INFERENTIAL STATISTICS& HYPOTHESES TESTING

CASE STUDY

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CASE STUDY:

The pharmaceutical company Sun Pharma is manufacturing a new batch of painkiller drugs, which are due for testing. Around 80,000 new products are created and need to be tested for their time of effect (which is measured as the time taken for the drug to completely cure the pain), as well as the quality assurance (which tells you whether the drug was able to do a satisfactory job or not).

Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.
Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.

Answer:

The probability distribution that would accurately portray the above scenario will be Binomial distribution as this event is of type success or failure, i.e., either the drug is able to produce a satisfactory result or unsatisfactory.

binomial distribution can be thought of as simply the probability of a SUCCESS or FAILURE outcome in an experiment or survey that is repeated multiple times. The binomial is a type of distribution that has two possible outcomes (the prefix "bi" means two, or twice). For example, a coin toss has only two possible outcomes: heads or tails and taking a test could have two possible outcomes: pass or fail.

- General formula :-

$$P(X=r)= nCr(p)r(1-p)n-r$$

Where

- n is no. of trials,
- p is probability of success
- and r is no. of successes after n trials.

Conditions to call a binomial distribution :

1. Total number of trials is fixed at n ,where n=10.
2. Each trial is binary, i.e., has only two possible outcomes - success or failure
3. Probability of success is same in all trials, denoted by p

(B)Calculate the required probability:

Given that

$n = 10$

$r = 3$

$P = 0.2$, $q = 0.8$

$P(X = r) = {}^{n}C_r p^r (1-p)^{n-r}$

at most 3

$P(X \leq 3)$

$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$P(X=0) = {}^{10}C_0 * (0.2)^0 * (0.8)^{10-0}$

$= 1 * 0.107374$

$= 0.107374$

$P(X=1) = {}^{10}C_1 * (0.2)^1 + (0.8)^{10-1}$

$= 10 \times 0.2 \times 0.134218$

$= 0.268436$

$P(X=2) = {}^{10}C_2 (0.2)^2 (0.8)^{10-2}$

$= 45 * 0.04 \times 0.16772$

$= 0.30198896$

$P(X=3) = {}^{10}C_3 (0.2)^3 (0.8)^7$

$= 120 \times 0.008 \times 0.2097$

$= 0.2013264$

so $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0.107374 + 0.268436 + 0.30198 + 0.2013$

$= 0.8791$

P(X<=3)=0.879126 =>87.91

Therefore,a probability that at most 3 drugs are unable to do satisfactory job is 87.91%..

Question 2

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the interval in which the population mean might lie — with a 95% confidence level.

a.)Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.

Answer ….

The main methodology that we would be using Central Limit Theorem to estimate the *population* mean in the form of an interval.

Hence based on the Central Limit Theorem, for a sampling distribution, we can say that

1. Sampling distribution's mean ($\mu\bar{x}$) = Population mean(μ) {unknown}

2. Sampling distribution's standard deviation

(Standard Error) = $\sigma/\sqrt{n}$.

Since we know only the samples standard deviation (S), we approximate the

population's standard deviation (σ) with that of sample. n is the sample size

3. For n > 30, sampling distribution becomes a normal distribution

Given the sample's size, mean and standard deviation, we can say that the confidence interval for μ

lies in the range of ($\bar{X}$ -Z*S/n,X+Z*S/n).

Here z* is the z-score associated to 95% of the confidence level, $\bar{X}$ is the sample's mean and S is the standard Deviations.

b.)Find the required interval.

Answer:

- Sample size n = 100
- Sample mean =207, standard deviation=65.

Here Given that

$n = 100$

$\bar{x} = 207$

S.D $= 65$

The confidence interval is given by

$$\left(\bar{x} - \frac{z^* s}{\sqrt{n}} , \bar{x} + \frac{z^* s}{\sqrt{n}} \right)$$

$z^* = 1.96$

$$= \left(207 - \frac{1.96 \times 65}{\sqrt{100}} , 207 + \frac{1.96 \times 65}{\sqrt{100}} \right)$$

$$= \left(207 - \frac{127.4}{10} , 207 + \frac{127.4}{10} \right)$$

$$= (207 - 12.74 , 207 + 12.74)$$

$$= (194.26, 219.74)$$

confidence interval = (194.26 seconds, 219.74 seconds)

Hence the margin of error corresponding to 95% confidence level is $\frac{z^* s}{\sqrt{n}} = 12.74$

The population mean between 194.26 seconds, 219.74 seconds.

