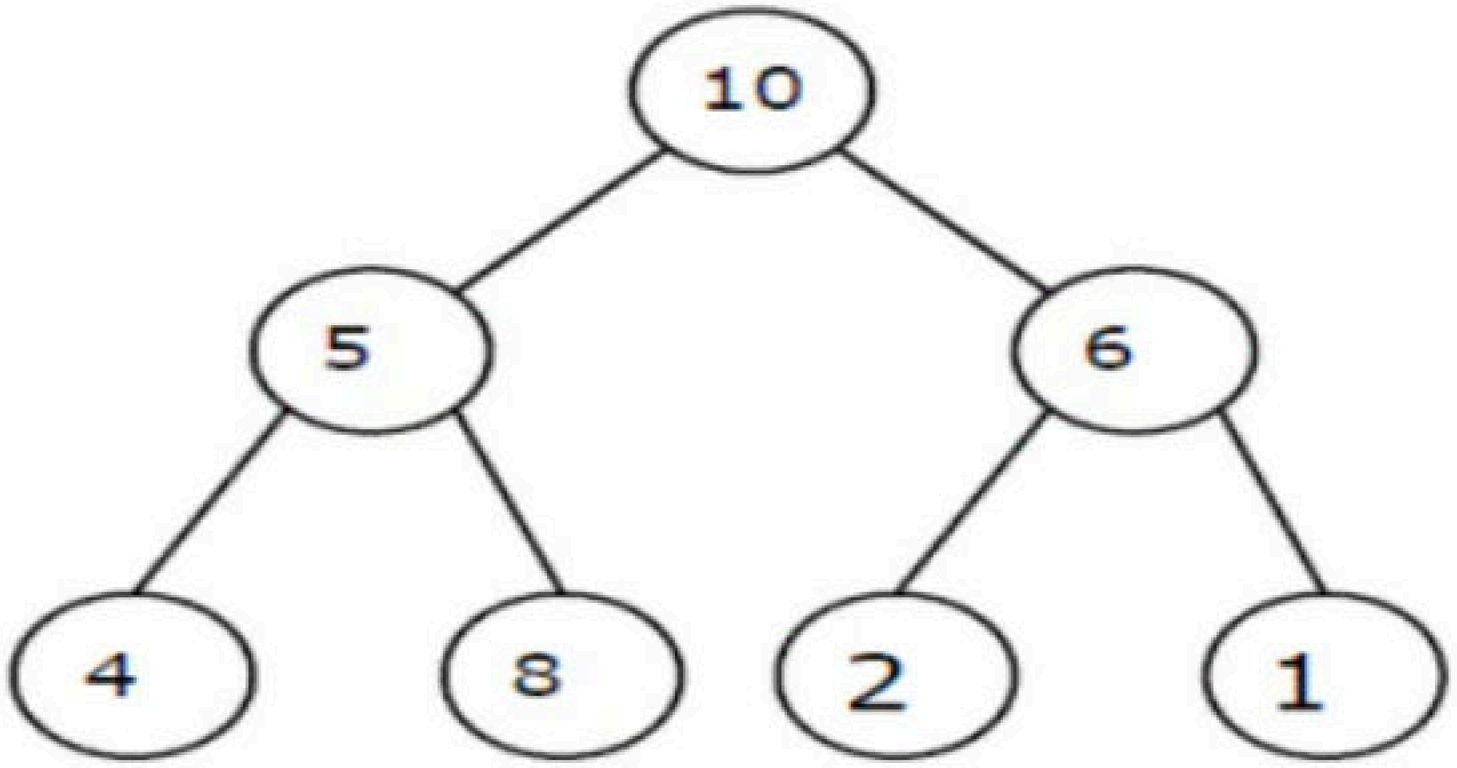
(A)



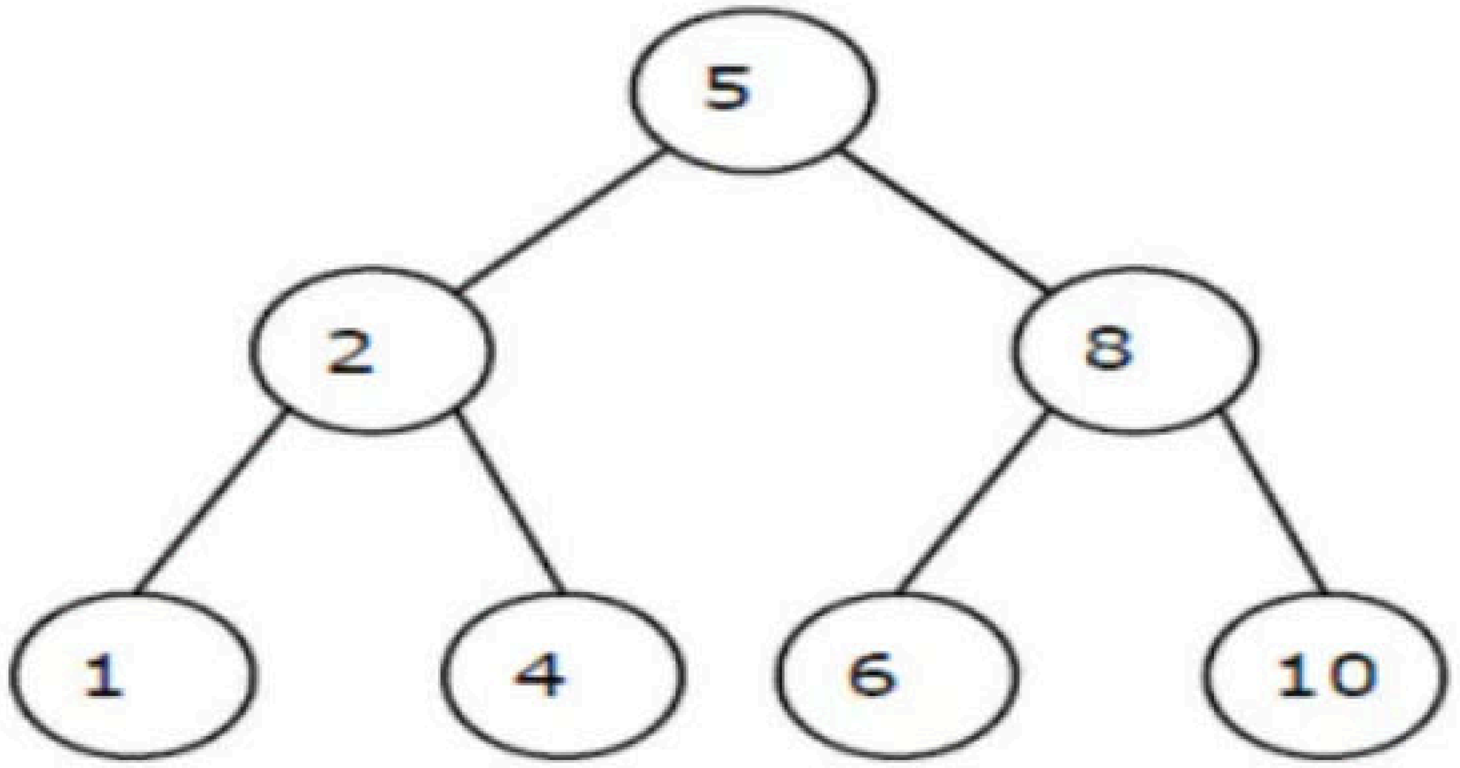
(B)



(C)



(D)



Q Consider a binary max-heap implemented using an array. Which one of the following arrays represents a binary max-heap? **(GATE - 2006) (Marks)**

(A) 23,17,14,6,13,10,1,12,7,5

(B) 23,17,14,6,13,10,1,5,7,12

(C) 23,17,14,7,13,10,1,5,6,12

(D) 23,17,14,7,13,10,1,12,5,7

Q Consider any array representation of an n element binary heap where the elements are stored from index 1 to index n of the array. For the element stored at index i of the array (i <= n), the index of the parent is

(GATE - 2001) (1 Marks)

(A) ~~i – 1~~ 11

(B) floor(i/2) 68

(C) ceiling(i/2) 13

(D) (i+~~1~~)/2 8

k

$2k$ $\quad$ $2k+1$

$\lfloor k/2 \rfloor$

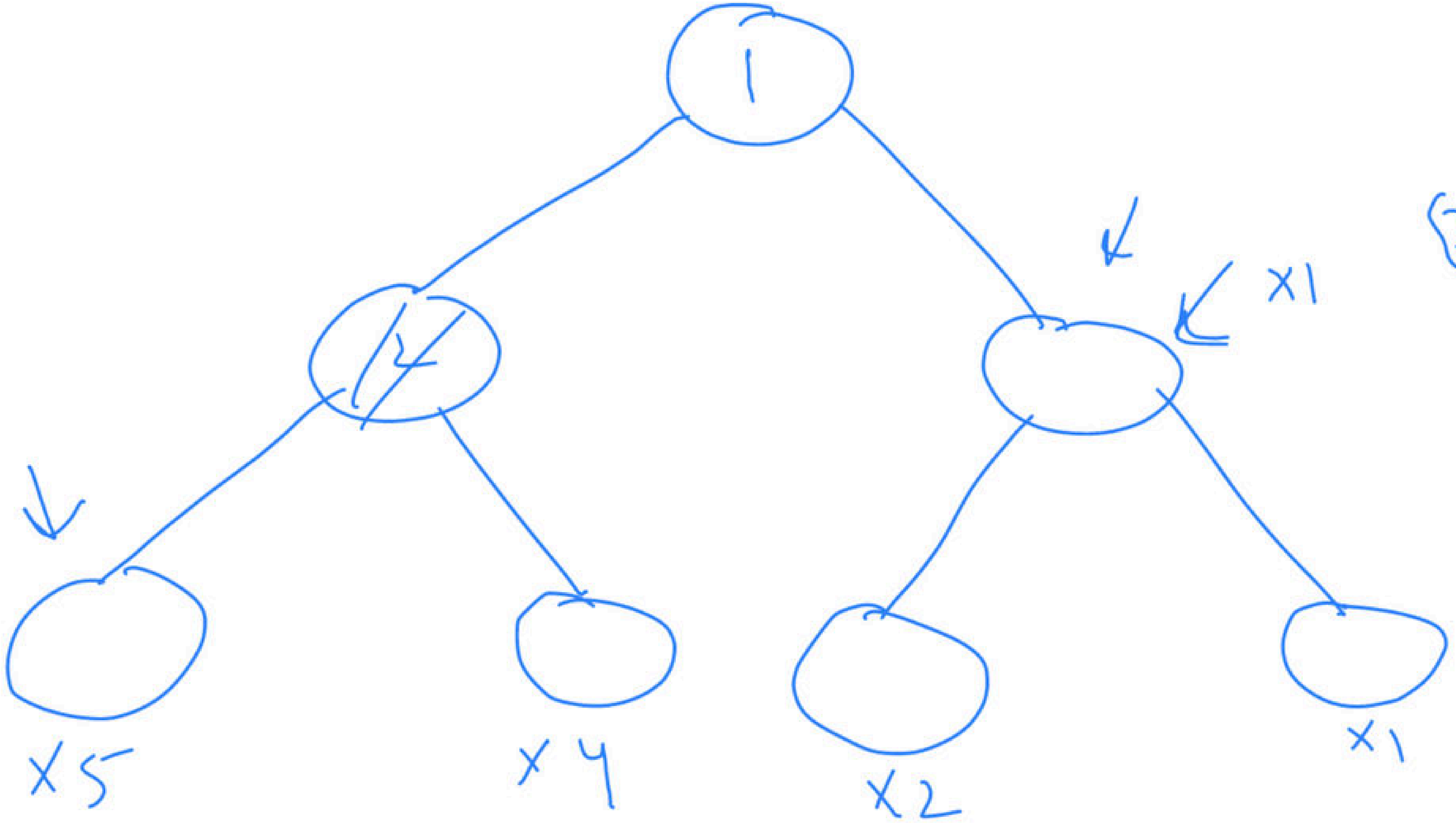
k

Break

Q The number of possible min-heaps containing each value from {1, 2, 3, 4, 5, 6, 7} exactly once is 80. **(Gate-2018) (1 Marks)**