- **Question** 3:

  a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

  b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of α and β come out to be 0.05 and 0.45 respectively.
  Now, a different sampling procedure(different sample size, mean and standard deviation) is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each.
  Explain under what conditions would either method be more preferred than the other, i.e., give an example of a situation where conducting the hypothesis test with α and β as 0.05 and 0.45 respectively would be preferred over conducting the same hypothesis test with α and β at 0.15 each. Similarly, give an example for the reverse scenario- where conducting the same hypothesis test with α and β at 0.15 each would be preferred over having them at 0.05 and 0.45 respectively.
  For each example, give suitable reasons for your particular choice using the given values of α and β only. (Assume that no other information is available. Also, the hypothesis test that you are conducting is the same as mentioned in the previous question - you need to test the claim whether the newer batch produces a satisfactory result.)

Answer

Hypothesis can be formulated as:

The Null and Alternate Hypothesis are

$H0 : \mu \leq 200$ seconds, the time of effect that the drug needs to do satisfactory job

$H1 : \mu > 200$ seconds, the time of effect that the drug needs to not do satisfactory job

The Type of Test can be deduced as:

Since > sign is used in alternate hypothesis, it would be a One-tailed test (upper-tailed test) and the

rejection region would be on the right side of the distribution.

Making Decision:

 Sample size n = 100

 Assumed Sample mean μ = 200

 Sample mean $\mu\bar{x}$= 207

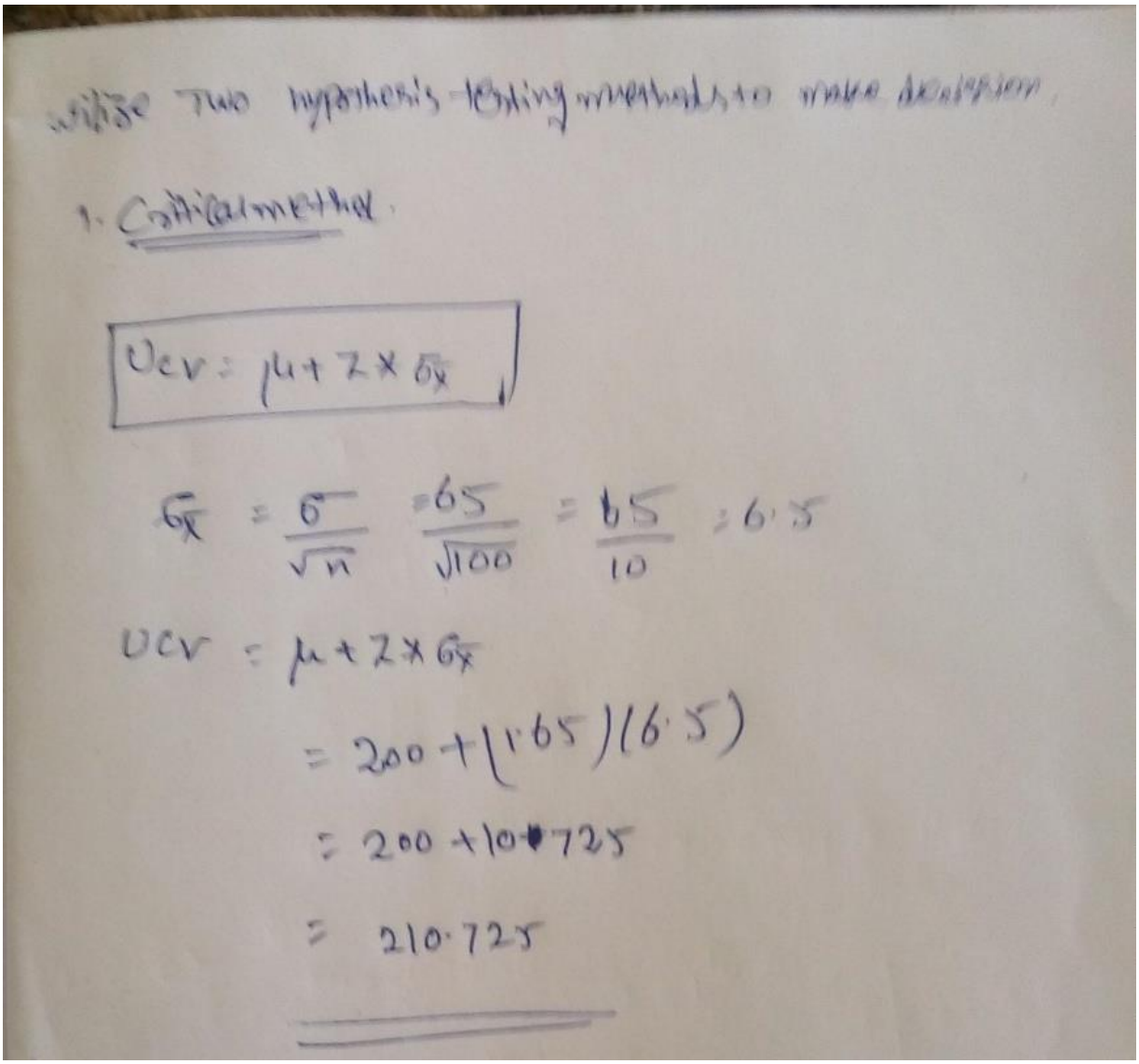
 Sample standard deviation $\sigma\bar{x}$= 65. (Since only the samples standard deviation is given, we approximate the population standard deviation to samples stand deviation=65.