80, 48, 32

{3 ~~4~~ ~~5~~ 6 7}

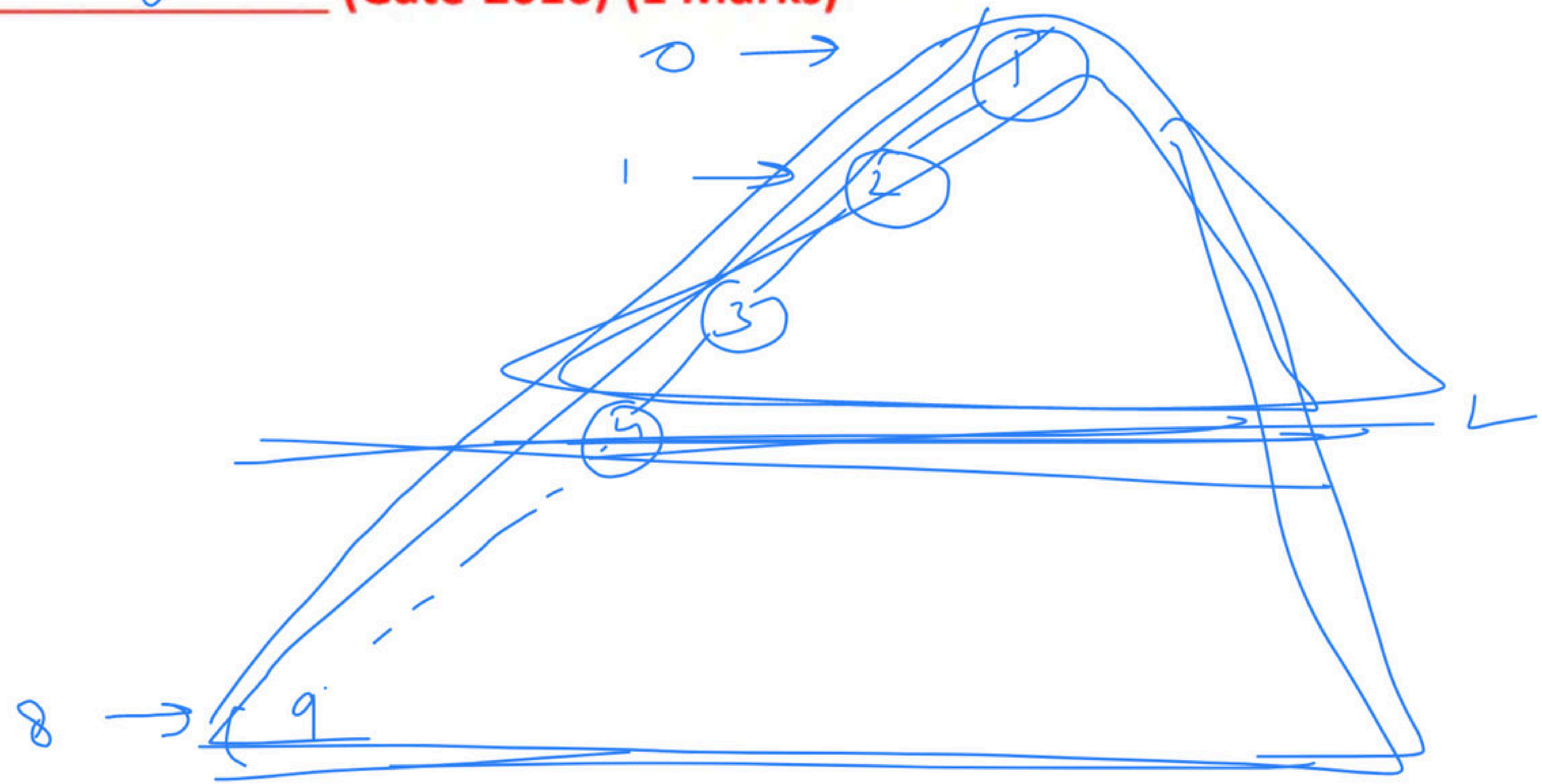


40 x2 = 80

Q A complete binary min-heap is made by including each integer in [1, 1023] exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is

____8____ **(Gate-2016) (1 Marks)**

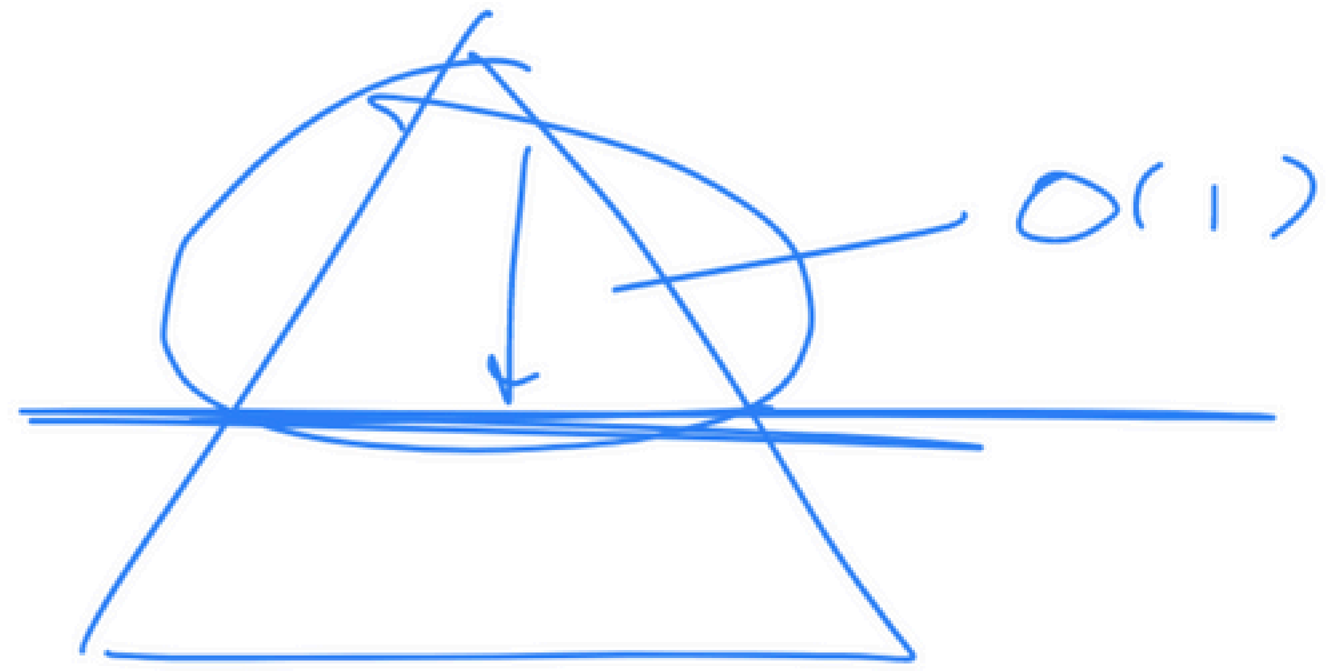
8, 4, 3, 2



$2^{h+1} - 1$

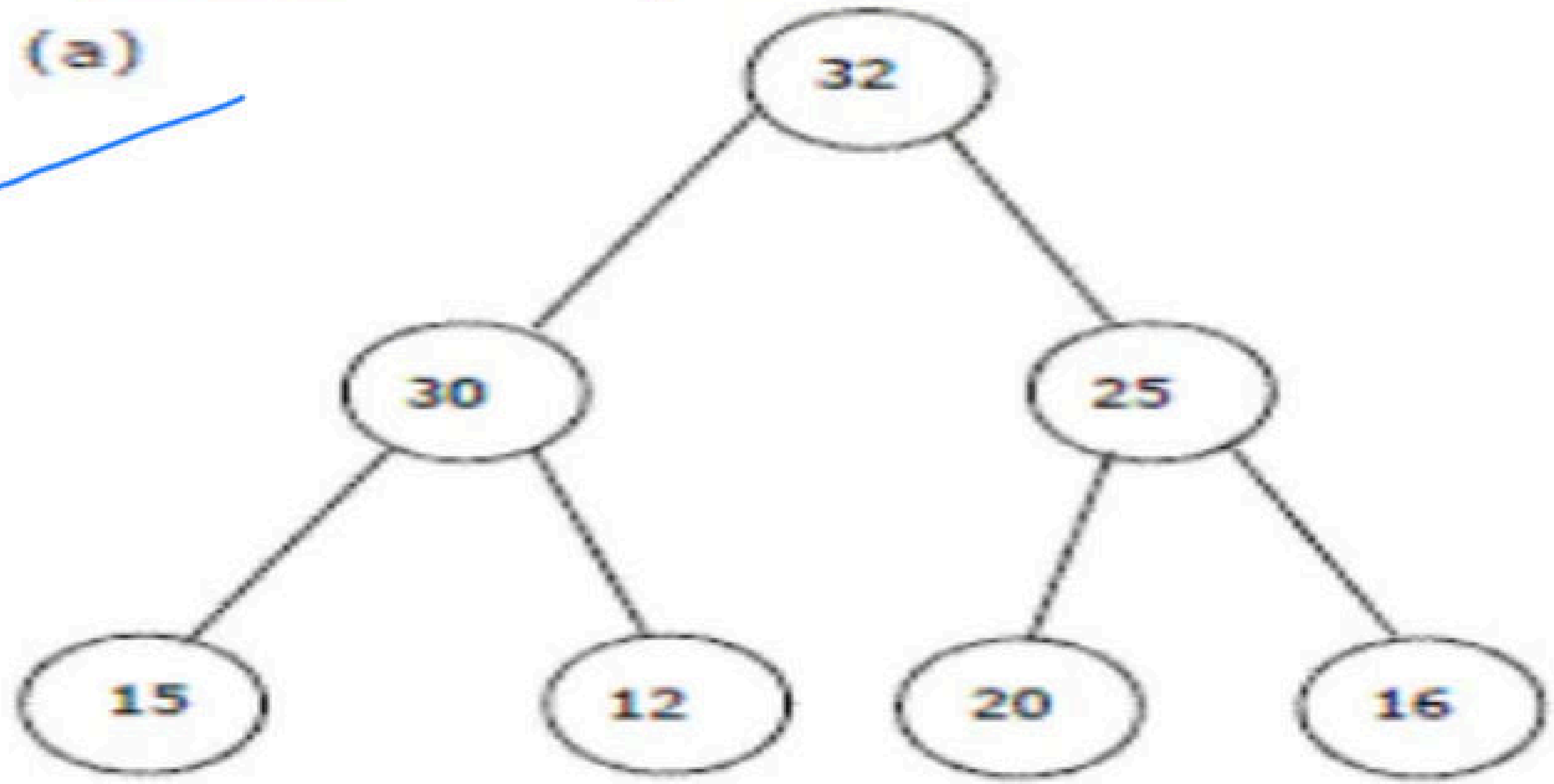
$2^{9} - 1 = 511$

Min-Heap   1, 2, 4, ——— 2^{i}; 0 ↑ ... ——— O(1)

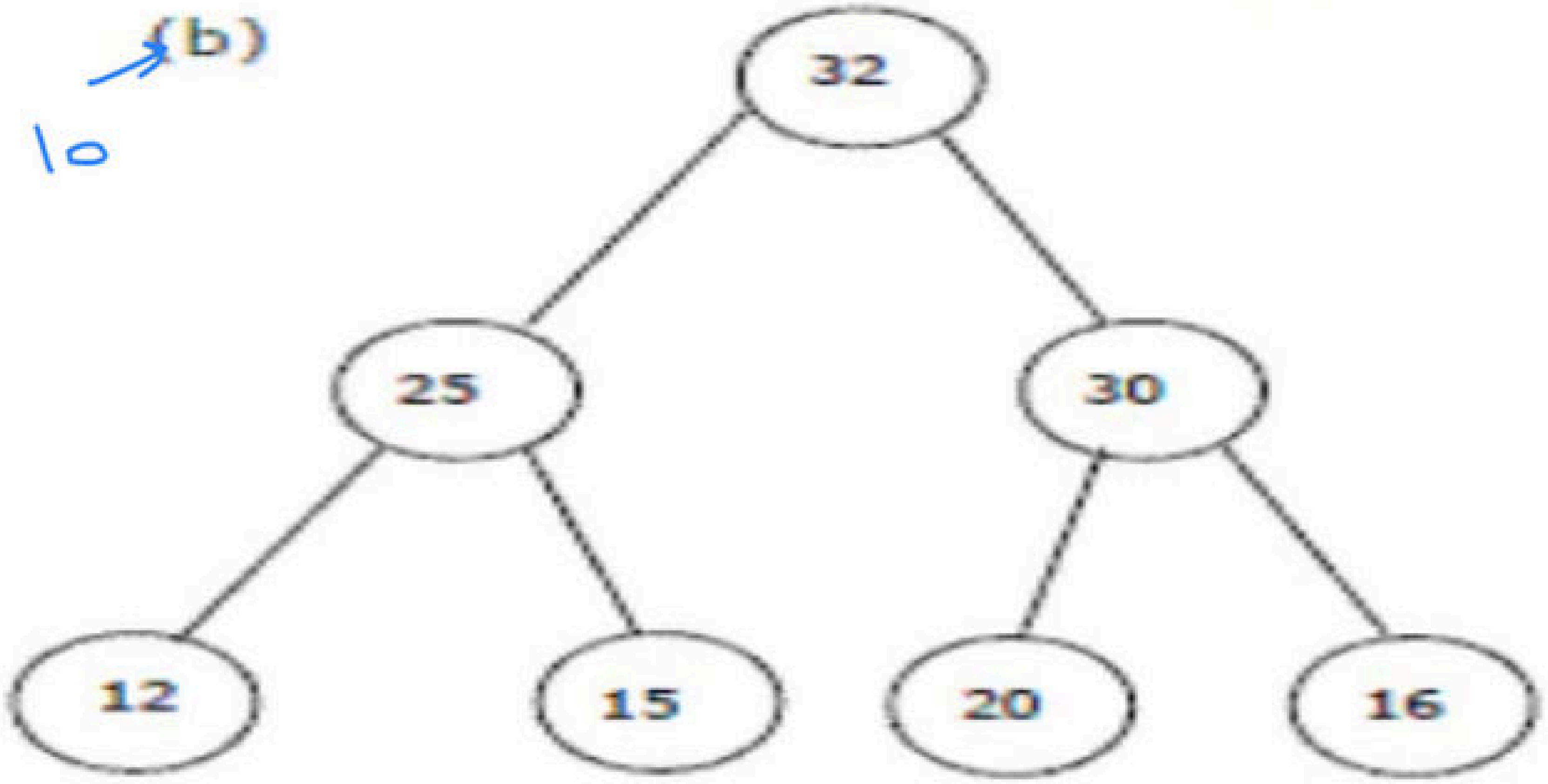


Q The elements 32, 15, 20, 30, 12, 25, 16 are inserted one by one in the given order into a Max Heap. The resultant Max Heap is. **(GATE - 2004) (2 Marks)**

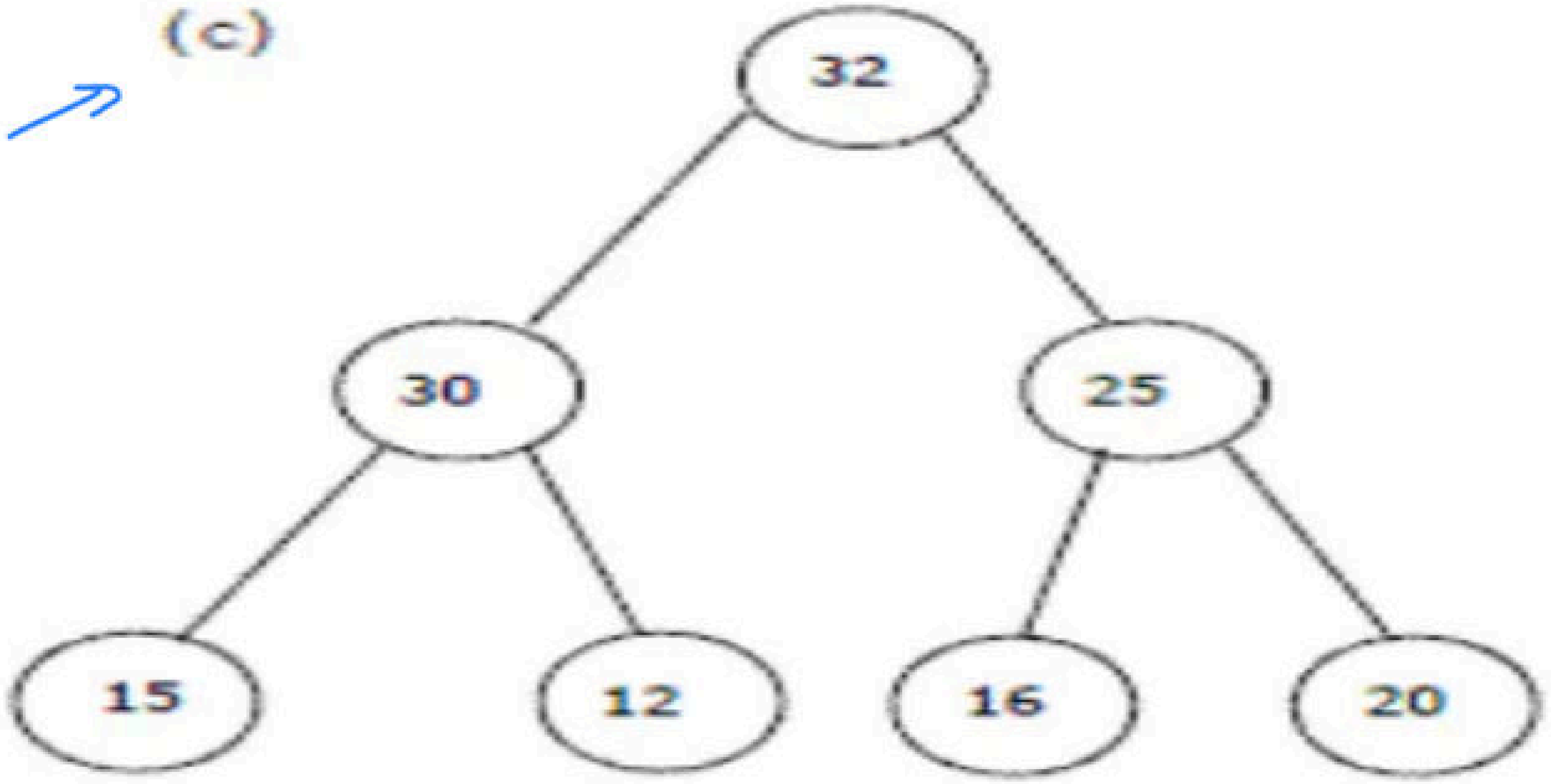
60 (a)



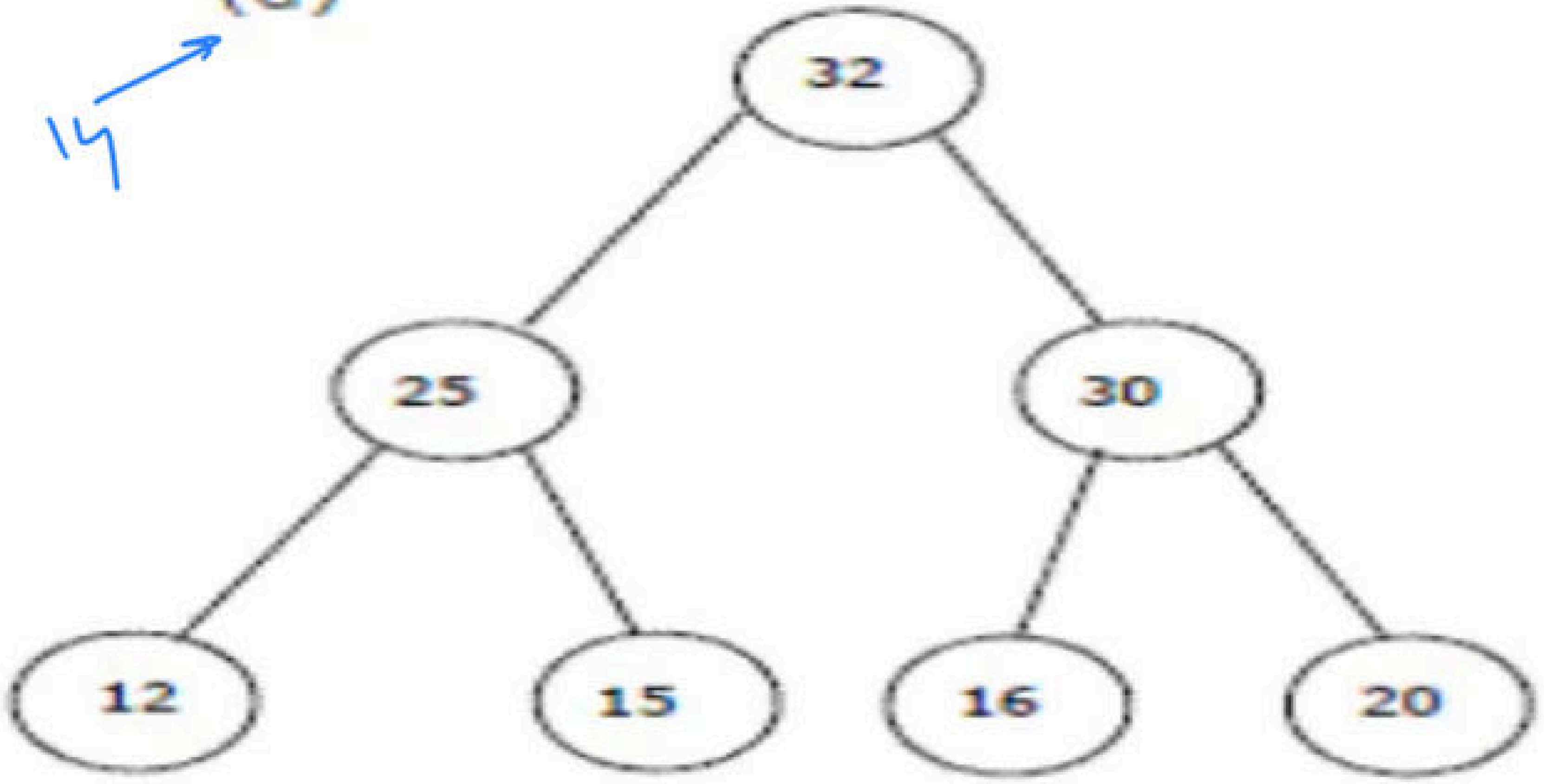
10 (b)



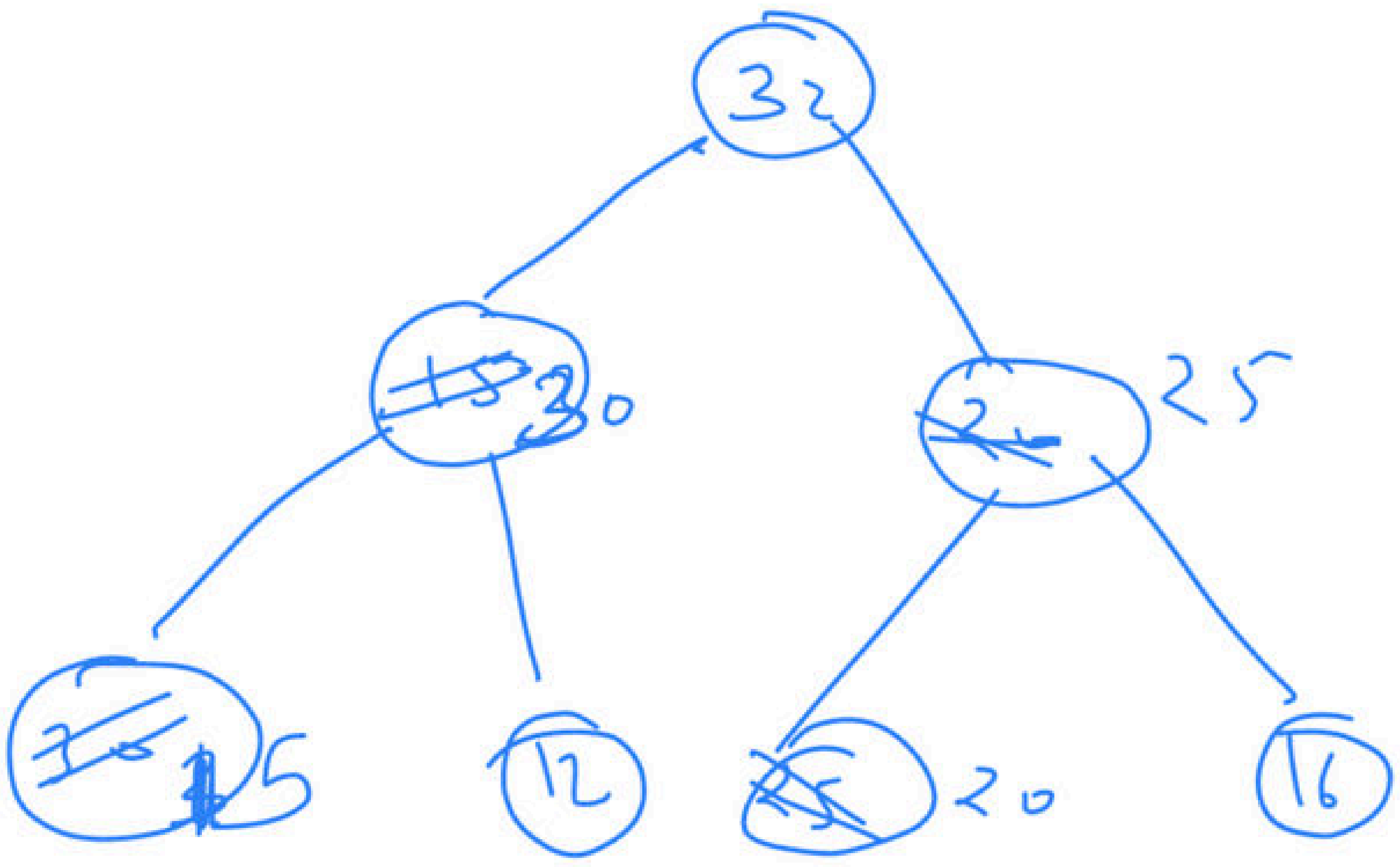
16 (c)