 Significance level α = 5% i.e. 0.05.

Utiluze Two hypothesis testing methods to make decisions:

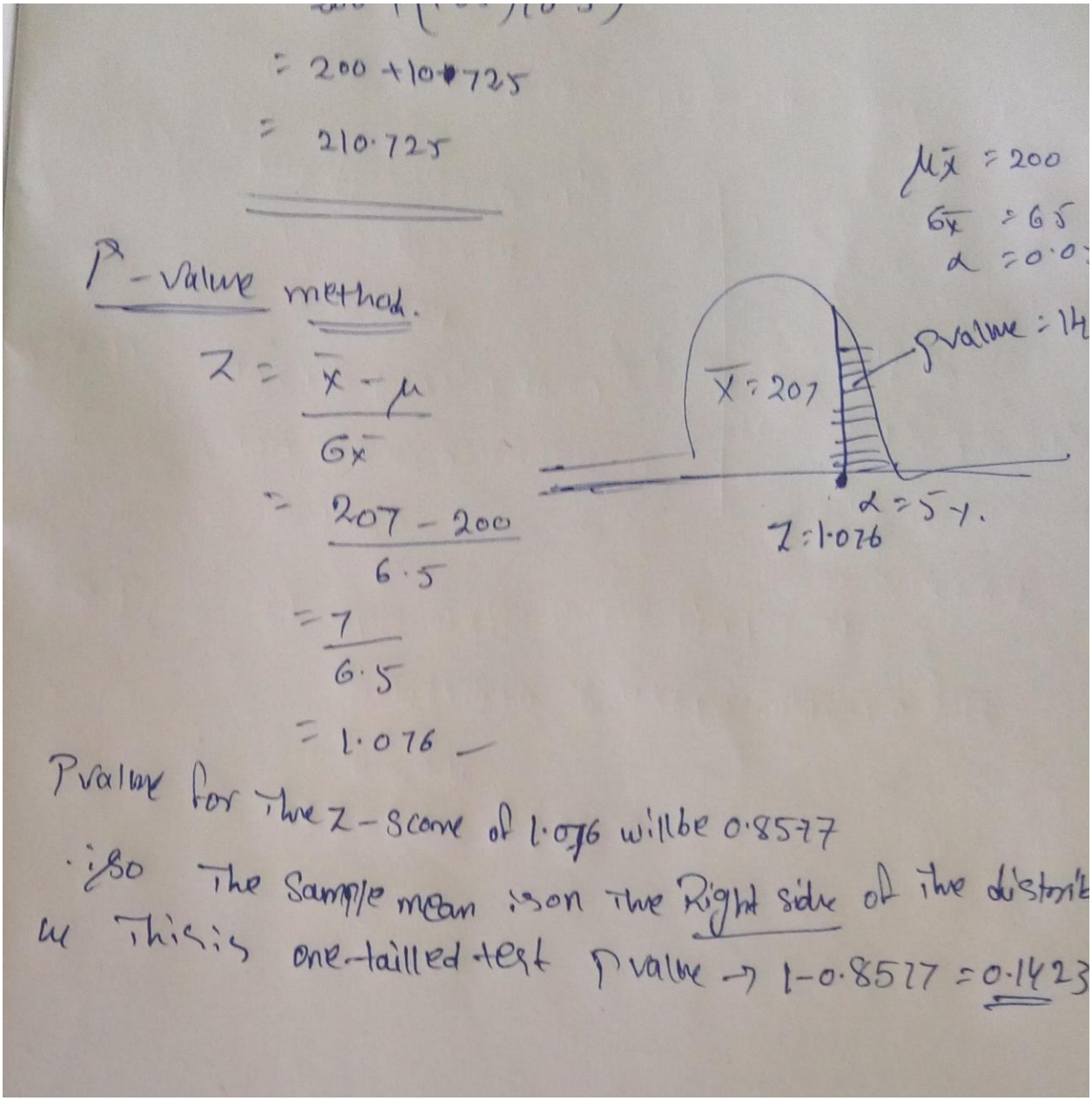
1. Critical value method
2. P_ Value method.