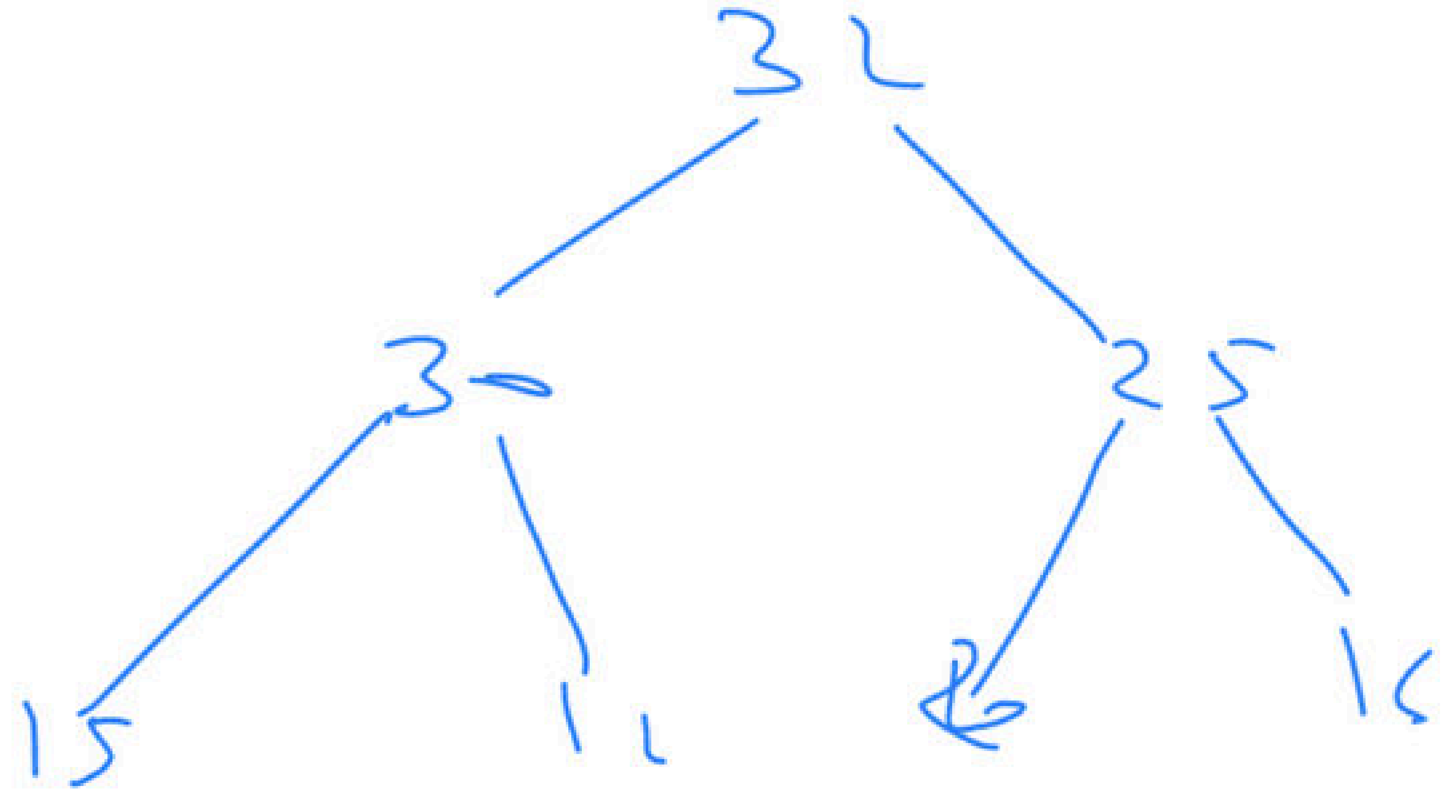


14 (d)



~~32~~, ~~18~~, ~~20~~, ~~30~~, ~~12~~, ~~25~~, 16





Q The elements 32, 15, 20, 30, 12, 25, 16 are inserted one by one in the given order into a Max Heap. The resultant Max Heap is. **(GATE - 2004) (2 Marks)**

Break

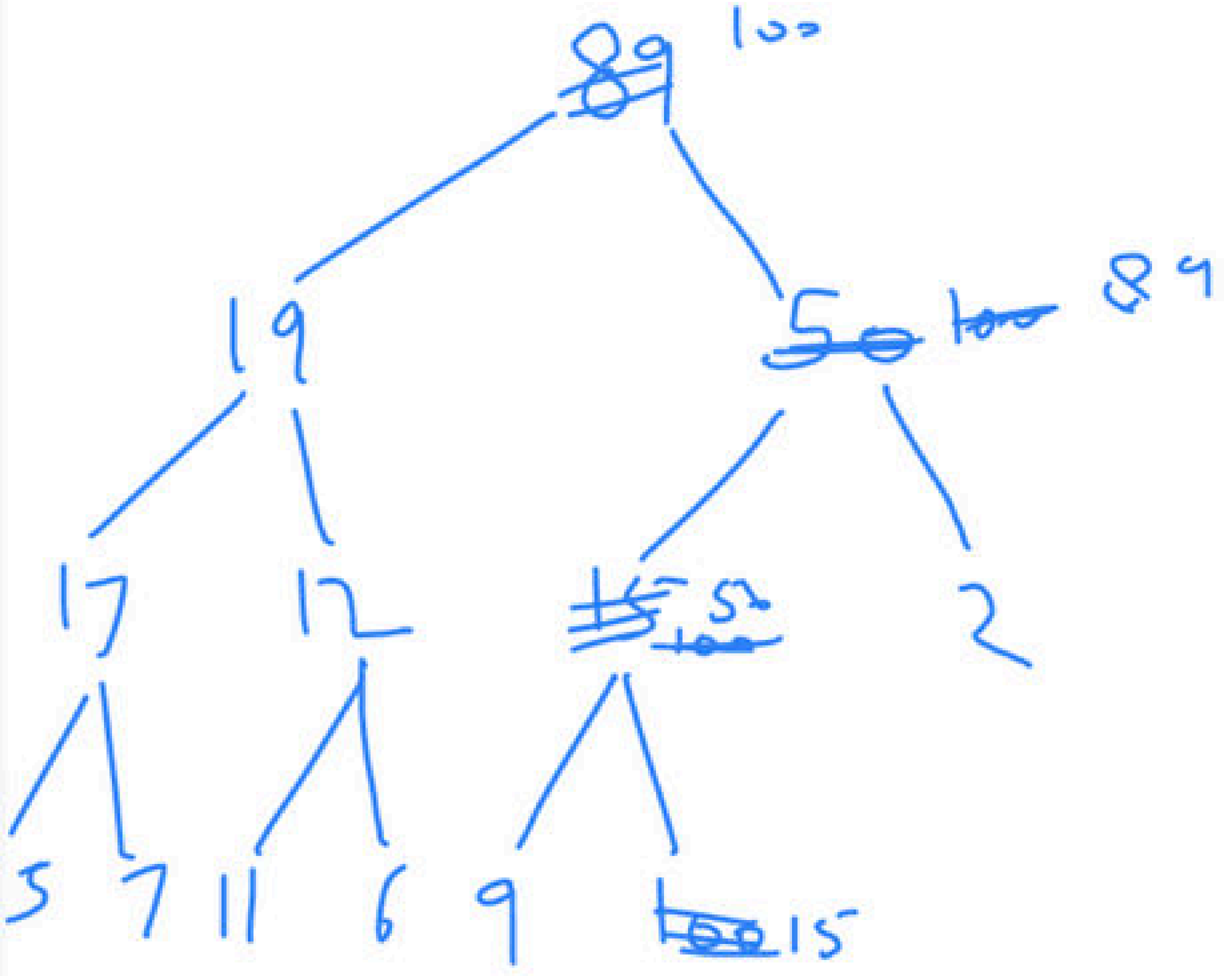
Q Consider the following array of elements. ⟨89, 19, 50, 17, 12, 15, 2, 5, 7, 11, 6, 9, 100⟩. The minimum number of interchanges needed to convert it into a max-heap is **(GATE - 2015) (2 Marks)**

(A) 4 — 8

(B) 5 — 6

(C) 2 — 6

(D) 3 — 81



Q Consider a max heap, represented by the array: 40, 30, 20, 10, 15, 16, 17, 8, 4.
Now consider that a value 35 is inserted into this heap. After insertion, the new heap is **(GATE - 2015) (2 Marks)**

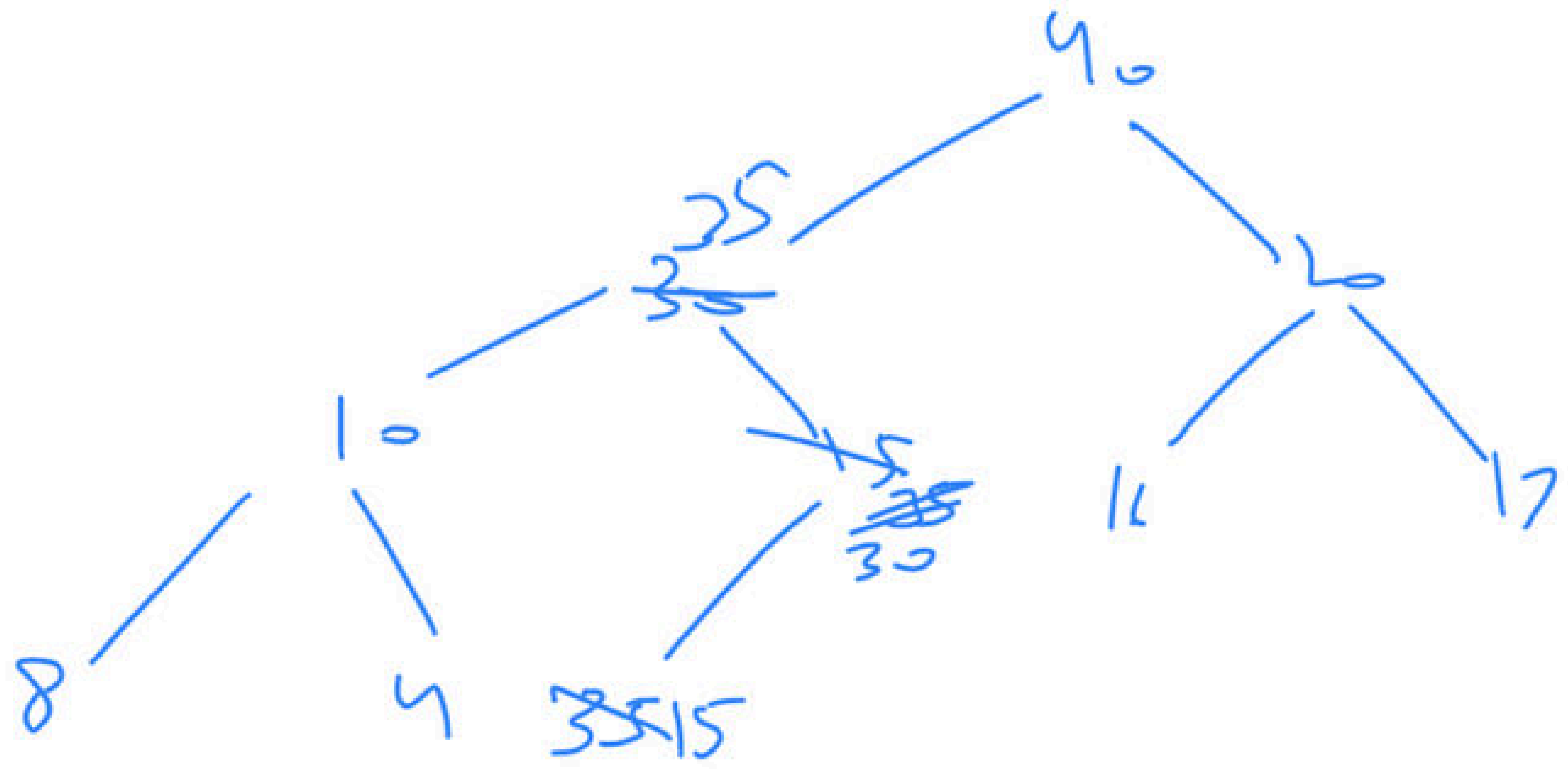
Array index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

a) 40,30,20,10,15,16,17,8,4,35

b) 40,35,20,10,30,16,17,8,4,15

c) 40,30,20,10,35,16,17,8,4,15

d) 40,35,20,10,15,16,17,8,4,30



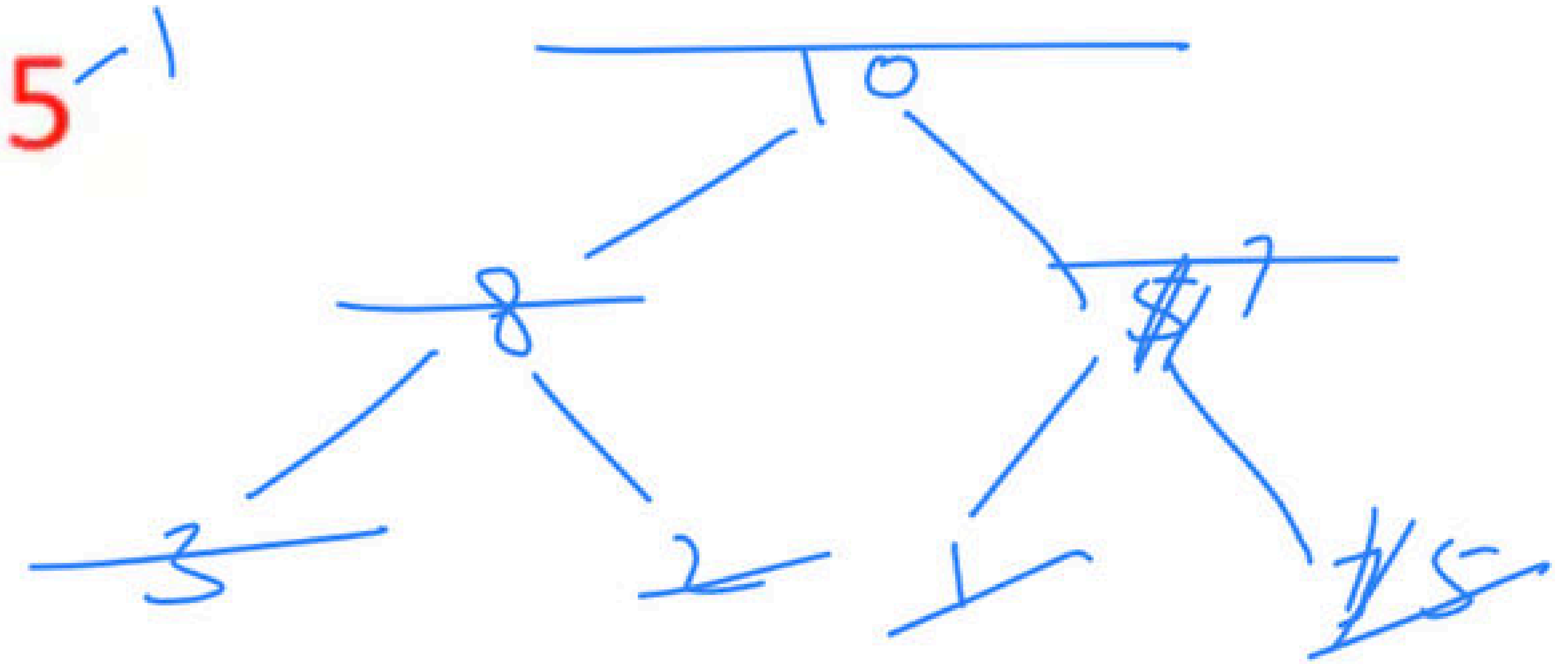
Q A priority queue is implemented as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is: ~~10, 8, 5, 3~~, 2. Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is: **(GATE - 2014) (2 Marks)**

~~**(A)** 10~~, 8, 7, 3, 2, 1, 5 — 89

(B) 10, 8, 7, 2, 3, 1, 5 1

(C) 10, 8, 7, 1, 2, 3, 5 1

(D) 10, 8, 7, 5, 3, 2, 1 9



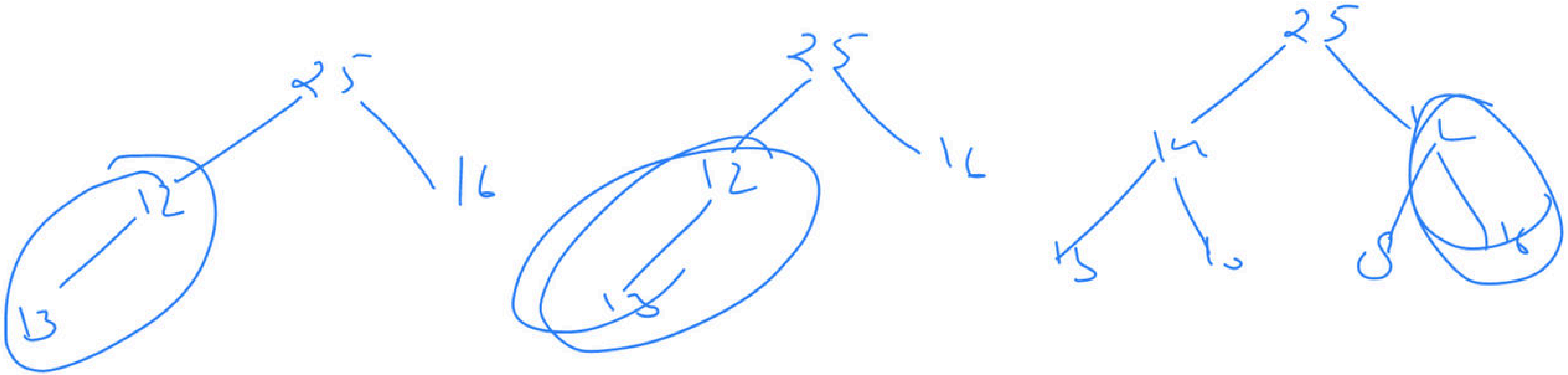
Q Consider a binary max-heap implemented using an array. Which one of the following arrays represents a binary max-heap? **(GATE - 2009) (2 Marks)**

(A) ~~25,12,16,13,10,8,14~~ 12

(B) ~~25,12,16,13,10,8,14~~ 2

(C) 25,14,16,13,10,8,12 82

(D) ~~25,14,12,13,10,8,16~~ 4



Q What is the content of the array after two delete operations on the correct answer to the previous question? **(GATE - 2009) (2 Marks)**

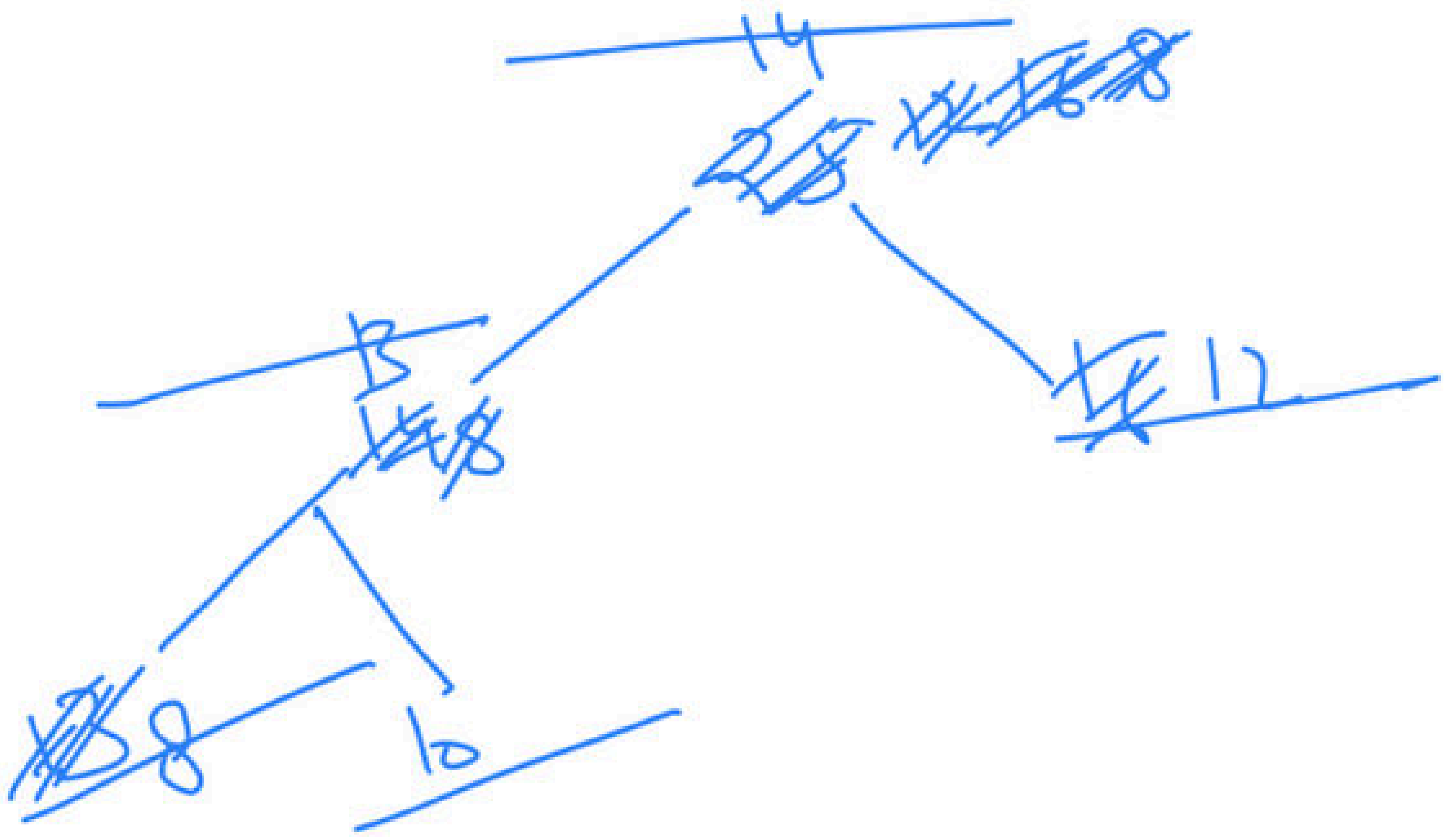
(A) 14,13,12,10,8 -48

(B) 14,12,13,8,10 -13

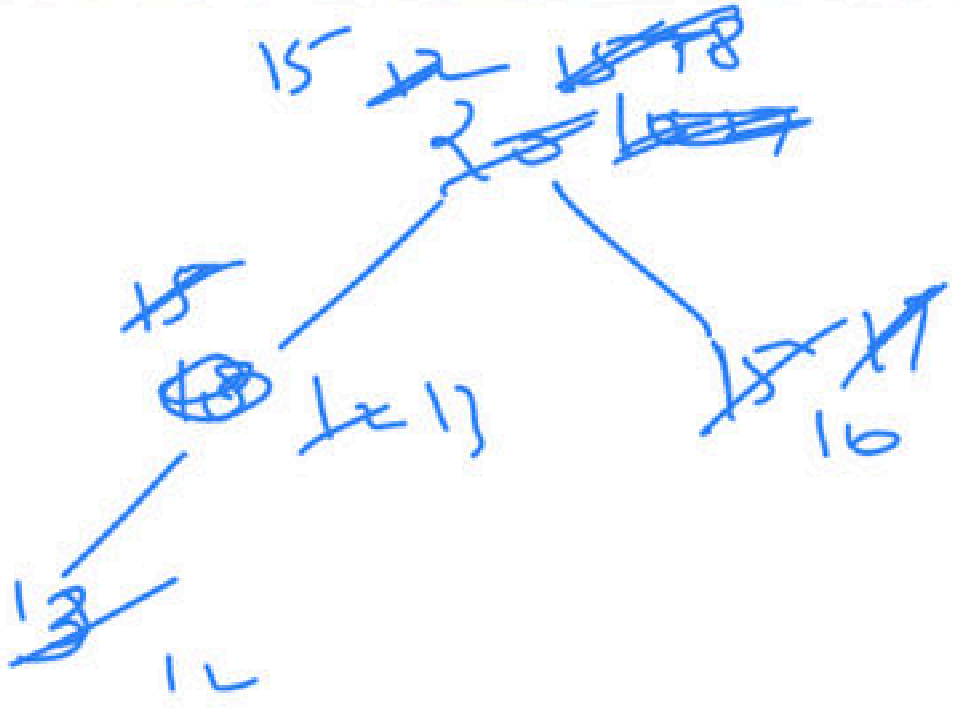
(C) 14,13,8,12,10 -15

(D) 14,13,12,8,10 -25

25, 14, 16, 13, 10, 8, 12



Q. A priority queue is implemented as a max-heap. Initially, it has five elements. The level-order traversal of the heap is as follows: 20, 18, 15, 13, 12 Two new elements '10' and '17' are inserted in the heap in that order. The level-order traversal of the heap after the insertion of the element is: **[Asked in TCS NQT 2019]**