1.Critical Value method:



Since the sample mean 207 seconds is less than the Upper critical value 210.69 seconds, we fail to reject the null hypothesis which states that the drug needs a time of effect <= 200 seconds to do a satisfactory job.

2) P_Value based Method.



Since the p-value is greater than the significance level (0.1423 > 0.05), we fail to reject the null hypothesis which states that the drug needs a time of effect <= 200 seconds to do a satisfactory job.

(B) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of α and β come out to be 0.05 and 0.45 respectively.
Now, a different sampling procedure(different sample size, mean and standard deviation) is

proposed so that the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each.
Explain under what conditions would either method be more preferred than the other, i.e., give an example of a situation where conducting the hypothesis test with α and β as 0.05 and 0.45 respectively would be preferred over conducting the same hypothesis test with α and β at 0.15 each. Similarly, give an example for the reverse scenario- where conducting the same hypothesis test with α and β at 0.15 each would be preferred over having them at 0.05 and 0.45 respectively.
For each example, give suitable reasons for your particular choice using the given values of α and β only. (Assume that no other information is available. Also, the hypothesis test that you are conducting is the same as mentioned in the previous question - you need to test the claim whether the newer batch produces a satisfactory result.)

Answer):

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new subscribers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.
Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer

A/B testing is a way to compare two versions of a single variable and determine which one performs better in improving the sales and revenue. In other words, A/B testing provides a way to test two different versions of the same element and see which one performs better. As there are two taglines proposed, we can show one tagline for a specific set of audience (controlled version) and another tagline for another set of audience (variant version). As different set of audience areshown different set of ads, their responses to these 2 ads can be

measured statistically and thereby used to determine which ad campaign gets converted into sales and generates revenue.

Why A/B Testing:

Below given are few of the reasons why A/B testing should be done from a generic perspective:

** Enables to solve visitor pain points. This can be done using data gathered through visitor behaviour analysis tools such as heat maps, Google analytics, and surveys to solve the visitors' pain points.**

** Get more conversion while investing less. ROI from A/B testing can be significant with minor changes resulting in a significant increase in conversions**

** A/B testing is completely data driven with no room for guesswork and hence helps to easily determine a "winner" and a "loser" based on various metrics.**

** A/B testing allows for maximum output with minimal modifications, resulting in increased ROI.**

** Any changes on product prices should be done after A/B testing as it will let know the customers receptiveness to the same.**

<u>Stepwise Procedure:</u>

** Performing Research:**

Make observations to identify the problem area using surveys, analytics tools etc. in the conversion funnel and find out what is stopping visitors from converting once they view the ads.

** Formulate Hypothesis:**

Based on the above research insights, a hypothesis should be built. The hypothesis can bearrived at by determining what should be the final result, statistics on the user behaviour towhich type of ad etc. The hypothesis in this case should be built with the main purpose ofincreasing conversions.

** Determine the test sample size:**

A An advance calculation of how many views need to be generated should be determined so that the test results are statistically significant.

Determine the duration of the experiment:

The test should last at least a week even if we have achieved the sample size in 2 days. The recommended testing time generally is around 10 – 14 days. Once the duration is decided,the test shouldn't be stopped before the duration ends. In this case since there are 2 banners, the test duration can be calculated keeping in mind the monthly visitors, current conversion rate, and the expected change in the conversion rate.

Testing, Analysing results and drawing conclusions:

Flag off the test and wait for the stipulated time for achieving statistically significant result.Once the testing is done, analyse the test results and, if it succeeds, deploy the winning variation. If the test remains inconclusive, draw insights from it and implement these in subsequent test.

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