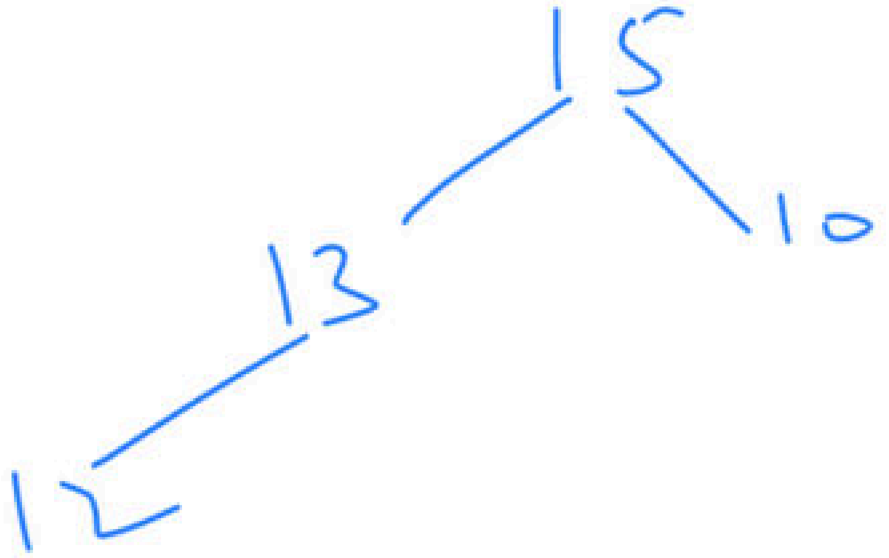


a. 20, 18, 17, 15, 13, 12, 10 – 8

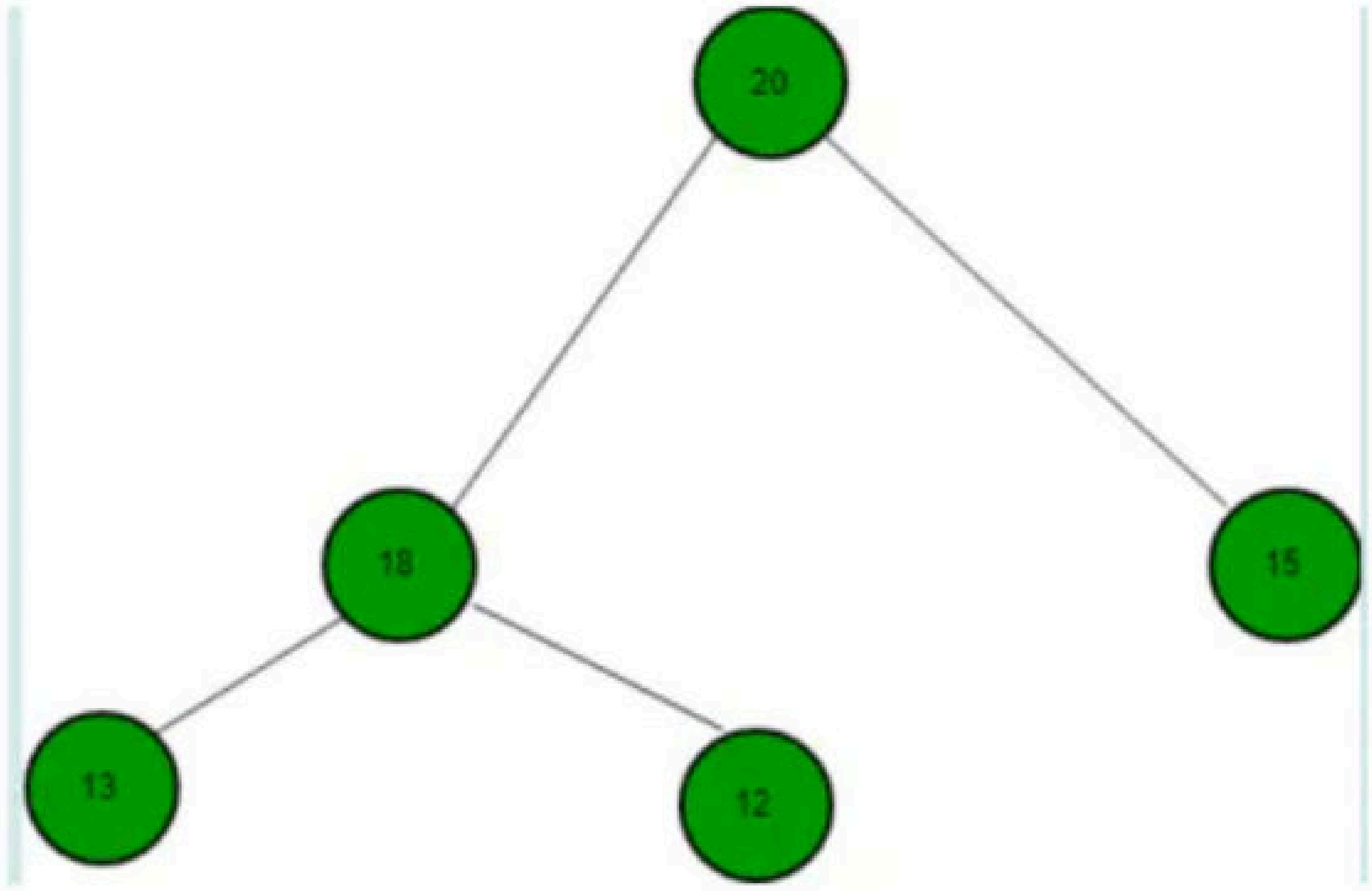
b. 20, 18, 17, 12, 13, 10, 15 – 2

c. 20, 18, 17, 10, 12, 13, 15 – 1

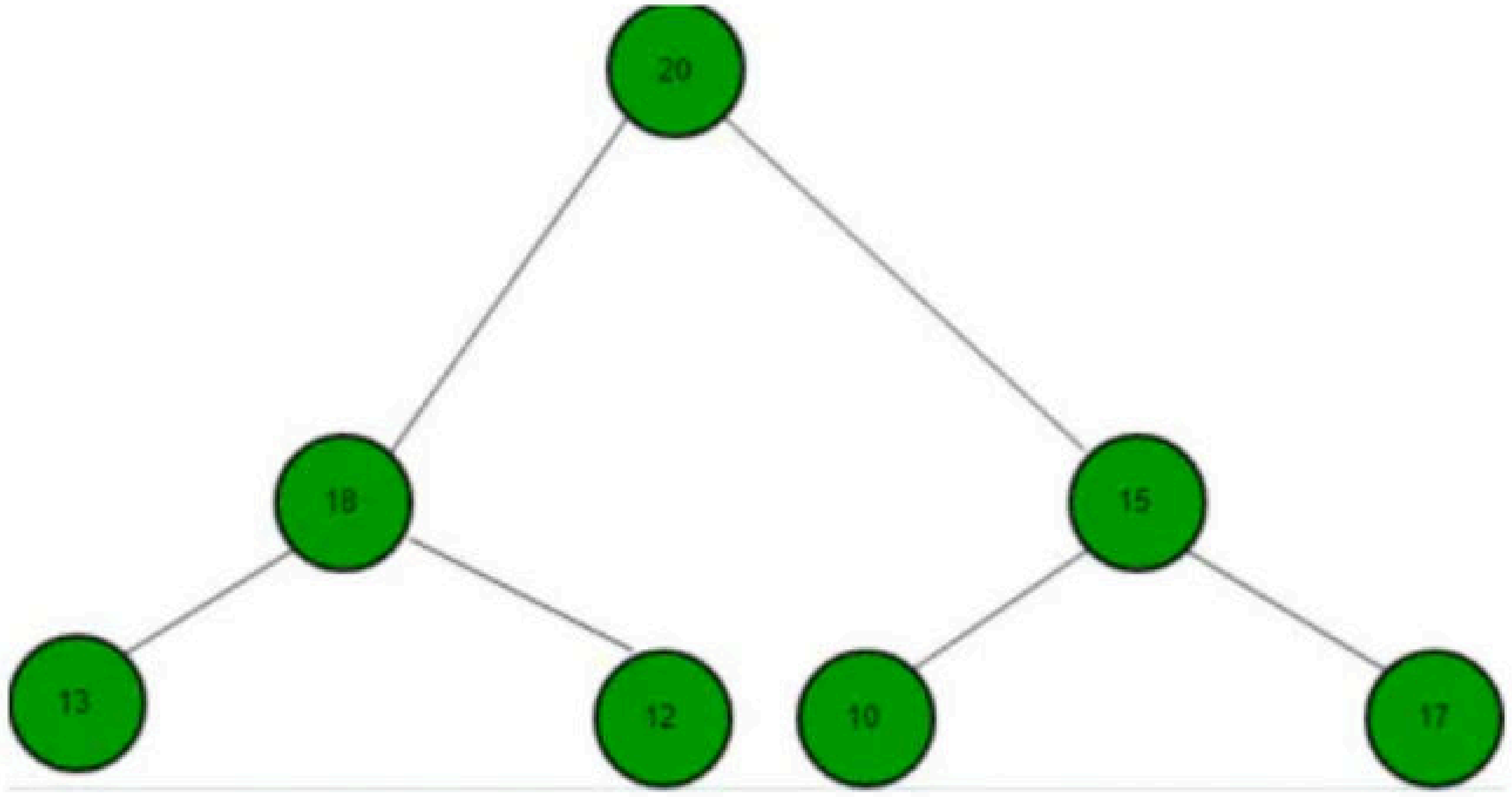
d. 20, 18, 17, 13, 12, 10, 15 – 90



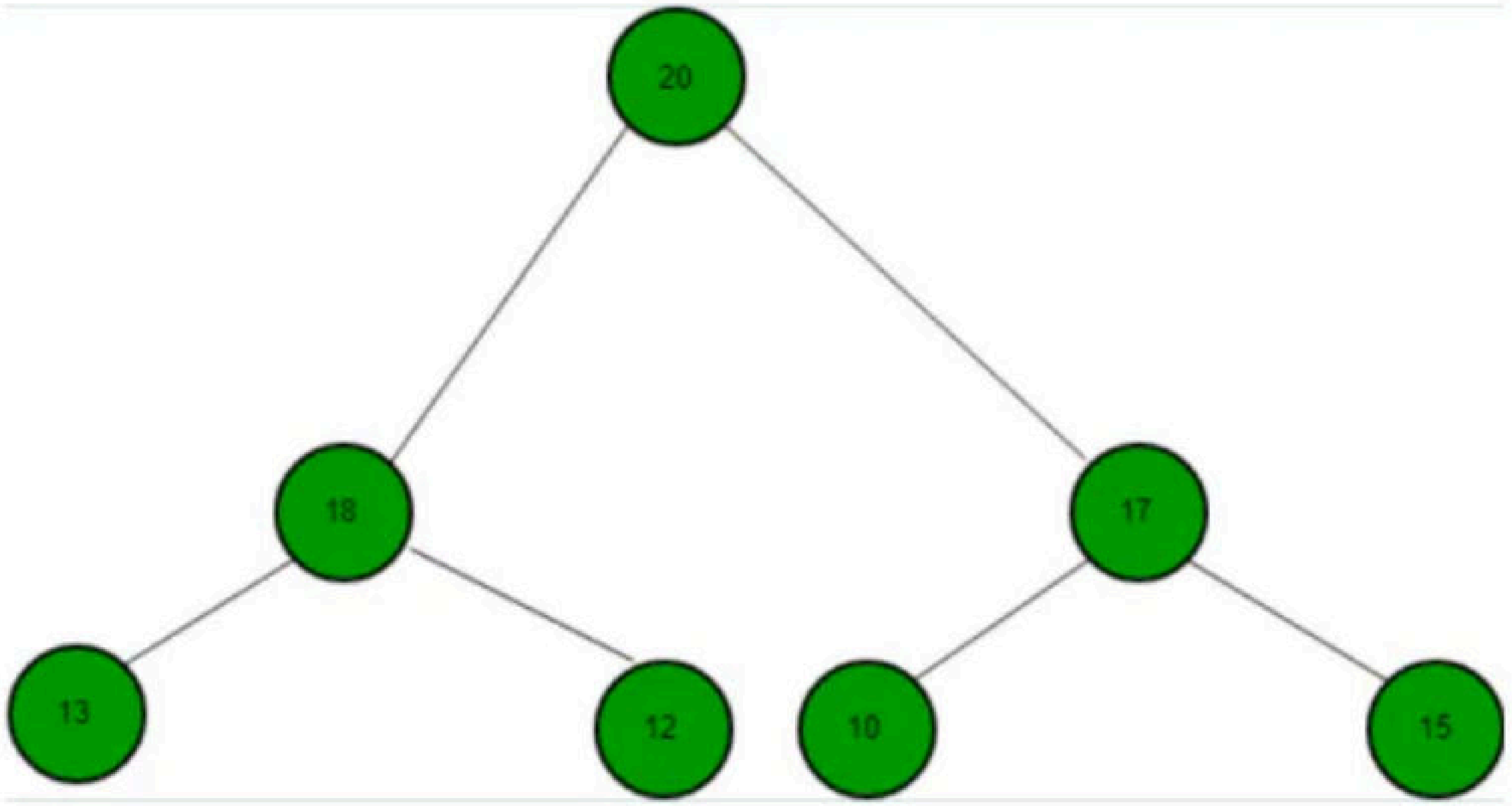
Initially we have:



When we insert 10 and 17:



We have to maintain max-heap, so:



The level-order traversal of the heap after the insertion of the element is 20, 18, 17, 13, 12, 10, 15 So, option (D) is correct.

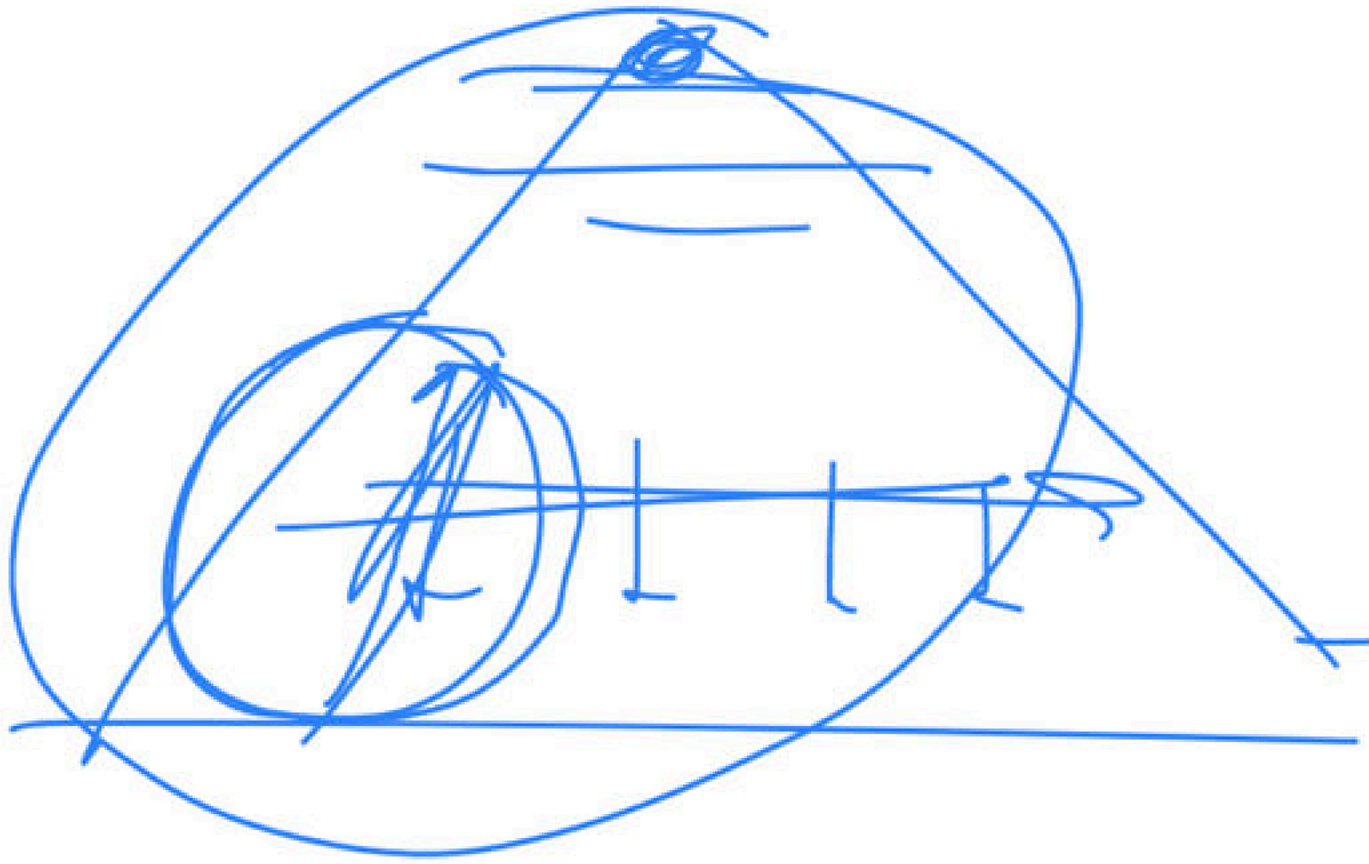
Break

Q Con~~sider a rooted Binary tree represented using poi~~nters. The best upper bound on the time required to determine the number of subtrees having exactly 4 nodes $O(n^a \operatorname{Log} n^b)$. Then the value of $a + 10b$ is ___1___ **(GATE-2015) (1 Marks)**

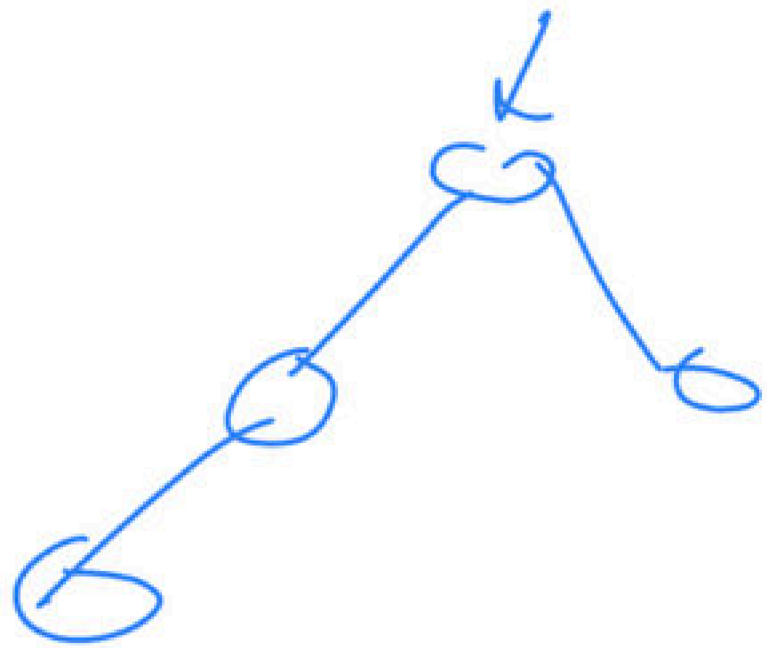
$a=1 \quad b=0$

$1 + 10 \times 0$

$O(n)$



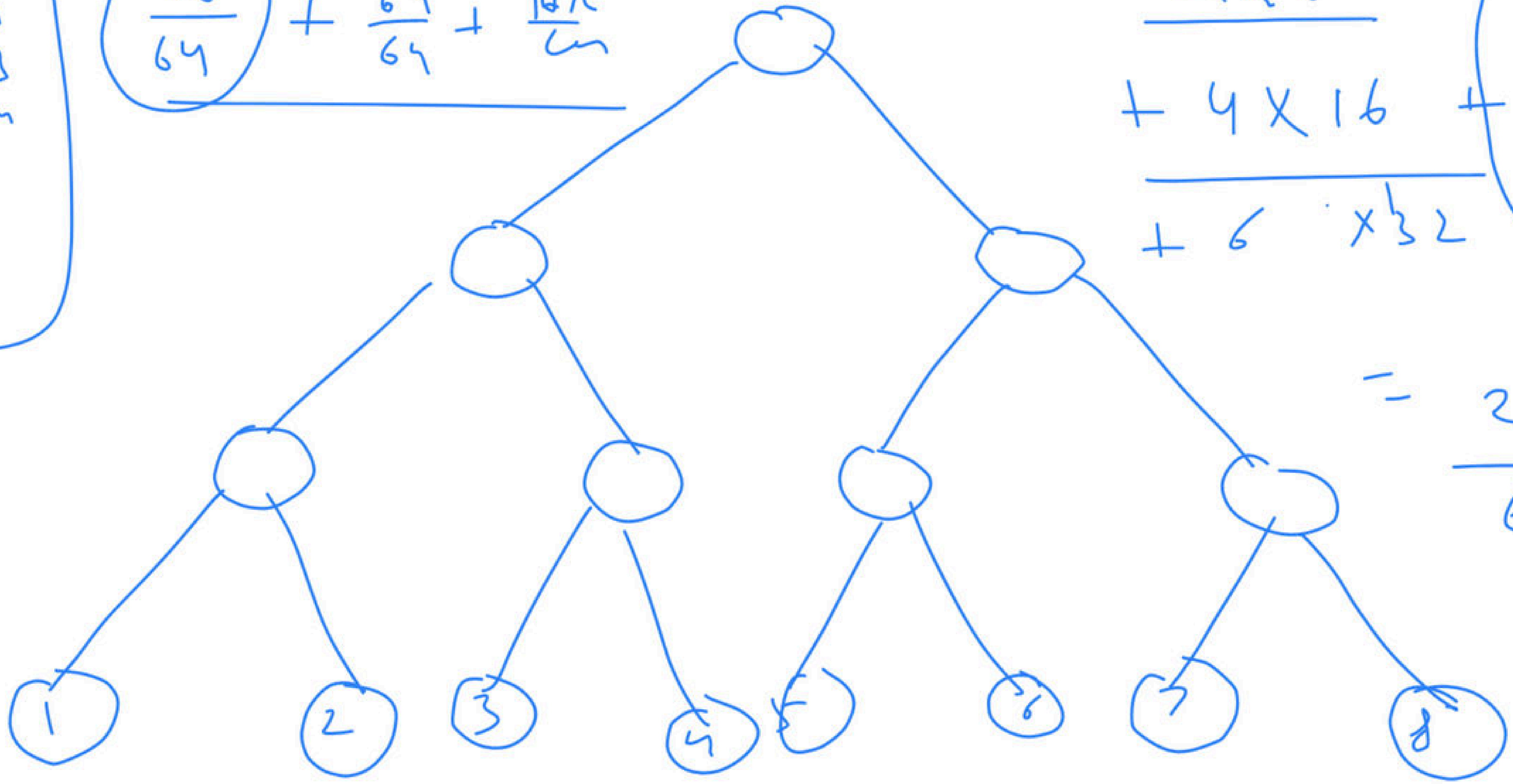
$O(n)$



Q Let T be a full binary tree with 8 leaves. (A full binary tree has every level full.) Suppose two leaves a and b of T are chosen uniformly and independently at random. The expected value of the distance between a and b in T (ie., the number of edges in the unique path between a and b) is (rounded off to 2 decimal places) 4.25. **(GATE-2019) (2 Marks)**

1–3
1–4
2 3
2 4
3 1
3 2
4 1
4 2

$\frac{16}{64} + \frac{64}{64} + \frac{192}{64}$



$\cancel{2 \times 8}$ 16

$+ 4 \times 16$ $+ 64$

$+ 6 \times 32$ $+ 192$

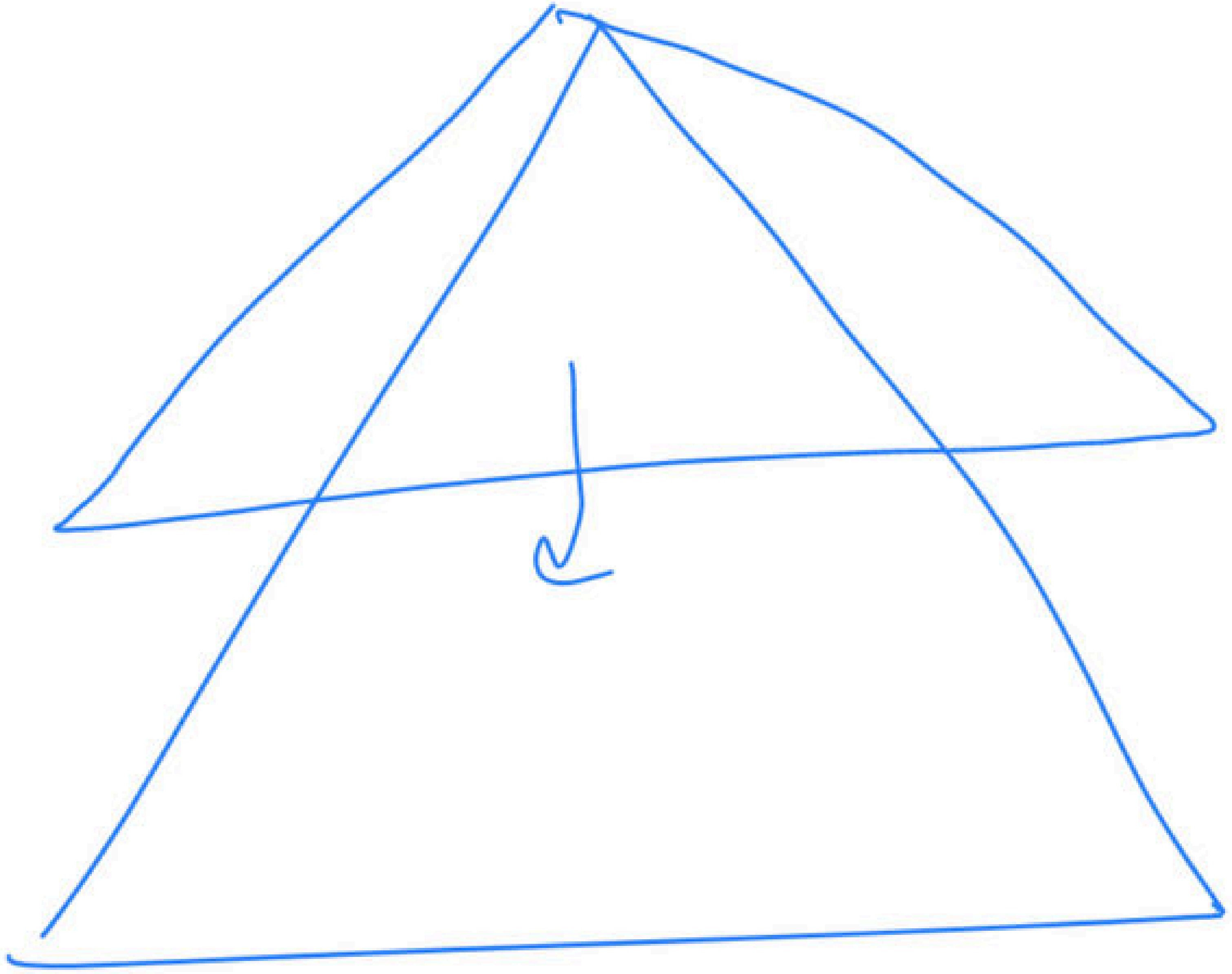
$= \frac{272}{64} = 4.25$

4
4
4
4
16

Q In a heap with n elements with the smallest element at the root, the 7th smallest element can be found in time **(GATE - 2008) (1 Marks)**

a) (n log n) **b)** (n) **c)** (log n) ~~**d)** (1)~~

2 14 4 81



Q In a binary max heap containing n numbers, the smallest element can be found in time **(GATE - 2006) (1 Marks)**

(A) ~~0(n)~~ **(B)** ~~O(logn)~~ **(C)** 0(loglogn) **(D)** 0(1)

57 14 3 29

$O(\frac{n}{2}) = O(n)$

Q A data structure is required for storing a set of integers such that each of the following operations can be done in O(logn) time, where n is the number of elements in the set.

I) Deletion of the smallest element

II) Insertion of an element if it is not already present in the set

Which of the following data structures can be used for this purpose? **(GATE - 2003) (2 Marks)**

a) A heap can be used but not a balanced binary search tree

b) A balanced binary search tree can be used but not a heap

c) Both balanced binary search tree and heap can be used

d) Neither balanced search tree nor heap can be used

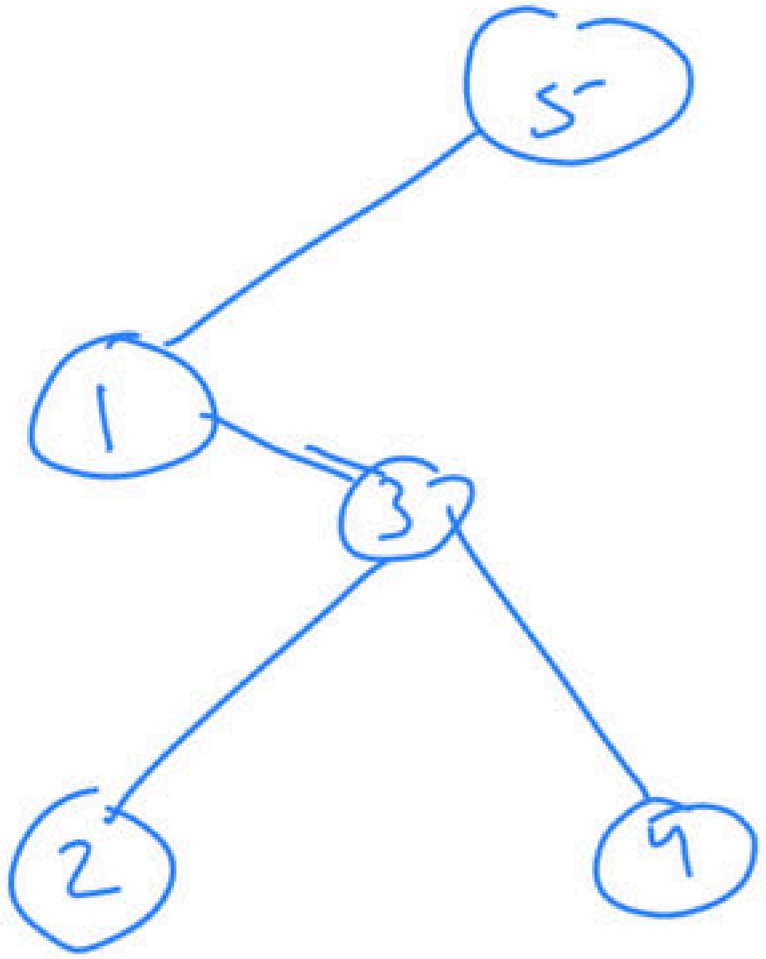
Break

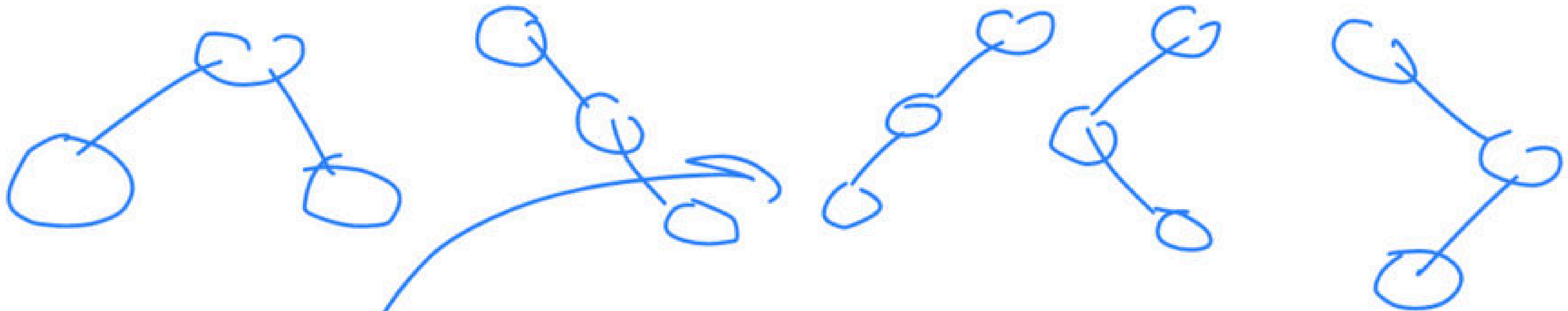
Q We are given a set of n distinct elements and an unlabeled binary tree with n nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree? **(GATE - 2011) (2 Marks)**

(A) 0 **(B)** 1 **(C)** n! **(D)** (1/(n+1)).2nCn

4!   3!   20

n=3





$\frac{1}{4} \times \frac{6!}{3!\,3!}$

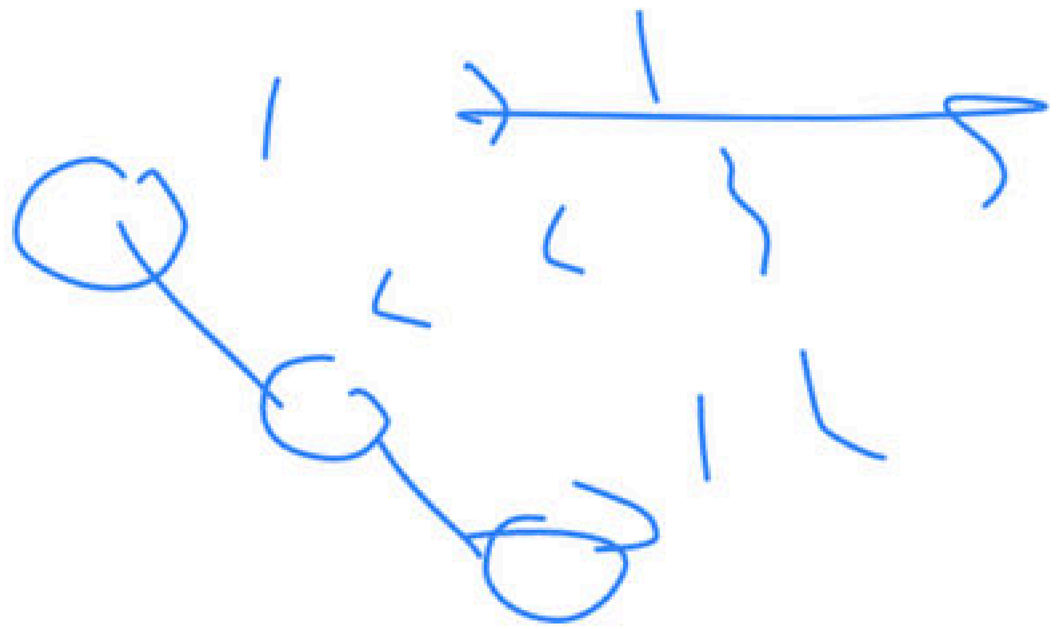
$= \frac{1}{4} \times \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} = 5$

$\frac{1}{n+1} \times {}^{2n}C_n$

$= \frac{1}{3+1} \cdot {}^{6}C_3$

$$n \longrightarrow \frac{{}^{2n}C_n}{n+1} \times n!$$

$$1 \times 1$$



Q The maximum number of binary trees that can be formed with three unlabelled nodes is: **(GATE-2007) (1 Marks)**

a) 1 **b)** 5 **c)** 4 **d)** 3

Q How many distinct binary search trees can be created out of 4 distinct keys?

(A) 4 **(B)** 14 **(C)** 24 **(D)** 42

$$\left(\frac{{}^{2n}C_n}{n+1}\right) \times n!$$

$\times \; 1$

2-3 CPU

n = 4

10-1

Q how many distinct BST can be constructed with 3 distinct keys?

a) 4 **b)** 5 **c)** 6 **d)** 9

Break

Q A complete n-ary tree is a tree in which each node has n children or no children. Let I be the number of internal nodes and L be the number of leaves in a complete n-ary tree. If L = 41, and I = 10, what is the value of n? **(GATE - 2007) (2 Marks)**

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

Q The number of leaf nodes in a rooted tree of n nodes, with each node having 0 or 3 children is: **(GATE - 2002) (2 Marks)**

a) $n/2$ **b)** $(n-1)/3$ **c)** $(n-1)/2$ **d)** $(2n+1)/3$

Q A complete n-ary tree is one in which every node has 0 or n sons. If x is the number of internal nodes of a complete n-ary tree, the number of leaves in it is given by **(GATE - 1998) (2 Marks)**

a) $x(n-1)+1$ **b)** $xn-1$ **c)** $xn+1$ **d)** $x(n+1)$

Q In a complete k-ary tree, every internal node has exactly k children or no child. The number of leaves in such a tree with an internal node is:

(A) n.k

(B) $(n - 1) k + 1$

(C) $n (k - 1) + 1$

(D) $n (k - 1)$

Break

Q A binary tree T has 20 leaves. The number of nodes in T having two children is ______. **(GATE - 2015) (1 Marks)**

Q In a binary tree, the number of internal nodes of degree 1 is 5, and the number of internal nodes of degree 2 is 10. The number of leaf nodes in the binary tree is **(GATE - 2006) (1 Marks)**

a) 10 **b)** 11 **c)** 12 **d)** 15

Q A binary tree T has n leaf nodes. The number of nodes of degree 2 in T is **(GATE-1995) (1 Marks)**

a) $\log_2 n$ **b)** $n-1$ **c)** n **d)** 2^n

Break

Q Consider the following nested representation of binary trees: (X Y Z) indicates Y and Z are the left and right subtrees, respectively, of node X. Note that Y and Z may be NULL, or further nested. Which of the following represents a valid binary tree? **(GATE - 2000) (1 Marks)**

a) (1 2 (4 5 6 7))

b) (1 (2 3 4) 5 6) 7)

c) (1 (2 3 4) (5 6 7))

d) (1 (2 3 NULL) (4 5))

Q Which of the following statements is false? **(GATE - 1998) (1 Marks)**

a) A tree with a n nodes has (n–1) edges

b) A labeled rooted binary tree can be uniquely constructed given its post order and preorder traversal results.

c) A complete binary tree with n internal nodes has (n+1) leaves.

d) The maximum number of nodes in a binary tree of height h is $2^{h+1}-1$

Q.11 Let H be a binary min-heap consisting of n elements implemented as an array. What is the worst case time complexity of an optimal algorithm to find the maximum element in H? **(GATE-2021)**

(a) $\Theta(\log n)$ (b) $\Theta(1)$

(c) $\Theta(n \log n)$ (d) $\Theta(n)$